Hough Transform for Circles

- Equation…
- Centered at (x_0, y_0) with radius r

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

- Three unknowns… x_0 y_0 r
- Three dimensional parameter space
- Conceptually… ?

How to simplify this algorithm

- Use of gradient direction, θ

$$x_0 = x - r\cos\theta$$
$$y_0 = y - r\sin\theta$$

- Algorithm?...

Hough transform

- Can be applied to any parametric representation $f(\mathbf{x}, \mathbf{a})=0$
- Initialize accumulator array, A to zeros…
- A is |a| dimensional
- For each pixel $\mathbf{x}$, and each $\mathbf{a}$ such that $f(\mathbf{x},\mathbf{a})=0$, $A[\mathbf{a}] = A[\mathbf{a}]+1$
- Local maxima of A corresponds to curves f in image.

Finding more than one curve

- Parameter space will have multiple maxima
- Threshold
- Or use better methods to find maximum points

Hough Transform

- Given parametric representation of a curve
  - LINE: $p = x \cos\theta + y \sin\theta$
  - CIRCLE: $x_0 = x - r \cos\theta$
    $y_0 = y - r \sin\theta$
  - ELLIPSE: $x_0 = x - a \cos\theta$
    $y_0 = y - b \sin\theta$
  - GENERAL: $f(\mathbf{x}, \mathbf{a}) = 0$

Hough Transform

- Initialize **A** (accumulator array) to all zeros
- **A** is |**a**| dimensional
- For each pixel **x**, and each **a** such that $f(\mathbf{x},\mathbf{a})=0$, $A[\mathbf{a}] = A[\mathbf{a}]+1$
- Local maxima of A corresponds to curves *f* in image.

Generalized Hough Transform

- To find arbitrary shapes in images
- Shapes which do not have an *easy* parametric representation

Centroid and Area

- The average location of all pixels of all pixels in a region *R*

$$\bar{r} = \frac{1}{A} \sum_{(r,c)\in R} r$$

$$\bar{c} = \frac{1}{A} \sum_{(r,c)\in R} c$$

- where A is the area of region *R*

$$A = \sum_{(r,c)\in R} 1$$

Example

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	0	0	0	0
3	0	0	0	1	0	0	0	0	1	0	0	0
4	0	0	1	0	0	0	0	0	0	1	0	0
5	0	0	1	0	0	0	0	0	0	1	0	0
6	0	0	0	1	0	0	0	0	1	0	0	0
7	0	0	0	0	1	1	1	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0

$A = 16$

$\mathbf{r} = 1/16 * (2+2+2+2+3+4+5+6$
$+7+7+7+7+6+5+4+3)$

$= 4.5$

$\mathbf{c} = 1/16 * (5+6+7+8+9+10+10+$
$9+8+7+6+5+4+3+3+4)$

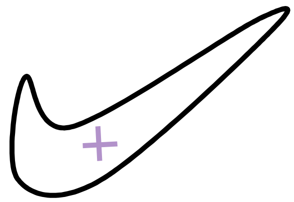
$= 6.5$

Generalized Hough Transform

- Training
  - A representation of shape of interest is built in the form of an R-Table
- Detection
  - Using R-Table, a given shape is matched to the shape of interest

GHT - Training

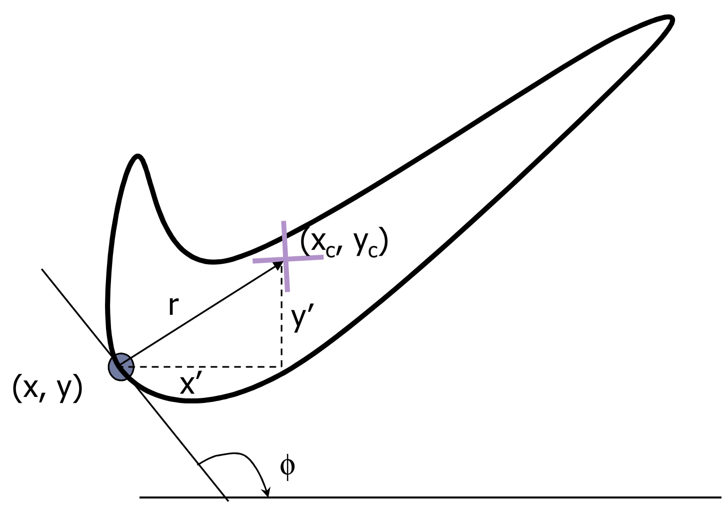
- Given the shape of interest



- Find Centroid (x_c, y_c) of shape

GHT - Training

- Find $r = (x', y')$ for each edge point
- $x_c = x + x'$
- $y_c = y + y'$
- ϕ is the angle tangent at (x,y) makes with x-axis

