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The diversification effects of volatility-related assets

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ABSTRACT

We examine whether investors can improve their investment opportunity sets through the addition of volatility-related assets into various groupings of benchmark portfolios. By first analyzing the weekly returns of three VIX-related assets over the period 1996–2008 and then applying mean–variance spanning tests, we find that adding VIX-related assets does lead to a statistically significant enlargement of the investment opportunity set for investors. Our empirical findings are robust and have two implications. First, there is scope for the further development of financial products relating to volatility indexes. Second, hedge fund managers can utilize VIX futures contracts or VIX-squared portfolios to enhance their equity portfolio performance, as measured by the Sharpe ratio.

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1. Introduction

Finance theory argues that non-redundant financial assets can help to improve the completeness of financial markets, as well as risk-sharing amongst investors (Huang and Litzenberger, 1988; Ingersoll, 1987). Thus, it is of importance to investors or hedge fund managers to determine whether VIX-related assets collectively, can as a new investment vehicle significantly expand the mean–variance frontier generated by the portfolios of equity, thereby leading to diversification benefits. Accordingly, this study suggests a portfolio diversification perspective in order to explore volatility-related assets by examining their diversification effects.

The idea of developing volatility indices and volatility-related assets has long been advocated by both academics (see Brenner and Galai (1989), Brenner et al. (2006), Whaley (1993), and Carr and Madan (1998), among others) and investment advisors. Nevertheless, volatility contracts such as Chicago Board Options Exchange (CBOE) Volatility Index (VIX) futures and options have been traded on exchanges only recently. VIX represents the 30-calendar-day volatility, which is calculated from a model-free formula using the prices of a portfolio of out-of-the-money S&P 500 index (SPX) calls and puts whose weights are inversely proportional to the squared strike price. Moreover, the VIX formula also implies that the VIX-squared is replicable with a static portfolio of SPX options and thus allows the implementation of a tradable strategy for volatility speculation and hedging. Therefore, VIX futures and VIX-squared portfolios offer investors opportunities to expose their investment positions to the volatility risk directly.[1]

The potential diversification effect of volatility-related assets comes from well-documented empirical findings that there exists a negative correlation between stock returns and volatility changes. For example, Dumas et al. (1998) find that the correlation between S&P 500 index returns and changes in the Black–Scholes implied volatility of S&P 500 index options is −0.57 from June 1, 1988 through December 31, 1993. Moreover, Whaley (2000) find not only a negative correlation but an asymmetric relation between stock market returns and changes in the VIX.[2] He shows that if the VIX falls by 1%, the S&P 100 index will rise by 0.47%, whereas when the VIX rises by 1%, the S&P 100 index will fall by −0.71%. The evidence indicates that volatility derivatives may offer different risk-return characteristics in comparison to the existing assets in the financial markets.

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[1] It is also possible to trade volatility using option trading strategies such as straddle and strangle. However these strategies are not pure instruments of volatility trading because their profits/losses are affected by changes in both the volatility and the underlying asset price.

[2] Originally VIX is the weighted average of the implied volatilities obtained from eight near-the-money, nearest, and second-nearest S&P 100 index (OEX) options. The implied volatility index of OEX options has a new ticker VXO.

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Our study contributes to the investment and volatility literature by examining whether some VIX-related assets (mainly VIX futures and VIX-squared portfolios) featuring tradable financial products that are readily available to investors can expand the mean–variance frontiers generated by size/book-to-market portfolios. If the outcome is affirmative, then there is room for further development of financial products relating to volatility indexes, since investors can gain diversification benefits through investing in such products. However, if the outcome does not support the attainment of diversification benefits through VIX investment, then the recent development of VIX-related products may have limited scope for equity portfolio diversification in the future. This paper presents an empirical test for determining whether VIX investment vehicles can provide diversification benefits to investors.[3]

Our specific method in this study is to apply portfolio selection analysis to explore the portfolio diversification effects of one VIX cash (non-tradable) asset and two tradable VIX-related assets, namely VIX futures and VIX-squared portfolios. Although this approach to portfolio selection, dating back to Markowitz (1952), has become the standard approach in many financial textbooks, no attempt has been made to address the portfolio diversification issue in the VIX literature. The question which can be examined empirically is whether a VIX-related asset significantly enlarges the investment opportunity set composed of currently traded stocks. In order to address this issue, we employ mean–variance spanning and intersection tests to examine whether the addition of a VIX-related asset can significantly expand the investment opportunity set for investors relative to different groupings of benchmark portfolios.

The above issues are intriguing in their own right to academics and also have pragmatic implications for the issuance of VIX investment vehicles. We summarize our findings as follows. First, we find the statistical and economic significance of the shifts from the original mean–variance frontier that is derived from a group of size/book-to-market benchmark portfolios alone to the new frontier that includes both a newly-introduced VIX-related asset and the same group of benchmark portfolios. The shifts in frontier indicate that adding VIX-related assets may provide diversification benefits to investors. Second, we decompose the variances of VIX-related assets based on a two-factor model and a three-factor model. We find that at least 40% of the variance for a VIX-related asset cannot be accounted for by either the variance of the market portfolio or the variance of any individual size/book-to-market benchmark portfolio in the group. Such findings further emphasize the likely positive impact of VIX-related assets on the diversification benefit. Finally, we verify that our empirical results are robust to out-of-sample tests. We conclude that the addition of VIX-related assets will produce significant benefits for mean–variance investors. This main finding has two implications. First, since our study shows that investors can gain diversification benefits from investing in VIX-related products, it provides a sound rationale for the further development of financial products associated with volatility indexes. Second, even though VIX cash is non-tradable, hedge fund managers who can trade in the derivatives markets can utilize VIX futures contracts or VIX-squared portfolios to enhance their equity portfolio performance, as measured by the Sharpe ratio.

The remainder of this paper is organized as follows. Section 2 describes the methodologies used for the mean–variance spanning and intersection tests. Section 3 presents the data, the analysis of the empirical results, and the robustness check using out-of-sample tests. Finally, Section 4 concludes this study.

[3] Another important aspect of research on diversification benefits is the tests of international diversification effect. Recent studies include Eun et al. (2008), You and Daigler (2010), Thapa and Poshakwale (2010), and Chollete et al. (2011).

2. Methodologies

2.1. Return calculations for VIX-related assets

In this study we investigate the diversification effect of three VIX-related assets, including VIX cash, VIX futures, and VIX-squared portfolios. In order to implement mean–variance spanning tests, we need to calculate the returns on these VIX-related assets. In the following we discuss how we compute the return for each asset.

We first define the return on VIX cash. Denote the VIX value at time t as VIX_t. The Δt-period return for holding VIX cash from time t to time $t + \Delta t$ is given by

$$R^{VC}_{t+\Delta t} = \frac{VIX_{t+\Delta t} - VIX_t}{VIX_t}. \tag{1}$$

Next we compute the return on VIX futures. Denote the price at time t of a VIX futures contract maturing at time Tas $VIXF^T_t$. This study defines the rate of return from holding a VIX futures contract from time t to $t + \Delta t$ as

$$R^{VF}_{t+\Delta t} = \frac{VIXF^T_{t+\Delta t} - e^{-r\Delta t}VIXF^T_t}{e^{-r\Delta t}VIXF^T_t}, \tag{2}$$

where r is the risk-free rate. Note that the above formula implicitly assumes that an investor buys a VIX futures contract, invests $e^{-r\Delta t}VIXF^T_t$ dollars in Treasury bills for the settlement of his futures position, and posts Treasury bills as margin.[4] Since the nearest-term futures contract is generally the most liquid contract, we utilize it to calculate the VIX futures return. When the time to maturity of the nearest-term VIX futures contract is less than Δt-period, we roll over to the second-nearest-term contract to compute the return of VIX futures.

Finally we compute the return for the VIX-squared portfolios. According to the CBOE VIX white paper, VIX-squared is the weighted average of two variances calculated from the nearest-term and next-nearest-term SPX put and call prices, respectively.[5] Thus, we form the VIX-squared portfolio at time t as follows:

$$VIXSQ_t = \frac{T_2 - \tau}{T_2 - T_1}\sigma_1^2 + \frac{\tau - T_1}{T_2 - T_1}\sigma_2^2, \tag{3}$$

where $\tau = 30/365$, T_1 and T_2 are the times to expiration of the nearest-term and second-nearest-term SPX options, respectively. We replicate the variances, σ_1^2 and σ_2^2, using out-of-the-money (OTM) SPX call and put options as follows[6]:

$$\begin{aligned} \sigma_1^2 &= \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT_1} Q(K_i), \\ \sigma_2^2 &= \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT_2} Q(K_i), \end{aligned} \tag{4}$$

where K_i is the strike price of the i_{th} OTM option, ΔK_i is the interval between strike prices (defined as half the distance between the strike price on either side of K_i, i.e., $\Delta K_i = (K_{i+1} - K_{i-1})/2$), and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike price K_i.

