Taylor Series Centered at *c*:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(c)}{n!}(x-c)^{n}$$

$$f(x)=\frac{f(c)}{0!}(x-c)^{0}+\frac{f'(c)}{1!}(x-c)^{2}+\frac{f''(c)}{2!}(x-c)^{2}$$

$$+\frac{f'''(c)}{3!}(x-c)^{3}\qquad+\frac{f^{(4)}(c)}{4!}(x-c)^{4}+\ldots$$

$$+\frac{f^{(n)}(c)}{n!}(x-c)^{n}+\ldots$$

Find Taylor series for the function $f(x) = e^{4x}$ centered at $c = 0$?

$$f(x) = e^{4x}$$

a) $f(0) = e^{4x} = e^{0} = 1 = 4^{0}$

b) $f'(x) = 4e^{4x}$

$$f'(0) = 4e^{4x} = 4e^{0} = 4 = 4^{1}$$

c) $f''(0) = 16e^{4x}$

$$f''(0) = 16e^{4x} = 16e^{0} = 16 = 4^{2}$$

d) $f'''(0) = 64e^{4x}$

$$f'''(0) = 64e^{4x} = 64e^{0} = 64 = 4^{3}$$

e) $f^{(4)}(0) = 256e^{4x}$

$$f^{(4)}(0) = 256e^{4x} = 256e^{0} = 256 = 4^{4}$$

f) $f^{(5)}(0) = 1024e^{4x}$

$$f^{(5)}(0) = 1024e^{4x} = 1024e^{0} = 1024 = 4^{5}$$

Find Taylor series for the function $f(x) = e^{4x}$ centered at $c = 0$?

g) Find Taylor series for the function $f(x) = e^{4x}$ centered at $c = 0$?

Taylor Series Centered at c:

$$f(x) = \frac{f(c)}{0!}(x-c)^0 + \frac{f'(c)}{1!}(x-c)^1 + \frac{f''(c)}{2}(x-c)^2$$

$$+ \frac{f'''(c)}{3!}(x-c)^3 + \frac{f^{(4)}(c)}{4!}(x-c)^4 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

$$f(x) = \frac{4^0}{0!}(x-c)^0 + \frac{4^1}{1!}(x-c)^1 + \frac{4^2}{2!}(x-c)^2 + \frac{4^3}{3!}(x-c)^3$$

$$+ \frac{4^4}{4!}(x-c)^4 + \ldots + \frac{4^n}{n!}(x-c)^n + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{4^n}{n!}(x-0)^n = \sum_{n=0}^{\infty} \frac{4^n}{n!}(x)^n$$

Find a Taylor series for the function $f(x) = \sin 2x$ centered at $c = \dfrac{\pi}{4}$?

a) $f\left(\pi/4\right) = \sin 2x = \sin\left(2\cdot\pi/4\right) = \sin\left(\pi/2\right) = 1$

b) $f'\left(\pi/4\right) = 2\cos(2x)$

$f'\left(\pi/4\right) = 2\cos(2x) = 2\cos(2\cdot\pi/4) = 2\cos(\pi/2) = 2\left(0\right) = 0$

c) $f''\left(\pi/4\right) = \text{-}4\sin(2x)$

$f''\left(\pi/4\right) = \text{-}4\sin(2x) = \text{-}4\sin(2\cdot\pi/4) = \text{-}4\sin(\pi/2) = \text{-}4$

d) $f'''\left(\pi/4\right) = \text{-}8\cos(2x)$

$f'''\left(\pi/4\right) = \text{-}8\cos(2x) \;\; = \text{-}8\cos(2\cdot\pi/4) \;\; = \text{-}8\cos(\pi/2) \;\; = 0$

e) $f^{(4)}\left(\pi/4\right) = 16\sin(2x)$

$f^{(4)}\left(\pi/4\right) = 16\sin(2x) = 16\sin(2\cdot\pi/4) = 16\sin(\pi/2) = 16$

f) $f^{(5)}\left(\pi/4\right) = 32\cos(2x)$

$f^{(5)}\left(\pi/4\right) = 32\cos(2x) \;\; = 32\cos(2\cdot\pi/4) \;\; = 32\cos(\pi/2) \;\; = \;\; 0$

g) Taylor series for the function $f(x) = \sin 2x$ centered at $c = \dfrac{\pi}{4}$?