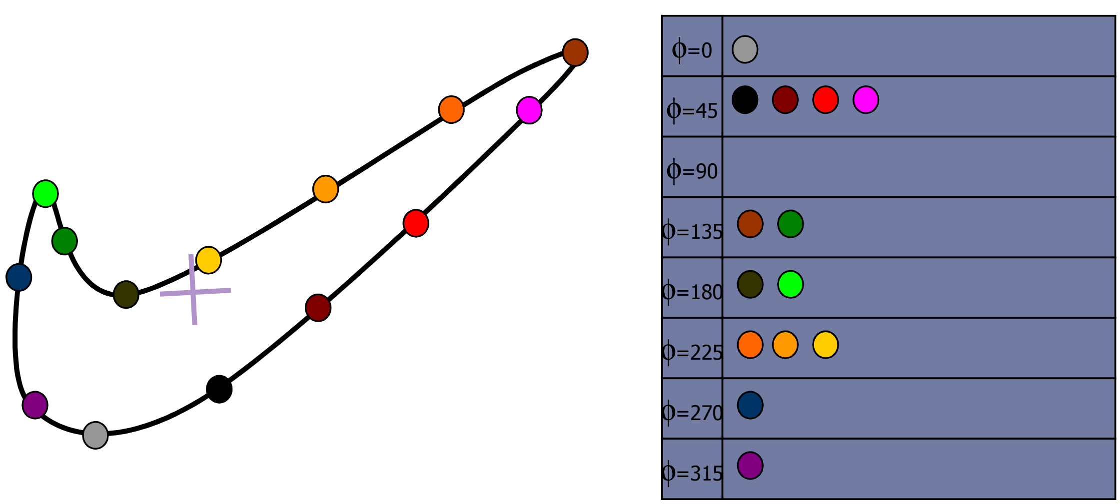


GHT - Training

- R-Table is indexed by ϕ

ϕ_1	$r_1^1, r_2^1, r_3^1, \ \ldots\ , r_{m_1}^1$
ϕ_2	$r_1^2, r_2^2, r_3^2, \ \ldots\ , r_{m_2}^2$
ϕ_3	$r_1^3, r_2^3, r_3^3, \ \ldots\ , r_{m_3}^3$
•	•
• •	• •
ϕ_n	$r_1^n, r_2^n, r_3^n, \ \ldots\ , r_{m_n}^n$

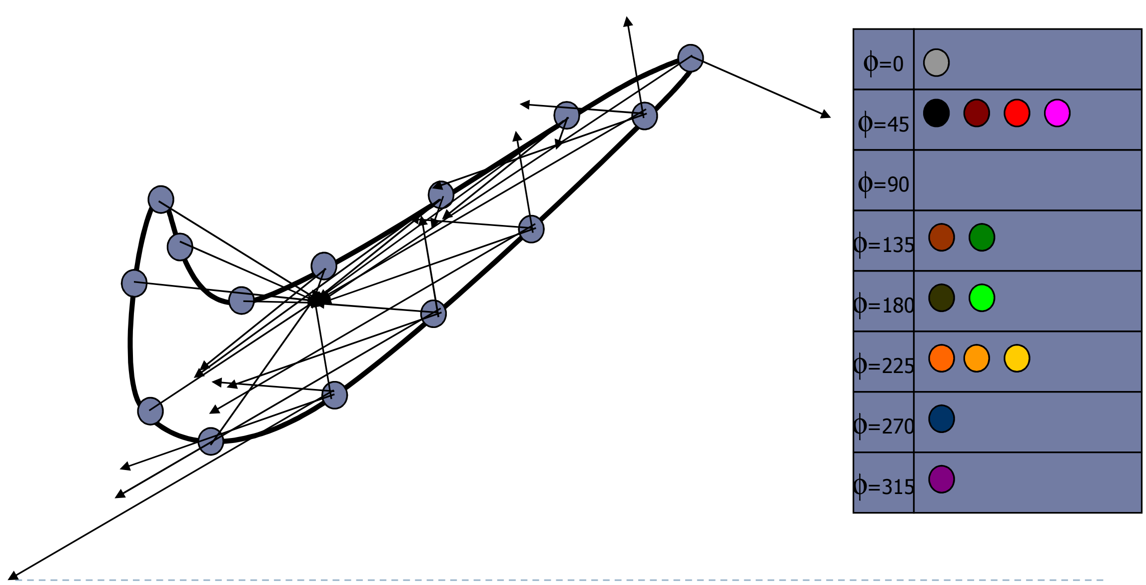
Example - Training



Detection

- Go to each (x,y) in image
- Find ϕ
- For corresponding entry in R Table
- Find all possible locations of centriods
  - $x_c = x + x'$
  - $y_c = y + y'$
- Increment centroid accumulator by 1

Detection



GHT - Algorithm

1. Quantize parameter space
   $P\,[x_{cmin}, \ldots , x_{cmax}, y_{cmin}, \ldots , y_{cmax}]$
2. For each edge point (x,y)
   Compute ϕ from gradient direction
   For each table entry in row ϕ
   $x_c = x + x'$
   $y_c = y + y'$
   $P\,[x_c, y_c] = P\,[x_c, y_c] + 1;$
3. Find local maxima in P

GHT - Questions

- Uniqueness of R-Table?
- Invariance to translation?
- Invariance to rotation?
- Invariance to scaling?

Rotation and Scaling Invariance

- Rotation invariance…

  $x'' = x' \cos\theta + y' \sin\theta$

  $y'' = -x' \sin\theta + y' \cos\theta$

- Rotation + Scaling invariance

  $x'' = s_x (x' \cos\theta + y' \sin\theta)$

  $y'' = s_y (-x' \sin\theta + y' \cos\theta)$

- Substitute in

  $x_c = x + x'$

  $y_c = y + y'$

- To get

  $x_c = x + s_x (x' \cos\theta + y' \sin\theta)$

  $y_c = y + s_y (-x' \sin\theta + y' \cos\theta)$

- Substitute these values of (x_c, y_c) in GHT algorithm