[4] Alternatively the rate of return of holding a futures contract can be defined as the percentage change in the futures price over the holding period. However, as argued by Bodie and Rosansky (1980), this definition does not take into account the interest the investor could have earned by posting Treasury bills as margin. Note that our definition of rate of return is the same as Eq. (5) of Bodie and Rosansky (1980).

[5] The VIX white paper is available from the CBOE website (http://www.cboe.com/micro/VIX/vixwhite.pdf).

[6] Our variance formulae differ from the CBOE's formulae by a small amount (typically far smaller than 0.0001) related to percentage difference between the forward index level and the first strike below it.

Eq. (4) indicates that the VIX-squared portfolio consists of out-of-the-money SPX calls and puts whose weights (e.g., $2e^{rT_2}\Delta K_i/(T_2 K_i^2)$) are inversely proportional to the squared strike price. The rate of return on the VIX-squared portfolio is simply the percentage change in the portfolio value over the holding period. Note that if using the above formulation, the investor needs to rebalance the VIX-squared portfolio at the end of each period, when the moneyness of the SPX options changes due to the changes in the SPX value, and when the near-term options are going to expire within 8 days.[7]

2.2. Mean–variance spanning and intersection tests

Huberman and Kandel (1987) are the first to introduce a mean–variance spanning test. This method statistically tests whether adding a group of new assets can improve the investment opportunity set of an existing group of basis assets used as a benchmark, by analyzing the effects of the added assets on the mean–variance frontier. For ease of illustration, we call the combined group of new assets and benchmark assets "augmented assets." If the mean–variance frontier of the benchmark assets coincides with that of the augmented assets, we refer to this result as "spanning," which indicates that investors will gain no benefit from the addition of the new assets. If, however, the mean–variance frontier of the benchmark assets is smaller than that of the augmented assets, this indicates an expanded opportunity set, showing that investors would gain diversification benefits from adding the new assets.

We go onto briefly describe several mean–variance spanning tests. For further details, readers should consult the comprehensive surveys by DeRoon and Nijman (2001), Kan and Zhou (2008), and Sentana (2009). We assume that there are K benchmark portfolios with return R_{1t} and one VIX-related asset with return R_{2t}.[8] Using the ordinary least squares approach, we estimate the following model:

$$R_{2t} = \alpha + \beta R_{1t} + \xi_t,\ t = 1, 2, \ldots, T. \tag{5}$$

Following Huberman and Kandel (1987), the null hypothesis of spanning is

$$H_{0S} : \alpha = 0,\ \delta = 1 - \beta 1_K = 0. \tag{6}$$

We can then calculate the Wald test statistic as[9]

$$W = T(\lambda_1 + \lambda_2) \overset{A}{\sim} \chi_2^2. \tag{7}$$

If we are unable to reject the null hypothesis, then the mean–variance frontier of the benchmark portfolios when combined with the added VIX-related asset is unlikely to be greater than the mean–variance frontier of the benchmark portfolios without the VIX-related asset. In other words, the mean–variance frontier of the original benchmark portfolios is likely to span the new mean–variance frontier of the combined portfolios-plus-new-asset. This implies that when the null hypothesis cannot be rejected, investors will probably not be able to enlarge their investment opportunity set by the addition of a VIX-related asset. Cheung et al. (2009) introduce an additional reason not to invest in new assets when spanning occurs, by proving that there is a residual risk from those new assets. However, if the null hypothesis is rejected, this implies that the addition of a VIX-related asset can improve the investment opportunity set.

In geometric terms, the test for mean–variance spanning can be divided into two elements: (i) the spanning of the global minimum-variance (GMV) portfolio and (ii) the spanning of the tangency portfolio. We can therefore rewrite the Wald test as

$$W = T\left(\frac{(\hat{\sigma}_{R_1})^2}{(\hat{\sigma}_R)^2} - 1\right) + T\left(\frac{1 + \hat{\theta}_R\left(R_1^{GMV}\right)^2}{1 + \hat{\theta}_{R_1}(R_1^{GMV})^2} - 1\right), \tag{8}$$

where $(\hat{\sigma}_{R_1})^2$ and $(\hat{\sigma}_R)^2$ are the global minimum-variance of the benchmark assets and the global minimum-variance of the augmented assets, respectively. $\hat{\theta}_{R_1}(R_1^{GMV})$ is the slope of the asymptote of the mean–variance frontier for the benchmark assets, and $\hat{\theta}_R(R_1^{GMV})$ is the slope of the asymptote of the mean–variance frontier for the augmented assets, using R_1^{GMV} as a reference point. The first term in Eq. (8) measures the change in the GMV portfolio due to the addition of a VIX-related asset. The second term measures whether there is any increase in the squared tangency slope from adding a VIX-related asset to the set of benchmark portfolios.

In order to identify the source of mean–variance frontier expansion, Kan and Zhou (2008) suggest a step-down procedure, which requires us to initially test $\alpha = 0$, followed by a test of $\delta = 0$, conditional on $\alpha = 0$. If the rejection of the null hypothesis is due to the first test, we know that this is because the two tangency portfolios are statistically very different. If the rejection is due to the second test, then it occurs because the two GMV portfolios are statistically very different.[10]

A further concept relevant to the assessment of movement, or change, in the tangency portfolio is the mean–variance intersection test proposed by Huberman and Kandel (1987). If the mean–variance frontier of the benchmark assets and the mean–variance frontier of the augmented assets have only one point in common, this case is known as intersection. Using Eq. (5), the null hypothesis of intersection is

$$H_{0I} : \alpha - \eta(1 - \beta 1_K) = 0, \tag{9}$$

where η is the risk-free rate. Following DeRoon and Nijman (2001), the test statistic for testing the intersection hypothesis can be rewritten in terms of the maximal Sharpe ratios as

$$W_I = T\left(\frac{1 + \hat{\theta}_R(\eta)^2}{1 + \hat{\theta}_{R_1}(\eta)^2} - 1\right) = T\left(\frac{\hat{\theta}_R(\eta)^2 - \hat{\theta}_{R_1}(\eta)^2}{1 + \hat{\theta}_{R_1}(\eta)^2}\right) \overset{A}{\sim} \chi_1^2, \tag{10}$$

where $\hat{\theta}_{R_1}(\eta)$ is the maximal Sharpe ratio attainable for the benchmark assets, and $\hat{\theta}_R(\eta)$ is the maximal Sharpe ratio attainable for the augmented assets.

Intuitively, we can expect that the empirical results from the intersection test would be very similar to the results from the first test of the step-down procedure described above. Furthermore, the test statistic specified in Eq. (10) indicates that the numerator of the right hand side statistic is related to the difference between the squared maximal Sharpe ratios attainable for benchmark assets and for augmented assets. The rejection of the null hypothesis in the intersection test implies that, based on the risk-free rate as the reference point, the mean–variance frontier of the augmented assets has no point in common with the mean–variance frontier of the benchmark assets. This lack of a point in common leads to the difference between the maximal Sharpe ratios of the augmented assets and the benchmark assets.

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[7] According to the CBOE VIX white paper, the new VIX "rolls" to the second and third contract months in order to minimize pricing anomalies that might occur close to expiration. We follow this guideline to mimic VIX-squared portfolios.

[8] The expected returns on $K+1$ assets are denoted as $\mu = E[R_t] \equiv \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$. The variance-covariance matrix of $K+1$ assets is $V = Var[R_t] \equiv \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$, where V is non-singular.

[9] We define $\widehat{G} = \begin{bmatrix} 1 + \hat{\mu}_1' \widehat{V}_{11}^{-1} \hat{\mu}_1 & \hat{\mu}_1' \widehat{V}_{11}^{-1} 1_K \\ \hat{\mu}_1' \widehat{V}_{11}^{-1} 1_K & 1_K' \widehat{V}_{11}^{-1} 1_K \end{bmatrix}$ and $\widehat{H} = \begin{bmatrix} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} & \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\delta} \\ \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\delta} & \hat{\delta}' \hat{\Sigma}^{-1} \hat{\delta} \end{bmatrix}$, where $\widehat{\Sigma}$ stands for residual variance. We then denote by λ_1 and λ_2 the two eigenvalues of the matrix $\widehat{H}\widehat{G}^{-1}$. Since there is only one test asset in our mean–variance spanning test, the smaller eigenvalue, λ_2, equals zero.

