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Answers are at the bottom of Question

Koç University
COMP341
Introduction to Artificial Intelligence
Written Assignment

Instructor: Barış Akgün
Due Date: December 21 2022, 23:59
Submission Through: Blackboard
Make sure you read and understand every part of this document

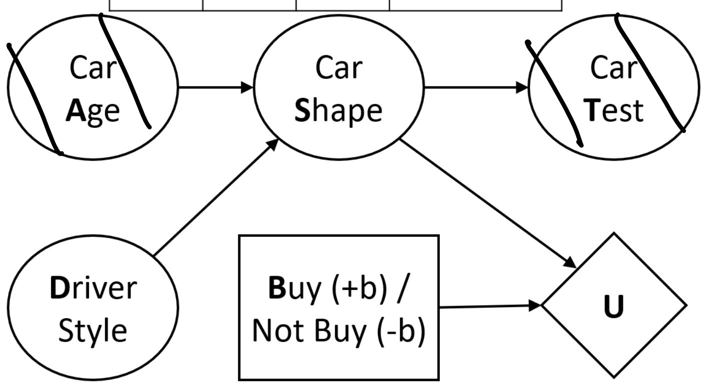
- This homework includes bayesian networks related problems.
- By submitting this homework, you **agree** to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply.
- You are expected to provide clear and concise answers. Gibberish will not receive any credit. Do not overly crowd your answers. Conciseness is a virtue. Write only what is relevant.
- Your answers need to be readable by a human, illegible writing is not gradable hence there is a strong chance that such answers will not get any credit.
- **Submit a single pdf with your solutions, your name and your ID.**

Second Hand Car Purchase

You want to buy a used car. You build a decision netwrok given below. In your decision network, the variable A is the car's age (old $A = +o$, not old $A = -o$), D is the current owner's driving style (aggressive $D = a$, neutral $D = n$, defensive $D = d$) and S is the shape of the car (good shape $S = +s$, bad shape $S = -s$). You further have the option of testing the car, which is represented by the variable T (success $T = +t$, fail $T = -t$). You decision is whether to buy the car or not. The price that the owner is asking is 5000 below the market value. However, if it is not in good shape, you will need to spend an additional 11000 on it. This is given in the utility node (not buying has no utility).

A	D	S	P(S|A, D)
+o	a	+s	0.2
+o	n	+s	0.5
+o	d	+s	0.7
-o	a	+s	0.4
-o	n	+s	0.7
-o	d	+s	0.9

A	P(A)
+o	0.6



S	T	P(T|S)
+s	+t	0.95
-s	+t	0.19

D	P(D)
aggressive	0.2
neutral	0.5
defensive	0.3

S	B	U
+g	+b	5000
+g	-b	0
-g	+b	-6000
-g	-b	0

Q1 (5 Points) What is the joint distribution of this Bayesian Network?

Q2 (25 Points) Calculate the probability of the car being in good shape if you get a positive test result (i.e. calculate $P(+s|+t)$), using variable elimination. Make sure to highlight all your factors at each time step. If you do not use variable elimination, there is a chance that you will not receive any points.

Q3 (10 points) Calculate $P(+s)$ however you want. You may use your result from the previous question.

Q4 (10 points) What is the expected utility of the buy $(+b)$ action without any evidence? You may use your results to the previous questions.

Q5 (15 points) What is the maximum amount of money you would be willing to pay to get the car tested? (You need to do calculations, this is not an essay question). You may use your results to the previous questions.

Q6 (10 points) You want to perform likelihood weighting to find $P(d|+o,+t)$. Calculate the weights of the samples given below.

A	D	S	T	Weight
$+o$	d	$+s$	$+t$	
$+o$	d	$-s$	$+t$	

Q7 (25 points) You want to perform Gibbs sampling. Given $\{A=-o, D=n, S=+s, T=-t\}$, calculate the probability distribution which will be used to sample S.

1)

$P(A,S,T,D)$

$= P(A) \times P(D) \times P(S|A,D) \times P(T|S)$

2)

$P(A), P(D), P(S|A,D), P(T|S)$

Query: +S
evidence: +t
hidden: A, D

Step 1: eliminate A

initial factors: $\{\cancel{P(A)}, P(D), \cancel{P(+s|A,D)}, P(+t|+s)\}$

$$f_1(+s,D) = \sum_{a \in A} P(a) \cdot P(+s|a,D)$$

A	D	S	
+o	a	+s	0.12
+o	n	+s	0.3
+o	d	+s	0.42
−o	a	+s	0.16
−o	n	+s	0.28
−o	d	+s	0.36

