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A Survey of Alternative Equity Index Strategies

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After reviewing the methodologies behind the more popular quantitative investment strategies offered to investors as passive equity indices, the authors devised an integrated evaluation framework. They found that the strategies outperform their cap-weighted counterparts largely owing to exposure to value and size factors. Almost entirely spanned by market, value, and size factors, any one of these strategies can be mimicked by combinations of the others. Thus, implementation cost is a better evaluation criterion than returns.

Recently, a number of alternative approaches to passive equity investing have gained popularity by claiming to offer risk-adjusted performance superior to that of traditional market-capitalization-weighted indices.[1] Some of these strategies, such as equal weighting and minimum-variance, have been around for decades but have only lately garnered meaningful interest. Other approaches—including the INTECH Diversity-Weighted Index,[2] the Research Affiliates Fundamental Index strategy, QS Investors' Diversification Based Investing,[3] TOBAM's Maximum Diversification Index,[4] and the EDHEC-Risk Efficient Equity Indices—are relatively new entrants to the world of passive investing. Arguably, the highly charged debates surrounding the fundamental indexation approach proposed in Arnott, Hsu, and Moore (2005) have spawned much of the recent movement to explore alternative passive equity strategies. If some investors have become convinced that a more intelligent passive equity portfolio is possible, most remain baffled by the array of available options.

The aim of our study was to produce an apples-to-apples comparison of the alternative beta strategies in a controlled backtest environment with full disclosure of data sources, parameters, and estimation methodologies; in particular, we wanted to examine the performance characteristics driven by the key assumptions for the strategies rather than by the implementation subtleties of the commercial products. We did not attempt to replicate the actual investment products derived from these strategies nor to provide investment recommendations of the commercial products based on the strategies; interested practitioners should conduct their own research on the commercial products, some of which will no doubt claim enhancements and refinements in addition to those reported in our study. Moreover, the commercial products are likely to differ in their asset management fees and expenses, which we did not analyze. We based all our backtests on the methodologies disclosed in the public domain (e.g., published journal articles and available research papers).

Methodology

For each alternative beta, we generated both a U.S. and a developed global backtest. For U.S. portfolios, we used the CRSP/Compustat Merged Database; for global portfolios, we used the merged Worldscope/Datastream database. We back tested each strategy with both annual and quarterly rebalancing to observe strategy robustness to two different rebalancing frequencies. We formed portfolios annually (quarterly) on the basis of market price data at the close of market on the last trading day of each year (quarter).

For both the U.S. and the global portfolios, we included eligible stocks from the largest 1,000 stocks.[5] All our portfolios could be classified as members of the large-cap core category on the basis of the final inclusion criteria and the portfolios' observed weighted average market capitalizations.

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We calculated the total monthly returns for each strategy over 1964–2009 for the U.S. strategies and over 1987–2009 for the global strategies. The choice of date ranges depended entirely on the breadth of available historical data for constructing the portfolios.[6]

Descriptions of Investment Strategies

The strategies that we studied can be classified into two categories: (1) heuristic-based weighting methodologies and (2) optimization-based weighting methodologies. Heuristic-based strategies are ad hoc weighting schemes established on simple and, arguably, sensible rules. Included in the heuristic category are equal weighting, risk-cluster equal weighting, cap weighting blended with equal weighting, and weighting by historical financial variables. Optimization-based strategies are predicated on an exercise to maximize the portfolio's *ex ante* Sharpe ratio, subject to practical investment constraints. In this category are minimum-variance strategies and a variety of maximum Sharpe ratio portfolios based on various expected return assumptions.

For our analysis, $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ represents a vector of portfolio weights such that the portfolio is fully invested $\left(\sum_{i=1}^{N} x_i = 1\right)$ and there are no short positions ($x_i \geq 0$ for all N stocks). The notation $\mathbf{x}$ signifies a portfolio. In briefly describing each alternative beta strategy, we focus on the investment intuition but provide enough detail about portfolio construction to illustrate the essence of a strategy without belaboring the technical subtleties.[7]

Heuristic-Based Weighting Strategies. Although the focus of our study was empirical, we took some liberties in interpreting the investment philosophies underlying the various heuristic-based weighting strategies. Specifically, we interpreted these strategies as extensions of an equal-weighted strategy—that is, each attempts to eliminate some undesirable portfolio characteristics associated with naive equal weighting.

■ *Equal weighting.* In an equal-weighted portfolio, constituents are selected from the largest N = 1,000 stocks sorted by descending market capitalization on the reconstitution date. The weight of each stock is set to $1/N$.

A notable feature of equal weighting is that the resulting portfolio risk–return characteristics are highly sensitive to the number of included stocks. Although the S&P 500 Index and the Russell 1000 Index have nearly identical risk–return characteristics over time, the equal-weighted S&P 500 portfolios and the equal-weighted Russell 1000 portfolios have dramatically different risk–return characteristics; the equal-weighted Russell 1000 has significantly greater exposure to small-cap names and is more volatile than the equal-weighted S&P 500.

■ *Risk-cluster equal weighting.* The equal-weighted portfolio strategy is too naive for some investors because the portfolio characteristics are dictated largely by the arbitrary choice of the stock universe to which the equal weighting is applied. The methodology of risk-cluster equal weighting (RCEW) improves upon the simple equal-weighted scheme by equally weighting risk clusters instead of individual stocks.

QS Investors' Diversification Based Investing (DBI) is related to RCEW. The DBI methodology has two distinguishing features: (1) It defines risk clusters on the basis of country and sector membership, and (2) it equally weights the country/sector portfolio within each risk-cluster portfolio. In our study, we examined a representative RCEW approach, which is comparable to the DBI construction methodology. Other adaptations in the marketplace are valid interpretations of the concept.

On each portfolio reconstitution date, we first acquired monthly time-series returns for $L \times M$ country/sector portfolios. These country/sector portfolios are weighted by market capitalization (e.g., U.K. Information Technology is one such country/sector portfolio).[8] We then specified the desired number of risk clusters, k,[9] and used a standard statistical technique[10] to partition the $L \times M$ country/sector portfolios into k mutually exclusive risk clusters.[11] The algorithm offers some insight by identifying correlations between all pairings of country/sector portfolios. Country/sector portfolios that co-moved strongly with other country/sector portfolios were all grouped together into a risk cluster. Once we had identified all k distinct risk clusters, we assigned the country/sector portfolios equal weights within each risk cluster. We generated the final portfolios by equally weighting each of the k risk clusters. In our backtest, we examined two portfolios with different k's to illustrate how variations of k affect portfolio performance. For the first case, we formed k = 20 clusters for the global RCEW portfolio and k = 7 clusters for the U.S. RCEW portfolio. For the second case, we used k = 10 and k = 4, respectively.

The advantage of the RCEW methodology over simple equal weighting is the robustness of the resulting portfolio to the size of the chosen stock universe. Recall that applying simple equal weighting to the S&P 500 and the Russell 1000 results in portfolios with different risk–return characteristics. The RCEW approach, however, produces portfolios with similar risk–return characteristics.

Diversity weighting. Two other potential concerns with equal weighting are relatively high tracking error against the cap-weighted benchmark and excess portfolio turnover. A simple solution is to blend portfolios on the basis of equal weighting and cap weighting in order to attenuate the levels of tracking error and turnover. Diversity weighting is one of the better-known portfolio heuristics that blend cap weighting and equal weighting.

Fernholz (1995) defined stock market diversity, D_p, as

$$D_p\left(\mathbf{x}_{Market}\right)=\left[\sum_{i=1}^{N}\left(x_{Market,i}\right)^{p}\right]^{1/p}, \tag{1}$$

where $p \in (0,1)$ and $x_{Market,i}$ is the weight of the ith stock in the cap-weighted market portfolio, and then proposed a strategy of portfolio weighting whereby portfolio weights are defined as

$$x_{Diversity,i}=\frac{\left(x_{Market,i}\right)^{p}}{\left[D_p\left(\mathbf{x}_{Market}\right)\right]^{p}}, \tag{2}$$

where $i = 1, \ldots, N$; $p \in (0,1)$; and the parameter p targets the desired level of portfolio tracking error against the cap-weighted index.

Intuitively, diversity weighting can be viewed as a method for interpolating between cap weighting and equal weighting.[12] Generally, this process redistributes weights from the larger names in the cap-weighted portfolio to the smaller names as p moves from 1 to 0. In the extreme case, when $p = 0$, the diversity-weighted portfolio is equivalent to the equal-weighted portfolio, and when $p = 1$, the diversity-weighted portfolio is equivalent to the cap-weighted portfolio.

In our replication, we started with the top N = 1,000 stocks sorted by descending market capitalization on each reconstitution date and then assigned weights according to Equation 2. We back tested two specifications of this strategy—one with $p = 0.76$ (the parameter chosen for the Diversity-Weighted S&P 500 Index in Fernholz, Garvy, and Hannon 1998) and the other with $p = 0.50$—to illustrate the effects of parameter p on the portfolio risk–return characteristics.

Fundamental weighting. Arnott et al. (2005) described a methodology for weighting stock indices by constituent companies' accounting size, measured by such reported financial variables as total sales and book value. Their aim was to propose weighting measures that are uncorrelated with the companies' market valuations. Hsu (2006) argued that if market prices contain nonpersistent pricing errors, portfolios weighted by such price-correlated measures as market capitalization are suboptimal.[13] In this framework, fundamental weighting and other price-uncorrelated weighting schemes achieve the same effect as equal weighting.[14] Arnott et al. (2005) argued that weighting by accounting-based measures of size improves upon equal weighting by reducing relative tracking error against the cap-weighted index and turnover while enhancing portfolio liquidity and capacity for equal weighting.