[10] For the purpose of brevity, we do not specify the exact test statistics for the step-down tests since these are discussed at length in Kan and Zhou (2008).

Table 1
Summary statistics of the weekly VIX-related asset and benchmark portfolio returns.

	Mean	Standard deviation	Skewness	Kurtosis	Median	Minimum	Maximum
Panel A: VIX-related assets							
VIX cash	0.0073	0.1169	0.9703	6.4620	0.0006	−0.3091	0.7590
VIX-squared portfolio	−0.0584	0.3487	3.9123	29.7016	−0.1458	−0.5811	3.3974
VIX futures	−0.0103	0.0710	0.9785	5.2868	−0.0160	−0.1636	0.3120
Panel B: benchmark portfolios							
6 EW Size/BM portfolios	0.0026	0.0235	−0.5252	6.3078	0.0037	−0.1350	0.0919
6 VW Size/BM portfolios	0.0021	0.0240	−0.5951	6.1168	0.0036	−0.1410	0.0854
25 EW Size/BM portfolios	0.0029	0.0250	−0.4493	6.5037	0.0040	−0.1460	0.1041
25 VW Size/BM portfolios	0.0022	0.0251	−0.6065	6.9543	0.0036	−0.1566	0.0993

The table presents the descriptive statistics of the weekly VIX-related asset and benchmark portfolio returns. The sample period for the VIX cash, VIX-squared portfolio, and the benchmark portfolios is from 1996/01/12 to 2008/04/18; the sample period for VIX futures is from 2004/04/02 to 2008/04/18 due to the fact that the VIX futures contracts were introduced to the market in 2004. All the statistics in Panel B are the average statistics of the whole 6 or 25 benchmark portfolios.

3. Empirical results

3.1. Data description

Our sample of VIX-related assets covers the period from January 1996 to April 2008. The sample period for VIX futures, from April 2004 to April 2008, is shorter, due to the fact that this product was only introduced to the market in 2004. Panel A of Table 1 presents the summary statistics of the three VIX-related assets, showing that the mean weekly return of the VIX cash/VIX-squared portfolio/VIX futures is 0.73%/−5.84%/−1.03%, with a standard deviation of 11.69%/34.87%/7.10%. We note that the VIX-squared portfolio returns are quite negative and very volatile. Since the VIX-squared portfolio is composed of out-of-the-money SPX options, our result echoes the findings of Bondarenko (2003) and Broadie et al. (2009) that out-of-the-money SPX put option returns are significantly negative. These negative returns may be due to the mispricing of SPX options (Jackwerth (2000)), the net buying pressure of SPX options (Bollen and Whaley (2004)), or the inefficiency of stock index options markets (Bates (2006)).

We select the benchmark portfolios for our tests from those on the Kenneth R. French website.[11] The portfolios, which all include NYSE, AMEX and Nasdaq stocks, are based on the size and book-to-market ratio of the firms and are grouped as follows:

- 6 equally-weighted (EW) portfolios;
- 6 value-weighted (VW) portfolios;
- 25 equally-weighted (EW) portfolios;
- 25 value-weighted (VW) portfolios.

We report the average summary statistics of each of the four sets of benchmark portfolios in Panel B of Table 1.[12] The mean returns and standard deviations for these benchmark portfolios are as follows:

- The average returns of the whole 6 EW-portfolio is 0.26% (range from 0.18% to 0.39%); the average standard deviations is 2.35% (range from 1.85% to 3.04%);
- The average returns of the whole 6 VW-portfolio is 0.21% (range from 0.13% to 0.29%); the average standard deviations is 2.40% (range from 2.09% to 3.15%);
- The average returns of the whole 25 EW-portfolio is 0.29% (range from 0.13% to 0.63%); the average standard deviations is 2.50% (range from 1.77% to 3.50%);
- The average returns of the whole 25 VW-portfolio is 0.22% (range from 0.04% to 0.31%); the average standard deviations is 2.51% (range from 2.11% to 3.49%).

The equity portfolios used in this set of tests are clearly much less risky than the VIX-related assets shown in Table 1. In addition, by observing the higher moments, i.e., skewness and kurtosis, of the VIX-related assets and equity portfolios, we find that VIX cash and VIX futures are skewed to the right, while the equity portfolios are skewed to the left. Nevertheless, they share similar characteristics in kurtosis with some evidence of fat tails. On the other hand, VIX-squared portfolio is highly positively skewed with very large kurtosis.

We further report the average correlation coefficients between each of the four sets of benchmark portfolios and one of the VIX-related assets in Table 2.[13] For VIX cash and VIX futures, the average correlation coefficient is around −0.60, ranging from −0.4991 to −0.7470. In the case of VIX-squared portfolio, the average correlation coefficient is around −0.35, with a range between −0.2678 and −0.4045. Overall, we find that all three VIX-related assets are highly negatively correlated with the benchmark portfolios. This fact provides us with a guidance of the potential diversification effect of the VIX-related assets for investors.

3.2. Mean–variance frontier expansion using VIX-related assets

We first test whether the addition of a VIX-related asset to the 6 and 25 Fama–French size/book-to-market portfolios enlarges the investment opportunity set for mean–variance investors. Panel A of Table 3 presents the empirical results from mean–variance spanning and intersection tests for VIX cash.[14] The asymptotic test W rejects the null hypothesis that the benchmark portfolios can span VIX cash at the 1% significance level. Further, both W_1 and W_2, the statistics obtained via the step-down spanning test, support this rejection of the null hypothesis. The fact that both W_1 and W_2 reject the null hypothesis indicates that the expansion comes from changes in both the tangency and the GMV portfolios.

Panel B of Table 3 presents the empirical results from mean–variance spanning and intersection tests for the VIX-squared portfolio. We find that the results are similar to those for VIX cash.

[11] We are grateful to Ken French for providing this data on his website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). See also Fama and French's original studies (1992, 1993) for further details.

[12] We do not report the complete summary statistics of the benchmark portfolios in the table, for the sake of brevity. The detailed statistics are available upon request.

[13] We do not report the complete correlation matrix of the VIX-related assets and the benchmark portfolios in the table, for the sake of brevity. The detailed statistics are available upon request.

[14] Within the mean–variance spanning tests, we also calculate the statistics for the finite sample, the likelihood ratio, the Lagrange multiplier tests, and tests under non-normality and heteroskedasticity (see Kan and Zhou (2008) for details). The results are qualitatively similar; thus, they are not reported here for the purpose of brevity. In addition, the risk-free rate used is the average 1-month T-bill return from the test periods, obtained from Ibbotson and Associates, Inc., details of which are also available from the French's website.

Table 2
Correlations between the benchmark portfolios and VIX-related assets.

	VIX cash	VIX-squared portfolio	VIX futures
6 EW Size/BM portfolios	−0.5788	−0.3400	−0.7036
	(−0.4991)	(−0.3253)	(−0.6613)
	[−0.6531]	[−0.3661]	[−0.7394]
6 VW Size/BM portfolios	−0.6398	−0.3677	−0.7106
	(−0.5867)	(−0.3215)	(−0.6798)
	[−0.7087]	[−0.4045]	[−0.7424]
25 EW Size/BM portfolios	−0.5741	−0.3317	−0.6854
	(−0.4239)	(−0.2804)	(−0.6315)
	[−0.6703]	[−0.3793]	[−0.7470]
25 VW Size/BM portfolios	−0.6007	−0.3545	−0.6938
	(−0.5012)	(−0.2678)	(−0.6515)
	[−0.6912]	[−0.4024]	[−0.7387]

The table reports the average, maximum (in the parenthesis), and minimum (in the bracket) correlation coefficients between the benchmark portfolios and VIX-related assets. There are three different VIX-related assets: VIX cash, VIX-squared portfolio, and VIX futures. Four sets of benchmarks are used, including 6 equally-weighted and value-weighted size/book-to-market portfolios and 25 equally-weighted and value-weighted size/book-to-market portfolios. Specifically, take the 6 EW size/BM portfolios as an example, the average correlation coefficient refers to the simple average correlation coefficient between each of the 6 benchmark portfolios and one of the VIX-related assets. The sample period for the VIX cash and VIX-squared portfolio is from 1996/01/12 to 2008/04/18; the sample period for VIX futures is from 2004/04/02 to 2008/04/18 due to the fact that the VIX futures contracts were introduced to the market in 2004.

