

Lab 11 AVL Trees Implementation

Learning Outcomes:

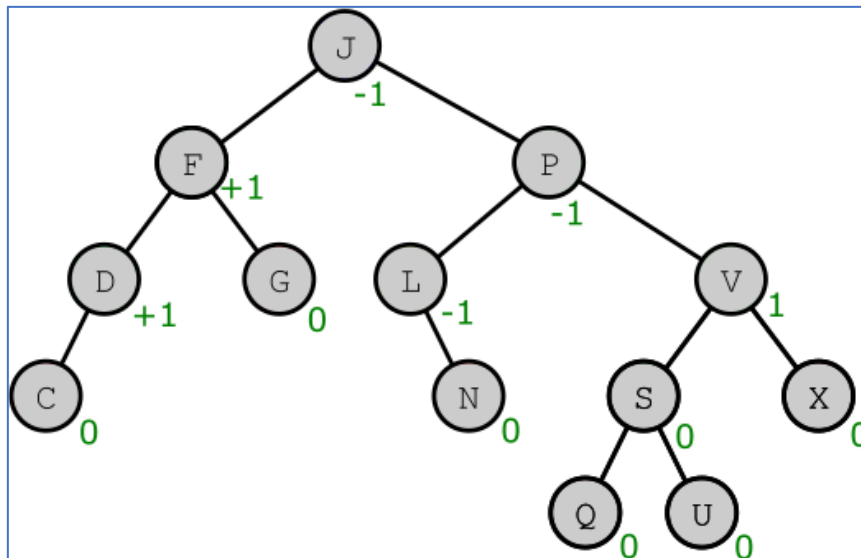
After successfully completing this lab the students will be able to:

1. Understand the properties of AVL Trees and their balancing features.
2. Develop C programs for implementing AVL Trees and their balancing features.

Pre-Lab Reading Task:

AVL Trees:

In an AVL tree, the heights of the two child subtrees of any node differ by at most one; if at any time they differ by more than one, re-balancing is done to restore this property. Lookup, insertion, and deletion all take $O(\log n)$ time in both the average and worst cases, where n is the number of nodes in the tree prior to the operation. Insertions and deletions may require the tree to be rebalanced by one or more tree rotations.



Why AVL Trees?

Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take $O(h)$ time where h is the height of the BST. The cost of these operations may become $O(n)$ for a skewed Binary tree. If we make sure that height of the tree remains $O(\log n)$ after every insertion and deletion, then we can guarantee an upper bound of $O(\log n)$ for all these operations. The height of an AVL tree is always $O(\log n)$ where n is the number of nodes in the tree

Balance Factor:

The balance factor of any node of an AVL tree is in the integer range $[-1, +1]$. If after any modification in the tree, the balance factor becomes less than -1 or greater than $+1$, the subtree rooted at this node is unbalanced, and a rotation is needed.

$$\text{Balance Factor} = \text{height}(\text{left subtree}) - \text{height}(\text{right subtree})$$

Insertion

To make sure that the given tree remains AVL after every insertion, we must augment the standard BST insert operation to perform some re-balancing. Following are two basic operations that can be performed to re-balance a BST without violating the BST property ($\text{keys}(\text{left}) < \text{key}(\text{root}) < \text{keys}(\text{right})$).

