The probability of multiple events

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So far, you've been learning about calculating the probability of single events. Many situations, both in daily life and in data work, involve more than one event. As a future data professional, you'll often deal with probability for multiple events.

In this reading, you'll learn more about multiple events. You'll learn three basic rules of probability: the complement rule, the addition rule, and the multiplication rule. These rules help you better understand the probability of multiple events. First, we'll discuss two different types of events that these rules apply to: mutually exclusive and independent. Then, you'll learn how to calculate probability for both types of events.

Two types of events

The three basic rules of probability apply to different types of events. Both the complement rule and the addition rule apply to events that are mutually exclusive. The multiplication rule applies to independent events.

Mutually exclusive events

Two events are **mutually exclusive** if they cannot occur at the same time.

For example, you can't be on the Earth and on the moon at the same time, or be sitting down and standing up at the same time.

Or, take two classic examples of probability theory. If you toss a coin, you cannot get heads and tails at the same time. If you roll a die, you cannot get a 2 and a 4 at the same time.

Independent events

Two events are **independent** if the occurrence of one event does not change the probability of the other event. This means that one event does not affect the outcome of the other event.

For example, watching a movie in the morning does not affect the weather in the afternoon. Listening to music on the radio does not affect the delivery of your new refrigerator. These events are separate and independent.

Or, take two consecutive coin tosses or two consecutive die rolls. Getting heads on the first toss does not affect the outcome of the second toss. For any given coin toss, the probability of any outcome is always 1 out of 2, or 50%. Getting a 2 on the first roll does not affect the outcome of the second roll. For any given die roll, the probability of any outcome is always 1 out of 6, or 16.7%.

Three basic rules

Now that you know more about the difference between mutually exclusive and independent events, let’s review three basic rules of probability:

- Complement rule
- Addition rule
- Multiplication rule

Complement rule

The complement rule deals with mutually exclusive events. In statistics, the complement of an event is the event not occurring. For example, either it snows or it does not snow. Either your soccer team wins the championship or it does not win the championship. The complement of snow is no snow. The complement of winning is not winning.

The probability of an event occuring and the probability of it not occuring must add up to 1. Recall that a probability of 1 is the same as a 100%.

Another way to think about it is that there is a 100% chance of one event or the other event occurring. There may be a 40% chance of snow tomorrow. However, there is a 100% chance that it will either snow or not snow tomorrow.

The **complement rule** states that the probability that event A does not occur is 1 minus the probability of A. In probability notation, you can write this as:

Complement rule

P(A’) = 1 - P(A)

Note: In probability notation, an apostrophe (‘) symbolizes negation. In other words, if you want to indicate the probability of event A NOT occurring, add an apostrophe after the letter A: P(A’). You can say this as “the probability of not A.”

So, if you know there is a 40% chance of snow tomorrow, or a probability of 0.4, you can use the complement rule to calculate the probability that it does not snow tomorrow. The probability of no snow equals one minus the probability of snow.

P(no snow) = 1 - P(snow) = 1 - 0.4 = 0.6.

So, the probability of no snow tomorrow is 0.6, or 60%.

Addition rule (for mutually exclusive events)

The **addition rule** states that if events A and B are mutually exclusive, then the probability of A or B occuring is the sum of the probabilities of A and B. In probability notation, you can write this as:

P(A or B) = P(A) + P(B)

Note that there is also an addition rule for mutually inclusive events. In this course, we focus on the rule for mutually exclusive events.

Let's explore our example of rolling a die.

Die roll (rolling either a 2 or a 4)

Say you want to find the probability of rolling either a 2 or a 4 on a single roll. These two events are mutually exclusive. You can roll a 2 or a 4, but not both at the same time.

The addition rule says that to find the probability of either event occurring, you sum up their probabilities. The odds of rolling any single number on a die are 1 out of 6, or 16.7%.

P(rolling 2 or rolling 4) = P(rolling 2) + P(rolling 4) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

So, the probability of rolling either a 2 or a 4 is one out of three, or 33%.

Multiplication rule (for independent events)

The **multiplication rule** states that if events A and B are independent, then the probability of both A and B occuring is the probability of A multiplied by the probability of B. In probability notation, you can write this as:

P(A and B) = P(A)×P(B)

Note that there is also a multiplication rule for dependent events. In this course, we focus on the rule for independent events.

Let's continue with our example of rolling a die.

Die roll (rolling a 1 and then rolling a 6)

Now imagine two consecutive die rolls. Say you want to know the probability of rolling a 1 and then rolling a 6. These are independent events as the first roll does not affect the outcome of the second roll.

The probability of rolling a 1 and then a 6 is the probability of rolling a 1 multiplied by the probability of rolling a 6. The probability of each event is $\frac{1}{6}$, or 16.7%. You can write this as:

P(rolling 1 on the first roll and rolling 6 on the second roll) = P(rolling 1 on the first roll)×P(rolling 6 on the second roll) = $\frac{1}{6} \times \frac{1}{6}$ = 1/36

So, the probability of rolling a 1 and then a 6 is one out of thirty-six, or about 2.8%.

Key takeaways

The basic rules of probability help you describe events that are mutually exclusive or independent. Understanding basic rules of probability is an essential foundation for more complex analyses you will perform as a future data professional.

Resources for more information

To learn more about the basic rules of probability, refer to the following resources:

> This online textbook from the University of Florida provides a detailed overview of the basic rules of probability from a more technical perspective.