Find a Taylor series for the function $f(x) = \sin 2x$ centered at $c = \dfrac{\pi}{4}$?

$$f(x) = \frac{f(c)}{0!}(x-c)^0 + \frac{f'(c)}{1!}(x-c)^2 + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3$$

$$+ \frac{f^{(4)}(c)}{4!}(x-c)^4 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

$$f(x) = \frac{2^0}{0!}(x-\pi/4)^0 + 0 + \frac{-2^2}{2!}(x-\pi/4)^2 + 0$$

$$+ \frac{2^4}{4!}(x-\pi/4)^4 + 0 + \frac{-2^6}{6!}(x-\pi/4)^4 + 0$$

$$+ \frac{2^8}{8!}(x-\pi/4)^4 + 0 + \frac{-2^{10}}{10!}(x-\pi/4)^4 + 0$$

$$+ \frac{2^{12}}{12!}(x-\pi/4)^4 + 0 + \frac{-2^{14}}{14!}(x-\pi/4)^4 + 0$$

$$+ \frac{2^{16}}{16!}(x-\pi/4)^4 + 0 + \frac{-2^{18}}{18!}(x-\pi/4)^4 + 0$$

$$+ \ldots$$

Find a Taylor series for the function $f(x) = \sin 2x$ centered at $c = \dfrac{\pi}{4}$:

$$f(x) = \frac{2^0}{0!}\left(x-\pi/4\right)^0 - \frac{2^2}{2!}\left(x-\pi/4\right)^2 + \frac{2^4}{4!}\left(x-\pi/4\right)^4$$

$$- \frac{2^6}{6!}\left(x-\pi/4\right)^6 + \frac{2^8}{8!}\left(x-\pi/4\right)^8 - \frac{2^{10}}{10!}\left(x-\pi/4\right)^{10}$$

$$+ \frac{2^{12}}{12!}\left(x-\pi/4\right)^{12} - \frac{2^{14}}{14!}\left(x-\pi/4\right)^{14} + \frac{2^{16}}{16!}\left(x-\pi/4\right)^{16}$$

$$- \frac{2^{18}}{18!}\left(x-\pi/4\right)^{18} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n}}{(2n)!}\left(x-\pi/4\right)^{2n}$$

Find Taylor series for the function $f(x)=\dfrac{1}{x-1}$ centered at $c = 2$?

Find $f\left(x\right)=\dfrac{1}{x-1}$

Find $f'\left(x\right)=-1\left(\dfrac{1}{x-1}\right)^{-2}=-\left(1!\right)\left(\dfrac{1}{x-1}\right)^{-2}$

Find $f''\left(x\right)=2\left(\dfrac{1}{x-1}\right)^{-3}=\left(2!\right)\left(\dfrac{1}{x-1}\right)^{-3}$

Find $f'''\left(x\right)=-6\left(\dfrac{1}{x-1}\right)^{-4}=-\left(3!\right)\left(\dfrac{1}{x-1}\right)^{-4}$

Find $f^{(4)}\left(x\right)=24\left(\dfrac{1}{x-1}\right)^{-5}=\left(4!\right)\left(\dfrac{1}{x-1}\right)^{-5}$

Find $f^{(5)}\left(x\right)=-120\left(\dfrac{1}{x-1}\right)^{-6}=-\left(5!\right)\left(\dfrac{1}{x-1}\right)^{-6}$

Find Taylor series for the function $f(x) = \dfrac{1}{x-1}$ centered at $c = 2$?

$$f(2) = \frac{1}{x-1} = 1 = 0!$$

$$f'(2) = -(1!)\left(\frac{1}{2-1}\right)^{-2} = -1!$$

$$f''(2) = (2!)\left(\frac{1}{2-1}\right)^{-3} = 2!$$

$$f'''(2) = -(3!)\left(\frac{1}{2-1}\right)^{-4} = -3!$$

$$f^{(4)}(2) = (4!)\left(\frac{1}{2-1}\right)^{-5} = 4!$$

$$f^{(5)}(2) = -(5!)\left(\frac{1}{2-1}\right)^{-6} = -5!$$

Find Taylor series for the function $f(x) = \dfrac{1}{x-1}$ centered at $c = 2$?