Panel C of Table 3 reports the results for VIX futures. Again, the asymptotic test W rejects the null hypothesis. However, while W_2, the second step-down test, also rejects the null hypothesis, the first step-down test, W_1, does not. This result indicates that for VIX futures, the expansion comes mainly from the change in the GMV portfolios. Table 3 indicates that the addition of VIX futures to the benchmark portfolios for the period 2004–2008 results in no improvement for the tangency portfolio. There is, however, significant improvement in the tangency portfolio from adding either the VIX cash or VIX-squared portfolio over the period 1996–2008.

The diversification benefit of the three VIX-related assets could also be illustrated clearly by the changes of the mean–variance frontier before and after adding a VIX-related asset, as shown in Figs. 1–3. By observing these figures, we find that the risk of GMV portfolio decreases significantly after the inclusion of all three VIX-related assets. We also observe that the apex of the frontiers with VIX futures is closer to the origin and the Y-axis than those with VIX cash and VIX-squared portfolio. This suggests that the expansion of the tangency part of the frontier after adding VIX cash and VIX-squared portfolio may be more pronounced than the expansion resulting from the addition of VIX futures. Overall, the results from the figures are consistent with those of statistical tests in Table 3.

The mean–variance spanning tests in Table 3 examine only whether the expansion of the mean–variance frontier is statistically significant. In order to assess the economic significance of this expansion, we evaluate the change in the Sharpe ratio, as Bekaert and Urias (1996) suggest. The Sharpe ratio, also known as the ‘reward to variability’ ratio, measures the slope of the line from the risk-free rate to any portfolio in the mean-standard deviation plane (Sharpe, 1994). The details of our evaluation of economic significance are given in Table 4.

As Table 4 indicates, the percentage change in the Sharpe ratio for the tangency portfolio due to the addition of VIX cash or a VIX-squared portfolio ranges from 3.61%/1.81% to 28.69%/19.04%, while the percentage change attributable to the addition of VIX futures is much smaller, ranging from 1.56% to 14.22%. We also find that, when adding VIX cash or a VIX-squared portfolio, the value-weighted benchmark portfolio cases yield larger improvements in the Sharpe ratio than do the equally-weighted portfolio cases. However, these results do not hold in the cases for VIX futures.

In Table 5, we show the optimal portfolio weights for an investor who adds VIX-related assets to each case of size/book-to-market benchmark portfolios. The table indicates that the weights for VIX cash are positive, ranging from 0.0427 to 0.1086. As for the VIX-squared portfolio and VIX futures, we find negative weights from −0.0104 to −3.9264. These results imply that investors would

Table 3
Mean–variance spanning and intersection tests for VIX-related assets.

	Wald tests							
	W		W_1 step-down		W_2 step-down		W_I	
	Statistics	p-Value	Statistics	p-Value	Statistics	p-Value	Statistics	p-Value
Panel A: VIX cash								
6 EW Size/BM portfolios	373.380	0.000***	23.266	0.000***	337.850	0.000***	15.920	0.000***
6 VW Size/BM portfolios	851.720	0.000***	28.791	0.000***	787.560	0.000***	18.305	0.000***
25 EW Size/BM portfolios	305.260	0.000***	19.012	0.000***	278.000	0.000***	12.735	0.000***
25 VW Size/BM portfolios	538.880	0.000***	27.195	0.000***	490.860	0.000***	17.510	0.000***
Panel B: VIX-squared portfolio								
6 EW Size/BM portfolios	123.594	0.000***	5.164	0.023**	117.483	0.000***	7.575	0.006***
6 VW Size/BM portfolios	156.608	0.000***	8.750	0.003***	145.867	0.000***	11.664	0.001***
25 EW Size/BM portfolios	87.672	0.000***	4.377	0.036**	82.730	0.000***	6.322	0.012**
25 VW Size/BM portfolios	114.672	0.000***	8.181	0.004***	105.149	0.000***	11.055	0.001***
Panel C: VIX futures								
6 EW Size/BM portfolios	295.008	0.000***	1.032	0.310	293.008	0.000***	3.202	0.074*
6 VW Size/BM portfolios	396.900	0.000***	2.459	0.117	389.940	0.000***	N/A	N/A
25 EW Size/BM portfolios	199.681	0.000***	0.104	0.747	199.478	0.000***	1.541	0.214
25 VW Size/BM portfolios	291.425	0.000***	1.324	0.250	288.301	0.000***	N/A	N/A

The table reports the mean–variance spanning and intersection test results arising from the addition of VIX-related assets into the benchmark portfolios. There are three different VIX-related asset returns: VIX cash, VIX-squared portfolio, and VIX futures. Four sets of benchmarks are used, including 6 equally-weighted and value-weighted size/book-to-market portfolios and 25 equally-weighted and value-weighted size/book-to-market portfolios. These portfolios are available from French's website. W refers to the asymptotic Wald test for spanning; W_1 and W_2 are the step-down Wald tests for spanning; W_I represents the asymptotic Wald test for the intersection hypothesis. The risk-free rate used is the weekly average rate for the sample period. For VIX cash and VIX-squared portfolio, the risk-free rate is 0.0007 in the intersection test, while for VIX futures, the risk-free rate for the intersection test is 0.0006. We have N/A in Panel C, because there is no solution for the optimal tangency portfolio.

* Indicates significance at the 10% level.

** Indicates significance at the 5% level.

*** Indicates significance at the 1% level.

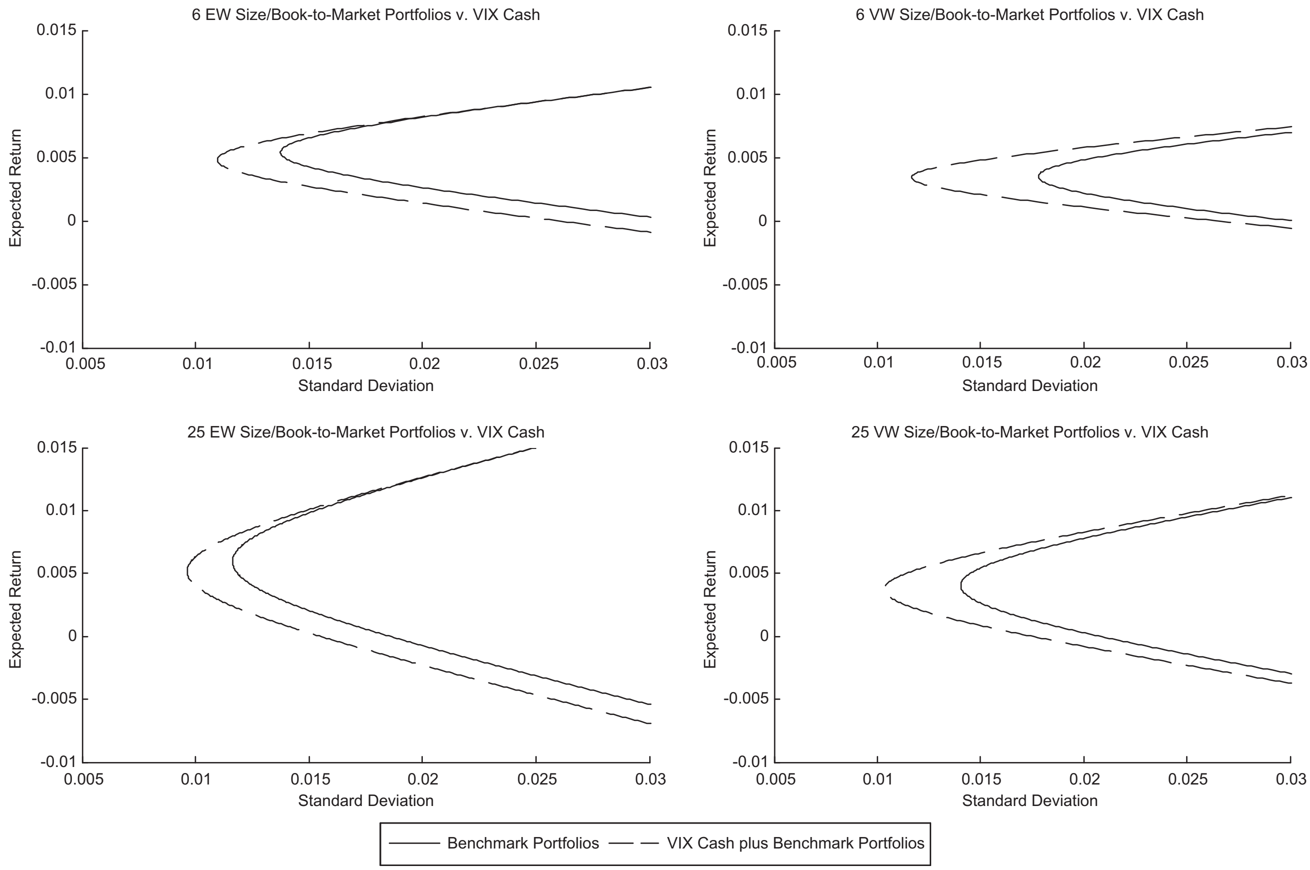

Fig. 1. Mean–variance frontiers before and after adding VIX cash. The figure plots the mean–variance frontier (the inner solid frontier) of the benchmark portfolios, i.e., four sets of Fama–French size/book-to-market portfolios which are based upon all firms trading on the NYSE, AMEX and Nasdaq, and the mean–variance frontier (the outer dashed frontier) of the augmented assets, comprising of the benchmark portfolios plus VIX cash.

