Filamentation instability of dust-acoustic wave in a collisional plasma with a variable charge

Abstract

The filamentation instability of dust-acoustic wave in a collisional plasma with variable-charge dusts is studied using fluid theory. The effect of ion drag force on the development of this instability is investigated. It is shown that the filamentary mode appears at the initial stages of void formation in dusty plasma, when the ion drag force acting on the dust particles is stronger than the external electric force. This instability is more easily excited when the percentage of free electrons is reduced and the dust particles are sufficiently large. Furthermore, the establishment time of the filamentation structure and the instability development threshold are obtained.

I. INTRODUCTION

Dusty or complex plasmas are partially ionized gases composed of neutral atoms, electrons, ions, and charged dust particles. Each dust grain is a solid particulate matter that usually acquire a large electric charge by collecting electrons and ions from the plasma.[1–3] The dust-charge variation due to electron and ion capture/release by the dust grains can affects various collective phenomena in dusty plasmas. Recently a lot of theoretical and experimental research has been conducted on the dust void and other structures in dusty plasmas.[1–14]

A dust void is a region without any dusty particle in the bulk of the plasma and is the result of a balance of forces acting on the dust particles. Samsonov and Goree[5] found that the spontaneous formation of voids will be developed by a sudden onset of a filamentary mode associated with increased local ionization which attribute to a depletion of the dust number density. They found that the boundary between void region and dusty cloud is typically quite sharp. Morfill et al.[6] experimentally investigated the effect of the plasma pressure on the void formation. They invoked the thermophoretic forces (a neutral temperature gradient force) to explain void formation. Thomas et al.[7] showed experimentally the formation of voids around negatively biased probes in a dusty plasma. The negative dust grains repel from the negatively biased probe and a dust void would form around the probe. Goree et al.[8] found that a minimum ionization rate is required to sustain a stable void equilibria. In a region of reduced dust density, a reduced depletion of the electrons by the dust particles leads to a higher electron density and a consequently a higher ionization rate. The dust density perturbation produces a positive space charge with respect to the surrounding medium, which yields to build up an electric field directed outward from the void center. This electric field gives rise to an inward electric force and an outward ion drag force on negatively charged dust particles which tends to expel them from their position. Therefore, in equilibrium state there is a balance between the outward ion drag force and the inward electric force on the dust particles that results in formation of dust-free region or void.[5,8]

The ion-drag force in dusty plasmas arises from the ion orbital motion around negatively charged dust particles as well as from the momentum transfer from all the ions which are collected by the dust grains.[1] The ion drag on the dust grains is important for the instability and can lead to the formation of the void.[1,2,5–8] This force is proportional to the square of the particle radius[1]. Hence, when the dust grains size exceeds a critical value, the outward ion drag force dominates over the inward electrostatic force. Thus, the fluctuation will grow in the dusty plasma and manifest itself as filamentation, which is valid only at the initial stage of the instability. For a small particle size, the electrostatic force will dominate, and the region of redused dust density will be filled up once again by dust, and the filamentation instability will disappear. Therefore, the threshold of the filamentary mode development is dependent on the particle size and the electric field strength.[2,8]

Filamentation instability is one of the fundamental instabilities that may arise in response of a plasma to an externally applied electric field. Electron and/or ion beam moving through a plasma will filament by a this instability that has a physical mechanism closely related to the Weibel instability. The filamentation instability convert the kinetic energy of beam into the electromagnetic energy. On the other hand, the filamentation instability and magnetic field generation attracts great attention.[14–25] Recently, we have investigated the filamentation instability of dust-acoustic wave[24] (DAW) and dust ion-acoustic wave[25] ($DIAW$) in a current-driven dusty plasma by using the kinetic approach. In the present work, using fluid theory we will investigate the filamentation instability of DAW at the initial stage of the void formation in dusty plasma. We consider the dust charge fluctuation as well as ion drag on the development of the filamentation instability at the dust-acoustic time scale. We also determine the threshold of the filamentation instability development and obtain the period of cross structure.

This work is organized into four sections. In Sec. II, the problem is formulated and the basic set of equations is given. In Sec. III, the dispersion relation describing the dust-acoustic instability in an external dc electric field is derived. Finally, a summary and conclusions are presented in Sec. IV.

