

*n*th Degree Taylor Polynomial:

$$\begin{aligned} P_n(x) = & f(c) + f'(x)(x - c) + \frac{f''(c)(x - c)^2}{2} + \frac{f'''(c)(x - c)^3}{3!} \\ & + + \frac{f^{(4)}(c)(x - c)^4}{4!} + \cdots + \frac{f^{(n)}(c)(x - c)^n}{n!} \end{aligned}$$

Let  $f(x) = \sin x$

Find a second-degree Taylor polynomial  $P_2(x)$  centered at  $c = \frac{\pi}{4}$

$$f(x) = \sin x \quad \Rightarrow \quad f\left(\frac{\pi}{4}\right) = \sin(\pi/4) = \sqrt{2}/2$$

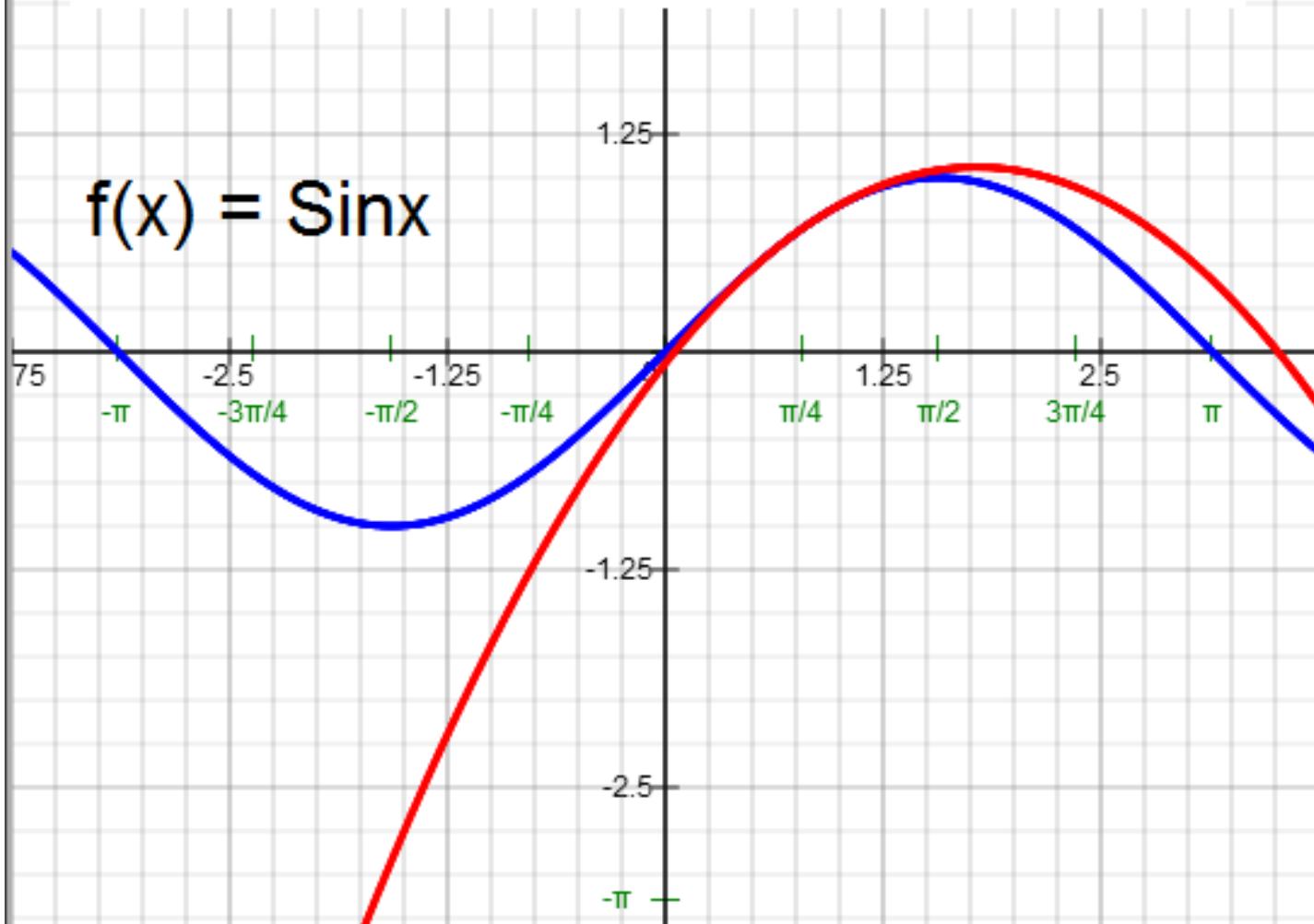
$$f'(x) = \cos x \quad \Rightarrow \quad f'\left(\frac{\pi}{4}\right) = \cos(\pi/4) = \sqrt{2}/2$$