We constructed a representative approach to fundamental weighting that replicated the four-factor fundamental indexation methodology of Arnott et al. (2005).[15] We defined the four accounting size metrics as the last five years' average sales, average cash flow, average total dividends paid, and the past year's book value. Following Arnott et al. (2005), we sampled constituents from the largest N = 1,000 companies for each fundamental-weighted portfolio, sorted by descending accounting size.[16]

We defined the portfolio weight of the ith stock as

$$x_{Accounting\,size,i}=\frac{Accounting\,size_i}{\sum_{i=1}^{N} Accounting\,size_i}. \tag{3}$$

We then constructed the final fundamental indexation portfolio by averaging the portfolios weighted by sales, cash flow, dividends, and book value. We also computed a portfolio on the basis of the prior-year dividend to illustrate the effect of using a single accounting variable without using five-year averaging.[17] All our accounting data represented annual financial performance and were lagged two years to prevent look-ahead bias.

Optimization-Based Weighting Strategies. In theory, mean–variance optimization (MVO) is a fantastic way to form passive portfolios, yet it frequently falls short of its target when applied in practice. The two inputs required to generate an optimal mean–variance portfolio—all the stocks' expected returns and their covariance matrix—are notoriously difficult to estimate.

Chopra and Ziemba (1993) showed that if an investor's return forecast contains errors—even if the errors are small in magnitude—the performance of the resulting MVO may be meaningfully reduced. Expected returns for individual stocks are very difficult to forecast accurately. The return covariance matrix is also difficult to estimate, because of its high dimensionality as well as potential issues with invertibility.[18] Michaud (1989) demonstrated that MVO can actually magnify errors in the empirical covariance matrix by overemphasizing (underemphasizing) assets with small (large) estimated variances and covariances. Several alternative beta strategies that apply MVO attempt to overcome obstacles associated with forecasting risks and returns for a large number of stocks.

Minimum-variance strategies. Because forecasting returns is so difficult and the potential for error so large, Chopra and Ziemba (1993) suggested that portfolio outcomes could be improved by assuming that all stocks have the same expected returns. Under this seemingly stark assumption, the optimal portfolio is the minimum-variance portfolio. Using historical backtests, Haugen and Baker (1991) and Clarke, de Silva, and Thorley (2006) demonstrated that minimum-variance strategies improve upon their cap-weighted counterparts by supplying better returns with reduced volatility. Note that a minimum-variance portfolio is mean–variance optimal only if stocks are assumed to have the same expected returns; although the minimum-variance portfolio is unlikely to be mean–variance optimal, outperformance against standard cap-weighted indices is certainly possible without mean–variance optimality if the cap-weighted indices are not on the efficient frontier.

To construct a minimum-variance strategy, we selected the largest N = 1,000 companies sorted by descending market capitalization on each reconstitution date. We estimated the covariance matrix by using monthly excess returns for the previous 60 months.[19] To reduce the influence of outliers in the empirical covariance matrix, we used a shrinkage estimator similar to that of Ledoit and Wolf (2004);[20] we also considered other covariance estimation techniques as tests for strategy robustness. Portfolio weights for a minimum-variance strategy can be expressed as the solution to the following optimization problem:

$$\min_{\mathbf{x}} \left(\mathbf{x}'\hat{\boldsymbol{\Sigma}}\,\mathbf{x}\right) \text{subject to} \begin{cases} \sum_{i=1}^{N} x_i = 1 \\ l \le x_i \le u \end{cases} \forall i, \tag{4}$$

where $\mathbf{x}$ is the vector of portfolio weights and $\hat{\boldsymbol{\Sigma}}$ is the estimated covariance matrix. We allowed no short selling by setting a lower bound of $l = 0$. In addition, we included a position cap of $u = 5$ percent to avoid excess concentration of weight in any particular stock.

Maximum Sharpe ratio I. Given that all stocks are unlikely to have the same expected returns, the minimum-variance portfolio—or any practical expression of its concept—is theoretically unlikely to be the portfolio with the maximum *ex ante* Sharpe ratio. To improve upon a minimum-variance strategy, investors need to incorporate useful information on future stock returns.

Choueifaty and Coignard (2008) proposed a simple linear relationship between the expected premium, $E(R_i) - R_f$, for a stock and its return volatility, σ_i:

$$E(R_i) - R_f = \gamma\sigma_i \;\forall i, \tag{5}$$

where $\gamma > 0$. Under this assumption, the constrained Sharpe ratio optimization problem can be written as

$$\max_{\mathbf{x}} \left(\frac{\mathbf{x}'\hat{\boldsymbol{\sigma}}}{\sqrt{\mathbf{x}'\hat{\boldsymbol{\Sigma}}\mathbf{x}}}\right) \text{subject to} \begin{cases} \sum_{i=1}^{N} x_i = 1 \\ l \le x_i \le u \end{cases} \forall i, \tag{6}$$

where $\mathbf{x}$ is the vector of portfolio weights, $\hat{\boldsymbol{\Sigma}}$ is the estimated covariance matrix, and $\hat{\boldsymbol{\sigma}}$ is the vector of estimated return volatilities.[21] Equation 6 encapsulates the general framework that we used to compute a proxy portfolio for TOBAM's Maximum Diversification Index.

In the backtests, we selected portfolio constituents from the largest N = 1,000 stocks sorted by descending market capitalization on each reconstitution date. We allowed no short selling ($l = 0$); following Choueifaty and Coignard (2008), we let u = 10 percent to limit excess concentration of weight in any given position. We estimated the covariance matrix and volatilities by using the same shrinkage technique described earlier to handle estimation errors.

Note that Equation 6 represents a departure from standard finance theory, which states that only the nondiversifiable component of volatility (systematic risk) should earn a premium. When applied to stocks and portfolios of stocks, Equation 6 becomes internally inconsistent; it suggests that all stocks and portfolios should have the same Sharpe ratio and, therefore, that volatilities are linearly additive in equilibrium, which cannot be correct.[22] Note also the conflicting empirical evidence regarding the relationship between diversifiable risk and expected return: Goyal and Santa-Clara (2003) found a positive relationship, whereas Ang, Hodrick, Xing, and Zhang (2006, 2009) found a negative relationship.

Maximum Sharpe ratio II. Amenc, Goltz, Martellini, and Retkowsky (2010) developed a related portfolio approach that assumes a stock's expected returns are linearly related to its downside semi-volatility. They argued that investors are more concerned with portfolio losses than with gains. Thus, risk premium should be related to downside risk (semi-deviation below zero) as opposed to volatility. This assumption serves as a foundation for the EDHEC-Risk Efficient Equity Indices.

To demonstrate how this strategy is constructed, we can define the downside semi-volatility for the ith stock as

$$\delta_i = \text{Downside semi-volatility}_i$$
$$= \sqrt{E\left[\min\left(R_{i,t},0\right)^2\right]}, \tag{7}$$

where $R_{i,t}$ is the return for stock i in period t. Under this assumption, the traditional MVO problem of maximizing a portfolio's Sharpe ratio can be expressed as

$$\max_{\mathbf{x}}\left(\frac{\mathbf{x}'\hat{\boldsymbol{\delta}}}{\sqrt{\mathbf{x}'\hat{\boldsymbol{\Sigma}}\mathbf{x}}}\right) \text{ subject to} \begin{cases} \sum_{i=1}^{N} x_i = 1 \\ l \le x_i \le u \end{cases} \forall i, \tag{8}$$

where $\mathbf{x}$ is the vector of portfolio weights, $\hat{\boldsymbol{\Sigma}}$ is the estimated covariance matrix,[23] and $\hat{\boldsymbol{\delta}}$ is the vector of estimated downside semi-volatilities.

Amenc, Goltz, Martellini, and Retkowsky (2010) used a two-stage estimation heuristic to estimate the semi-volatility of stocks. Under their method, one first computes empirical semi-volatilities and sorts stocks by these estimates into deciles; then one sets the semi-volatility of stocks in the same decile equal to the median value of the containing decile.[24] This methodology also imposes strong restrictions on single-stock weights, with a lower limit of $l = 1/(\lambda N)$ and an upper limit of $u = \lambda/N$. Amenc et al. used $\lambda = 2$, whereby portfolio weights vary between 0.05 percent and 0.2 percent. These position constraints shrink the unconstrained EDHEC-Risk Efficient Equity Indices portfolio weights toward equal weighting; note that as λ tends toward 1, the position constraints approach $1/N$, or equal weighting.

To assess the impact of the λ restriction, we back tested two portfolios (N = 1,000 largest-cap stocks), one with $\lambda = 2$ and the other with $\lambda = 50$, restricting the portfolio weights to a range of 0.002 percent to 5 percent, which is quite comparable to the other methodologies. In the backtests, we also implemented a turnover restriction (required by Amenc et al. 2010) that suppresses rebalancing upon reconstitution if weights have not deviated significantly from the new model weights. We acknowledge that we were unable to measure the impacts of the various external restrictions separately from the key underlying investment philosophy, which assumes that stock returns should be related to downside volatility.

Empirical Results and Discussion

We then analyzed the simulated time-series returns for the alternative betas as previously described. To assess the accuracy of our simulations, we compared our backtest time-series results with the time-series results reported by the strategy provider, when available, or with the time-series summary statistics reported in published papers. Unless otherwise stated, the reported returns are annualized and geometric. Again, we stress that our aim in conducting the backtests was not to replicate the commercial products marketed by various asset managers but, rather, to illustrate the performance associated with the key investment philosophies stated in these alternative betas.[25] Using identical datasets, stock universes, rebalancing dates, rebalancing frequency, optimization algorithms, and estimation methods for distributional characteristics, we carefully compared these alternative beta concepts. In this controlled environment, we were able to understand how the core features and philosophies of the considered methodologies influence portfolio performance. We were also able to ascertain how often-overlooked details of the methodologies (e.g., rebalancing frequency, number of constituents included, and position constraints) influence a strategy's performance.

