MATH001PB Section 1 Problem Set A

November 14, 2025

Part I
Functions

1 Introduction to Functions

For Questions 1.1 to 1.4, prove the following statements.

Question 1.1 *The composition of two injective functions is injective.*

Question 1.2 *A strictly monotonic function is injective.*

Question 1.3 *The image of a continuous function on a closed interval is bounded.*

Question 1.4 *Every function can be written as the sum of an even and an odd function.*

For questions 1.5-1.8, either prove that it is true or provide a counterexample.

Question 1.5

(a) If $f \circ g$ is injective, then both f and g are injective.

(b) If $f \circ g$ is surjective, then both f and g are surjective.

Question 1.6 *The inverse of a strictly increasing function is strictly increasing.*

Question 1.7 *A bounded function must achieve its maximum and minimum values.*

Question 1.8 *If f and g are both unbounded, then $f + g$ is unbounded.*

Question 1.9 *Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is bijective, then f is injective and g is surjective.*