$$f''(x) = -\sin x \quad \Rightarrow \quad f''\left(\frac{\pi}{4}\right) = -\sin(\pi/4) = -\sqrt{2}/2$$

$$P_2(x) = f(\pi/4) + f'(\pi/4)(x - \pi/4) + \frac{f''(\pi/4)(x - \pi/4)^2}{2}$$

$$P_2(x) = \sqrt{2}/2 + (\sqrt{2}/2)(x - \pi/4) + \frac{(-\sqrt{2}/2)(x - \pi/4)^2}{2}$$

$$P_2(x) = \sqrt{2}/2 + (\sqrt{2}/2)\left(x - \frac{\pi}{4}\right) + \frac{(-\sqrt{2}/2)\left(x - \frac{\pi}{4}\right)^2}{2}$$



Let  $f(x) = \cot x$

Find a second-degree polynomial  $P_2(x)$  centered at  $c = \frac{\pi}{3}$

$$f(\pi/3) = \cot x = \cot(\pi/3) = 1/\sqrt{3}$$

$$f'(x) = -(\csc x)^2$$

$$f'(\pi/3) = -(\csc x)^2 = -(\csc(\pi/3))^2 = -(2/\sqrt{3})^2 = -4/3$$

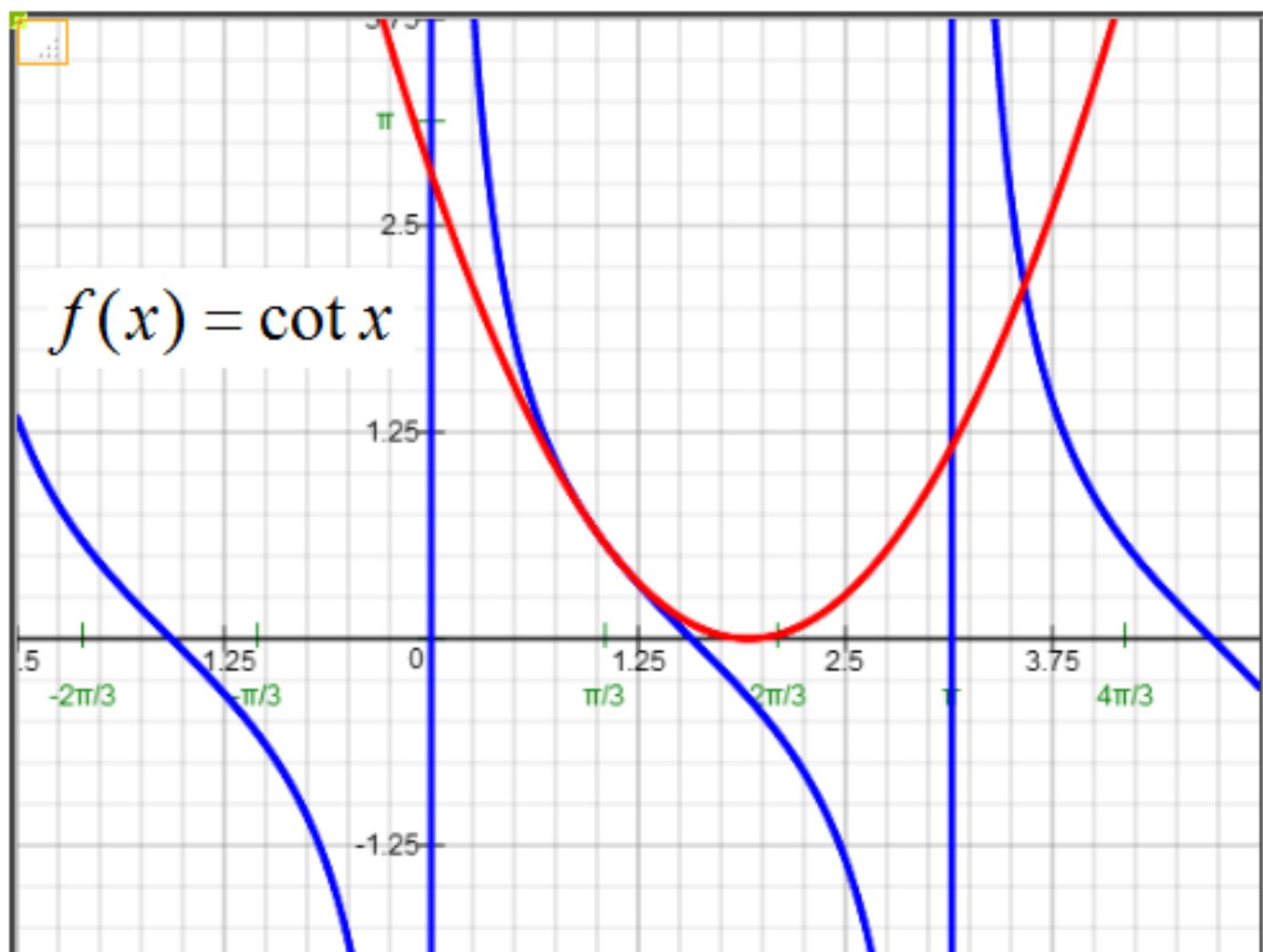
$$f''(x) = -2(\csc x)(-\csc x \cdot \cot x) = 2(\csc x)(\csc x \cdot \cot x)$$

