Chapter 3

Momentum

Figure 3.1: Launch of ESA's Ariane 5 rocket, carrying rather important payload to space – the James Webb Space Telescope – on Christmas Day in 2021. Rockets use momentum, in principle, to get into space.

Assignment 4: Momentum

Fri, 10 Jan 19:00

3.1 Momentum and Impulse

Definition 10 (Momentum and Impulse).

Linear Momentum:

$$\mathbf{p} = m \cdot \mathbf{v}$$

Here, $\mathbf{p}$ is the linear momentum, m is the mass, and $\mathbf{v}$ is the velocity.

Conservation of Linear Momentum:

$$\sum m_i \cdot \mathbf{v_i} = \sum m_f \cdot \mathbf{v_f}$$

This equation expresses the conservation of linear momentum in a system, where m_i and $\mathbf{v_i}$ are the initial mass and velocity, and m_f and $\mathbf{v_f}$ are the final mass and velocity of the system.

Impulse-Momentum Theorem:

$$\mathbf{F} \cdot \Delta t = \Delta \mathbf{p}$$

This theorem relates the impulse ($\mathbf{F} \cdot \Delta t$) applied to an object to the change in its momentum ($\Delta \mathbf{p}$). This is true for a constant force over time. We can generalise this theorem for a variable force, one that changes with time, by considering an infinitesimal time period, such that: $\Delta t \to \delta t$. One can write the macroscopic momentum, $\Delta \mathbf{p}$ as:

$$\Delta \mathbf{p} = \int_{p_1}^{p_2} \delta \mathbf{p}(t) = \underbrace{\int_{t_1}^{t_2} \mathbf{F}(t) \, \mathrm{d}t}_{\text{via. Impulse-Momentum Theorem}}$$

This is a more general result of the Impulse-Momentum Theorem, it accounts for the Force potentially being time dependent.

Angular Momentum:

$$\mathbf{L} = I \cdot \boldsymbol{\omega}$$

Angular momentum ($\mathbf{L}$) is the product of moment of inertia (I) and angular velocity ($\boldsymbol{\omega}$).

3.2 Collisions

Via the principle of the conservation of linear momentum: the momentum describing the state of a system before a collision must equal the momentum describing the state of the system after the collision. Kinetic energy, though, needn't be conserved. If it is conserved after a collision event, it is an elastic collision. If kinetic energy is not conserved after a collision event, it is an inelastic collision.

Example. In an elastic collision between two equal masses, assuming they both collide, and one of the masses is at rest, the angle of separation after the collision is 90 degrees.

Proof. Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. An elastic collision conserves internal kinetic energy.

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2.$$

Because the masses are equal ($m_1 = m_2 = m$), algebraic manipulation of conservation of momentum in the x- and y-directions can show that

$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2'\cos(\theta_1 - \theta_2).$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv_1'v_2'\cos(\theta_1 - \theta_2) = 0.$$

There are three ways that this term can be zero. They are

- $v_1' = 0$: head-on collision; incoming ball stops
- $v_2' = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation ($\theta_1 - \theta_2$) is 90° after the collision. Thus, as we are interested solely in the last case, the angle of separation between the two balls will be 90 degrees.