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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Info
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% Version: 1.0
% Created: 5/3/2023
%
% Description
%   Script to model the hanging down of a pendulum
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
close all;
% pkg load control
% pkg load signal
set(gcf, 'Visible', 'on')

% Measured Parameters
mass_of_copter = 0.0132;
mass_of_rod = 0.0234;
length_of_rod = 0.5021;
length_of_rod_to_pivot = 0.29;
radius_of_copter = 0.01;

% Damping Coefficient
b = 0.60;

% Constants and Basic Calculated Parameters
g = 9.81;
density_of_rod = mass_of_rod/length_of_rod;
length_of_back_rod = length_of_rod - length_of_rod_to_pivot;
mass_of_rod_to_pivot = density_of_rod * length_of_rod_to_pivot;
total_mass_of_rod = density_of_rod * length_of_rod;
mass_of_back_rod = density_of_rod * length_of_back_rod;

% Moment of Inertia
inertia = 1/3*mass_of_rod_to_pivot*length_of_rod_to_pivot^2 + 1/3*mass_of_back_rod*length_of_back_rod^2 + 2/5 * mass_of_copter * radius_of_copter^2 + mass_of_copter

% State Space Model
A = [[0,1];[-(g*length_of_rod_to_pivot*(mass_of_copter+mass_of_rod_to_pivot/2))/inertia,-b]];
B = [0; length_of_rod_to_pivot/inertia];
C = [[1, 0]; [0, 1]];
D = 0;

CopterSys = ss(A, B, C, D);
dCopterSys = c2d(CopterSys,.005);

% Full Order Observer

Vd = .1*eye(2);
Vn = 1*eye(2);

% Disturbance and noise included
Bf= [B Vd 0*B];

disp("Kf")
[kf,p,e] = lqe(A,Vd,C,Vd,Vn)
kf = (lqr(A',C',Vd,Vn))
disp("Discrete Kf")
[dkf,dp,de] = dlqe(dCopterSys.a,Vd,C,Vd,Vn)

disp("Simulation")
sysC = ss(A,Bf,C,[0*Vn Vn])

disp("System without noise")
sysFull = ss(A,Bf,eye(2),zeros(2,size(Bf,2)))

disp("System with noise")
sysKF = ss(A-kf*C,[B kf],eye(2),0*[B kf])

disp("Discrete system simulation")
dsysC = c2d(sysC,.005)

disp("Discrete system without noise")
dsysFull = c2d(sysFull,.005)

disp("Discrete system with noise")
dsysKF = c2d(sysKF,.005)

dt = .01;
t = dt:dt:50;
disturbance = randn(2,size(t,2));
noise = randn(size(t));
u=0*t;
u(100:105) = 3.14;
u(1500:1505) = -1.32;
augment = [u; Vd*Vd*disturbance;noise]';

[y,t,x] = lsim(sysC,augment,t);
[xtrue,t] = lsim(sysFull,augment,t);
[x,t] = lsim(sysKF,[u;y'],'',t);

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plot(t,y(:,2))
hold on
plot(t,xtrue(:,2),'b','LineWidth',2.0)
plot(t,x(:,2),'k--','LineWidth',2.0)
set(gca, "fontsize", 12)
title("Full Order Observer")
xlabel("Time (s)");
ylabel("Thetadot (rads/s)");
legend("Noise", "Actual", "Estimate")
legend("show")

```

Kf

kf =

```

    0.0008    -0.0005
   -0.0005     0.0277

```

p =

```

    0.0008    -0.0005
   -0.0005     0.0277

```

e =

```

-0.3143 + 5.7456i
-0.3143 - 5.7456i

```

kf =

```

    0.0423    -0.0480
   -0.0480     1.3057

```

Discrete Kf

dkf =

```

    0.0167    -0.0209
   -0.0209     0.0817

```

dp =

```

    0.0175    -0.0232
   -0.0232     0.0895

```

de =

```

    0.0167    -0.0209
   -0.0209     0.0817

```

Simulation

sysC =

```

A =
      x1      x2
x1      0      1
x2 -33.09    -0.6

```

```

B =
      u1      u2      u3      u4
x1      0      0.1      0      0
x2 169      0      0.1      0

```

```

C =
      x1      x2
y1      1      0
y2      0      1

```

```

D =
      u1      u2      u3      u4
y1      0      0      1      0
y2      0      0      0      1

```

Continuous-time state-space model.

System without noise

sysFull =

```

A =
      x1      x2
x1      0      1
x2 -33.09    -0.6

```

```

B =
      u1  u2  u3  u4
x1      0  0.1  0   0
x2 169    0  0.1  0

```

```

C =
      x1  x2
y1      1  0
y2      0  1

```

```

D =
      u1  u2  u3  u4
y1      0  0  0  0
y2      0  0  0  0

```

Continuous-time state-space model.

System with noise

```

sysKF =

A =
      x1      x2
x1 -0.04228  1.048
x2 -33.05    -1.906

B =
      u1      u2      u3
x1      0  0.04228 -0.04796
x2 169 -0.04796  1.306

C =
      x1  x2
y1      1  0
y2      0  1

D =
      u1  u2  u3
y1      0  0  0
y2      0  0  0

```

Continuous-time state-space model.

Discrete system simulation

```

dsysC =

A =
      x1      x2
x1  0.9996  0.004992
x2 -0.1652  0.9966

B =
      u1      u2      u3      u4
x1 0.002111 0.0004999 1.249e-06 0
x2 0.8438 -4.132e-05 0.0004992 0

C =
      x1  x2
y1      1  0
y2      0  1

D =
      u1  u2  u3  u4
y1      0  0  1  0
y2      0  0  0  1

```

Sample time: 0.005 seconds
Discrete-time state-space model.

Discrete system without noise

```

dsysFull =

A =
      x1      x2
x1  0.9996  0.004992
x2 -0.1652  0.9966

B =
      u1      u2      u3      u4
x1 0.002111 0.0004999 1.249e-06 0
x2 0.8438 -4.132e-05 0.0004992 0

C =
      x1  x2
y1      1  0
y2      0  1

D =
      u1  u2  u3  u4

```

```

y1  0  0  0  0
y2  0  0  0  0

```

Sample time: 0.005 seconds
Discrete-time state-space model.

Discrete system with noise

```

dsysKF =

A =
      x1      x2
x1  0.9994  0.005214
x2 -0.1644  0.9901

B =
      u1      u2      u3
x1  0.002207  0.0002107 -0.0002227
x2   0.841  -0.000256  0.006517

C =
      x1  x2
y1  1  0
y2  0  1

D =
      u1  u2  u3
y1  0  0  0
y2  0  0  0

```

Sample time: 0.005 seconds
Discrete-time state-space model.

Warning: Simulation will start at a nonzero initial time.
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