Universe Construction and Stock Selection Rules. For the U.S. equity universe, we used the CRSP universe, which includes all stocks listed on the NYSE, Amex, and NASDAQ. We excluded all exchange-traded funds (ETFs), title records, and American Depositary Receipts (ADRs). We obtained financial reporting data of U.S. companies from Capital IQ Compustat. We first excluded companies not incorporated in the United States and then excluded companies without two full years of reported financials or stock return data. For the global universe, we used the Thomson Reuters Datastream stock universe and constrained our country definition to match the MSCI classification for developed countries. We determined a company's country of origin by a variety of factors: the location of its primary operations, head offices, incorporation, and auditing, as well as the stock exchange where its shares were most liquidly traded.[26] Again, we excluded all ETFs and ADRs. We obtained global company financial reporting data from the Worldscope database. To determine inclusion in a strategy in which market capitalization is a discriminating variable, we used the market capitalization tied to a company's primary share class prior to rebalancing; we did not float-adjust the market capitalization. Although our constituent universe and stock selection rules did not match the Standard & Poor's and MSCI rules exactly, we did not expect this discrepancy to bias our results.

Standard Performance Characteristics. We report summary statistics of the time-series returns on the basis of parameters that are most similar to the official specifications supplied by the provider of the methodology. To ensure a controlled environment for performance comparison, we synchronized the rebalancing time and frequency[27] in both our tests and our constituent universe. We calculated the monthly total returns for each portfolio strategy over 1964–2009 for U.S. portfolios and over 1987–2009 for global portfolios. Unless otherwise specified, we used annual rebalancing on the last day of the calendar year. We report the performance characteristics for the alternative betas for global strategies in **Table 1** and **Figure 1** and for U.S. strategies in **Table 2** and **Figure 2**.

Note that all the strategies in our study produced meaningfully higher returns than their cap-weighted benchmarks over the full sample period. Because we studied only strategies that have achieved commercial or publication success, the observed in-sample outperformance may be explained by selection bias.[28] To verify the robustness of our results, we report the subsample period returns for the strategies in Appendix A and other robustness analyses based on variations in the portfolio construction parameters and index membership in Appendix B. We found no evidence that the long-horizon results are dominated by one particular subsample period. We also found that all variations in strategy specification continue to outperform the cap-weighted benchmark, suggesting that these alternative betas do indeed provide a reliable avenue for improved performance. Later in the article, we will explore the potential sources of the observed outperformance when we present the Carhart (1997) and Fama–French (1993) factor analyses.

Table 1. Return Characteristics of Annually Rebalanced Global Strategies for 1,000 Stocks, 1987–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
MSCI World Index[a]	7.58%	15.65%	0.22	—	—	—	8.36%
Heuristic-based weighting							
Equal weighting	8.64%	15.94%	0.28	1.05%	3.02%	0.35	21.78%
RCEW (*k* clusters)	10.78	16.57	0.40	3.20	6.18	0.52	32.33
Diversity weighting (p = 0.76)	7.75	15.80	0.22	0.16	1.60	0.10	10.39
Fundamental weighting	11.13	15.30	0.45	3.54	4.77	0.74	14.93
Optimization-based weighting							
Minimum-variance	8.59%	11.19%	0.39	1.01%	8.66%	0.12	51.95%
Maximum diversification	7.77	13.16	0.27	0.18	7.41	0.02	59.72
Risk-efficient (λ = 2)	8.94	14.90	0.32	1.35	3.58	0.38	36.40

[a]For the MSCI World Index, we report the turnover of a simulated global developed cap-weighted index of the top 1,000 stocks rebalanced annually on 31 December.

Table 2. Return Characteristics of Annually Rebalanced U.S. Strategies for 1,000 Stocks, 1964–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
S&P 500[a]	9.46%	15.13%	0.26	—	—	—	6.69%
Heuristic-based weighting							
Equal weighting	11.78%	17.47%	0.36	2.31%	6.37%	0.36	22.64%
RCEW (*k* clusters)	10.91	14.84	0.36	1.45	4.98	0.29	25.43
Diversity weighting (p = 0.76)	10.27	15.77	0.30	0.81	2.63	0.31	8.91
Fundamental weighting	11.60	15.38	0.39	2.14	4.50	0.47	13.60
Optimization-based weighting							
Minimum-variance	11.40%	11.87%	0.49	1.94%	8.08%	0.24	48.45%
Maximum diversification	11.99	14.11	0.45	2.52	7.06	0.36	56.02
Risk-efficient (λ = 2)	12.46	16.54	0.42	3.00	6.29	0.48	34.19

[a]For the S&P 500, we report the turnover of a simulated U.S. cap-weighted index of the top 500 stocks rebalanced annually on 31 December. Actual S&P 500 turnover is generally lower owing to committee-based stock selection rules.

Figure 1. Return Characteristics of Annually Rebalanced Global Strategies for 1,000 Stocks, 1987–2009

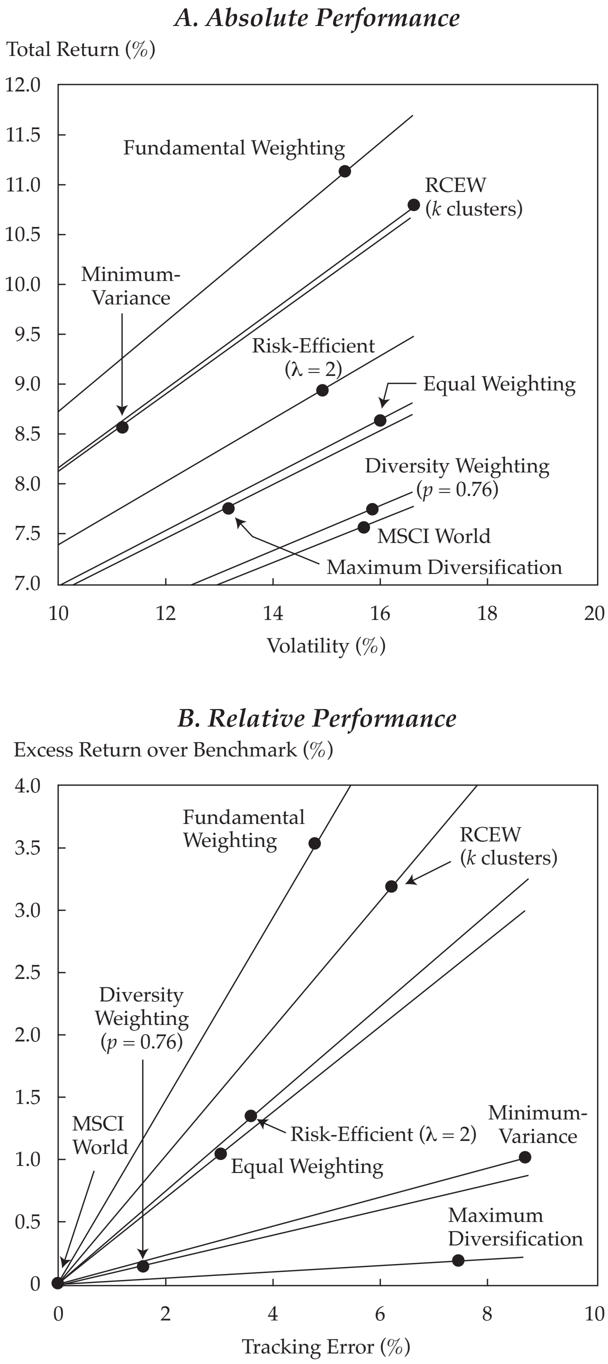
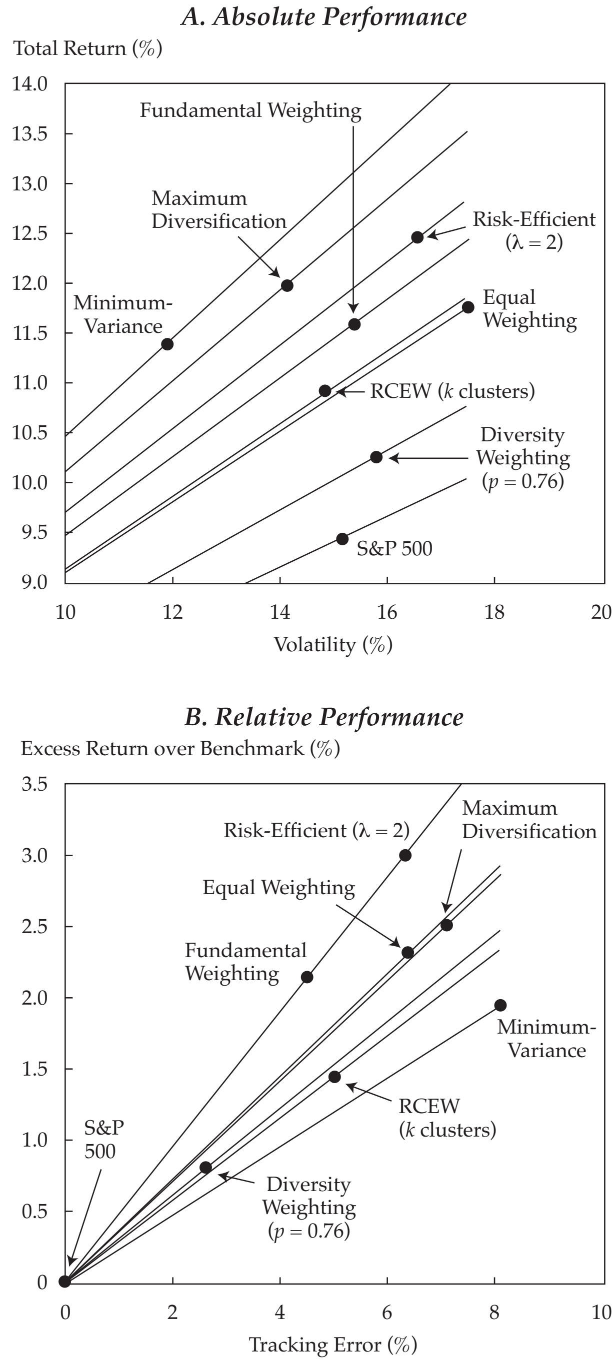


