Comp 341 - Assignment 3

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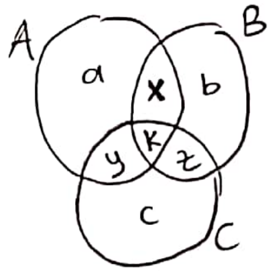
Q1) 1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$0.8 = 0.4 + 0.7 - P(A \cap B)$

$P(A \cap B) = 0.3 = x + k$

$$\begin{array}{r} x+k+z = 0.4 \\ - \quad x+k = 0.3 \\ \hline z = 0.1 \end{array} \qquad \begin{array}{r} z+k = 0.2 \\ - \quad z = 0.1 \\ \hline k = 0.1 \end{array} \qquad \begin{array}{r} x+k+z = 0.4 \\ - \quad z+k = 0.2 \\ \hline x = 0.2 \end{array} \qquad \begin{array}{r} y+k+z = 0.2 \\ - \quad k+z = 0.2 \\ \hline y = 0 \end{array}$$

$\underline{x + y + z = 0.3}$

2) $1 - P(A \cup B \cup C) = 1 - (P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C))$

$= 1 - (0.4 + 0.7 + 0.3 - 0.3 - 0.1 - 0.2 + 0.1)$

$= 1 - 0.9 = \underline{\underline{0.1}}$

Q2) • A & B independent $\Rightarrow P(A, B) = P(A) \cdot P(B)$

• $P(+a) > P(+b)$

$P(+x_1) = 0.277$
$P(+x_2) = 0.11$
$P(+x_3) = 0.296$

$P(+x_1, +x_2) = 0.091 \neq P(+x_1) \cdot P(+x_2)$
$P(+x_1, +x_3) = 0.212 \neq P(+x_1) \cdot P(+x_3)$
$P(+x_2, +x_3) = 0.034 \approx P(+x_2) \cdot P(+x_3)$
$P(-x_2, -x_3) = 0.628 \approx P(-x_2) \cdot P(-x_3)$
$P(+x_2, -x_3) \approx P(+x_2) \cdot P(-x_3)$, $P(-x_2, +x_3) \approx P(-x_2) \cdot P(+x_3)$

x_2 and x_3 are independent
$P(+x_3) > P(+x_2)$

So ; $x_3 = A$, $x_2 = B$, $x_1 = C$

Q3) 1. True 2. False 3. False 4. False 5. True 6. True

Q4) a) $P(C,A,P,J,m,w,S,H) = P(C).P(A).P(P).P(J|C,A).P(m|C,A).P(w|A,P).P(S|J,w).P(H|S,m,P)$

b) $2+2+2+8+8+8+8+16 = \underline{54}$

c) $P(+c), P(+a), P(P), P(J|+c,+a), P(m|+c,+a), P(W|+a,P), P(S|J,W), P(H|S,m,P)$

d) $P(C).P(A).P(+p).P(J|C,A).P(m|C,A).P(+w|A,+p).P(S|J,+w).P(H|S,M,+p)$

$\Rightarrow$ The largest factors are $P(H|S,M,+p)$, $P(J|C,A)$ and $P(m|C,A)$. Each has 3 non-evidence variables.
So, size of largest factor is $\underline{2^3 = 8}$

e) The initial factors that are needed:

$f_1(C) = P(C)$, $f_2(A) = P(A)$, $f_3(J,C,A) = P(J|C,A)$, $f_4(m,C,A) = P(m|C,A)$
$f_5(S,J,+w) = P(S|J,+w)$, $f_6(H,S,m,+p) = P(H|S,M,+p)$

Step 1: Join f_1, f_3 and f_4 to get $f_7(J,M,C,A)$
Step 2: Sum out C from f_7 to get $f_8(J,m,A)$
Step 3: Join f_2 and f_8 to get $f_9(J,M,A)$
Step 4: Sum out A from f_9 to get $f_{10}(J,M)$
Step 5: Join f_5 and f_{10} to get $f_{11}(S,J,M,+w)$
Step 6: Sum out J from f_{11} to get $f_{12}(S,M,+w)$
Step 7: Join f_6 and f_{12} to get $f_{13}(H,S,M,+w,+p)$
Step 8: Sum out S from f_{13} to get $f_{14}(H,M,+w,+p)$
Step 9: Sum out m from f_{14} to get $f_{15}(H,+w,+p)$
Step 10: Normalize f_{15} to get the desired probability $(P(H|+w,+p))$

Q5) a) $P(B,m,G,I,S) = P(B).P(m).P(I|B).P(G).P(S|m,I,G)$

b) $P(S=\text{true}) = \frac{\# +S}{\# \text{all}} = \underline{\frac{14}{39}}$

c) $P(S=\text{true} | B=\text{full}, M=\text{true}) = \frac{\#(+s,+b,+m)}{\#(+b,+m)} = \underline{\frac{6}{13}}$

d) All weights are the same and are equal to $0{,}7 \times 0{,}6 = \underline{0{,}42}$

$P(S=\text{true} | B=\text{full}, M=\text{true}) = \frac{0{,}42 \times (66+1+31+0)}{0{,}42 \times (66+1+31+0+14+19+25+44)} = \underline{0{,}49}$

Q6) a) $P(S \mid B=full, M=true, I=true, G=low) = \dfrac{P(S, B=full, M=true, I=true, G=low)}{P(B=full, M=true, I=true, G=low)}$

$$= \frac{\cancel{P(B=full)} \cdot \cancel{P(M=true)} \cdot \cancel{P(I=true \mid B=full)} \cdot \cancel{P(G=low)} \cdot P(S \mid G=low, I=true, M=true)}{\cancel{P(B=full)} \cdot \cancel{P(M=true)} \cdot \cancel{P(I=true \mid B=full)} \cdot \cancel{P(G=low)} \cdot \sum_S P(s \mid G=low, I=true, M=true)}$$

$$= \frac{P(S \mid G=low, I=true, M=true)}{0,2+0,8} = P(S \mid G=low, I=true, M=true)$$

b) $P(M \mid B=full, I=true, G=low, S=false) = \dfrac{P(M, B=full, I=true, G=low, S=false)}{P(B=full, I=true, G=low, S=false)}$

$$= \frac{\cancel{P(B=full)} \cdot P(M) \cdot \cancel{P(I=true \mid B=full)} \cdot \cancel{P(G=low)} \cdot P(S=false \mid G=low, I=true, M)}{\cancel{P(B=full)} \cdot \cancel{P(I=true \mid B=full)} \cdot \cancel{P(G=low)} \cdot \sum_m P(m) \cdot P(S=false \mid G=low, I=true, m)}$$

$$= \frac{P(m) \cdot P(S=false \mid G=low, I=true, m)}{\sum_m P(m) \cdot P(S=false \mid G=low, I=true, m)} = \frac{P(M) \cdot P(S=false \mid G=low, I=true, M)}{0,6 \times 0,8 + 0,4 \times 0,95}$$

$$= \frac{P(M) \cdot P(S=false \mid G=low, I=true, M)}{0,86}$$

c) $P(M=true \mid B=full, I=true, G=low, S=false)$

$$= \frac{P(M=true) \cdot P(S=false \mid G=low, I=true, M=true)}{0,86} = \frac{0,6 \times 0,8}{0,86} \approx 0,558$$

then $P(M=false \mid B=full, I=true, G=low, S=false) = 1 - 0,558 = 0,442$

and $0,4 \leq 0,442$

so choose $M=false$