II. BASIC EQUATIONS

We consider the effect of dust-charge variation on DAW propagation in a uniform, unmagnetized, partially ionized current-carrying dusty plasma, whose constituents are negative electrons, positive ions, micron-sized extremely massive negatively charged dust grains, and a fraction of neutral atoms. For very low frequency waves such as DAWs, the electron, ion, and dust grain temperatures satisfy $T_e, T_i \gg T_d$ and the DA wave occurs in the frequency range $kv_{Td} \ll \omega \ll kv_{Ti}, kv_{Te}$, where k and ω are wave number and frequency, respectively. v_{Tj} denotes thermal velocity of the particle species j where $j = e, i, d$ for electrons, ions and dust particles, respectively. In the absence of charge fluctuation the quasineutrality condition $n_{i0} = n_{e0} + Z_{d0}n_{d0}$ is satisfied, where n_{j0} is the unperturbed number densities of the particle species j and Z_{d0} represents the equilibrium number of charges residing on the negatively charged dust grains. The dust particles are assumed to be spherical and are of the same radius a. Charging of the dust grains are considered to be connected to attachment of the electrons and ions on the dust grains due to electron and ion currents entering the dust grains.

For one-dimensional wave propagation along the x-axis, the dynamics of plasma and dust particles are obtained from the fluid equations.[15] For the plasma particles, we have

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j v_j)}{\partial x} = -\nu_{jd} n_j + \zeta, \tag{1}$$

$$p v_j \frac{\partial v_j}{\partial x} + \nu_j^{eff} v_j + \frac{T_j}{n_j m_j} \frac{\partial n_j}{\partial x} = \frac{q_j}{m_j} E, \tag{2}$$

where E is the electric field of the DAW, n_j stands for the sum of the equilibrium and the perturbed number densities of the species j (where $j = e, i$ for electron and ion respectively). v_j, q_j and m_j denote fluid velocity, charge and mass of the species j respectively. ν_{jd} is the collection rate of species j by the dust grains. p is equal to unity for ion species and equal to zero for electrons. We have defined $\zeta = \nu_{ion} n_e - \rho n_e^2$, where ν_{ion} is the ionization rate, and ρ is the volume recombination coefficient. We have also defined the effective electron (ion) collision frequency $\nu_e^{eff} = \nu_e + \nu_e^{el} + \nu_e^{ch}$ ($\nu_i^{eff} = \nu_i + \nu_i^{el} + \nu_i^{ch}$) where ν_e (ν_i) is the rate of electron (ion) collisions with neutral atoms and plasma particles, ν_e^{el} (ν_i^{el}) is the rate of elastic Coulomb electron-dust (ion-dust) collisions, and ν_e^{ch} (ν_i^{ch}) is the effective rate of collection of electrons (ions) by the dusts.

For the dust particles, we have

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_d)}{\partial x} = 0, \tag{3}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} + \nu_{dn} v_d + \frac{T_d}{n_d m_d} \frac{\partial n_d}{\partial x} + \mu_{drag}^i (v_d - v_i) = -\frac{Z_d e}{m_d} E, \tag{4}$$

where n_d stands for the sum of the equilibrium and the perturbed number density of the dust particle. v_d, Z_d and m_d denote fluid velocity, charge number and mass of the dust grain respectively.

In Eq. (4) the term $\mu^i_{drag}(v_d - v_i)$ corresponds to the ion drag force acting on the dust grains, where $\mu^i_{drag} \sim 4m_i n_{0i} b^2 v_s/m_d$ is the ion drag coefficient. $v_s = \sqrt{T_e/m_i}$ is the ion-acoustic velocity and $b \sim a\sqrt{\pi}(1 - \Delta\varphi_g/T_i)$ is the ion collection impact parameter where a is the radius of spherical dust grains. $\Delta\varphi_g = (\phi_g - \phi_0) = q_{d0}/C$ denotes the grain surface potential relative to the ambient plasma potential where q_{d0} is the equilibrium value of the dust charge and $C = a(1 + a/r_D)$ is the grain capacitance. Clearly the ion drag force, which is proportional to the square of particle radius, dominates over the electrostatic force when dust grains have sufficiently large sizes.[1]

The dust charge fluctuation is governed by the current balance equation[1]

$$\frac{\partial q_{d1}}{\partial t} + v_d \frac{\partial q_{d1}}{\partial x} + \nu_d^{ch} q_{d1} = -|I_{e0}| \frac{n_{e1}}{n_{e0}} + |I_{i0}| \frac{n_{i1}}{n_{i0}} \tag{5}$$

where $q_{d1} = Z_{d1} e$ is the perturbation of average charge, Z_{d1} is the perturbation of dust-charge number, n_{e1} (n_{i1}) is the variation of electron (ion) density. ν_d^{ch} is the dust charging rate by the equilibrium electron and ion microscopic currents.

