Poisson_MGWR_univariate_check

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Notebook Outline:

0.0.1 Set up Cells

In [1]:
```
import sys
sys.path.append("C:/Users/msachde1/Downloads/Research/Development/mgwr")
```

In [2]:
```
import warnings
warnings.filterwarnings("ignore")
import pandas as pd
import numpy as np

from mgwr.gwr import GWR
from spglm.family import Gaussian, Binomial, Poisson
from mgwr.gwr import MGWR
from mgwr.sel_bw import Sel_BW
import multiprocessing as mp
pool = mp.Pool()
from scipy import linalg
import numpy.linalg as la
from scipy import sparse as sp
from scipy.sparse import linalg as spla
from spreg.utils import spdot, spmultiply
from scipy import special
import libpysal as ps
import seaborn as sns
import matplotlib.pyplot as plt
from copy import deepcopy
import copy
from collections import namedtuple
import spglm
```

0.0.2 Fundamental equations for Poisson MGWR

$$y = Poisson[E_i exp(\sum \beta_k x_{k,i})] - (1) \tag{1}$$
$$E_i = Offset - (2) \tag{2}$$
$$\hat{y} = predicted(y) - (3) \tag{3}$$
$$z = (\sum \beta_k x_{k,i}) + ((y - \hat{y})/\hat{y}) - (4) \tag{4}$$
$$\tag{5}$$

0.0.3 Example Dataset

```
In [3]: data_p = ps.io.open(ps.examples.get_path('Tokyomortality.csv'))
        coords = list(zip(data_p.by_col('X_CENTROID'),data_p.by_col('Y_CENTROID')))
        off = np.array(data_p.by_col('eb2564')).reshape((-1,1))
        y = np.array(data_p.by_col('db2564')).reshape((-1,1))
        occ = np.array(data_p.by_col('OCC_TEC')).reshape((-1,1))
        own = np.array(data_p.by_col('OWNH')).reshape((-1,1))
        pop = np.array(data_p.by_col('POP65')).reshape((-1,1))
        unemp = np.array(data_p.by_col('UNEMP')).reshape((-1,1))
        X = np.hstack([occ,own,pop,unemp])
        x = unemp

        X_std = (X-X.mean(axis=0))/X.std(axis=0)
        x_std = (x-x.mean(axis=0))/x.std(axis=0)
        y_std = (y-y.mean(axis=0))/y.std(axis=0)
```

0.0.4 Helper functions

Hardcoded here for simplicity in the notebook workflow

Please note: A separate bw_func_p will not be required when changes will be made in the repository

```
In [5]: kernel='bisquare'
        fixed=False
        spherical=False
        search_method='golden_section'
        criterion='AICc'
        interval=None
        tol=1e-06
        max_iter=200
        X_glob=[]
```

```
In [6]: def gwr_func(y, X, bw,family=Gaussian(),offset=None):
            return GWR(coords, y, X, bw, family,offset,kernel=kernel,
                       fixed=fixed, constant=False,
                       spherical=spherical, hat_matrix=False).fit(
                           lite=True, pool=pool)
```

```
        def bw_func_p(coords,y, X):
            selector = Sel_BW(coords,y, X,family=Poisson(),offset=off, X_glob=[],
                              kernel=kernel, fixed=fixed,
                              constant=False, spherical=spherical)
            return selector

        def bw_func(coords,y,X):
            selector = Sel_BW(coords,y,X,X_glob=[],
                              kernel=kernel, fixed=fixed,
                              constant=False, spherical=spherical)
            return selector

        def sel_func(bw_func, bw_min=None, bw_max=None):
            return bw_func.search(
                search_method=search_method, criterion=criterion,
                bw_min=bw_min, bw_max=bw_max, interval=interval, tol=tol,
                max_iter=max_iter, pool=pool, verbose=False)
```

0.0.5 Univariate example

GWPR model with independent variable, x = unemployment

```
In [7]: bw_gwpr=Sel_BW(coords,y_std,x_std,family=Poisson(),offset=off,constant=False).search()

In [8]: gwpr_model=GWR(coords,y_std,x_std,bw=bw_gwpr,family=Poisson(),offset=off,constant=Fals
```

