Course: Math 2414 Calculus II

Trigonometry Formulas

Derivative Formulas

Integration Formulas

Angles

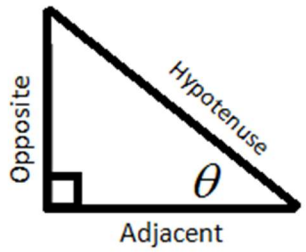
One grad equals 9/10 of a degree or $\frac{\pi}{200}$ of a radian

1 full circle $= 360$ degrees $= 2\pi$ radians $=$ 400 grads.

Degrees	Radians	Gradients	Sin(x)	Cos(x)	Tan(x)
30°	$\pi/6$	33⅓ grad	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\pi/4$	50 grad	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\pi/3$	66⅔ grad	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\pi/2$	100 grad	1	0	Undefined
120°	$2\pi/3$	133⅓ grad	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$3\pi/4$	150 grad	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
150°	$5\pi/6$	166⅔ grad	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
180°	π	200 grad	0	-1	0
210°	$7\pi/6$	233⅓ grad	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
225°	$5\pi/4$	250 grad	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
240°	$4\pi/3$	266⅔ grad	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	$3\pi/2$	300 grad	-1	0	Undefined
300°	$5\pi/3$	333⅓ grad	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$7\pi/4$	350 grad	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
330°	$11\pi/6$	366⅔ grad	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
360°	2π	400 grad	0	1	0

Trignometry Formulas

Right Triange Definitions

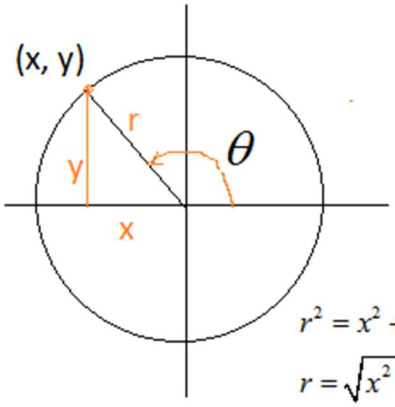


$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Circular Function Definitions



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Even Identities

$$\cos(-x) = \cos(x)$$

$$\sec(-x) = \sec(x)$$

Odd Identities

$$\sin(-x) = -\sin(x) \qquad \csc(-x) = -\csc(x)$$

$$\tan(-x) = -\tan(x) \qquad \cot(-x) = -\cot(x)$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$$

Triple-Angle Formula

$$\sin 3\theta = -\sin^3\theta + 3\cos^2\theta\sin\theta$$

$$\sin 3\theta = -4\sin^3\theta + 3\sin\theta$$

$$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$$\cot 3\theta = \frac{3\cot\theta - \cot^3\theta}{1-3\cot^2\theta}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right]$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

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$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

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$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

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$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$$

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$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$$\cot 3\theta = \frac{3\cot\theta - \cot^3\theta}{1-3\cot^2\theta}$$

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$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$

Definitions of Inverse Trigonometric Functions

1) $y = \sin^{-1}(x) = \arcsin(x)$ if and only if $\sin(y) = x$

Domain: $-1 \le x \le 1$ Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

2) $y = \cos^{-1}(x) = \arccos(x)$ if and only if $\cos(y) = x$

Domain: $-1 \le x \le 1$ Range: $0 \le y \le \pi$

3) $y = \tan^{-1}(x) = \arctan(x)$ if and only if $\tan(y) = x$

Domain: $-\infty < x < \infty$ Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

4) $y = \cot^{-1}(x) = \operatorname{arccot}(x)$ if and only if $\cot(y) = x$

Domain: $-\infty < x < \infty$ Range: $0 < y < \pi$

5) $y = \sec^{-1}(x) = \operatorname{arcsec}(x)$ if and only if $\sec(y) = x$

Domain: $|x| \ge 1 \Leftrightarrow x \le -1$ or $x \ge 1$ Range: $0 \le y \le \pi;\ y \ne \frac{\pi}{2}$

6) $y = \csc^{-1}(x) = \operatorname{arccsc}(x)$ if and only if $\csc(y) = x$

Domain: $|x| \ge 1 \Leftrightarrow x \le -1$ or $x \ge 1$ Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2};\ y \ne 0$