Equation (5) is valid for grain charging arising from plasma currents due to electrons and ions reaching the grain surface. When the streaming velocities of the electrons and ions are much smaller than their corresponding thermal velocities and the thermal velocity of the electrons are higher than the thermal velocity of the ions, the surface potential of an isolated dust particle will be negative and the electron and ion currents reaching to dust grains are determined by

$$I_{e0} = -\pi a^2 e (8T_e/\pi m_e)^{1/2} n_{e0} \exp\left[e(\phi_g - \phi_0)_g/T_e\right], \tag{6}$$

$$I_{i0} = -\pi a^2 e (8T_i/\pi m_i)^{1/2} n_{i0} \left[1 - e(\phi_g - \phi_0)/T_i\right], \tag{7}$$

In the next section, using the dispersion relation of DAW and considering that the ion drag force exceeds the electrostatic force, we will investigate the filamentation instability, which takes place at the initial stage of the formation of stable dust voids.

IV. DUST-ACOUSTIC FILAMENTATION

By using appropriate expressions for collision frequencies, considering quasi-neutrality condition for perturbed densities, assuming monochromatic form for perturbed quantities and then linearizing Eqs (1-5), one can obtain dispersion relation of DAW for one-dimensional wave propagation along the x-axis in aforementioned system as follow:*****

$$\frac{\omega_{pe}^2}{\eta_e \nu_e^{eff}}\left(1+\frac{\nu_{ed}}{\nu_d^{ch}}\right)+\frac{\omega_{pi}^2}{\eta_i \nu_i^{eff}}\left(1+\frac{n_{e0}\nu_{ed}}{n_{i0}\nu_d^{ch}}\right)$$
$$+\frac{i\omega_{pd}^2}{\Omega_{d0}[\Omega_{d0}+i(\nu_{dn}+\mu_{drag}^i)]}\left(1-\frac{\Omega_{i0}+i\nu_{id}}{\eta_i\beta}\right)=0 \qquad (8)$$

where we have defined $\Omega_{d0}=\Omega-kv_{d0}$, $\Omega_{j0}=\Omega_{d0}-k\vartheta_j$ and $\vartheta_j=v_{j0}-v_{d0}$ for $j=e,i$. $\beta=Z_{d0}m_i\nu_i^{eff}/m_d\mu_{drag}^i$ represents the ratio of electric force to ion drag force and $\omega_{p\alpha}=\sqrt{4\pi n_\alpha q_\alpha^2/m_\alpha}$ is the plasma frequency of the α species. $\eta_i=\Omega_{i0}+i\left(\nu_{id}+k^2V_{Ti}^2/\nu_i^{eff}\right)$ and $\sqrt{}$ is the ion thermal velocity.

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It is seen that the most unstable situations occur when the percentage of free electrons is reduced.[16] Thus, we assume most of background electrons are stick onto immobile dust grain surface during charging processes. As a result, we have a significant electron density depletion in dusty plasma, i.e., $n_{e0}\ll n_{i0}$. In this case, Eq. (8) reduces to

$$\frac{\omega_{pi}^2}{\eta_i\nu_i^{eff}}+\frac{i\omega_{pd}^2}{\Omega_{d0}[\Omega_{d0}+i(\nu_{dn}+\mu_{drag}^i)]}\left(1-\frac{\Omega_{i0}+i\nu_{id}}{\eta_i\beta}\right)=0 \qquad (9)$$

By invoking the quasi-neutrality approximation for negatively charged dust grains ($Z_{d0}n_{d0} \approx n_{i0}$), Eq (2) can be rewritten into a simpler form

$$\frac{\Omega_d^3}{m_i} + i\frac{Z_{d0}k^2 v_{Ti}^2}{m_d}\left(i\Omega_d + \nu_{dn} + \mu^i_{drag}\right) + \frac{\mu^i_{drag}}{m_i}$$

$$\times\left[(\beta - 1)(\Omega_d - k\vartheta_i + i\nu_{id})(i\Omega_d + \nu_{dn} + \mu^i_{drag})\right] = 0 \tag{10}$$

where $v_{Ti} = \sqrt{T_i/m_i}$ is the ion thermal velocity.

We now consider the situation when the ion drag force acting on the dust particles is stronger than the external electric force, that is, when $\beta < 1$. When the outward ion drag force exceeds inward electrostatic force, the particles in that region repulse the surrounding particles, until a stable circular void appears. In this case, the dust grains are pushed out by the ion drag force and move in the same direction with the ions. Thus, the relative ion-to-dust drift is small and we can neglect of $k\vartheta_i$ in contrast with ν_{id}. Therefore, assuming $k\vartheta_i \ll \nu_{id}$, we expand the dispersion equation (3) in the static limit, i.e., $\Omega_d \to 0$ to find the spatial structure

$$k_0^2 = \frac{m_d \mu^i_{drag} \nu_{id}(1-\beta)}{m_i Z_{d0} v_{Ti}^2} \tag{11}$$

