HW 6: Dynamic Programming (No Deadline, Just Practice)

Q – 1

A *contiguous subsequence* of a list S is a subsequence made up of consecutive elements of S. For instance, if S is

$$5, 15, -30, 10, -5, 40, 10,$$

then $15, -30, 10$ is a contiguous subsequence but $5, 15, 40$ is not. Give a linear-time algorithm for the following task:

> *Input:* A list of numbers, $a_1, a_2, \ldots, a_n$.
> *Output:* The contiguous subsequence of maximum sum (a subsequence of length zero has sum zero).

For the preceding example, the answer would be $10, -5, 40, 10$, with a sum of 55.
(*Hint:* For each $j \in \{1, 2, \ldots, n\}$, consider contiguous subsequences ending exactly at position j.)

Q – 2

You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.

You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty—that is, the sum, over all travel days, of the daily penalties.

Q – 3

Yuckdonald's is considering opening a series of restaurants along Quaint Valley Highway (QVH). The n possible locations are along a straight line, and the distances of these locations from the start of QVH are, in miles and in increasing order, $m_1, m_2, \ldots, m_n$. The constraints are as follows:

- At each location, Yuckdonald's may open at most one restaurant. The expected profit from opening a restaurant at location i is p_i, where $p_i > 0$ and $i = 1, 2, \ldots, n$.
- Any two restaurants should be at least k miles apart, where k is a positive integer.

Give an efficient algorithm to compute the maximum expected total profit subject to the given constraints.

Q – 4

Given two strings $x = x_1x_2\cdots x_n$ and $y = y_1y_2\cdots y_m$, we wish to find the length of their *longest common substring*, that is, the largest k for which there are indices i and j with $x_ix_{i+1}\cdots x_{i+k-1} = y_jy_{j+1}\cdots y_{j+k-1}$. Show how to do this in time $O(mn)$.

Q – 5

Given two strings $x = x_1x_2\cdots x_n$ and $y = y_1y_2\cdots y_m$, we wish to find the length of their *longest common subsequence*, that is, the largest k for which there are indices $i_1 < i_2 < \cdots < i_k$ and $j_1 < j_2 < \cdots < j_k$ with $x_{i_1}x_{i_2}\cdots x_{i_k} = y_{j_1}y_{j_2}\cdots y_{j_k}$. Show how to do this in time $O(mn)$.