have to sell the VIX-squared portfolio or VIX futures short in order to gain a diversification benefit.

3.3. Two-factor model analysis and variance decomposition

In an attempt to gain a complete understanding of the pattern of returns generated by VIX-related assets in our study, we employ a simple two-factor model (similar to the model adopted by Eun et al. (2008)), the results of which are given in Table 6 below. This model assumes that VIX-related asset returns are driven by a market portfolio plus any one of the 6 size/book-to-market portfolios used in our cases. This two-factor model is estimated as

$$R_{VIX,t} = \alpha + \beta^{CRSP} R_{CRSP,t} + \beta^{FF} R_{FF,t} + \xi_t, \tag{11}$$

where R_{VIX} is the return on the VIX cash, VIX-squared portfolio or VIX futures, R_{CRSP} is the return on the value-weighted CRSP market index, and R_{FF} is the residual obtained by regressing each of the 6 Fama–French value-weighted size/book-to-market portfolios on R_{CRSP}.[15]

Based on the estimated market and size/book-to-market portfolio betas, we can decompose the variance of a VIX-related asset into three components: (i) $(\beta^{CRSP})^2 \times Var(R_{CRSP})$, the component attributable to the volatility of the market portfolio, (ii) $(\beta^{FF})^2 \times Var(R_{FF})$, the component attributable to the volatility of each of the 6 size/book-to-market portfolios, and (iii) $Var(\xi)$, the idiosyncratic volatility of the VIX-related asset itself. The variance of a VIX-related asset is written as

$$Var(R_{VIX}) = (\beta^{CRSP})^2 \times Var(R_{CRSP}) + (\beta^{FF})^2 \times Var(R_{FF}) + Var(\xi). \tag{12}$$

Each part of the decomposition can be calculated as follows:

$$(\beta^{CRSP})^2 \times Var(R_{CRSP}) \Big/ Var(R_{VIX}) \text{ gives the market portfolio proportion;} \tag{13}$$

$$(\beta^{FF})^2 \times Var(R_{FF}) \Big/ Var(R_{VIX}) \text{ gives the size/book-to-market portfolio proportion;} \tag{14}$$

$$Var(\xi)/Var(R_{VIX}) \text{ gives the idiosyncratic proportion of the VIX-related asset.} \tag{15}$$

Since the dependent variable is the same for each VIX-related asset, the market portfolio beta (β^{CRSP}), the coefficient for the first factor in the model, is the same for each of the 6 regressions. β^{CRSP} is −3.615 for VIX cash, −5.865 for the VIX-squared portfolio, and −3.058 for VIX futures. All of these negative coefficients are statistically significant, which is consistent with other evidence (see Dumas et al. (1998) and Whaley (2000)) that there exists a negative correlation between stock returns and volatility changes.

Table 6 indicates that, for about half of the size/book-to-market portfolio cases, beta (β^{FF}), the coefficient for the second factor in the model, is statistically significant for VIX cash and for the VIX-squared portfolio. By contrast, beta (β^{FF}) is not significant for VIX futures in any cases. Note that the explanatory power of the

[15] We also conduct the two-factor model analysis and variance decomposition based on the 6 equally-weighted size/book-to-market portfolios and the 25 equally-weighted and value-weighted size/book-to-market portfolios. Since the results are qualitatively similar to those given in Table 6, we do not report them for the sake of brevity.

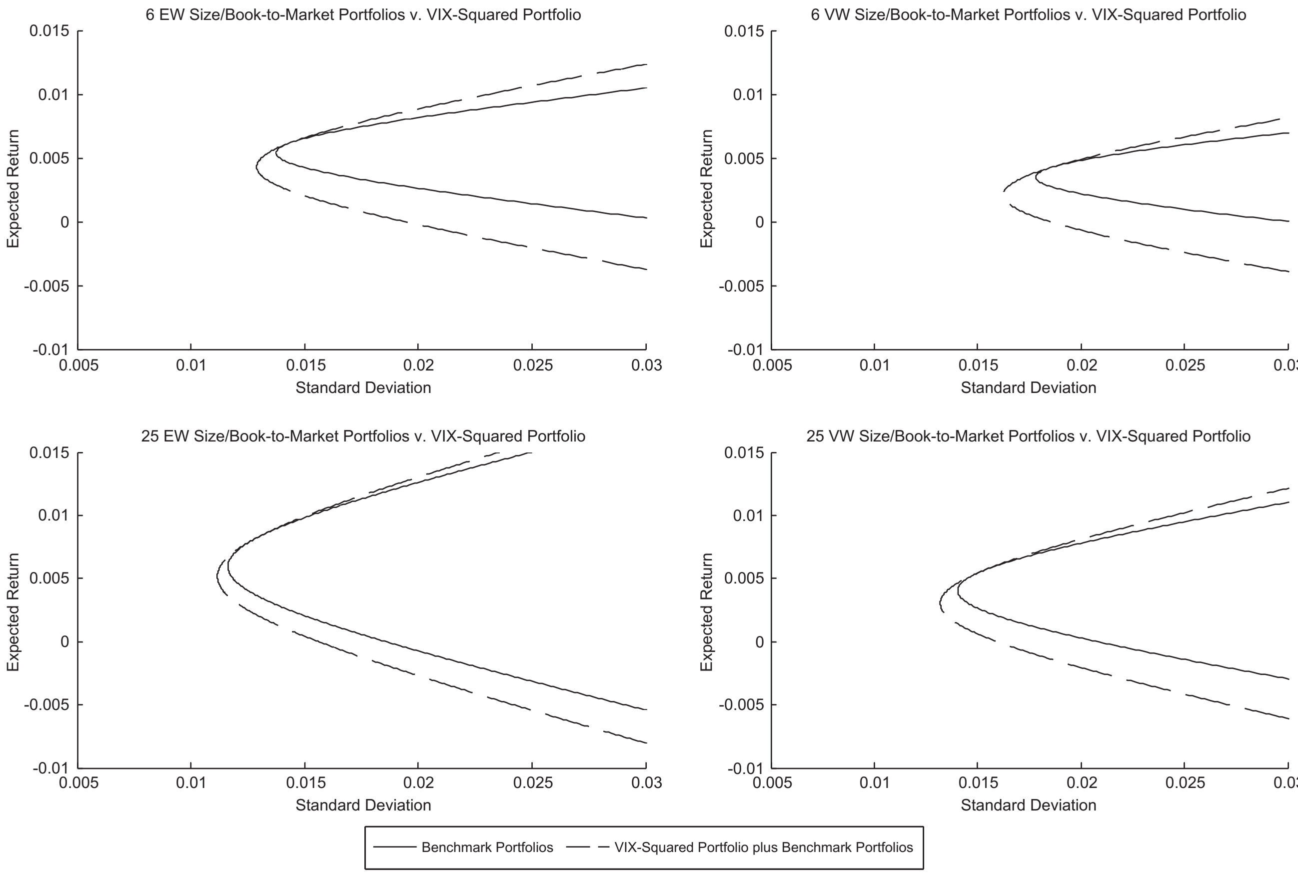

Fig. 2. Mean–variance frontiers before and after adding a VIX-squared portfolio. The figure plots the mean–variance frontier (the inner solid frontier) of the benchmark portfolios, i.e., four sets of Fama–French size/book-to-market portfolios which are based upon all firms trading on the NYSE, AMEX and Nasdaq, and the mean–variance frontier (the outer dashed frontier) of the augmented assets, comprising of the benchmark portfolios plus a VIX-squared portfolio.

two-factor model is modest, with an average adjusted R^2 of 15.94% for the VIX-squared portfolio, 52.06% for VIX cash, and 57.72% for VIX futures.