Figure 2. Return Characteristics of Annually Rebalanced U.S. Strategies for 1,000 Stocks, 1964–2009



As advertised, the minimum-variance portfolios show the lowest volatilities of all the strategies that we considered. Consistent with DeMiguel, Garlappi, and Uppal (2009), the MVO-based strategies do not outperform other, more simplistic strategies. In general, the optimization-based weighting strategies tend to have higher tracking errors and lower volatilities; the heuristic-based weighting strategies, in contrast, tend toward relatively higher volatilities and lower tracking errors. In the absence of portfolio optimization, most long-only portfolios naturally tend toward a market beta of 1, whereas optimized portfolios naturally tend to give low or zero weights to higher-beta stocks, resulting in a portfolio beta significantly below 1. The risk-efficient portfolio construction with position constraints (between 0.05 percent and 0.2 percent) is the exception; it has empirical market betas of 0.94 (global) and 1.0 (U.S.) over the full sample periods. This result is largely driven by the strategy's imposed position constraints, which tilt the final portfolio weights toward equal weighting. Our intuition is that the performance of the MVO-based strategies is largely dictated by their constraints. Indeed, Jagannathan and Ma (2003) found that the problems associated with traditional Markowitz MVO methodology are largely ameliorated when any weighting constraints are imposed, even random ones without economic rationales.

With the exception of the RCEW strategy, the heuristic-based portfolios realized significantly lower turnover than the optimization-based portfolios. This outcome is related to the observation that MVO can be quite sensitive to small changes in estimates of expected returns and covariances, resulting in volatile portfolio weights from one rebalancing to the next. The RCEW methodology involves a quasi optimization when country/sector pairings are partitioned into clusters, which can lead to relatively higher turnover. Whether portfolio optimization leads to a meaningfully higher degree of diversification than merely naive diversification is unclear. The full-sample-period Sharpe ratios from a number of the heuristic-based strategies are comparable to or greater than those of the optimization-based strategies. In our factor analyses (discussed later in the article), we found evidence that the reduced volatility of optimized portfolios is driven by a reduction in the exposure to the market beta rather than by a further reduction in idiosyncratic volatility.

Robustness of Strategies. We examined the robustness of the various alternative strategies to changes in some of the "user" and "technical" parameters. Specifically, we considered the total return in excess of the benchmarks for (1) rebalancing quarterly (instead of annually), (2) constructing portfolios with the 500 largest stocks (instead of the 1,000 largest stocks), and (3) imposing 5 percent and 1 percent limits on single-stock concentration. We compared the impacts on the optimized portfolios of using different techniques for estimating the return covariance matrix. Generally, we found that strategy variants often have different in-sample performances; *ex ante*, however, we cannot conclude whether one particular variant would have a stronger out-of-sample performance than another.[29]

Table 3 shows that a strategy's performance does not materially depend on the rebalancing frequency. Quarterly rebalancing, however, increases the turnover nearly twofold.[30] Although some vendors of alternative beta strategies use quarterly rebalancing, we found no benefit from more frequent rebalancing, and the elevated turnover will prima facie erode performance in implementation.

Reducing the constituent universe from the top 1,000 largest stocks to the top 500 systematically reduced performance for every tested strategy. This result is intuitive; for the heuristic-based weighting strategies, a narrower universe of larger stocks reduces exposure to the size (small-cap) factor, which reduces portfolio returns over time. For the optimization-based strategies, the smaller universe diminishes the opportunity set, which reduces performance.

Enforcing a 5 percent limit on single-stock concentration does not have a significant impact on portfolio return and risk characteristics. As the limit becomes smaller, strategy portfolio weights begin to converge toward equal weighting. Investors should understand whether a strategy's performance is driven by an investment philosophy or by external constraints and be cognizant of when external constraints are overriding the investment methodology.

Table 4 shows that the covariance matrix estimation methodology can have a significant impact on the optimized portfolio's performance. We used the shrinkage methods proposed by Clarke, de Silva, and Thorley (2006; Bayesian shrinkage A) and by Ledoit and Wolf (2004; Bayesian shrinkage B). We also used principal component analysis as a third alternative for estimating the return covariance matrix. *Ex ante*, investors have no reason to expect one of the three techniques to offer better portfolio performance than the others. What Table 4 does demonstrate is that optimized strategies are quite sensitive to the covariance matrix estimation technique used. Investors should understand this sensitivity when examining backtests associated with optimized portfolios.[31] Note that the same criticism applies to all the strategies that we considered: Variations in parameters and specifications lead to different levels of outperformance. The more sensitive a methodology is to trivial perturbation in parameters or specifications, the less reliable the methodology's back-tested outperformance.

Four-Factor Risk-Adjusted Performance. **Table 5** provides the results of a Carhart (1997) four-factor return decomposition for the various strategies.[32] After adjusting the strategies for market, size, value, and momentum factor loadings, we found that only one strategy—the global fundamental indexation strategy—displays a statistically significant, positive alpha (at the 5 percent level).

Table 3. Excess Return over Benchmark for Various User Parameters

Strategy	Global (1987–2009): 1,000 Stocks Rebalanced Annually	Global (1987–2009): 5% Weight Limit	Global (1987–2009): 1% Weight Limit	Global (1987–2009): 500 Stocks Rebalanced Annually	Global (1987–2009): 1,000 Stocks Rebalanced Quarterly	U.S. (1964–2009): 1,000 Stocks Rebalanced Annually	U.S. (1964–2009): 5% Weight Limit	U.S. (1964–2009): 1% Weight Limit	U.S. (1964–2009): 500 Stocks Rebalanced Annually	U.S. (1964–2009): 1,000 Stocks Rebalanced Quarterly
Heuristic-based weighting										
Equal weighting	1.05%	1.05%	1.05%	0.55%	1.19%	2.31%	2.31%	2.31%	1.26%	2.11%
RCEW (k clusters)	3.20	3.20	2.82	2.86	3.17	1.45	1.45	1.34	1.43	1.29
Diversity weighting (p = 0.76)	0.16	0.16	0.21	–0.24	0.26	0.81	0.81	0.95	0.38	0.75
Fundamental weighting	3.54	3.54	3.50	3.29	3.39	2.14	2.16	2.47	1.84	1.97
Optimization-based weighting										
Minimum-variance	1.01%	1.01%	1.44%	–0.79%	0.38%	1.94%	1.94%	1.80%	1.12%	1.54%
Maximum diversification	0.18	0.16	–0.04[a]	–0.94	–0.14	2.52	2.53	2.17	1.87	1.82
Risk-efficient[b]	1.35	1.02	1.18	1.25	1.32	3.00	2.23	2.36	2.06	2.40

[a]For the simulations, the number of securities with nonzero weights was less than 100 in 5 out of 23 years, which made the maximum weight exceed 1 percent.
[b]Maximum weight limits of 1 percent and 5 percent were achieved by λ choices of 10 and 50, respectively.

Table 4. Optimization-Based Strategies with Different Covariance Matrix Estimation Methods

Strategy	Bayesian Shrinkage (A): Excess Return over Benchmark	Bayesian Shrinkage (A): Volatility	Bayesian Shrinkage (A): Tracking Error	Bayesian Shrinkage (B): Excess Return over Benchmark	Bayesian Shrinkage (B): Volatility	Bayesian Shrinkage (B): Tracking Error	Principal Component Analysis: Excess Return over Benchmark	Principal Component Analysis: Volatility	Principal Component Analysis: Tracking Error
Annually rebalanced global portfolios of 1,000 stocks, 1987–2009									
Minimum-variance	1.01%	11.19%	8.66%	1.29%	9.85%	11.00%	0.31%	10.76%	9.42%
Maximum diversification	1.44	15.63	8.45	0.18	13.16	7.41	–0.40	13.17	8.60
Risk-efficient (λ = 2)	1.44	16.39	3.56	1.44	15.69	3.37	1.35	14.90	3.58
Annually rebalanced U.S. portfolios of 1,000 stocks, 1964–2009									
Minimum-variance	1.94%	11.87%	8.08%	1.54%	11.61%	8.10%	2.25%	11.59%	9.13%
Maximum diversification	3.72	17.64	10.02	2.52	14.11	7.06	2.92	14.55	9.16
Risk-efficient (λ = 2)	3.02	18.19	7.41	2.93	16.87	6.32	3.00	16.54	6.29

Table 5. Four-Factor Model Risk Decomposition

Strategy	Annual Alpha	Alpha p-Value	Market ($Mkt - R_f$)	Size (SMB)	Value (HML)	Momentum (MOM)	R^2
Annually rebalanced global strategies for 1,000 stocks, 1987–2009							
MSCI World Index	0.00%	—	1.000	0.000	0.000	0.000	1.00
Equal weighting	0.77	(0.131)	1.015**	0.259**	0.025*	–0.008	0.98
RCEW (k clusters)	0.68	(0.547)	1.071**	0.338**	0.232**	0.045**	0.90
Diversity weighting (p = 0.76)	0.38	(0.173)	1.001**	0.087**	–0.058**	0.011*	0.99
Fundamental weighting	2.18	(0.000)	0.970**	0.040*	0.332**	–0.090**	0.97
Minimum-variance	1.25	(0.329)	0.628**	0.001	0.138**	–0.013	0.73
Maximum diversification	0.49	(0.716)	0.760**	0.097*	0.004	0.029	0.78
Risk-efficient (λ = 2)	0.97	(0.154)	0.947**	0.176*	0.056**	–0.003	0.96
Annually rebalanced U.S. strategies for 1,000 stocks, 1964–2009							
S&P 500	0.00%	—	1.000	0.000	0.000	0.000	1.00
Equal weighting	0.15	(0.786)	1.043**	0.482**	0.144**	–0.012	0.96
RCEW (k clusters)	–0.13	(0.846)	0.954**	0.116**	0.185**	0.040**	0.91
Diversity weighting (p = 0.76)	0.07	(0.798)	1.012**	0.173**	0.029**	0.002	0.99
Fundamental weighting	0.50	(0.193)	1.010**	0.128**	0.338**	–0.076**	0.97
Minimum-variance	0.30	(0.713)	0.708**	0.198**	0.344**	0.011	0.81
Maximum diversification	–0.02	(0.977)	0.844**	0.342**	0.264**	0.061**	0.87
Risk-efficient (λ = 2)	0.19	(0.732)	1.002**	0.465**	0.250**	0.004	0.95

Notes: For the global strategies, we used the MSCI World Index for the market factor; we simulated the *HML* and *SMB* factors by following the methodology outlined on Kenneth French's website, with two exceptions: (1) We rebalanced factor portfolios in September to guarantee no look-ahead bias in the global accounting data, and (2) instead of using the NYSE median breakpoint, we used the top 20th global universe percentile as a cutoff point between the small-cap and large-cap portfolios. For the U.S. strategies, we used the S&P 500 for the market factor; we downloaded the *SMB*, *HML*, and *MOM* factor portfolios from Kenneth French's website.

