Integration Formulas

1. $\int kf(u)du = k\int f(u)du$

2. $\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$

3. $\int 1 \cdot du = u + C$

4. $\int u^n du = \frac{u^{n+1}}{n+1} + \mathrm{C},\ n \neq -1$

5. $\int \frac{1}{u} du = \ln|u| + C$

6. $\int e^u du = e^u + \mathrm{C}$

7. $\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$

8. $\int \sin u\ du = -\cos u + \mathrm{C}$

9. $\int \cos u\ du = \sin u + C$

10. $\int \tan u\ du = -\ln|\cos u| + \mathrm{C}$

11. $\int \cot u\ du = \ln|\sin u| + C$

12. $\int \sec u\ du = \ln|\sec u + \tan u| + \mathrm{C}$

13. $\int \csc u\ du = -\ln|\csc u + \cot u| + \mathrm{C}$

14. $\int \sec^2 u\ du = \tan u + \mathrm{C}$

15. $\int \csc^2 u\ du = -\cot u + C$

16. $\int \sec u \cdot \tan u \cdot du = \sec u + \mathrm{C}$

17. $\int \csc u \cdot \cot u \cdot du = -\csc u + C$

18. $\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin\left(\frac{u}{a}\right) + \mathrm{C}$

19. $\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + \mathrm{C}$

20. $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + \mathrm{C}$

Hyperbolic Functions

21) $\int \cosh u du = \sinh u + C$

22) $\int \sinh u du = \cosh u + C$

23) $\int \operatorname{sech}^2 u du = \tanh u + C$

24) $\int \operatorname{csch}^2 u du = -\coth u + C$

25) $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$

26) $\int \operatorname{csc} h u \coth u du = -\operatorname{csc} h u + C$

Integrals Containing $a + bu$

27) $\int \frac{u}{a+bu} du = \frac{1}{b^2}\left[bu - a\ln|a+bu|\right] + C$

28) $\int \frac{u^2}{a+bu} du = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2 \ln|a+bu|\right] + C$

29) $\int \frac{u}{(a+bu)^2} du = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

30) $\int \frac{u^2}{(a+bu)^2} du = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

31) $\int \frac{u^2}{(a+bu)^3} du = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

32) $\int \frac{1}{u(a+bu)}du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

33) $\int \frac{1}{u^2(a+bu)}du = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

34) $\int \frac{1}{u(a+bu)^2}du = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

Integrals Containing $\sqrt{a+bu}$

35) $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$

36) $\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(15b^2u^2 - 12abu + 8a^2)(a+bu)^{3/2} + C$

37) $\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$

38) $\int \frac{u}{\sqrt{a+bu}}du = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

39) $\int \frac{u^2}{\sqrt{a+bu}}du = \frac{2}{15b^2}(3b^2u^2 - 4abu + 8a^2)\sqrt{a+bu} + C$

40) $\int \frac{u^n}{\sqrt{a+bu}}du = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}}{\sqrt{a+bu}}du$

41) $$\int \frac{1}{u\sqrt{a+bu}}\,du = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C & \text{if } a > 0 \\ \dfrac{2}{\sqrt{-a}}\tan^{-1}\left(\sqrt{\dfrac{a+bu}{-a}}\right) + C & \text{if } a < 0 \end{cases}$$

42) $$\int \frac{1}{u^n\sqrt{a+bu}}\,du = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{1}{u^{n-1}\sqrt{a+bu}}\,du$$

43) $$\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{1}{u\sqrt{a+bu}}\,du$$

44) $$\int \frac{\sqrt{a+bu}}{u^n}\,du = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bu}}{u^{n-1}}\,du$$

Integrals Containing $a^2 \pm u^2$ $\quad (a > 0)$

45) $$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

46) $$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\left[\ln|u-a| - \ln|u+a|\right] + C$$

47) $$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\left[\ln|u+a| - \ln|u-a|\right] + C$$