Our results show that idiosyncratic volatility accounts for 83.62% of the variance in the VIX-squared portfolio, for about 47.79% of the variance in VIX cash, and for around 41.88% of the variance in VIX futures. The combination of the weak explanatory power of the two-factor model and the high proportion of variance accounted for by idiosyncratic volatility seems to indicate that the significant expansion of the mean–variance frontier following the addition of a VIX-related asset to the 6 benchmark portfolios reflects an innovative or unexplained component of VIX-related asset returns.

3.4. The quadratic characteristic model and variance decomposition

In Section 3.3 we adopt the two-factor model for decomposing the variance of the test asset returns. While the explanatory power of the two-factor model used in Eun et al. (2008) is usually high, the explanatory power of the two-factor model presented in the previous section is modest for our three volatility-related assets. This result casts doubt on the choice of the model and/or explanatory variables. One alternative specification is to consider the quadratic characteristic model which takes into account both the systematic risk and systematic skewness (see Kraus and Litzenberger (1976) for more details).[16]

We use the following orthogonal variant of the quadratic characteristic line as a new specification to analyze the variance of the test asset returns. This model assumes that VIX-related asset returns are driven by a market portfolio return, the squared return of the market portfolio, and any one of the 6 size/book-to-market portfolios used in our cases. The quadratic characteristic model is specified as

$$R_{VIX,t} = \alpha + \beta^{CRSP} R_{CRSP,t} + \beta^{SQ} R_{SQ,t} + \beta^{FF} R_{FF,t} + \xi_t, \tag{16}$$

where R_{VIX} is the return on the VIX cash, VIX-squared portfolio or VIX futures, R_{CRSP} is the return on the value-weighted CRSP market index, R_{SQ} is the residual obtained by regressing the squared return of the value-weighted CRSP market index on R_{CRSP}, and R_{FF} is the residual obtained by regressing each of the 6 Fama–French value-weighted size/book-to-market portfolios on R_{CRSP} and R_{SQ}.[17]

Based on the estimated coefficients, we can decompose the variance of a VIX-related asset into four components, similarly discussed in Section 3.3. The variance of a VIX-related asset can be written as

$$\begin{aligned} Var(R_{VIX}) = {} & (\beta^{CRSP})^2 \times Var(R_{CRSP}) + (\beta^{SQ})^2 \times Var(R_{SQ}) + (\beta^{FF})^2 \\ & \times Var(R_{FF}) + Var(\xi). \end{aligned} \tag{17}$$

Since the dependent variable is the same for each VIX-related asset, the market portfolio beta (β^{CRSP}), the coefficient for the first factor in the model, is the same for each of the 6 regressions. β^{CRSP} is −3.615 for VIX cash, −5.865 for the VIX-squared portfolio, and −3.058 for VIX futures. Similarly, the coefficient for the second

[16] We thank the referee for this suggestion.

[17] We also examine variance decomposition based on the 6 equally-weighted size/book-to-market portfolios and the 25 equally-weighted and value-weighted size/book-to-market portfolios. Since the results are qualitatively similar to those given in Table 7, we do not report them for the sake of brevity.

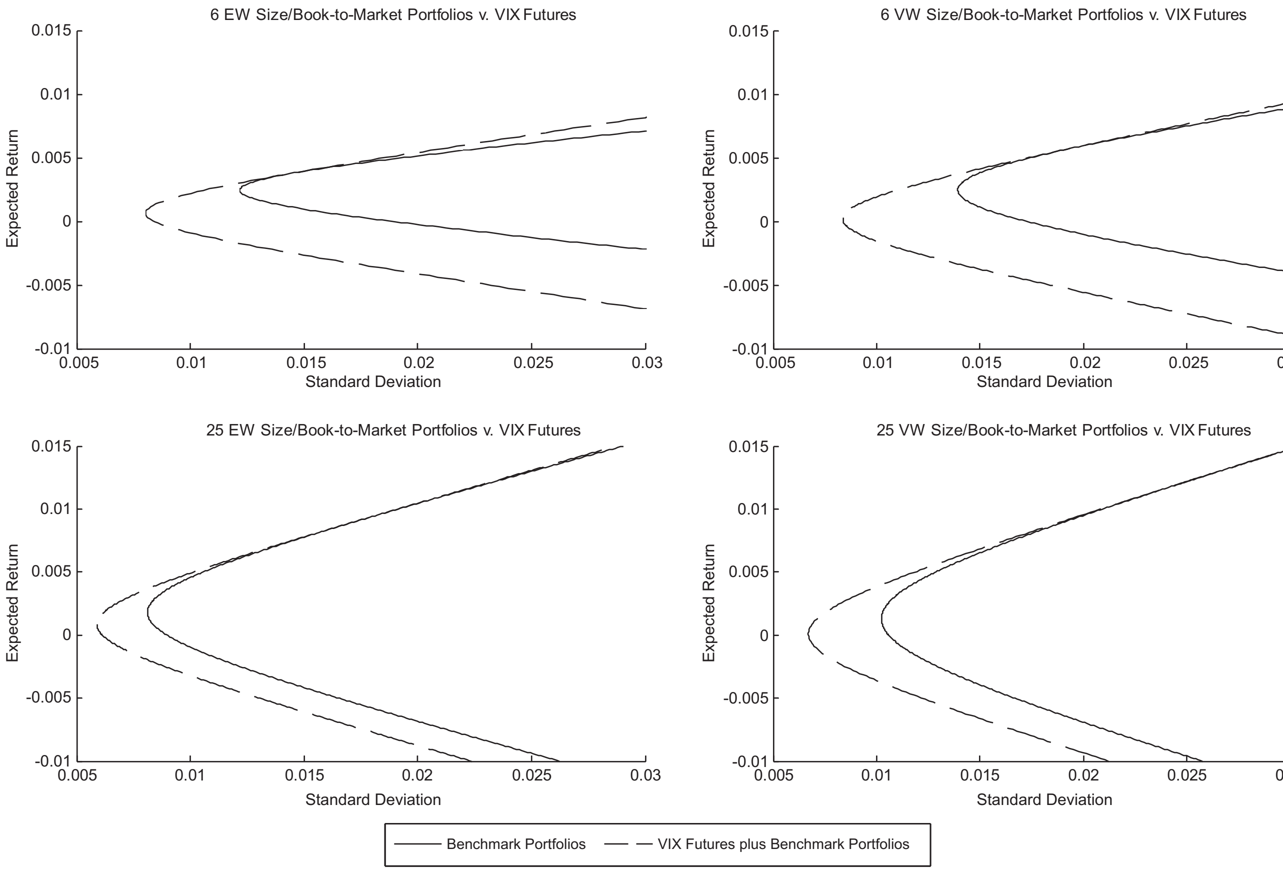

Fig. 3. Mean–variance frontiers before and after adding VIX futures. The figure plots the mean–variance frontier (the inner solid frontier) of the benchmark portfolios, i.e., four sets of Fama–French size/book-to-market portfolios which are based upon all firms trading on the NYSE, AMEX and Nasdaq, and the mean–variance frontier (the outer dashed frontier) of the augmented assets, comprising of the benchmark portfolios plus VIX futures.

factor in the model is also the same for each of the 6 regressions. β^{SQ} is 2.891 for VIX cash, 137.49 for the VIX-squared portfolio, and 8.832 for VIX futures.

Table 4
Sharpe ratios and changes in the Sharpe ratios for adding VIX-related assets.