*Significant at the 10 percent level.
**Significant at the 1 percent level.

Note that the U.S. fundamental indexation strategy does not show a significant alpha. Because only 1 of the 20 strategies/variants that we tested shows a significant alpha, we interpret that alpha as an outlier in our experiment and thus not meaningful.[33] Almost all the examined strategies display positive and significant exposure to the size and value factors. We conclude that all the strategies we examined outperform because of the positive value and size loadings. In a way, none of these strategies are different from naive equal weighting in their investment insights. In Appendix C, we show the same analysis by using the Fama–French three-factor model and reach a similar conclusion.

The absence of any Carhart four-factor alpha is not surprising. None of the considered strategies use nonpublic information, contain useful or uncommon insights, or deliberately seek exposure to *other* factors/anomalies. We conclude that the investment insights of the various strategies, such as expected returns, are linearly related to volatility (Choueifaty and Coignard 2008) or to downside semi-deviation (Amenc et al. 2010) and do not offer a comparative return advantage over traditional quant factor tilting. Recall that for a well-diversified portfolio, volatility is approximately the portfolio's market beta multiplied by the market portfolio's volatility. From the linear relationship that we observe between the market beta reported in Table 5 and the portfolio volatility reported in Tables 1 and 2, lower portfolio volatility (i.e., as provided by the minimum-variance strategy) results largely from a lower market beta. Whether portfolio optimization measurably improves diversification more than the naive diversification strategy of holding a large number of stocks is unclear.

Even without a statistically significant Carhart alpha, these alternative betas can still be valuable to investors. Offering access to the size and value premiums, they should be judged on their ability to facilitate investors' access to these factors efficiently. Hsu, Kalesnik, and Surti (2010) found that traditional value and small-cap indices exhibit negative Fama–French alphas, which suggests that they may be suboptimal portfolios for providing value and small-cap tilts.[34] Fama–French factor portfolios are also difficult to invest in because they require shorting, experience high turnover at rebalancing, and contain many illiquid stocks. Alternative beta strategies, which provide efficient long-only access to value and size factors, represent improvements over existing value and small-cap indices. Insofar as one can efficiently apply alternative beta portfolios as tools to improve an investor's portfolio Sharpe ratios and information ratios, these strategies are valuable.

Finally, we explored why these alternative betas result in value and small-cap biases. First, most of the strategies load significantly on the value factor. This observation is consistent with Arnott and Hsu (2008) and Arnott, Hsu, Liu, and Markowitz (2010), who found that any portfolio that rebalances regularly toward non-price-based weights naturally incurs a positive value loading. We also found high small-cap loading for diversity weighting and RCEW, which are directly related to equal weighting, and for risk-efficient weighting, which is indirectly related to equal weighting through the weight constraints. This finding is also not surprising because equal weighting, by construction, systematically overweights smaller stocks relative to the comparable cap-weighted index. Moreover, note that optimized strategies generally have a loading on the market portfolio of much less than 1. MVO tends to favor stocks with low average covariance relative to other stocks in the selection universe (unless the expected return for stocks depends on covariance, as in the capital asset pricing model); thus, MVO often results in large weights to lower-beta stocks and, therefore, "low-beta" portfolios.

Alternative Beta Portfolios and Mimicking Portfolios. Each alternative beta strategy offers different performance advantages over the others, but none of them dominate in all categories. An alternative beta portfolio represents a mapping to the market, value, and size factors—that is, the three Fama–French factors span the investment opportunity set for the alternative betas. Therefore, they can generally be linearly combined with one another (and/or cash) to mimic each other. To illustrate, we replicated the minimum-variance portfolio with other alternative betas. The minimum-variance portfolio is attractive because of its high Sharpe ratio; historically, it offers a higher return and lower volatility than the cap-weighted benchmark. From the Carhart four-factor decomposition, we can see that the minimum-variance portfolio is *special* in its low-market-beta loading; it suggests that we can construct a minimum-variance-mimicking portfolio by holding x percent in an alternative beta portfolio and ($1 - x$ percent) in cash, where x is set to ensure that the mimicking portfolio has the same volatility as the minimum-variance portfolio. **Table 6** shows the characteristics of the various minimum-variance-mimicking portfolios.

For the global comparison, mimicking portfolios based on RCEW and fundamental weighting outperform the actual minimum-variance portfolios as measured by the Sharpe and information ratios. For the U.S. results, maximum diversification weighting turns in a performance that is nearly identical to that of the minimum-variance strategy. Even if the performances were similar but not statistically better, these mimicking portfolios might be considered more-capital-efficient alternatives to traditional minimum-variance portfolios because they free up cash for other investments.

Turnover, Trading Costs, Capacity, and Liquidity

In addition to comparing strategy performances, we also examined the costs of rebalancing the alternative betas. We computed a number of portfolio characteristics, such as turnover and average bid–ask spread, that are related to portfolio transaction costs. The results are shown in **Table 7**. Turnover measures only one part of the total rebalancing cost; stocks with different liquidity characteristics generally incur different trading costs. Using a transaction cost model proposed by Keim and Madhavan (1997), we estimated annual portfolio turnover costs.[35] The reported trading cost estimate is based on a $100 million portfolio; larger portfolios will incur higher-percentage costs as buy-and-sell orders start to approach the average daily trading volume of some of the smaller and less liquid stocks. Strategies with higher weighted average daily trading volume would likely face a smaller cost increase at a higher asset level. Owing to the lack of stationarity in the Keim–Madhavan calibrated model (a problem with other calibrated cost models as well), the results for more recent trades may have a downward bias (and the results for earlier trades in the sample may have an upward bias); thus, we consider our cost estimates rough references only. We suggest that investors retain independent execution service providers to estimate transaction costs on alternative betas of interest.[36]

Note that the trading cost estimates are naturally lowest for market capitalization and are economically higher for the other strategies. From our estimation, however, we can see that the transaction costs for most strategies generally do not erode the entire return in excess of benchmark (we stress that we did not factor in asset management fees and expenses because we did not have such information). Diversity weighting and fundamental weighting generally have lower annual turnover and trading costs than the other strategies. Their

Table 6. Minimum-Variance-Mimicking Portfolios

Strategy	% Invested in Strategy (x)	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio
Annually rebalanced global strategies for 1,000 stocks, 1987–2009							
MSCI World Index	71.53%	6.89%	11.19%	0.24	–0.70%	4.45%	–0.16
Heuristic-based weighting							
Equal weighting	70.22%	7.60%	11.19%	0.30	0.01%	5.12%	0.00
RCEW (k clusters)	67.54	8.95	11.19	0.42	1.37	6.71	0.20
Diversity weighting (p = 0.76)	70.84	6.99	11.19	0.25	–0.60	4.65	–0.13
Fundamental weighting	73.16	9.49	11.19	0.47	1.90	6.04	0.32
Optimization-based weighting							
Minimum-variance	100.00%	8.59%	11.19%	0.39	1.01%	8.66%	0.12
Maximum diversification	85.02	7.35	11.19	0.28	–0.24	7.83	–0.03
Risk-efficient (λ = 2)	75.11	7.97	11.19	0.34	0.39	5.39	0.07
Annually rebalanced U.S. strategies for 1,000 stocks, 1964–2009							
S&P 500	78.46%	8.82%	11.87%	0.27	–0.64%	3.26%	–0.20
Heuristic-based weighting							
Equal weighting	67.94%	10.12%	11.87%	0.38	0.66%	5.87%	0.11
RCEW (k clusters)	79.97	10.02	11.87	0.38	0.56	5.51	0.10
Diversity weighting (p = 0.76)	75.28	9.35	11.87	0.32	–0.11	3.94	–0.03
Fundamental weighting	77.17	10.42	11.87	0.41	0.96	5.12	0.19
Optimization-based weighting							
Minimum-variance	100.00%	11.40%	11.87%	0.49	1.94%	8.08%	0.24
Maximum diversification	84.11	11.09	11.87	0.47	1.63	7.19	0.23
Risk-efficient (λ = 2)	71.76	10.79	11.87	0.44	1.33	6.13	0.22

Note: This table reports the performances of portfolios that invest x percent in the alternative beta strategies and (1 – x percent) in one-month T-bills.

turnover levels are half those of the other heuristic-based portfolios and one-third those of the optimization-based portfolios. The estimated transaction costs for fundamental weighting show a similar advantage over those for other strategies, whereas the costs for diversity weighting (p = 0.76) are only slightly higher than those for cap weighting. Note that the commercial products are likely to have not only additional turnover constraints but also management during rebalancing to address the excess turnover associated with alternative betas; we did not capture these factors in our study.

