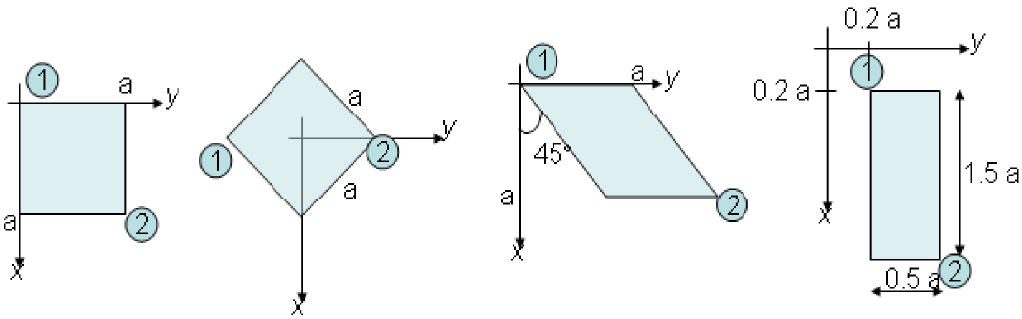
PUCIT

Computer Vision (MSCS/BIT) fall 2012

Home Work # 1

Start Date	24 Oct 2012 Wednesday
End Date	**8 Nov 2012 Thursday 4:30 PM**
Marks (Q-1)	20
Marks (Q-2)	5
Marks (Q-3)	5
Marks (Q-4)	30
Marks (Q-5)	30
Marks (Q-6)	30
Marks (Q-7)	15
Marks (Q-8)	15
Marks (Q-9)	30
Marks (Q-10)	30
Marks (Q-11)	50
Marks (Q-12)	30
Note:	***MS problems can be attempted by BS students for Bonus marks***

1. Consider the shapes shown in the figure. Give affine parameters to transform the first shape into the other three.



2. What is the effect of applying the following transformation matrix? Describe its effect in words.

$$\begin{bmatrix} -0.97166 & -0.23638 & 35 \\ -0.23638 & 0.97166 & 24 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Consider the 2D affine transformation matrix given below in homogeneous form:

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 100 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Give two different explanations of this matrix in terms of its constituent transformations of the following form:

i. This matrix denotes a translation of _______ followed by a counter clockwise rotation of 45º.

ii. This matrix denotes a counter-clockwise rotation of 45º degrees followed by a translation of ______

4. *Equivalence of least squares and pseudo-inverse solutions*:

Show that minimizing the following error term

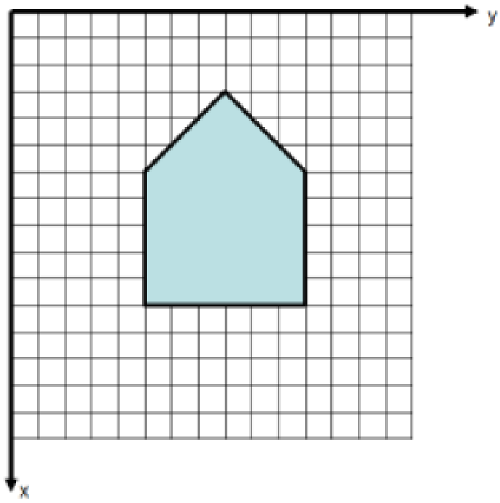
$$E(\mathbf{a}) = \sum_{j=1}^{n}\left((a_1x_j + a_2y_j + a_3 - x_j')^2 + (a_4x_j + a_5y_j + a_6 - y_j')^2\right)$$