	Sharpe ratio		Change in Sharpe ratio (%)
	Before	After	
Panel A: VIX cash			
6 EW Size/BM benchmarks	0.3909	0.4259	8.96
6 VW Size/BM benchmarks	0.2132	0.2743	28.69
25 EW Size/BM benchmarks	0.6088	0.6308	3.61
25 VW Size/BM benchmarks	0.3549	0.3958	11.53
Panel B: VIX-squared portfolio			
6 EW Size/BM benchmarks	0.3909	0.4079	4.35
6 VW Size/BM benchmarks	0.2132	0.2538	19.04
25 EW Size/BM benchmarks	0.6088	0.6198	1.81
25 VW Size/BM benchmarks	0.3549	0.3812	7.41
Panel C: VIX futures			
6 EW Size/BM benchmarks	0.2279	0.2603	14.22
6 VW Size/BM benchmarks	N/A	N/A	N/A
25 EW Size/BM benchmarks	0.4955	0.5045	1.82
25 VW Size/BM benchmarks	N/A	N/A	N/A

The table reports the Sharpe ratios and percentage changes in the Sharpe ratios before and after the addition of the VIX-related asset returns. There are three different VIX-related asset returns: VIX cash, VIX-squared portfolio, and VIX futures. Four sets of benchmarks are used, including 6 equally-weighted and value-weighted size/book-to-market portfolios and 25 equally-weighted and value-weighted size/book-to-market portfolios. These portfolios are available from French's website. The risk-free rates used for computing the Sharpe ratios are the same as those reported in Table 2 for each VIX-related asset return. We have N/A in Panel C, because there is no solution for the optimal tangency portfolio. Thus we could not calculate the Sharpe ratio and the change in Sharp ratio.

For both VIX cash and futures, the residual of squared CRSP index return, the second factor, has almost no effect when the results of the new specification in Table 7 is compared to those in the two-factor model presented in Table 6. First, the explanatory power for the regression models (adjusted R-squared) is quite close. Second, the volatility attributable to the residual of squared market return is less than 0.4%. By contrast, the volatility attributable to the market return is greater than 51% for these two VIX-related assets.

This new factor, the residual of squared CRSP index return, improves the explanatory power in the case of VIX-squared portfolio by more than 22%, increasing from the average of 15.94% in Panel B of Table 6 to the average of 38.32% in Panel B of Table 7. It is consistent with the fact that VIX-squared portfolio has very different higher moment characteristics compared to those of the

Table 5
Optimal portfolio weights for VIX-related assets.

	VIX cash	VIX-squared portfolio	VIX futures
6 EW Size/BM benchmarks	0.0579	−0.0167	−3.9264
6 VW Size/BM benchmarks	0.1086	−0.0813	N/A
25 EW Size/BM benchmarks	0.0427	−0.0104	−0.3005
25 VW Size/BM benchmarks	0.0789	−0.0338	N/A

The table reports the optimal portfolio weights for the VIX-related assets contained in the tangency portfolios. Each tangency portfolio consists of one of the VIX-related assets and the benchmark portfolios specified in each row. There are three different VIX-related asset returns: VIX cash, VIX-squared portfolio, and VIX futures. Four sets of benchmarks are used, including 6 equally-weighted and value-weighted size/book-to-market portfolios and 25 equally-weighted and value-weighted size/book-to-market portfolios. These portfolios are available from French's website. We have two cases with N/A, because there is no solution for the optimal tangency portfolio.

Table 6
Two-factor model and variance decomposition of VIX-related assets.

(Size, book-to-market) Portfolios	Two-factor model			Variance decomposition	
	β^{FF}		Adjusted R^2 (%)	Volatility attributable to size/BM portfolio (%)	Volatility attributable to VIX portfolio (%)
	Coefficient	*t*-Statistics			
Panel A: (dependent variable: VIX Cash)					
$\beta^{CRSP} = -3.615^{***}$					
Variance of VIX cash = 0.0137; Volatility attributable to market portfolio = 51.95%					
Small-Low	0.411	2.10	52.13	0.33	47.72
Small-Medium	−0.278	−1.06	51.89	0.08	47.96
Small-High	−0.405	−1.64	52.00	0.20	47.85
Large-Low	−0.421	−0.66	51.84	0.03	48.01
Large-Medium	−0.951	−2.84	52.40	0.60	47.45
Large-High	−0.518	−1.95	52.09	0.28	47.76
Panel B: (dependent variable: VIX-squared portfolio)					
$\beta^{CRSP} = -5.865^{***}$					
Variance of VIX-squared portfolio = 0.1216; volatility attributable to market portfolio = 15.38%					
Small-Low	−1.597	−2.06	14.67	0.56	84.06
Small-Medium	−3.845	−3.74	16.93	1.81	82.81
Small-High	−3.952	−4.08	17.27	2.15	82.47
Large-Low	8.250	3.28	16.52	1.41	83.21
Large-Medium	0.840	0.63	15.16	0.05	84.57
Large-High	0.032	0.03	15.11	0.00	84.62
Panel C: (dependent variable: VIX futures)					
$\beta^{CRSP} = -3.058^{***}$					
Variance of VIX futures = 0.0050; Volatility attributable to market portfolio = 57.95%					
Small-Low	0.480	1.41	57.95	0.39	41.65
Small-Medium	0.506	1.32	57.90	0.35	41.70
Small-High	0.100	0.27	57.56	0.01	42.03
Large-Low	−0.542	−0.76	57.67	0.11	41.93
Large-Medium	0.551	0.81	57.68	0.13	41.92
Large-High	−0.149	−0.24	57.56	0.01	42.04

The table reports the results of the two-factor model estimation and variance decomposition for the VIX-related assets. The two-factor model is specified as:

$$R_{VIX,t} = \alpha + \beta^{CRSP} R_{CRSP,t} + \beta^{FF} R_{FF,t} + \xi_t,$$

where R_{VIX} is the return on the VIX cash, VIX-squared portfolio or VIX futures, R_{CRSP} is the return on the value-weighted CRSP market index, and R_{FF} is the residual obtained from regressing each of the 6 Fama–French value-weighted size/book-to-market portfolios on R_{CRSP}. The variance decomposition for the VIX-related assets consists of three components: (i) the proportion attributable to the volatility of the market portfolio, (ii) the proportion attributable to the volatility of each of the 6 size/book-to-market portfolios, and (iii) the idiosyncratic volatility of the VIX portfolio itself.

* Indicates significance at the 10% level.
** Indicates significance at the 5% level.
*** Indicates significance at the 1% level.

benchmark portfolios as presented in Table 1. Furthermore, the volatility attributable to the residual of squared market return is greater than 23% in Panel B of Table 7.

Our results show that idiosyncratic volatility accounts for around 61.39% of the variance in the VIX-squared portfolio, for about 47.69% of the variance in VIX cash, and for approximately 41.51% of the variance in VIX futures. Similar to the results presented in Table 6, the combination of the weak explanatory power of the quadratic characteristic model and the high proportion of variance accounted for by idiosyncratic volatility again indicate that the significant expansion of the mean–variance frontier following the addition of a VIX-related asset to the 6 benchmark portfolios reflects an unexplained component of VIX-related asset returns.

3.5. Out-of-sample robustness check

We now conduct an out-of-sample test to examine whether our empirical results in the earlier sections are robust. We first use a 240-week window to estimate the optimal portfolio weights; we then use the estimated weights to design an investment portfolio for the next 4 weeks, after which we calculate the average weekly return on the investment.[18] The out-of-sample test is implemented on a rolling basis, which allows us to calculate the mean and standard deviation of the out-of-sample investment portfolio and its Sharpe ratio as well. Finally, we only implement the test for the VIX cash and VIX-squared portfolio, since the sample period is too short for VIX futures.

Table 8 reports the results of the out-of-sample test. Panel A of Table 8 uses the set of 6 size/book-to-market benchmark portfolios. We find that for all four cases, the Sharpe ratio improves with the addition of VIX-related assets. The most impressive result occurs when the VIX-squared portfolio is included in value-weighted benchmark portfolios. In that case, the Sharpe ratio increases by almost 60% over the ratio for the portfolios without the VIX-squared portfolio. Panel B of Table 8 uses the set of 25 size/book-to-market portfolios. We again find an improvement in the Sharpe ratio for all four cases. Since these out-of-sample results are similar to the in-sample results shown in Table 4, we conclude that our empirical results supporting the diversification benefit of VIX-related assets are robust.

4. Conclusion

In this paper, we employ mean–variance spanning and intersection tests to determine whether the addition of VIX-related assets can significantly enlarge the investment opportunity set, relative to currently traded stocks. To the best of our knowledge, no prior VIX research has attempted to address this issue.

[18] We also repeat the analysis using the different investment periods for 12, 24, and 48 weeks. The results are qualitatively similar to those based on the 4-week investment horizon.

Table 7
The quadratic characteristic model and variance decomposition of VIX-related assets.