Because we estimated the trading costs for a $100 million portfolio, which may not be representative for larger investors, we included other portfolio characteristics to illustrate investability and potential transaction costs for more sizable portfolios. In Table 7, we show each portfolio's investment capacity by computing its weighted average market capitalization and each portfolio's liquidity by examining its weighted average bid–ask spread and daily trading volume. The measure of average market capitalization is representative of portfolio capacity and gives investors a rough approximation of the dollar investment a strategy can accommodate.

Both fundamental weighting and diversity weighting have two to three times the average market capitalization of the other alternative betas; thus, by that measure, they have the greatest portfolio capacity, which means that they face relatively smaller increases in portfolio transaction costs because investors deploy these strategies at higher asset levels. Fundamental weighting and diversity weighting also generally have lower bid–ask spreads and higher average daily trading volumes than the other alternative beta strategies. By construction, fundamental weighting uses accounting size variables that are integrated with market capitalization, thus emphasizing larger-cap companies. Diversity weighting is parameterized to partially mimic cap weighting, which naturally allocates more to larger-cap companies. Because capitalization is correlated in the cross section with both narrow bid–ask spread and high daily trading volume, diversity weighting and fundamental weighting score unsurprisingly well on these three liquidity measures.

Table 7. Transaction Cost Analysis

	Global (1987–2009)						U.S. (1964–2009)					
Strategy	Excess Return over Benchmark	One-Way Turnover	Market Cap (US$ billions)	Avg. Bid–Ask Spread	Adj. Daily Volume (US$ millions)	Trading Costs[a,b,c]	Excess Return over Benchmark	One-Way Turnover	Market Cap (US$ billions)	Avg. Bid–Ask Spread	Adj. Daily Volume (US$ millions)	Trading Costs[a,c]
Cap-weighted benchmark		8.4%[d]	66.34	0.11%	464.91	0.10%[d]		6.69%[e]	80.80	0.03%	735.40	0.03%[e]
Heuristic-based weighting												
Equal weighting	1.05%	21.8%	23.90	0.16%	174.96	0.31%	2.31%	22.6%	11.48	0.06%	132.49	0.22%
RCEW (*k* clusters)	3.20	32.3	37.47	0.17	189.12	0.69	1.45	25.4	37.14	0.04	312.04	0.12
Diversity weighting (p = 0.76)	0.16	10.4	52.37	0.12	368.16	0.13	0.81	8.9	50.53	0.04	477.87	0.06
Fundamental weighting	3.54	14.9	59.14	0.14	397.81	0.28	2.14	13.6	66.26	0.05	617.47	0.13
Optimization-based weighting												
Minimum-variance	1.01%	52.0%	23.97	0.35%	128.43	0.49%	1.94%	48.4%	19.63	0.05%	136.37	0.43%
Maximum diversification	0.18	59.7	20.08	0.45	122.50	0.57	2.52	56.0	14.77	0.06	124.08	0.53
Risk-efficient (λ = 2)	1.35	36.4	26.90	0.15	193.53	0.33	3.00	34.2	12.06	0.06	140.07	0.25

Note: Market capitalization, bid–ask spread, and adjusted daily volume are estimated for rebalancing at the end of 2009.

[a]Trading costs are estimated with the model proposed by Keim and Madhavan (1997), which accounts for (1) different exchanges, (2) size of trade, (3) market capitalization, (4) price per share, and (5) style of investment. Portfolio size is fixed at US$100 million; style of investment is set as indexed.

[b]We modified the Keim–Madhavan model to reflect additional costs for trading on the London Stock Exchange (50 bps for selling) and the Hong Kong Stock Exchange (10 bps for buying and selling).

[c]Trading costs include portfolio rebalancing only, not the costs of entering and exiting the strategies.

[d]Turnover and trading costs are based on a simulated cap-weighted index of the top 1,000 stocks in the global developed market.

[e]Turnover and trading costs are based on a simulated cap-weighted index of the top 500 stocks in the U.S. market.

Conclusion

In this article, we offer empirical evidence that the popular alternative betas outperform cap-weighted indexing unadjusted for risk factors. Using the Carhart four-factor model, we identified the sources of outperformance as exposure to the value and size factors, with risk-adjusted alpha not statistically different from zero. Because these strategies are essentially spanned by the same return factors (market, value, and size), they can be carefully combined to mimic one another.[37] This finding leads us to conclude that, despite the unique investment insights and technological sophistication claimed by the purveyors of these strategies, the performances are directly related to a strategy of naive equal weighting, which produces outperformance by tilting toward value and size factors. Nonetheless, the alternative betas represent an efficient and potentially low-cost way to access the value and size premiums because traditional style indices tend to have negative Fama–French alpha and direct replication of Fama–French factors is often impractical and costly. Moreover, combining alternative betas with one another (and with cash and equity index futures) would allow investors to better target desired levels of value and size tilt in their equity allocations.

We found that mean–variance optimization, which is required for a number of the alternative betas, does not appear to result in a meaningful diversification improvement over nonoptimized portfolios, despite the added complexity. Investment insights about the relationship between a stock's expected excess return and volatility, or downside semi-volatility, do not appear to produce performance benefits that are otherwise not present in other simple portfolio heuristics, which are derived from equal weighting. This finding is consistent with the extensive literature documenting the puzzling underperformance of MVO approaches.

Lastly, we caution investors to pay special attention to the potential implementation costs of these alternative betas relative to the cap-weighted benchmark. The excess turnover, reduced portfolio liquidity, and decreased investment capacity, in addition to the fees and expenses associated with managing a more complex index portfolio strategy, may erode much of the anticipated performance advantage. Because less costly alternative betas may be combined to mimic the more costly strategies, implementation costs should be one of the evaluation criteria.

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This article qualifies for 1 CE credit.

Appendix A. Subsample Period Returns

In **Table A1** and **Table A2**, we report the subsample period returns for the various strategies to verify the robustness of our results.

Table A1. Subsample Returns for Global Alternative Equity Indices of 1,000 Stocks, 1987–2009

	1987–2009		1987–1989		1990–1999		2000–2009	
Strategy	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha
MSCI World Index	7.58%	0.00%	19.26%	0.00%	11.96%	0.00%	0.23%	0.00%
Equal weighting	8.64	0.77	20.35	–0.05	10.57	–0.51	3.51	1.92*
RCEW (*k* clusters)	10.78	0.68	18.64	–2.93	13.96	–0.47	5.50	3.86*
Diversity weighting (p = 0.76)	7.75	0.38	18.89	–0.23	11.39	0.03	1.19	0.82*
Fundamental weighting	11.13	2.18**	20.52	2.15	13.87	1.15	5.84	3.05**
Minimum-variance	8.59	1.25	18.73	0.74	8.94	–0.36	5.38	1.67
Maximum diversification	7.77	0.49	17.00	–2.26	8.60	–1.06	4.34	0.85
Risk-efficient (λ = 2)	8.94	0.97	20.86	0.36	9.73	–1.01	4.83	2.13*

*Significant at the 10 percent level.
**Significant at the 1 percent level.

Table A2. Subsample Returns for U.S. Alternative Equity Indices of 1,000 Stocks, 1964–2009

	1964–2009		1964–1969		1970–1979		1980–1989		1990–1999		2000–2009	
Strategy	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha	Total Return	Carhart 4-Factor Alpha
MSCI World Index	9.46%	0.00%	6.75%	0.00%	5.88%	0.00%	17.55%	0.00%	18.21%	0.00%	–0.95%	0.00%
Equal weighting	11.78	0.15	14.57	0.12	7.77	–0.68	17.71	0.48	16.26	–0.80	4.28	2.21
RCEW (*k* clusters)	10.91	–0.13	6.76	0.04	8.11	–0.06	19.52	1.13	12.02	–3.34*	6.97	4.91**
Diversity weighting (p = 0.76)	10.27	0.07	9.02	0.04	6.46	–0.16	17.50	0.07	17.61	–0.18	1.18	1.11*
Fundamental weighting	11.60	0.50	7.54	–0.12	8.81	0.11	19.38	0.25	16.89	0.31	4.44	1.23
Minimum-variance	11.40	0.30	10.31	0.54	8.35	–0.10	21.00	1.68	11.78	–0.66	5.73	1.46
Maximum diversification	11.99	–0.02	13.63	0.38	7.98	–0.83	21.01	1.40	13.85	–1.37	4.80	1.12
Risk-efficient (λ = 2)	12.46	0.19	13.93	0.26	9.55	0.02	18.52	0.37	15.43	–0.55	5.92	2.32*

*Significant at the 10 percent level.
**Significant at the 1 percent level.

Appendix B. Parameter Variation Analyses

In **Table B1** and **Table B2**, we report the parameter variation analyses to verify the robustness of our results.