Integrals Containing $\sqrt{u^2 \pm a^2}$ $\quad (a > 0)$

48) $\displaystyle\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

49) $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

50) $\displaystyle\int u\sqrt{u^2 \pm a^2}\,du = \frac{1}{3}\left(u^2 \pm a^2\right)^{3/2} + C$

51) $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}\left(2u^2 \pm a^2\right)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

52) $\displaystyle\int \frac{\sqrt{u^2 + a^2}}{u}\,du = \sqrt{u^2 + a^2} - a\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

53) $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\,du = \sqrt{u^2 - a^2} - a\sec^{-1}\left(\frac{u}{a}\right) + C$

54) $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\,du = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

55) $\displaystyle\int \frac{u^2}{\sqrt{u^2 \pm a^2}}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2}$ [illegible] $\displaystyle\frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

56) $\int \frac{1}{u\sqrt{u^2+a^2}}\,du = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

57) $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = -\frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$

58) $\int \frac{1}{u^2\sqrt{u^2\pm a^2}}\,du = \mp\frac{\sqrt{u^2\pm a^2}}{a^2 u} + C$

59) $\int \left(u^2\pm a^2\right)^{3/2}\,du = \frac{u}{8}\left(2u^2\pm 5a^2\right)\sqrt{u^2\pm a^2} + \frac{3a^4}{8}\ln\left|u+\sqrt{u^2\pm a^2}\right| + C$

60) $\int \frac{1}{\left(u^2\pm a^2\right)^{3/2}}\,du = \pm\frac{u}{a^2\sqrt{u^2\pm a^2}} + C$

Integrals Containing $\sqrt{a^2-u^2}$ $\quad (a>0)$

61) $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + C$

62) $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

63) $\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}\left(2u^2-a^2\right)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{u}{a}\right)+C$

64) $\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|+C$

65) $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1}\left(\frac{u}{a}\right)+C$

66) $\int \frac{u^2}{\sqrt{a^2-u^2}}\,du = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+C$

67) $\int \frac{1}{u\sqrt{a^2-u^2}}\,du = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|+C$

68) $\int \frac{1}{u^2\sqrt{a^2-u^2}}\,du = -\frac{\sqrt{a^2-u^2}}{a^2u}+C$

69) $\int \left(a^2-u^2\right)^{3/2}\,du = -\frac{u}{8}\left(2u^2-5a^2\right)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\left(\frac{u}{a}\right)+C$

70) $\int \frac{1}{\left(a^2-u^2\right)^{3/2}}\,du = \frac{u}{a^2\sqrt{a^2-u^2}}+C$

Integrals Containing $2au-u^2$

71) $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

72) $\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

73) $\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right) + C$

74) $\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \sin^{-1}\left(\frac{u-a}{a}\right) + C$

75) $\int \frac{1}{\sqrt{2au-u^2}}\,du = \sin^{-1}\left(\frac{u-a}{a}\right) + C$

76) $\int \frac{u}{\sqrt{2au-u^2}}\,du = -\sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right) + C$

77) $\int \frac{u^2}{\sqrt{2au-u^2}}\,du = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

78) $\int \frac{1}{u\sqrt{2au-u^2}}\,du = -\frac{\sqrt{2au-u^2}}{au} + C$

79) $\int \frac{1}{\left(2au-u^2\right)^{3/2}}\,du = \frac{u-a}{a^2\sqrt{2au-u^2}} + C$

80) $\int \frac{u}{\left(2au-u^2\right)^{3/2}}\,du = \frac{u}{a\sqrt{2au-u^2}} + C$

Integrals Containing Trigonometric Functions

81) $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

82) $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

83) $\int \tan^2 u du = \tan u - u + C$

84) $\int \cot^2 u du = -\cot u - u + C$

85) $\int \sin^n u du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u du + C$

86) $\int \cos^n u du = -\frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u du + C$

87) $\int \tan^n u du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u du + C$

88) $\int \cot^n u du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u du + C$

89) $\int \sec^n u du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u du + C$