Derivative Formulas

Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' + uv'}{v^2}$
5. $\frac{d}{dx}[v] = 0$
6. $\frac{d}{dx}\left[u^n\right] = nu^{n-1} \cdot u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}\left[|u|\right] = \frac{u}{|u|}(u')$
9. $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$
10. $\frac{d}{dx}\left[e^u\right] = e^u \cdot u'$
11. $\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \cdot u'$
12. $\frac{d}{dx}\left[a^u\right] = (\ln a)a^u \cdot u'$
13. $\frac{d}{dx}[\sin u] = (\cos u) \cdot u'$
14. $\frac{d}{dx}[\cos u] = (-\sin u) \cdot u'$
15. $\frac{d}{dx}[\tan u] = (\sec u)^2 \cdot u'$
16. $\frac{d}{dx}[\cot u] = -(\csc u)^2 \cdot u'$
17. $\frac{d}{dx}[\sec u] = (\sec u)(\tan u) \cdot u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u)(\cot u) \cdot u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}} \cdot u'$
20. $\frac{d}{dx}[\arccos u] = \frac{-1}{\sqrt{1-u^2}} \cdot u'$
21. $\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2} \cdot u'$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-1}{1+u^2} \cdot u'$

21. $\frac{d}{dx}\left[\arctan u\right] = \frac{1}{1+u^2}\cdot u'$

22. $\frac{d}{dx}\left[\operatorname{arccot} u\right] = \frac{-1}{1+u^2}\cdot u'$

23. $\frac{d}{dx}\left[\operatorname{arcsec} u\right] = \frac{1}{|u|\sqrt{u^2+1}}\cdot u'$

24. $\frac{d}{dx}\left[\operatorname{arccsc} u\right] = \frac{-1}{|u|\sqrt{u^2-1}}\cdot u'$

25. $\frac{d}{dx}\left[\sinh u\right] = \left(\cosh u\right)\cdot u'$

26. $\frac{d}{dx}\left[\cosh u\right] = \left(\sinh u\right)\cdot u'$

27. $\frac{d}{dx}\left[\tanh u\right] = \left(\operatorname{sech} u\right)^2\cdot u'$

28. $\frac{d}{dx}\left[\coth u\right] = -\left(\operatorname{csch} u\right)^2\cdot u'$

29. $\frac{d}{dx}\left[\operatorname{sech} u\right] = -\left(\operatorname{sech} u\right)\left(\tanh u\right)\cdot u'$

30. $\frac{d}{dx}\left[\operatorname{csch} u\right] = -\left(\operatorname{csch} u\right)\left(\coth u\right)\cdot u$

31. $\frac{d}{dx}\left[\sinh^{-1} u\right] = \frac{1}{\sqrt{u^2+1}}\cdot u'$

32. $\frac{d}{dx}\left[\cosh^{-1} u\right] = \frac{1}{\sqrt{u^2-1}}\cdot u'$

33. $\frac{d}{dx}\left[\tanh^{-1} u\right] = \frac{1}{1-u^2}\cdot u'$

34. $\frac{d}{dx}\left[\coth^{-1} u\right] = \frac{1}{1-u^2}\cdot u'$

35. $\frac{d}{dx}\left[\operatorname{sech}^{-1} u\right] = \frac{-1}{u\sqrt{1-u^2}}\cdot u'$

36. $\frac{d}{dx}\left[\operatorname{csch}^{-1} u\right] = \frac{-1}{|u|\sqrt{1+u^2}}\cdot u'$

Integration Formulas

Integration Formulas

1. $\int kf(u)du = k\int f(u)du$
2. $\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$
3. $\int 1 \cdot du = u + C$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
5. $\int \frac{1}{u} du = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u \, du = -\cos u + C$
9. $\int \cos u \, du = \sin u + C$
10. $\int \tan u \, du = -\ln|\cos u| + C$
11. $\int \cot u \, du = \ln|\sin u| + C$
12. $\int \sec u \, du = \ln|\sec u + \tan u| + C$
13. $\int \csc u \, du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u \, du = \tan u + C$
15. $\int \csc^2 u \, du = -\cot u + C$
16. $\int \sec u \cdot \tan u \cdot du = \sec u + C$
17. $\int \csc u \cdot \cot u \cdot du = -\csc u + C$
18. $\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin\left(\frac{u}{a}\right) + C$
19. $\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$
20. $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