MGWR Poisson loop with one independent variable, x = unemployment

Edited multi_bw function - original function in https://github.com/pysal/mgwr/blob/master/mgwr/search.p

```
In [9]: def multi_bw(init,coords,y, X, n, k, family=Gaussian(),offset=None, tol=1e-06, max_ite
                             verbose=False):

            if isinstance(family,spglm.family.Poisson):
                bw = sel_func(bw_func_p(coords,y,X))
                optim_model=gwr_func(y,X,bw,family=Poisson(),offset=offset)
                err = optim_model.resid_response.reshape((-1, 1))
                param = optim_model.params
                #This change for the Poisson model follows from equation (1) above
                XB = offset*np.exp(np.multiply(param, X))

            else:
                bw=sel_func(bw_func(coords,y,X))
                optim_model=gwr_func(y,X,bw)
                err = optim_model.resid_response.reshape((-1, 1))
                param = optim_model.params
                XB = np.multiply(param, X)
```

```
bw_gwr = bw
XB=XB

if rss_score:
    rss = np.sum((err)**2)
iters = 0
scores = []
delta = 1e6
BWs = []
bw_stable_counter = np.ones(k)
bws = np.empty(k)

try:
    from tqdm.auto import tqdm  #if they have it, let users have a progress bar
except ImportError:

    def tqdm(x, desc=''):  #otherwise, just passthrough the range
        return x

for iters in tqdm(range(1, max_iter + 1), desc='Backfitting'):
    new_XB = np.zeros_like(X)
    params = np.zeros_like(X)

    for j in range(k):
        temp_y = XB[:, j].reshape((-1, 1))
        temp_y = temp_y + err
        temp_X = X[:, j].reshape((-1, 1))

        #The step below will not be necessary once the bw_func is changed in the r
        if isinstance(family,spglm.family.Poisson):

            bw_class = bw_func_p(coords,temp_y, temp_X)

        else:
            bw_class = bw_func(coords,temp_y, temp_X)


        if np.all(bw_stable_counter == bws_same_times):
            #If in backfitting, all bws not changing in bws_same_times (default 3)
            bw = bws[j]
        else:
            bw = sel_func(bw_class, multi_bw_min[j], multi_bw_max[j])
            if bw == bws[j]:
                bw_stable_counter[j] += 1
            else:
                bw_stable_counter = np.ones(k)

        #Changed gwr_func to accept family and offset as attributes
```

```
                    optim_model = gwr_func(temp_y, temp_X, bw,family,offset)
                    err = optim_model.resid_response.reshape((-1, 1))
                    param = optim_model.params.reshape((-1, ))
                    new_XB[:, j] = optim_model.predy.reshape(-1)
                    params[:, j] = param
                    bws[j] = bw

                num = np.sum((new_XB - XB)**2) / n
                den = np.sum(np.sum(new_XB, axis=1)**2)
                score = (num / den)**0.5
                XB = new_XB

                if rss_score:
                    predy = np.sum(np.multiply(params, X), axis=1).reshape((-1, 1))
                    new_rss = np.sum((y - predy)**2)
                    score = np.abs((new_rss - rss) / new_rss)
                    rss = new_rss
                scores.append(deepcopy(score))
                delta = score
                BWs.append(deepcopy(bws))

                if verbose:
                    print("Current iteration:", iters, ",SOC:", np.round(score, 7))
                    print("Bandwidths:", ', '.join([str(bw) for bw in bws]))

                if delta < tol:
                    break

            print("iters = "+str(iters))
            opt_bws = BWs[-1]
            print("opt_bws = "+str(opt_bws))
            return (opt_bws, np.array(BWs), np.array(scores), params, err, bw_gwr)
```

Running the function with family = Poisson() and offset

```
In [10]: bw_mgwpr = multi_bw(init=None,coords=coords,y=y_std, X=x_std, n=262, k=x.shape[1], fa

iters = 2
opt_bws = [178.]
```

Running without family and offset attributes runs the normal MGWR loop

```
In [11]: bw_mgwr = multi_bw(init=None,coords=coords,y=y_std, X=x_std, n=262, k=x.shape[1])

iters = 1
opt_bws = [73.]
```

0.0.6 Parameter check

Difference in parameters from the GWPR model and MGWPR model

```
In [12]: max(bw_mgwpr[3]-gwpr_model.params)

Out[12]: array([1.89357983e-05])
```

The parameters are not identical but the maximum difference in the parameters is to the order of 1e-05

0.0.7 Bandwidths check

```
In [13]: bw_gwpr

Out[13]: 178.0

In [14]: bw_mgwpr[0]

Out[14]: array([178.])
```

Bandwidths from both models is the same