We can express the frequency spectrum in a weakly ionized current-driven dusty plasma, as follows:

$$\Omega_d = i\frac{m_i Z_{d0} k^2 v_{Ti}^2}{m_d \mu^i_{drag}(1-\beta)}\left[1 - \frac{k_0^2}{k^2}\right] \tag{12}$$

This equation shows that the current-carrying dusty plasma is unstable when $k_0^2 < k^2$. Also, this spectrum shows that a transverse structure with a characteristic period π/k_0 can exist in the static limit and filamentation instability may arise when $k_0^2 < k^2$. As already mentioned, with reduction of ion drag force the the filamentation will disappear, and the frequency spectrum Eq. (5) will become aperiodic. Also, this frequency spectrum indicates that the filamentation instability will arise when the particle diameter are sufficiently large. On the other hand, when the dust grains are small the filamentation instability thershold cannot be reached. According to Eq. (4), the parameters involved in k_0^2, the dimensions of filaments can be high or low and the number of filaments dependent to plasma dimensions and width filaments.

As we have a ion drift in the dusty plasma, a magnetic field $B_0 \approx 4\pi e n_i u x_0/c$ arises around its axis, where x_0 is the lateral distance from current axis and n_i is the ion current density. If the transverse dusty plasma dimension is greater than the the lateral distance from the current axis, the magnetic pressure is larger than gas kinetic pressure and the filamentation instability can be result i.e.,

$$\frac{B_0^2}{8\pi} = \frac{1}{8\pi}\left[\frac{4\pi e n_i u x_0}{c}\right]^2 > n_i T_i \tag{13}$$

where n_i is the ion plasma density. From Eq. (4) we can find

$$x_0 = \frac{2 v_{Ti}\sqrt{m_i Z_{d0}}}{\sqrt{m_d \mu^i_{drag} \nu_{id}(1-\beta)}} \cong l_0 \tag{14}$$

Thus, the self-pressing (pinch effect) is possible.

The time needed for the establishment of this structure can be determined from the time that is necessary for instability development. To obtain this value we consider a system close to the threshold $k \approx k_0$, where its minimal value can be determined from Eq. (5):

$$\tau \cong \frac{1}{Im(\omega)} \approx \frac{m_d \mu^i_{drag}(1-\beta)}{m_i Z_{d0} k_0^2 v_{Ti}^2} \approx \frac{1}{\nu_{id}} \tag{15}$$

From Eq. (8) we can find that the particle size or charge is a critical parameter for the onset of the instability. Also, the establishment time of the filamentation instability is proportional to the reverse of collision frequency between ion and dust particles. This means that ion collisions have a destabilizing effect on the dust acoustic waves.

V. NUMERICAL RESULTS

In this section numerical solution of the dispersion relation Eq. (5) are obtained. In all of the results that follow we use the following set of parameters:

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For a typical dusty argon plasma,[15] the parameter values used in the numerical computation are: $T_i \sim 0.15\ ev$, $v_{Ti} \sim 5.7 \times 10^4\ cm/s$, $Z_{d0}m_i/m_d = 1.95 \times 10^{-6}$, $\beta = 0.01$, and $\mu^i_{drag} \sim 1.65 \times 10^3\ s^{-1}$. Hence, if $\nu_{id} \sim 10\ s^{-1}$, we find $l_0 \sim 1.2\ cm$ and $\tau \sim\ 10^{-1}\ s$.

IV. SUMMARY AND CONCLUSION

In this work, using fluid theory and taken into account the dust grain charge fluctuations, we investigated the fiilamentation instability of dust-acoustic wave in a collisional dusty plasma. The dust-acoustic wave was excited by a relative drift of the ions produced by a steady-state electric field externally applied to the dusty plasma. We assumed that most electrons are attached to the immobile dust grains, i.e., $n_i \gg n_e$, because the dust acoustic waves becomes more easily excited when the relative concentration of negatively charged dust is increased. Then, we obtained the dispersion relation for low-frequency dust-acoustic wave and studied the filamentation instability in the aforementioned system. We showed that this instability takes place when the ion drag force acting on the dust particles is stronger than the external electric force. In this case the ion drag force pushes the dust grains in the same direction of the ion drift. Thus, the filamentary mode appeared abruptly in the initial stages of void formation in dusty plasma. It was shown that if the dust particles become small the electrostatic force will dominate, and the region of redused dust density will be filled up once again by dust, and the filamentation instability will disappear. Also, it was shown that close to a threshold, $k \cong k_0$, which dependence on the particle size and the electric field strength, the current layer will be subdivided into separate current filaments with a establishment time of the order of $\tau \approx 1/\nu_{id}$.

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