Table B1. Comparing Key Alternative Equity Indices and Their Variants for Annually Rebalanced Global Portfolios, 1987–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover[g]
MSCI World Index	7.58%	15.65%	0.22	—	—	—	9.32%
Equal weighting (N = 1,000)[a]	8.64	15.94	0.28	1.05%	3.02%	0.35	21.78
Equal weighting (N = 500)[a]	8.14	15.89	0.25	0.55	2.15	0.26	22.06
RCEW (k clusters)[b]	10.78	16.57	0.40	3.20	6.18	0.52	32.33
RCEW ($k/2$ clusters)[b]	10.76	16.55	0.40	3.17	6.42	0.49	30.16
RCEW (k clusters, N = 500)[b]	10.44	16.67	0.37	2.86	6.45	0.44	33.44
Diversity weighting (p = 0.76)[c]	7.75	15.80	0.22	0.16	1.60	0.10	10.39
Diversity weighting (p = 0.5)[c]	8.16	15.83	0.25	0.58	2.07	0.28	14.02
Diversity weighting (p = 0.76, N = 500)[c]	7.34	15.86	0.20	–0.24	1.70	–0.14	11.32
Fundamental weighting (composite factors)[d]	11.13	15.30	0.45	3.54	4.77	0.74	14.93
Fundamental weighting (single unsmoothed factor)[d]	11.68	14.82	0.50	4.10	6.20	0.66	19.30
Fundamental weighting (composite factors, N = 500)[d]	10.87	15.23	0.44	3.29	4.78	0.69	14.55
Minimum-variance (Bayesian shrinkage)[e]	8.59	11.19	0.39	1.01	8.66	0.12	51.95
Minimum-variance (PCA)[e]	7.89	10.76	0.34	0.31	9.42	0.03	56.11
Minimum-variance (Bayesian shrinkage, N = 500)[e]	6.79	11.49	0.23	–0.79	9.42	–0.08	54.91
Maximum diversification (Bayesian shrinkage)[e]	7.77	13.16	0.27	0.18	7.41	0.02	59.72
Maximum diversification (PCA)[e]	7.18	13.17	0.23	–0.40	8.60	–0.05	60.14
Maximum diversification (Bayesian shrinkage, N = 500)[e]	6.64	13.07	0.19	–0.94	7.24	–0.13	59.45
Risk-efficient (λ = 2)[f]	8.94	14.90	0.32	1.35	3.58	0.38	36.40
Risk-efficient (λ = 50)[f]	8.60	13.67	0.32	1.02	4.95	0.21	76.31
Risk-efficient (λ = 2, N = 500)[f]	8.83	14.97	0.31	1.25	2.68	0.47	40.16

[a]Equally weighting the 1,000 and 500 largest stocks by market capitalization.
[b]Grouping and equally weighting 20 and 10 risk clusters of sector portfolios for the 1,000 largest stocks by market capitalization and 20 clusters for the 500 largest stocks by market capitalization.
[c]Setting the blending factor to 0.76 (as chosen by INTECH for its U.S. simulations) and to 0.5 for a stronger tilt toward small-cap and value for the 1,000 largest stocks by market capitalization and setting the blending factor to 0.76 for the 500 largest stocks by market capitalization.
[d]Weighting by a composite of four fundamental factors (as defined by Arnott et al. 2005) and by a one-year dividend (single unsmoothed factor) for the 1,000 largest stocks by fundamental value and weighting by a composite of four fundamental factors for the 500 largest stocks by fundamental value.
[e]Computing the covariance matrix by Bayesian shrinkage and by principal component analysis (PCA) for the 1,000 largest stocks by market capitalization and by Bayesian shrinkage for the 500 largest stocks by market capitalization.
[f]Setting the weight restriction factor λ to 2 (as defined by Amenc et al. 2010) and to 50 to allow a maximum single-stock concentration of 5 percent for the 1,000 largest stocks by market capitalization and setting the weight restriction factor λ to 2 for the 500 largest stocks by market capitalization.
[g]Turnover is based on a simulated cap-weighted index.

Table B2. Comparing Key Alternative Equity Indices and Their Variants for Annually Rebalanced U.S. Portfolios, 1964–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover[g]
S&P 500	9.46%	15.13%	0.26	—	—	—	6.69%
Equal weighting (N = 1,000)[a]	11.78	17.47	0.36	2.31%	6.37%	0.36	22.64
Equal weighting (N = 500)[a]	10.72	16.48	0.31	1.26	4.27	0.29	20.27
RCEW (k clusters)[b]	10.91	14.84	0.36	1.45	4.98	0.29	25.43
RCEW ($k/2$ clusters)[b]	9.82	15.55	0.27	0.36	4.79	0.08	29.08
RCEW (k clusters, N = 500)[b]	10.89	14.76	0.36	1.43	4.60	0.31	25.72
Diversity weighting (p = 0.76)[c]	10.27	15.77	0.30	0.81	2.63	0.31	8.91
Diversity weighting (p = 0.5)[c]	10.87	16.37	0.32	1.41	4.11	0.34	13.12
Diversity weighting (p = 0.76, N = 500)[c]	9.84	15.45	0.28	0.38	1.78	0.21	8.82
Fundamental weighting (composite factors)[d]	11.60	15.38	0.39	2.14	4.50	0.47	13.60
Fundamental weighting (single unsmoothed factor)[d]	10.95	14.34	0.38	1.49	5.18	0.29	13.53
Fundamental weighting (composite factors, N = 500)[d]	11.30	15.19	0.38	1.84	4.40	0.42	12.95
Minimum-variance (Bayesian shrinkage)[e]	11.40	11.87	0.49	1.94	8.08	0.24	48.45
Minimum-variance (PCA)[e]	11.71	11.59	0.53	2.25	9.13	0.25	51.68
Minimum-variance (Bayesian shrinkage, N = 500)[e]	10.58	12.43	0.40	1.12	7.38	0.15	47.13
Maximum diversification (Bayesian shrinkage)[e]	11.99	14.11	0.45	2.52	7.06	0.36	56.02
Maximum diversification (PCA)[e]	12.38	14.55	0.47	2.92	9.16	0.32	59.91
Maximum diversification (Bayesian shrinkage, N = 500)[e]	11.33	14.41	0.40	1.87	5.83	0.32	51.32
Risk-efficient (λ = 2)[f]	12.46	16.54	0.42	3.00	6.29	0.48	34.19
Risk-efficient (λ = 50)[f]	11.69	15.07	0.41	2.23	6.33	0.35	74.21
Risk-efficient (λ = 2, N = 500)[f]	11.52	15.75	0.38	2.06	4.39	0.47	34.72

[a]Equally weighting the 1,000 and 500 largest stocks by market capitalization.

[b]Grouping and equally weighting 7 and 4 risk clusters of sector portfolios for the 1,000 largest stocks by market capitalization and 20 clusters for the 500 largest stocks by market capitalization.

[c]Setting the blending factor to 0.76 (as chosen by INTECH for its U.S. simulations) and to 0.5 for a stronger tilt toward small-cap and value for the 1,000 largest stocks by market capitalization and setting the blending factor to 0.76 for the 500 largest stocks by market capitalization.

[d]Weighting by a composite of four fundamental factors (as defined by Arnott et al. 2005) and by a one-year dividend (single unsmoothed factor) for the 1,000 largest stocks by fundamental value and weighting by a composite of four fundamental factors for the 500 largest stocks by fundamental value.

[e]Computing the covariance matrix by Bayesian shrinkage and by principal component analysis (PCA) for the 1,000 largest stocks by market capitalization and by Bayesian shrinkage for the 500 largest stocks by market capitalization.

[f]Setting the weight restriction factor λ to 2 (as defined by Amenc et al. 2010) and to 50 to allow a maximum single-stock concentration of 5 percent for the 1,000 largest stocks by market capitalization and setting the weight restriction factor λ to 2 for the 500 largest stocks by market capitalization.

[g]Turnover is based on a simulated cap-weighted index. Actual S&P 500 turnover is generally lower owing to committee-based stock selection rules.

Appendix C. Fama–French Three-Factor Model Risk Decomposition

In **Table C1**, we report the results of a Fama–French three-factor risk decomposition for the various strategies.

Table C1. Three-Factor Model Risk Decomposition

Strategy	Annual Alpha	Alpha *p*-Value	Market ($Mkt - R_f$)	Small-Cap (*SMB*)	Value (*HML*)	R^2
Annually rebalanced global strategies for 1,000 stocks, 1987–2009						
MSCI World Index	0.00%	—	1.000	0.000	0.000	1.00
Equal weighting	0.68	(0.175)	1.018**	0.260**	0.023*	0.98
RCEW (*k* clusters)	1.19	(0.287)	1.053**	0.332**	0.241**	0.90
Diversity weighting (p = 0.76)	0.51	(0.071)	0.996**	0.085**	–0.056**	0.99
Fundamental weighting	1.15	(0.100)	1.005**	0.053*	0.315**	0.96
Minimum-variance	1.10	(0.384)	0.633**	0.003	0.135**	0.73
Maximum diversification	0.82	(0.536)	0.749**	0.093*	0.010	0.78
Risk-efficient (λ = 2)	0.94	(0.162)	0.949**	0.176**	0.055**	0.96
Annually rebalanced U.S. strategies for 1,000 stocks, 1964–2009						
S&P 500	0.00%	—	1.000	0.000	0.000	1.00
Equal weighting	0.01	(0.984)	1.046**	0.482**	0.148**	0.96
RCEW (*k* clusters)	0.33	(0.630)	0.946**	0.115**	0.171**	0.91
Diversity weighting (p = 0.76)	0.10	(0.718)	1.012**	0.173**	0.028**	0.99
Fundamental weighting	–0.37	(0.369)	1.026**	0.131**	0.364**	0.97
Minimum-variance	0.42	(0.591)	0.706**	0.197**	0.341**	0.81
Maximum diversification	0.68	(0.389)	0.830**	0.340**	0.243**	0.86
Risk-efficient (λ = 2)	0.24	(0.659)	1.001**	0.465**	0.249**	0.95

Note: See notes to Table 5.

*Significant at the 10 percent level.
**Significant at the 1 percent level.