90) $\int \csc^n u du = -\frac{1}{n-1}\csc^{n-2} u \cot u + \frac{n-2}{n-1}\int \csc^{n-2} u du + C$

91) $\int \sin mu \cdot \cos nu du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

92) $\int \cos mu \cdot \cos nu du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C + C$

93) $\int \sin mu \cdot \cos nu du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

94) $\int u \sin u du = \sin u - u\cos u + C$

95) $\int u \cos u du = \cos u - u\sin u + C$

96) $\int u^2 \sin u du = 2u\sin u + \left(2-u^2\right)\cos u + C$

97) $\int u^2 \cos u du = 2u\cos u + \left(u^2-2\right)\sin u + C$

98) $\int u^n \sin u du = -u^n \cos u + n\int u^{n-1}\cos u du + C$

99) $\int u^n \cos u du = u^n \sin u - n\int u^{n-1}\sin u du + C$

100) $\int \sin^m u \cdot \cos^n u du = -\frac{\sin^{m-1} u \cdot \cos^{n+1} u}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} u \cos^n u du$

Integrals Containing Inverse Trigonometric Functi

101) $\int \sin^{-1} u du = u \cdot \sin^{-1} u + \sqrt{1-u^2} + C$

102) $\int \cos^{-1} u du = u \cdot \cos^{-1} u - \sqrt{1-u^2} + C$

103) $\int \tan^{-1} u du = u \cdot \tan^{-1} u - \ln \sqrt{1+u^2} + C$

104) $\int \cot^{-1} u du = u \cdot \cot^{-1} u + \ln \sqrt{1+u^2} + C$

105) $\int \sec^{-1} u du = u \cdot \sec^{-1} u - \ln \left| u + \sqrt{u^2-1} \right| + C$

106) $\int \csc^{-1} u du = u \cdot \csc^{-1} u + \ln \left| u + \sqrt{u^2-1} \right| + C$

Integrals Containing Exponential and Logarithmic

107) $\int e^{au}du = \frac{1}{a}e^{au} + C$ Note: $\int e^{u}du = e^{u} + C$

108) $\int a^{bu}du = \frac{1}{b}\frac{1}{\ln a}a^{u} + C$ Note: $\int a^{u}du = \frac{1}{\ln a}a^{u} + C$

109) $\int \frac{1}{au+b}du = \frac{1}{a}\cdot \ln|au+b| + C$ Note: $\int \frac{1}{u}du = \ln|u| + C;$

110) $\int ue^{u}du = e^{u}(u-1) + C$

111) $\int u^{n}e^{u}du = u^{n}e^{u} - n\int u^{n-1}e^{u}du + C$

112) $\int u^{n}a^{u}du = \frac{u^{n}a^{u}}{\ln a} - \frac{n}{\ln a}\int u^{n-1}a^{u}du + C$

113) $\int \frac{e^{u}}{u^{n}}du = -\frac{e^{u}}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^{u}}{u^{n-1}}du + C$

114) $\int \frac{a^u}{u^n} du = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1} \int \frac{a^u}{u^{n-1}} du + C$

115) $\int \ln u du = u \ln u - u + C$

116) $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} \left[(n+1) \ln u - 1 \right] + C$

117) $\int \frac{1}{u \ln u} du = \ln \left| \ln u \right| + C$

118) $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$

119) $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$

Wallis Formulas

120) $\int_0^{\pi/2} \sin^n u du = \int_0^{\pi/2} \cos^n u du = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot (n-2)(n-1)}{2 \cdot 4 \cdot 6 \cdot 8 \cdot \cdots \cdot (n-1)(n)}$ if n is even; $n \geq$

121) $\int_0^{\pi/2} \sin^n u du = \int_0^{\pi/2} \cos^n u du = \frac{2 \cdot 3 \cdot 6 \cdot 8 \cdot \cdots \cdot (n-2)(n-1)}{3 \cdot 5 \cdot 7 \cdot 9 \cdot \cdots \cdot (n-1)(n)}$ if n is odd; $n \geq$