$$f''(\pi/3) = 8/(\sqrt{3})^3$$

$$P_2(x) = f(\pi/3) + f'(\pi/3)(x - \pi/3) + \frac{f''(\pi/3)(x - \pi/3)^2}{2}$$

$$P_2(x) = 1/\sqrt{3} + (-4/3)(x - \pi/3) + \frac{8/(\sqrt{3})^3(x - \pi/3)^2}{2}$$

$$P_1(x) = 1/\sqrt{3} + (-4/3)(x - \pi/3) + \frac{8/(\sqrt{3})^3 (x - \pi/3)^2}{2}$$



$$\text{Let } f(x) = \frac{4}{\sqrt{x^3}} = 4x^{-3/2}$$

Find a second-degree polynomial  $P_2(x)$  centered at  $c = 1$ .

$$f'(x) = 4 \left( \frac{-3}{2} x^{-5/2} \right) = -6x^{-5/2}; \quad f''(x) = -6 \left( \frac{-5}{2} x^{-7/2} \right) = 15x^{-7/2}$$

$$f(1) = \frac{4}{\sqrt{x^3}} = \frac{4}{\sqrt{1^3}} = 4; \quad f'(1) = -6x^{-5/2} = -6(1)^{-5/2} = -6$$

$$f''(1) = 15x^{-7/2} = 15(1)^{-7/2} = 15$$

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2}$$

$$P_2(x) = 4 - 6(x-1) + \frac{15(x-1)^2}{2}$$

$$f(x) = \frac{4}{\sqrt[3]{x}} = 4x^{-3/2}$$

-10 -5 0 5 10

$$P_2(x) = 4 - 6(x-1) + \frac{15(x-1)^2}{2}$$

$$\text{Let } f(x) = 5xe^x$$

Find a second-degree polynomial  $P_2(x)$  centered at  $c = 0$ .