$$f(x) = \frac{f(c)}{0!}(x-c)^0 + \frac{f'(c)}{1!}(x-c)^2 + \frac{f''(c)}{2!}(x-c)^2$$

$$+ \frac{f'''(c)}{3!}(x-c)^3 + \frac{f^{(4)}(c)}{4!}(x-c)^4 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

$$f(x) = \frac{0!}{0!}(x-2)^0 + \frac{-1!}{1!}(x-2)^1 + \frac{2!}{2!}(x-2)^2 + \frac{-3!}{3!}(x-2)^3$$

$$+ \frac{4!}{4!}(x-2)^4 + \frac{-5!}{5!}(x-2)^5 + \frac{6!}{6!}(x-2)^6 + \ldots$$

$$f(x) = (x-2)^0 - (x-2)^1 + (x-2)^2 - (x-2)^3 + (x-2)^4$$

$$-(x-2)^5 + (x-2)^6 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n (x-2)^n$$

Find Taylor series for the function $f(x) = \ln(x+1)$ centered at $c = 0$.

a) $f(x) = \ln(x+1)$

b) $f'(x) = \dfrac{1}{x+1} = \dfrac{(1-1)!}{x+1}$

c) $f''(x) = -\dfrac{1}{(x+1)^2} = -\dfrac{(2-1)!}{(x+1)^2}$

d) $f'''(x) = \dfrac{2}{(x+1)^3} = \dfrac{(3-1)!}{(x+1)^3}$

e) $f^{(4)}(x) = \dfrac{-6}{(x+1)^4} = \dfrac{-(4-1)!}{(x+1)^4}$

f) $f^{(5)}(x) = \dfrac{24}{(x+1)^5} = \dfrac{(5-1)!}{(x+1)^5}$

Find Taylor series for the function $f(x) = \ln\left(x+1\right)$ centered at $c = 0.$

a) $f\left(0\right) = \ln\left(x+1\right) = \ln\left(0+1\right) = 0$

b) $f'\left(1\right) = \dfrac{1}{x+1} = \dfrac{\left(1-1\right)!}{1+1} = \dfrac{\left(1-1\right)!}{2^1}$

c) $f''\left(1\right) = -\dfrac{\left(2-1\right)!}{\left(x+1\right)^2} = -\dfrac{\left(2-1\right)!}{\left(1+1\right)^2} = -\dfrac{\left(2-1\right)!}{\left(2\right)^2} =$

d) $f'''\left(1\right) = \dfrac{\left(3-1\right)!}{\left(x+1\right)^3} = \dfrac{\left(3-1\right)!}{\left(1+1\right)^3} = \dfrac{\left(3-1\right)!}{\left(2\right)^3}$

e) $f^{(4)}\left(1\right) = \dfrac{-\left(4-1\right)!}{\left(x+1\right)^4} = \dfrac{-\left(4-1\right)!}{\left(1+1\right)^4} = \dfrac{-\left(4-1\right)!}{\left(2\right)^4}$

f) $f^{(5)}\left(1\right) = \dfrac{\left(5-1\right)!}{\left(x+1\right)^5} = \dfrac{\left(5-1\right)!}{\left(1+1\right)^5} = \dfrac{\left(5-1\right)!}{\left(2\right)^5}$

Find Taylor series for the function $f(x) = \ln(x+1)$ centered at $c = 0$.

$$f(x) = \frac{f(c)}{0!}(x-c)^0 + \frac{f'(c)}{1!}(x-c)^2 + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \frac{f^{(4)}(c)}{4!}(x-c)^4 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

$$= \frac{0}{0!}(x-0)^0 + \frac{\frac{(1-1)!}{2^1}}{1!}(x-0)^1 + \frac{-\frac{(2-1)!}{(2)^2}}{2!}(x-0)^2 + \frac{\frac{(3-1)!}{(2)^3}}{3!}(x-0)^3 + \frac{\frac{-(4-1)!}{(2)^4}}{4!}(x-0)^4 + \frac{\frac{(5-1)!}{(2)^5}}{5!}(x-0)^5 + \ldots$$

Find Taylor series for the function $f(x) = \ln\left(x+1\right)$ centered at $c = 0.$

$$f(x) = \ln\left(x+1\right)$$

$$= \frac{\left(1-1\right)!}{2^1 1!}\left(x\right)^1 - \frac{\left(2-1\right)!}{\left(2\right)^2 2!}\left(x\right)^2 + \frac{\left(3-1\right)!}{\left(2\right)^3 3!}\left(x\right)^3 - \frac{\left(4-1\right)!}{\left(2\right)^4 4!}\left(x\right)^4$$

$$+ \frac{\left(5-1\right)!}{\left(2\right)^5 5!}\left(x\right)^5 + \ldots$$

$$= \sum_{n=1}^{\infty}\left(-1\right)^{n+1}\frac{\left(n-1\right)!}{\left(2\right)^n n!}\left(x\right)^n$$