Hyperbolic Functions

21) $\int \cosh u du = \sinh u + C$
22) $\int \sinh u du = \cosh u + C$
23) $\int \operatorname{sech}^2 u du = \tanh u + C$
24) $\int \operatorname{csch}^2 u du = -\coth u + C$
25) $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$
26) $\int \operatorname{csc} h u \coth u du = -\operatorname{csc} h u + C$

27) $\int \frac{u}{a+bu} du = \frac{1}{b^2}\left[bu - a\ln|a+bu|\right] + C$

28) $\int \frac{u^2}{a+bu} du = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln|a+bu|\right] + C$

29) $\int \frac{u}{(a+bu)^2} du = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

30) $\int \frac{u^2}{(a+bu)^2}du = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

31) $\int \frac{u^2}{(a+bu)^3}du = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

32) $\int \frac{1}{u(a+bu)}du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

33) $\int \frac{1}{u^2(a+bu)}du = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

34) $\int \frac{1}{u(a+bu)^2}du = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

34b) $\int \frac{1}{u^2(a+bu)^2}du = \frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

Integrals Containing $\sqrt{a+bu}$

35) $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$

36) $\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}\left(15b^2u^2 - 12abu + 8a^2\right)(a+bu)^{3/2}$

37) $\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}$

38) $\int \frac{u}{\sqrt{a+bu}}du = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

39) $\int \frac{u^2}{\sqrt{a+bu}}du = \frac{2}{15b^2}\left(3b^2u^2 - 4abu + 8a^2\right)\sqrt{a+bu} + C$

40) $\int \frac{u^n}{\sqrt{a+bu}}du = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}}{\sqrt{a+bu}}du$

41) $\int \frac{1}{u\sqrt{a+bu}}du = \begin{cases} \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C & \text{if } a>0 \\ \frac{2}{\sqrt{-a}}\tan^{-1}\left(\sqrt{\frac{a+bu}{-a}}\right) + C & \text{if } a<0 \end{cases}$

42) $\int \frac{1}{u^n\sqrt{a+bu}}du = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{1}{u^{n-1}\sqrt{a+bu}}du$

44) $\int \frac{\sqrt{a+bu}}{u^n}du = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bu}}{u^{n-1}}du$

Integrals Containing $a^2 \pm u^2$ $\quad (a>0)$

45) $\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$

46) $\int \frac{1}{a^2-u^2}du = \frac{1}{2a}\left[\ln|u-a| - \ln|u+a|\right]+C$

47) $\int \frac{1}{u^2-a^2}du = -\frac{1}{2a}\left[\ln|u-a| - \ln|u+a|\right]+C$

Integrals Containing $\sqrt{u^2 \pm a^2}$ $\quad (a>0)$

48) $\int \frac{1}{\sqrt{u^2 \pm a^2}}du = \ln\left|u+\sqrt{u^2 \pm a^2}\right|+C$

49) $\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|u+\sqrt{u^2 \pm a^2}\right|+C$

50) $\int u\sqrt{u^2 \pm a^2}\,du = \frac{1}{3}\left(u^2 \pm a^2\right)^{3/2}+C$

51) $\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}\left(2u^2 \pm a^2\right)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2 \pm a^2}\right|+C$

52) $\int \frac{\sqrt{u^2+a^2}}{u}du = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|+C$

53) $\int \frac{\sqrt{u^2-a^2}}{u}du = \sqrt{u^2-a^2} - a\sec^{-1}\left(\frac{u}{a}\right)+C$

54) $\int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

55) $\int \frac{u^2}{\sqrt{u^2 \pm a^2}} du = \frac{u}{2}\sqrt{u^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

56) $\int \frac{1}{u\sqrt{u^2 + a^2}} du = -\frac{1}{a}\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

57) $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

58) $\int \frac{1}{u^2\sqrt{u^2 \pm a^2}} du = \mp\frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

59) $\int \left(u^2 \pm a^2\right)^{3/2} du = \frac{u}{8}\left(2u^2 \pm 5a^2\right)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

60) $\int \frac{1}{\left(u^2 \pm a^2\right)^{3/2}} du = \pm\frac{u}{a^2\sqrt{u^2 \pm a^2}} + C$