(Size, book-to-market) Portfolios	The quadratic characteristic model			Variance decomposition	
	β^{FF}		Adjusted R^2 (%)	Volatility attributable to size/BM portfolio (%)	Volatility attributable to VIX portfolio (%)
	Coefficient	*t*-Statistics			
Panel A: (dependent variable: VIX cash)					
$\beta^{CRSP} = -3.615^{***}$; $\beta^{SQ} = 2.891$; Variance of VIX Cash = 0.0137					
Volatility Attributable to Market Portfolio (Squared Market Portfolio) = 51.95% (0.09%)					
Small-Low	0.447	2.27	52.20	0.38	47.57
Small-Medium	−0.238	−0.89	51.88	0.06	47.90
Small-High	−0.365	−1.45	51.98	0.16	47.80
Large-Low	−0.522	−0.81	51.87	0.05	47.91
Large-Medium	−0.992	−2.95	52.47	0.65	47.31
Large-High	−0.528	−1.98	52.11	0.29	47.66
Panel B: (dependent variable: VIX-squared portfolio)					
$\beta^{CRSP} = -5.865^{***}$; $\beta^{SQ} = 137.49^{***}$; variance of VIX-squared portfolio = 0.1216					
Volatility attributable to market portfolio (squared market portfolio) = 15.38% (23.06%)					
Small-Low	−0.254	−0.38	38.16	0.01	61.55
Small-Medium	−1.673	−1.86	38.49	0.33	61.22
Small-High	−1.455	−1.71	38.43	0.28	61.28
Large-Low	3.956	1.82	38.47	0.32	61.24
Large-Medium	−0.760	−0.66	38.19	0.04	61.51
Large-High	−0.486	−0.54	38.18	0.03	61.53
Panel C: (dependent variable: VIX futures)					
$\beta^{CRSP} = -3.058^{***}$; $\beta^{SQ} = 8.832$; variance of VIX futures = 0.0050					
Volatility attributable to market portfolio (squared market portfolio) = 57.95% (0.37%)					
Small-Low	0.475	1.39	58.12	0.39	41.29
Small-Medium	0.469	1.23	58.03	0.30	41.37
Small-High	0.092	0.25	57.74	0.01	41.66
Large-Low	−0.589	−0.82	57.86	0.13	41.54
Large-Medium	0.573	0.84	57.87	0.14	41.53
Large-High	−0.188	−0.31	57.74	0.02	41.65

The table reports the results of the quadratic characteristic model estimation and variance decomposition for the VIX-related assets. This model is specified as:

$$R_{VIX,t} = \alpha + \beta^{CRSP} R_{CRSP,t} + \beta^{SQ} R_{SQ,t} + \beta^{FF} R_{FF,t} + \xi_t,$$

where R_{VIX} is the return on the VIX cash, VIX-squared portfolio or VIX futures, R_{CRSP} is the return on the value-weighted CRSP market index, R_{SQ} is the return on the residual obtained by regressing the squared value-weighted CRSP market index on R_{CRSP}, and R_{FF} is the residual obtained from regressing each of the 6 Fama–French value-weighted size/book-to-market portfolios on R_{CRSP} and R_{SQ}. The variance decomposition for the VIX-related assets consists of four components: (i) the proportion attributable to the volatility of the market portfolio, (ii) the proportion attributable to the volatility of the squared market portfolio, (iii) the proportion attributable to the volatility of each of the 6 size/book-to-market portfolios, and (iv) the idiosyncratic volatility of the VIX portfolio itself..

* Indicates significance at the 10% level.

** Indicates significance at the 5% level.

*** Indicates significance at the 1% level.

Table 8
Out-of-sample test for VIX-related assets.

	VIX cash/EW benchmarks		VIX cash/VW benchmarks		VIX-squared portfolio/EW benchmarks		VIX-squared portfolio/VW benchmarks	
	Without VIX	With VIX	Without VIX	With VIX	Without VIX	With VIX	Without VIX	With VIX
Panel A: 6 size/book-to-market benchmark portfolios								
Mean Return	0.0056	0.0052	0.0055	0.0041	0.0056	0.0070	0.0043	0.0133
Standard Deviation	0.0108	0.0086	0.0160	0.0092	0.0108	0.0128	0.0082	0.0182
Mean Difference	−0.0004		−0.0013		0.0014		0.0091	
(P-value)	(0.0187)		(0.0015)		(0.0000)		(0.0000)	
Sharpe Ratio	0.4517	0.5279	0.2961	0.3727	0.4517	0.4900	0.4352	0.6941
Difference Sharp Ratio	0.0762		0.0765		0.0384		0.2589	
(% Change)	(16.88%)		(25.85%)		(8.49%)		(59.49%)	
Panel B: 25 size/book-to-market benchmark portfolios								
Mean Return	0.0070	0.0065	0.0063	0.0048	0.0070	0.0083	0.0059	0.0100
Standard Deviation	0.0108	0.0094	0.0171	0.0105	0.0108	0.0122	0.0167	0.0267
Mean Difference	−0.0005		−0.0012		0.0013		0.0041	
(P-value)	(0.0000)		(0.0003)		(0.0000)		(0.0000)	
Sharpe Ratio	0.5833	0.6212	0.3267	0.3903	0.5833	0.6217	0.3112	0.3476
Difference Sharp Ratio	0.0379		0.0636		0.0384		0.0363	
(% Change)	(6.50%)		(19.47%)		(6.59%)		(11.67%)	

The table reports the out-of-sample test for VIX-related asset returns in a tangency portfolio consisting of one of the VIX-related assets and the benchmark portfolios. We first estimate the in-sample optimal portfolio weights based on the previous 240-week returns; we then use these weights for asset allocation for the next 4 weeks and calculate the average weekly return for such investment. The out-of-sample test is implemented on a rolling basis. There are two different VIX-related asset returns: VIX cash and VIX-squared portfolio. Four sets of benchmarks are used, including 6 equally-weighted and value-weighted size/book-to-market portfolios and 25 equally-weighted and value-weighted size/book-to-market portfolios. These portfolios are available from French's website. The risk-free rate used is the weekly average risk-free rate over the sample period.

We summarize our empirical results as follows. For this study, we use two sets of size/book-to-market benchmark portfolios, one containing 6 portfolios and the other containing 25. Each set involves two types of portfolios, either equally-weighted or value-weighted, which results in four cases for our mean–variance spanning tests. In each case, we either add a VIX-related asset or do not; the portfolios to which no VIX-related asset is added provided our benchmark investment opportunity set. The VIX-related assets include VIX cash, a VIX-squared portfolio, and VIX futures.

The results of our study have demonstrated that investors who invest in a VIX-related asset are able to enlarge their investment opportunity set. Additional support for this conclusion is implied by our finding that at least 41% of the variance of a VIX-related asset cannot be explained by the market portfolio and the size/book-to-market benchmark portfolios. In fact, our two-factor model and quadratic characteristic model demonstrate that the proportion of idiosyncratic volatility for the VIX-squared portfolio can be higher than 61%. Out-of-sample tests indicating that the Sharpe ratios for the tangency portfolios which include VIX-related assets are much larger than for those not including VIX-related assets support the robustness of our empirical results.

Our study contributes to the investment and volatility literature by providing evidence that the availability of VIX futures and VIX-squared portfolios (featuring tradable financial products) can improve the investment opportunity set generated by size/book-to-market portfolios. Based on our findings, we therefore offer the following two conclusions. First, there is potential for the further development of financial products associated with volatility indexes because investors can gain diversification benefits from investing in such assets. Second, hedge fund managers who can trade in the derivatives markets can utilize VIX futures contracts or VIX-squared portfolios to enhance their equity portfolio performance, as measured by the Sharpe ratio.

Since traditional mean–variance spanning test ignore higher order moments, one can extend our analyses to mean–variance-skewness spanning tests on the diversification benefits of VIX-related assets in future research.[19] This issue becomes more important when various approaches estimating efficient portfolios with higher moments, such as Lai (1991), Gourieroux and Monfort (2005), Adler and Kritzman (2006), Jondeau and Rockinger (2006), Briec et al. (2007), and Zakamouline and Koekebakker (2009) gain popularity recently. Another direction of future research as mentioned in Sentana (2009) is to investigate how a dynamic asset allocation strategy based on the VIX information can improve the mean–variance efficiency. We refer the readers to Ferson and Siegel (2001), Abhyankar et al. (2007), Penaranda and Sentana (2008), and Horneff et al. (2010) for the details of such active strategies.

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[19] Please see Sentana (2009) for the details of mean–variance–skewness efficiency and spanning tests. We thank the referee for drawing our attention to this direction of future research.

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