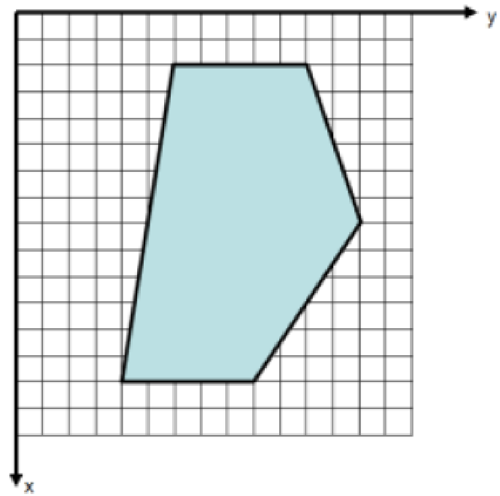
with respect to **a** yields the following system:

$$\begin{bmatrix} \sum_j x_j^2 & \sum_j x_jy_j & \sum_j x_j & 0 & 0 & 0 \\ \sum_j x_jy_j & \sum_j y_j^2 & \sum_j y_j & 0 & 0 & 0 \\ \sum_j x_j & \sum_j y_j & \sum_j 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_j x_j^2 & \sum_j x_jy_j & \sum_j x_j \\ 0 & 0 & 0 & \sum_j x_jy_j & \sum_j y_j^2 & \sum_j y_j \\ 0 & 0 & 0 & \sum_j x_j & \sum_j y_j & \sum_j 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \sum_j x_jx_j' \\ \sum_j y_jx_j' \\ \sum_j x_j' \\ \sum_j x_jy_j' \\ \sum_j y_jy_j' \\ \sum_j y_j' \end{bmatrix}$$

Futhermore, show that the pseudo-inverse solution also yields the same system.

5.





The shape shown in the left figure is being transformed into the shape shown in the right figure.

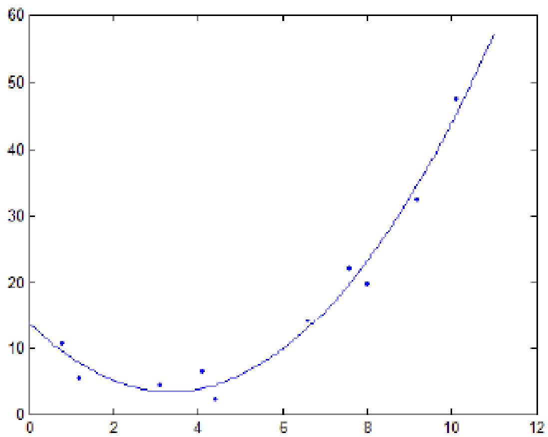
a. What type of displacement model exists between the two transformations? [*Provide a clear argument for your answer. Answer alone is worth zero points*]

b. Provide a sequence of simple 2D transformation matrices which, when concatenated, will transform the shape shown in the left figure to the one shown in the right figure.

6. Consider the following set of 10 data points:

x	0.8	1.2	3.1	4.1	4.4	6.6	7.6	8	9.2	10.1
y	10.6	5.4	4.3	6.4	2.2	14.1	22.1	19.8	32.4	47.5

Fit a quadratic polynomial to these points as shown in the figure below, using least squared error approach. Use both the derivative method and the pseudo inverse method. Use your result to compute the quadratically interpolated value of the data at x = 5. Show all steps of the solution.



7. What are the properties of a 2-D rotation matrix? Rotation about an arbitrary point in 2-D is achieved by translation followed by rotation followed by inverse translation. Does the over all transformation matrix is a rotation matrix?

8. What are the properties of a reflection matrix in 2-D? Can a 2-D reflection be achieved by applying several 2-D rotations? Can a 2-D rotation be achieved by applying several 2-D reflections?

9. Prove that parallel lines remain parallel under 2-D affine transformation

10. [MS students Only] List and prove all the invariants of 2-D affine transformation

11. [MS students Only] List and prove all the invariants of 2-D projective transformation

12. [MS students Only] In 2-D affine warping, why the extents of the resultant image can always be estimated by only knowing the extents of the source image? i.e why do extents map to extents? Do the extents map to extents in the case of 2-D projective warping?