D	S	$f_1(+s,D)$
a	+s	0.28
n	+s	0.58
d	+s	0.78

Step 2: eliminate D

factors left: $\cancel{P(D)}$, $P(+t|+s)$, $\cancel{f_1(+s,D)}$

$$f_2(+s) = \sum_{d \in D} P(d) \cdot f_1(+s,d)$$

D	S	
a	+s	0.056
n	+s	0.29
d	+s	0.234

S	$f_2(+s)$
+s	0.58

→ answer to Q3

factors left: $P(+t \mid +s)$, $f_2(+s)$

Join remaining factors

$$f_3(+s,+t) = P(+t \mid +s) \cdot f_2(+s) = P(+s,+t)$$

S	T	$f_3(+s,+t)$
+s	+t	0.559
-s	+t	0.0798

table gives $P(+s,+t)$. To find $P(+s \mid +t)$, we should normalize over +t

To normalize, we should make probabilities sum up to 1 (no selection needed because ther is no $-t$ values)

$$\alpha \times (0.551 + 0.0798) = 1$$

$$\alpha = \frac{1}{0.6308}$$

S	T	P(S|+t)
+s	+t	0.873
-s	+t	0.127

So, $P(+s|+t) = 0.873$

3) Actually, f_2 is $P(S)$ and we found its table.

To prove f_2 is $P(S)$,

$$f_1(+s, D) = \sum_{a \in A} P(a) \cdot P(+s|a, D) = P(+s|D)$$

$$f_2(+s) = \sum_{d \in D} P(d) \cdot f_1(+s, d)$$

$$= \sum P(d) \cdot P(+s|d)$$

$\overline{\partial} \in D$

$= P(+s) = $

S	$f_2(+S)$
+S	0.58

$= 0.58$

4) $EU(+b) = \sum_{s \in S} P(s) \times U(s, +b) = P(+s) \times U(+s, +b) + P(-s) \times U(-s, +b)$

$= 0.58 \times 5000 + 0.42 \times (-6000) = 380$

5)

a) $MEU(\varnothing) = 380$ (from above)

b) $MEU(T = +t) = \max_b EU(b \mid t = +t)$

$EU(+b \mid t = +t) = \sum_s P(s \mid +t) \times U(s, +b) = 0.873 \times 5000 + 0.127 \times (-6000) = 3603$

$EU(-b \mid t = +t) = \sum_s P(s \mid +t) \times U(s, -b) = 0$

So, $MEU(T = +t) = 3603$

c) $MEU(T = -t) = \max_b EU(b \mid t = -t)$

$EU(+b \mid t = -t) = \sum_s P(s \mid -t) \times U(s, +b)$

to find this, recall the table

which $f_3(+s,+t) = P(+s,+t)$.

S	T	$f_3(+s,+t)$
+s	+t	0.551
-s	+t	0.0798

if we marginalize over t ($\sum_s P(S,+t)$) we find

$P(+t) = 0.6308$, $P(-t) = 0.3692$. we also

need to invert $P(+t|S)$ table.

$0.551 + 0.0798$

S	T	$P(-t\|S)$
+s	-t	0.05
-s	-t	0.81

$$P(+s|-t) = \frac{P(-t|+s) \times P(+s)}{P(-t)} = \frac{0.05 \times 0.58}{0.3692}$$

$= 0.078$

$$P(-s|-t) = \frac{P(-t|-s) \times P(-s)}{P(-t)} = \frac{0.81 \times 0.42}{0.3692}$$

$= 0.922$

then back to: $EU(+b \mid t = -t) = \sum_{s} P(s \mid -t) \times U(s, +b)$

$= 0.078 \times 5000 + 0.922 \times (-6000)$

$= -5142$

$EU(-b \mid t = -t) = 0$

So, $MEU(T = +t) = 0$

b) c) a)

$VPI(T) = 0.6308 \times 3603 + 0.3692 \times 0 - 380$

$= 1892.7724$

b)

$$w(z, e) = \prod_{i=1}^{m} P(e_i \mid \text{parents}(E_i))$$

A	D	S	T	Weight
$+o$	d	$+s$	$+t$	
$+o$	d	$-s$	$+t$	

$\to P(+o) \times P(+t \mid +s) = 0.6 \times 0.95 = 0.57$

$\to P(+o) \times P(+t \mid -s) = 0.6 \times 0.19 = 0.114$

7)

Q7 (25 points) You want to perform Gibbs sampling. Given $\{A = -o, D = n, S = +s, T = -t\}$, calculate the probability distribution which will be used to sample S.

$$P(+s \mid -o, n, -t) = \frac{P(+s, -o, n, -t)}{P(-o, n, -t)}$$

$$= \frac{P(+s, -o, n, -t)}{\sum_{s} P(s, -o, n, -t)} = \frac{\cancel{P(-o)} \times \cancel{P(n)} \times P(+s \mid -o, n) \times P(-t \mid +s)}{\cancel{P(-o)} \times \cancel{P(n)} \times P(+s \mid -o, n) \times P(-t \mid +s) + \cancel{P(-o)} \times \cancel{P(n)} \times P(-s \mid -o, n) \times P(-t \mid -s)}$$

$$= \frac{P(+s \mid -o, n) \times P(-t \mid +s)}{P(+s \mid -o, n) \times P(-t \mid +s) + P(-s \mid -o, n) \times P(-t \mid -s)}$$

$$= \frac{0.7 \times 0.05}{0.7 \times 0.05 + 0.3 \times 0.81}$$

$$= 0.126$$