Notes

1. We are using the language of many investment consultants, who classify quantitative indices with transparent methodology disclosure as *passive indices*. These strategies are referred to as *alternative equity betas* or, simply, *alternative betas*.
2. Diversity-Weighted is a service mark of DiversityPatent, LLC.
3. Formerly DB Advisors, QS Investors is owned by Deutsche Bank.
4. TOBAM was formerly Lehman Brothers' QAM (Quantitative Asset Management). Maximum Diversification is a registered trademark of TOBAM. The Maximum Diversification Index is a long-only version of TOBAM's anti-benchmark strategy, which is based on Choueifaty and Coignard (2008).
5. The various strategies use different definitions of "large." For most, large is measured by year-end market capitalization; for the fundamental indexation strategy, it is measured by company financial variables (e.g., book value).
6. We acknowledge that our backtest results generally cover longer time spans and more stocks than those used by some alternative beta providers, which can result in Sharpe ratios and information ratios that are different from their self-reported performance statistics. We selected our sample ranges in order to use the available data from the CRSP/Compustat and Worldscope/Datastream databases and not to favor one strategy over another.
7. Interested readers may obtain the exact construction methodologies and the resulting time series from the authors upon request.
8. For the U.S. application, we used 30 industry sectors and the industry portfolio returns from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french). For the global application, we used all the stocks listed in Thomson Reuters Datastream from 24 developed countries and 12 industry sectors to construct 288 country/sector portfolio returns.
9. The number of risk clusters may be chosen arbitrarily or by a variety of quantitative methodologies, such as principal component analysis.
10. Popular clustering methodologies include agglomerative hierarchical clustering, divisive hierarchical clustering, and *k*-medoid partitioning (for details, see Kaufman and Rousseeuw 1990). Intuitively, clustering methodologies seek to group similar data items together (in our case, equity industry portfolios) on the basis of some definition of *similarity* (we defined similarity by correlation—that is, if two portfolios are highly correlated, they are considered

similar and should be grouped together). Although we observed differences in the resulting portfolios from applying different clustering methodologies, there are no *ex ante* reasons to believe that one clustering methodology should lead to better portfolios than another methodology. We report only the results based on *k*-medoid partitioning, which is the most robust to outlier effects in the data and allows the user to create any number of clusters with ease.

11. The *dissimilarity* measure between two country/sector time-series returns is computed by $\hat{d}_{i,j} = \sqrt{(1-\hat{\rho}_{i,j})/2}$, where $\hat{\rho}_{i,j}$ is the sample return correlation between country/sector portfolios i and j.
12. Kaplan (2008) presented a portfolio construction methodology that blends accounting size weighting with cap weighting. This approach can be viewed as similar to the Fernholz (1995) methodology, which blends equal weighting with cap weighting. Like the Fernholz approach, Kaplan's methodology allows the investor to control the tracking error of the resulting blended portfolio relative to traditional cap-weighted benchmarks.
13. Perold (2007) challenged the findings of Arnott et al. (2005). First, he pointed out that if prices follow a random walk without mean reversion, the cap-weighted index does not experience return drag. Because Vuolteenaho (2002), among others, documented mean reversion in prices, we maintain our view that mean reversion is a valid assumption. Perold also pointed out that cap weighting would not have a return drag if fair value were randomly distributed around the price, which is a valid theoretical argument. Practically, however, we believe it is more intuitive to think that the company's price is distributed around the fair value, not the other way around.
14. Arnott et al. (2005) claimed that the exact choice of weighting metrics and the number of financial metrics chosen do not result in statistically different performances in the long run.
15. Although the FTSE RAFI Index Series, the FTSE GWA Index Series, the Russell Fundamental Index Series, the MSCI Value Weighted Indices, and the WisdomTree dividend- and earnings-weighted indices use accounting-based measures of size in their methodologies, they use different weighting variables and different selection universes, which leads to different in-sample performances and can lead to different out-of-sample returns. We believe that our replication captures the salient features of these indices and provides a valuable reference point. For a more detailed comparison of these strategies, see Hsu, Kalesnik, and Xie (2011).
16. Note that the security selection in this approach is different from that in other approaches, which generally operate on the constituent stocks of the benchmark index (the cap-weighted 1000 index). Hsu, Kalesnik, and Xie (2011) showed that security selection based on the fundamental variables can lead to better portfolio performance versus selection by market capitalization. The Fama–French alphas, however, are statistically zero for both constructions and thus do not affect the main message of our analysis.
17. Motivated by the WisdomTree dividend-weighted exchange-traded funds, this choice is often referred to as a Fundamental Index–based methodology.
18. The size of the covariance matrix for a 1,000-stock portfolio is 1,000 × 1,000, with 500,500 unique parameters that must be estimated.
19. We required at least 60 months of past returns for a stock to be included in our simulation. Although we could also use daily data to estimate the covariance matrix, we found that using daily data did not create materially different backtested results.
20. We calculated the shrinkage target and intensity as defined in Appendix A of Clarke, de Silva, and Thorley (2006); in their study of minimum-variance portfolios, they used a shrinkage target that had a smaller degree of freedom than that of Ledoit and Wolf (2004).
21. Choueifaty and Coignard (2008) proposed an extremely clever and efficient way to perform the equivalent constraint maximization. They created synthetic securities by first dividing a stock's original time-series returns by the estimated volatility; they could then compute the optimal Sharpe ratio portfolio as a minimum-variance portfolio, which is a relatively easy computation. Although one could choose to solve the optimal Sharpe ratio problem directly, the computational complexity becomes significantly greater. Using Bayesian shrinkage as defined in Ledoit and Wolf (2004), we estimated the covariance matrix with the last five years' monthly return data. Yves Choueifaty pointed out to us that our global backtest might be improved by estimating the covariance matrix on a shorter window of historical data at a daily frequency and with market information across different time zones synchronized by a "plesiochronous" estimator (see Choueifaty, Froidure, and Reynier 2011).
22. Yves Choueifaty pointed out to us that he intended Equation 6 to apply only to stocks and not to portfolios in Choueifaty and Coignard (2008). Choueifaty, Froidure, and Reynier (2011) argued that if risky assets have expected returns proportional to their diversification ratio times volatility, then volatility would not need to be additive.
23. See Appendix C of FTSE (2009).
24. This approach can be considered a simple but extreme form of the Bayesian shrinkage technique.
25. Noting that the actual products include carefully calibrated parameters, constraints, and other thoughtful designs that our replication does not properly capture, EDHEC-Risk Institute asked us to use its reported time series instead of replicating them. We acknowledge these concerns and reiterate our disclaimer that commercial products may differ from our simulated results for a variety of reasons. We are happy to provide interested readers with a careful comparison of our simulated time series and the time series offered by EDHEC-Risk Institute across multiple regions and periods. We did not observe meaningful differences in the factor exposures and long-horizon returns and turnovers between our simulated time series and the indices of EDHEC-Risk Institute.
26. Although we considered such companies as Accenture and Schlumberger to be U.S.-listed companies in our global sample (though incorporated offshore, they have major businesses operating in the United States and are traded most liquidly in the United States), they are excluded from the Compustat database because they do not file financials in the United States. Therefore, we did not include these companies in our U.S. sample.
27. For the risk-efficient strategy, we followed the turnover control proposed by Amenc et al. (2010). The annually "rebalanced" risk-efficient portfolio is reassessed quarterly, and the portfolio weights are overhauled if they deviate from targets by a predefined threshold. Empirically, this quarterly reassessed efficient index rebalances approximately once a year. We constructed our "quarterly rebalanced" risk-efficient strategy by raising the reassessing frequency and lowering the rebalancing threshold.
28. Phil Tindall of Towers Watson suggested to us that the selection bias problem can be addressed by considering a strategy for which one has no *ex ante* knowledge of its

historical performance and for which one could not accidentally data-mine its control parameter for in-sample outperformance, such as weighting by random weights.

29. We provide what we consider the most interesting variants in the text and appendices. Additional variants are available upon request.
30. Turnovers are not shown in Table 3 in order to provide a cleaner presentation; that frequent rebalancing leads to higher turnover is intuitive. Interested readers can obtain the turnovers, the volatilities, and the other performance measures upon request.
31. Note that the risk-efficient portfolio is the exception; its results are relatively immune to changes in the covariance matrix estimation technique. We suspect that this outcome is also driven by the imposed stock weight constraints.
32. The Carhart (1997) four-factor approach uses the market factor and the Fama–French size and value factors in conjunction with a momentum factor. We downloaded the U.S. Fama–French size and value factors from Kenneth French's website; we created the global Fama–French factors by following the methodology described on his website. The market factor that we used in our analysis is the cap-weighted benchmark return minus the risk-free rate, R_f. We constructed the momentum factor on the basis of the methodology described in Carhart (1997). Cremers, Petajisto, and Zitzewitz (2008) found that the Fama–French three-factor analysis and the Carhart four-factor analysis can have downward biases in alpha estimation. We did not address this finding in our empirical analysis.
33. A Bonferroni correction for multiple comparisons would fail to reject the null hypothesis that the alpha is zero.
34. Hsu, Kalesnik, and Surti (2010) attributed the negative Fama–French alpha for traditional style indices to the cap-weighting construction, whereby the more expensive value and small stocks take up larger weights than the cheaper value and small stocks.
35. Keim and Madhavan (1997) estimated transaction costs by using order-level data from $83 billion worth of equity transactions initiated by various institutional traders. Their model accounts for costs associated with type of trade (buy or sell), style of investment (indexed versus active; we fixed our trades as indexed), price per share, market capitalization, size of trade, and exchange (NASDAQ is more expensive than the NYSE). We used their model to estimate trading costs at a stock-by-stock level and then aggregated them to obtain portfolio-level estimates. Because Keim and Madhavan's model was estimated for U.S. trades only, we modified it to adjust for additional charges for the London Stock Exchange (50 bps for selling) and the Hong Kong Stock Exchange (10 bps for both buying and selling). We acknowledge that our model is likely to be more robust for estimating trade costs for U.S. equity transactions than for global transactions.
36. Because the Keim–Madhavan model (1997) is not stationary (it takes market capitalization and price per share over 1992–1993 as factors), its results can be downward biased for estimating costs on portfolio schemes that use more recent data. We attempted to partially correct this deficiency by adjusting the market capitalization of stocks to the 1992 level. Using this adjustment with a US$500 million portfolio, we found only an insignificant increase in estimated portfolio transaction costs.
37. See Arnott, Kalesnik, Moghtader, and Scholl (2010) for an example that uses the fundamental indexation strategy, a minimum-variance portfolio, and an equal-weighted portfolio to span the three-factor space of market, value, and size.

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