$$f'(x) = (5x)D_x(e^x) + (e^x)D_x(5x) = 5xe^x + e^x(5) = 5xe^x + 5e^x$$

$$f''(x) = 5xe^x + 5e^x + 5e^x$$

$$f(0) = 5xe^x = 5(0)e^0 = 0(1) = 0$$

$$f'(0) = 5xe^x + 5e^x = 0 + 5 = 5$$

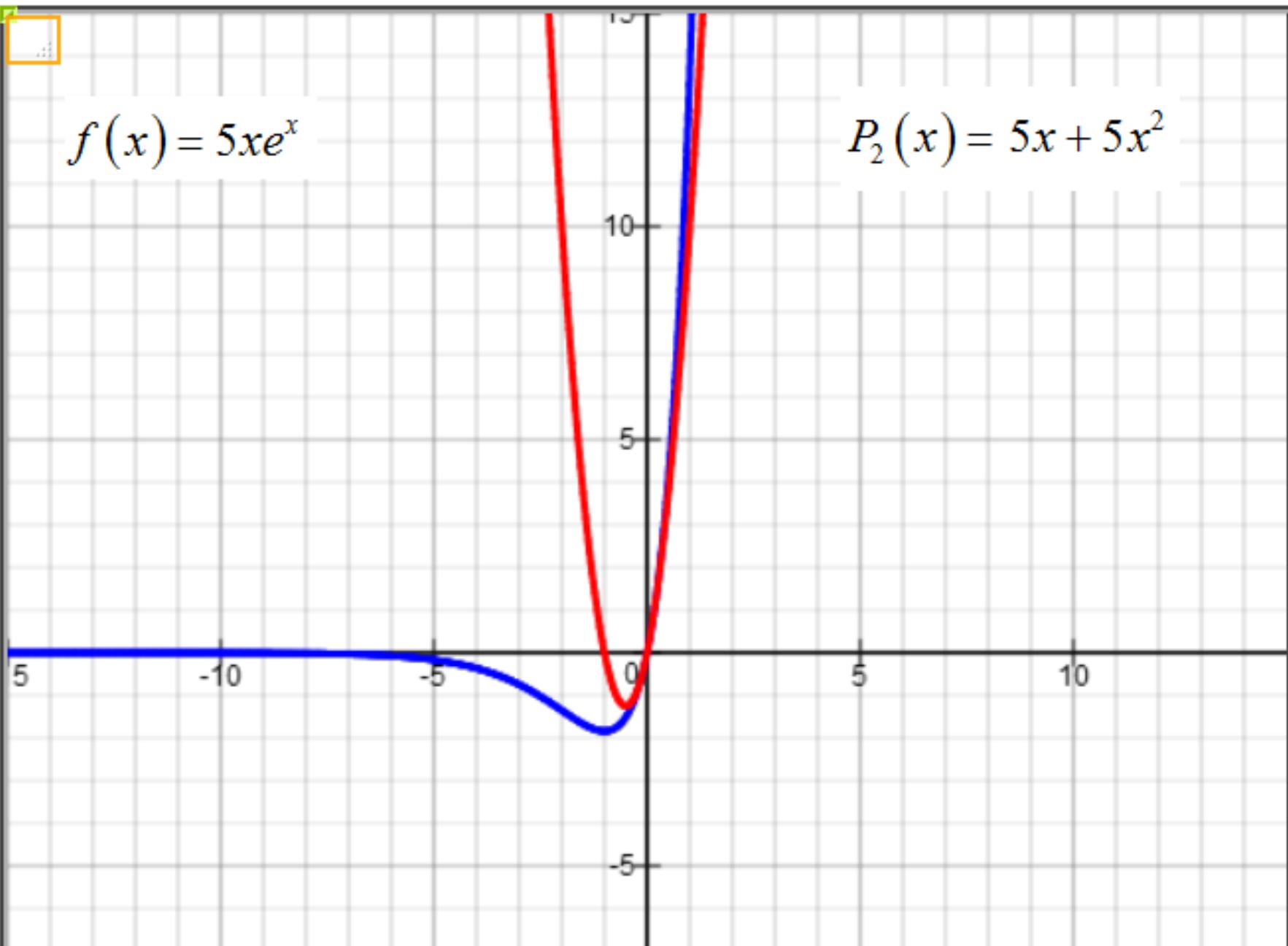
$$f''(0) = 5xe^x + 10e^x = 0 + 10 = 10$$

$$P_2(x) = f(0) + f'(0)(x - 0) + \frac{f''(0)(x - 0)^2}{2}$$

$$P_2(x) = 0 + 5(x) + \frac{10(x)^2}{2} = 5x + 5x^2$$

$$f(x) = 5xe^x$$

$$P_2(x) = 5x + 5x^2$$



$$\text{Let } f(x) = \frac{4}{x+2} = 4(x+2)^{-1}$$

Find a second-degree polynomial  $P_2(x)$  centered at  $c=1$

$$f'(x) = 4 \left[ -1(x+2)^{-2} \cdot D(x-2) \right] = 4 \left[ -1(x+2)^{-2} \cdot (1) \right] = -4(x+2)^{-2}$$

$$f''(x) = -4 \left[ -2(x+2)^{-3} \cdot D(x-2) \right] = -4 \left[ -2(x+2)^{-3} \cdot (1) \right] = 8(x+2)^{-3}$$

$$f(1) = \frac{4}{x+2} = \frac{4}{1+2} = 4/3$$

$$f'(1) = -4(x+2)^{-2} = -4(1+2)^{-2} = -4(3)^{-2}$$

$$f''(1) = 8(x+2)^{-3} = 8(1+2)^{-3} = 8(3)^{-3}$$

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2}$$

$$P_2(x) = 4/3 - 4(3)^{-2}(x-1) + \frac{8(3)^{-3}(x-1)^2}{2}$$

$$f(x) = \frac{4}{x+2}$$

$$P_2(x) = \frac{4}{3} - 4(3)^{-2}(x-1) + \frac{8(3)^{-3}(x-1)^2}{2}$$