60b) $\int \frac{1}{\left(a^2 + u^2\right)^2} du = \frac{1}{2a^3}\left[\frac{au}{a^2 + u^2} + \tan^{-1}\left(\frac{u}{a}\right)\right] + C$

Integrals Containing $\sqrt{a^2 - u^2}$ $\quad (a > 0)$

61) $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1}\left(\frac{u}{a}\right) + C$

62) $\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

63) $\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}\left(2u^2 - a^2\right)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{u}{a}\right) + C$

64) $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

65) $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1}\left(\frac{u}{a}\right) + C$

66) $\int \frac{u^2}{\sqrt{a^2-u^2}}du = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

67) $\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

68) $\int \frac{1}{u^2\sqrt{a^2-u^2}}du = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$

69) $\int \left(a^2-u^2\right)^{3/2}du = -\frac{u}{8}\left(2u^2-5a^2\right)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\left(\frac{u}{a}\right) + C$

70) $\int \frac{1}{\left(a^2-u^2\right)^{3/2}}du = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

Integrals Containing $2au - u^2$

71) $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

72) $\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

73) $\int \frac{\sqrt{2au-u^2}}{u}du = \sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right) + C$

74) $\int \frac{\sqrt{2au-u^2}}{u^2}du = -\frac{2\sqrt{2au-u^2}}{u} - \sin^{-1}\left(\frac{u-a}{a}\right) + C$

75) $\int \frac{1}{\sqrt{2au-u^2}}du = \sin^{-1}\left(\frac{u-a}{a}\right) + C$

76) $\int \frac{u}{\sqrt{2au-u^2}}du = -\sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right) + C$

77) $\int \frac{u^2}{\sqrt{2au-u^2}}du = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$

78) $\int \frac{1}{u\sqrt{2au-u^2}}du = -\frac{\sqrt{2au-u^2}}{au} + C$

79) $\int \frac{1}{\left(2au-u^2\right)^{3/2}}du = \frac{u-a}{a^2\sqrt{2au-u^2}} + C$

80) $\int \frac{u}{\left(2au-u^2\right)^{3/2}}du = \frac{u}{a\sqrt{2au-u^2}} + C$

Integrals Containing Trigonometric Functions

81) $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

82) $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

83) $\int \tan^2 u du = \tan u - u + C$

84) $\int \cot^2 u du = -\cot u - u + C$

85) $\int \sin^n u du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u du + C$

86) $\int \cos^n u du = -\frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u du + C$

87) $\int \tan^n u du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u du + C$

88) $\int \cot^n u du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u du + C$

89) $\int \sec^n u du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u du + C$

90) $\int \csc^n u du = -\frac{1}{n-1}\csc^{n-2} u \cot u + \frac{n-2}{n-1}\int \csc^{n-2} u du + C$

91) $\int \sin mu \cdot \cos nu du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

92) $\int \cos mu \cdot \cos nu du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C + C$

93) $\int \sin mu \cdot \cos nu du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

94) $\int u \sin u du = \sin u - u\cos u + C$

95) $\int u \cos u du = \cos u - u\sin u + C$

96) $\int u^2 \sin u du = 2u\sin u + (2-u^2)\cos u + C$

97) $\int u^2 \cos u du = 2u\cos u + (u^2 - 2)\sin u + C$

98) $\int u^n \sin u du = -u^n \cos u + n\int u^{n-1}\cos u du + C$

99) $\int u^n \cos u du = u^n \sin u - n\int u^{n-1}\sin u du + C$

100) $\int \sin^m u \cdot \cos^n u du = -\frac{\sin^{m-1} u \cdot \cos^{n+1} u}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} u \cos^n u du$

For $m = 1$ and $n = 1$: $\int \sin u \cos u du = \frac{(\cos u)^2}{2} + C$

100b) $\int \frac{1}{1 \pm \tan u} du = \frac{1}{2}\left(u \pm \ln\left|\cos u \pm \sin u\right|\right) + C$

100c) $\int \frac{1}{1 \pm \cot u} du = \frac{1}{2}\left(u \mp \ln\left|\sin u \pm \cos u\right|\right) + C$