1. Left Rotation
2. Right Rotation

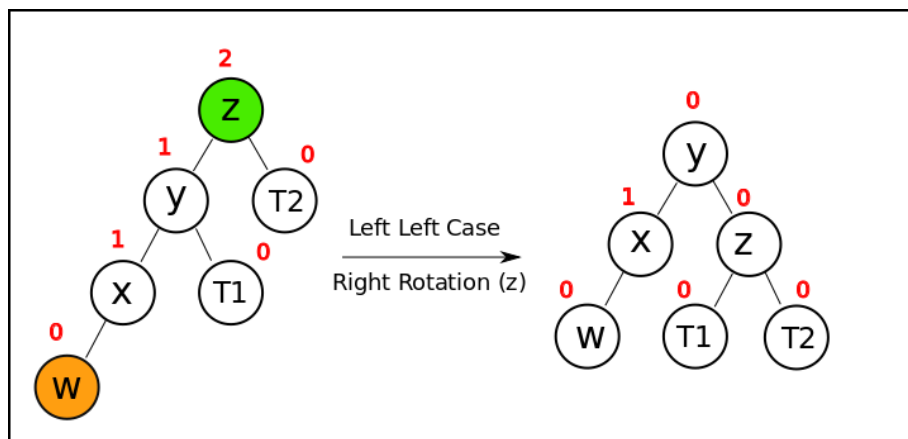
Steps to follow for insertion

Let the newly inserted node be w

1. Perform standard BST insert for w .
2. Starting from w , travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the child of z that comes on the path from w to z and x be the grandchild of z that comes on the path from w to z .
3. Re-balance the tree by performing appropriate rotations on the subtree rooted with z . There can be 4 possible cases that needs to be handled as x , y and z can be arranged in 4 ways. Following are the possible 4 arrangements:
 - y is left child of z and x is left child of y (Left Left Case)
 - y is left child of z and x is right child of y (Left Right Case)
 - y is right child of z and x is right child of y (Right Right Case)
 - y is right child of z and x is left child of y (Right Left Case)

Following are the operations to be performed in above mentioned 4 cases. In all of the cases, we only need to re-balance the subtree rooted with z and the complete tree becomes balanced as the height of subtree (After appropriate rotations) rooted with z becomes same as it was before insertion.

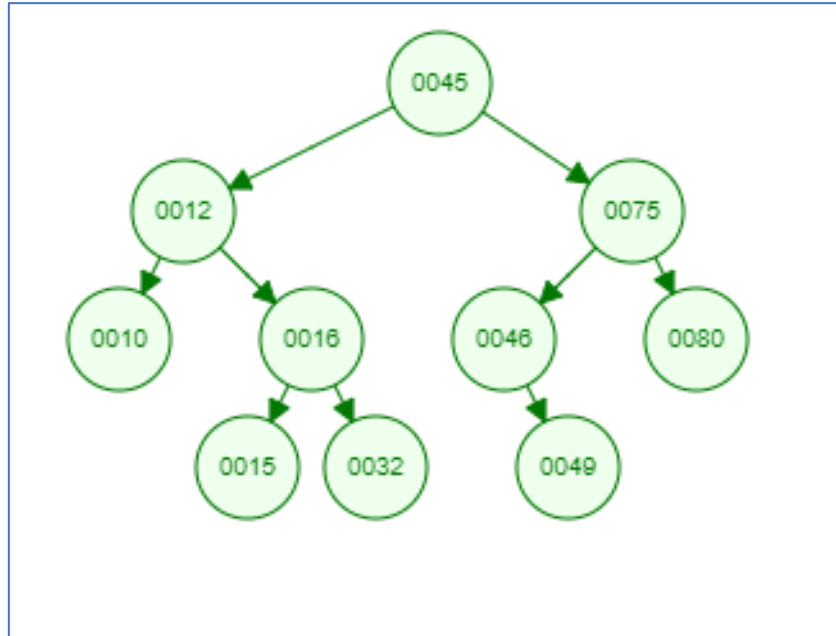
Left Left Case: (We will need to perform a right rotation)



For more information read **Chapter 10.4** from the book: *“Data Structures using C”* by Reema Thareja.

In-Lab Tasks:

You are provided with skeleton code that builds a Binary Search Tree by adding 10 nodes to it. Functions for node insertion and printing the tree (in-order traversal only) are already implemented. Your task is to **modify the insert** function to incorporate AVL insertion. You will find Programming Example on Page 324 of the above-mentioned book useful.



1. Implement the functions **rotateLeft()** and **rotateRight()**.
2. Implement the 4 cases of balancing the tree.

Post-Lab Tasks:

Complete the following functions for the BST:

1. Add the ***delete node*** functionality.