100d) $\int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + C$

100e) $\int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + C$

Integrals Containing Inverse Trigonometric Functions

101) $\int \sin^{-1} u du = u \cdot \sin^{-1} u + \sqrt{1 - u^2} + C$

101b) $\int u \sin^{-1} u du = \frac{2u^2 - 1}{4} \sin^{-1} u + \frac{u\sqrt{1 - u^2}}{4} + C$

102) $\int \cos^{-1} u du = u \cdot \cos^{-1} u - \sqrt{1 - u^2} + C$

102b) $\int u \cos^{-1} u du = \frac{2u^2 - 1}{4} \cos^{-1} u - \frac{u\sqrt{1 - u^2}}{4} + C$

103) $\int \tan^{-1} u du = u \cdot \tan^{-1} u - \ln\sqrt{1 + u^2} + C$

103b) $\int u \tan^{-1} u du = \frac{u^2 + 1}{2} \tan^{-1} u - \frac{u}{2} + C$

104) $\int \cot^{-1} u du = u \cdot \cot^{-1} u + \ln\sqrt{1 + u^2} + C$

105) $\int \sec^{-1} u du = u \cdot \sec^{-1} u - \ln\left|u + \sqrt{u^2 - 1}\right| + C$

106) $\int \csc^{-1} u du = u \cdot \csc^{-1} u + \ln\left|u + \sqrt{u^2 - 1}\right| + C$

Integrals Containing Exponential and Logarithmic Functions

107) $\int e^{au}du = \frac{1}{a}e^{au} + C$ Note: $\int e^{u}du = e^{u} + C$

108) $\int a^{bu}du = \frac{1}{b}\frac{1}{\ln a}a^{u} + C$ Note: $\int a^{u}du = \frac{1}{\ln a}a^{u} + C$

109) $\int \frac{1}{au+b}du = \frac{1}{a}\cdot \ln|au+b| + C$ Note: $\int \frac{1}{u}du = \ln|u| + C;$

110) $\int ue^{u}du = e^{u}(u-1) + C$

111) $\int u^{n}e^{u}du = u^{n}e^{u} - n\int u^{n-1}e^{u}du + C$

112) $\int u^{n}a^{u}du = \frac{u^{n}a^{u}}{\ln a} - \frac{n}{\ln a}\int u^{n-1}a^{u}du + C$

113) $\int \frac{e^{u}}{u^{n}}du = -\frac{e^{u}}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^{u}}{u^{n-1}}du + C$

114) $\int \frac{a^{u}}{u^{n}}du = -\frac{a^{u}}{(n-1)u^{n-1}} + \frac{\ln a}{n-1}\int \frac{a^{u}}{u^{n-1}}du + C$

115) $\int \ln u du = u\ln u - u + C$

$\int (\ln u)^{2}du = u\left[2 - 2\ln u + (\ln u)^{2}\right] + C$

$\int (\ln u)^{n}du = u(\ln u)^{n} - n\int (\ln u)^{n-1}du + C$

116) $\int u^{n}\ln u du = \frac{u^{n+1}}{(n+1)^{2}}\left[(n+1)\ln u - 1\right] + C$

117) $\int \frac{1}{u\ln u}du = \ln|\ln u| + C$

118) $\int e^{au}\sin bu du = \frac{e^{au}}{a^{2}+b^{2}}(a\sin bu - b\cos bu) + C$

119) $\int e^{au}\cos bu du = \frac{e^{au}}{a^{2}+b^{2}}(a\cos bu + b\sin bu) + C$

119b) $\int \frac{1}{1+e^{u}}du = u - \ln(1+e^{u}) + C$

Wallis Formulas

120) $\int_0^{\pi/2} \sin^n u\,du = \int_0^{\pi/2} \cos^n u\,du = \dfrac{1\cdot 3\cdot 5\cdot 7\cdot \cdots \cdot (n-2)(n-1)}{2\cdot 4\cdot 6\cdot 8\cdot \cdots \cdot (n-1)(n)}$ if n is even; $n \geq 2$

121) $\int_0^{\pi/2} \sin^n u\,du = \int_0^{\pi/2} \cos^n u\,du = \dfrac{2\cdot 3\cdot 6\cdot 8\cdot \cdots \cdot (n-2)(n-1)}{3\cdot 5\cdot 7\cdot 9\cdot \cdots \cdot (n-1)(n)}$ if n is odd; $n \geq 3$