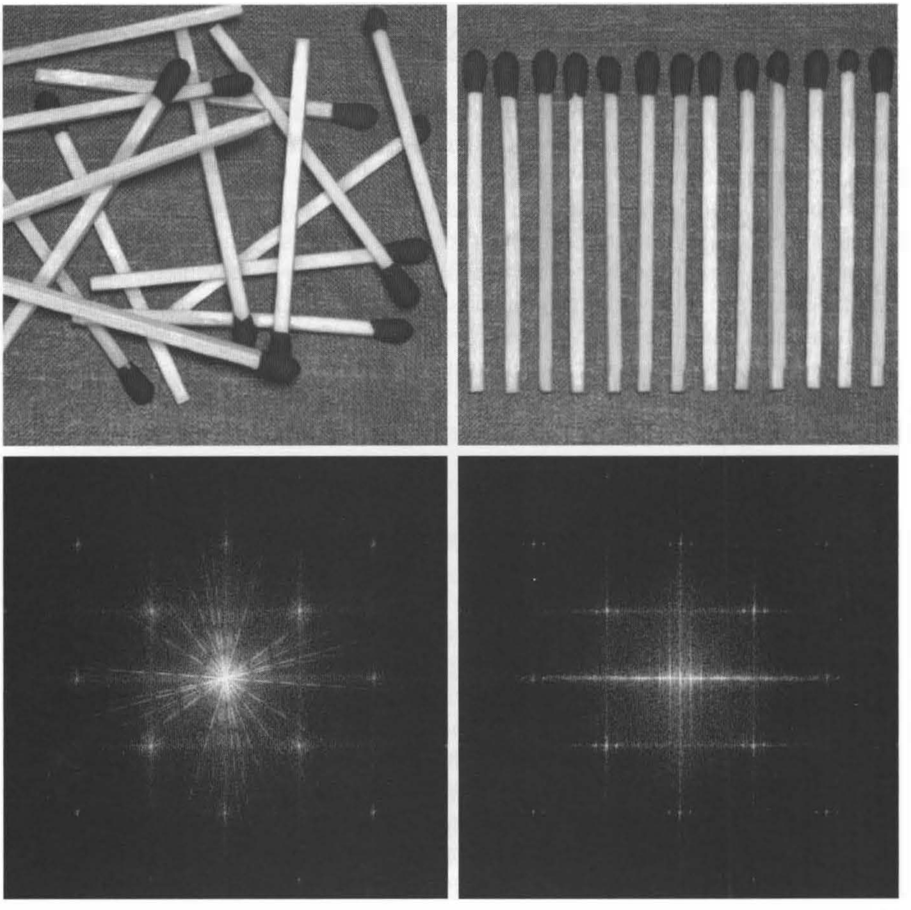


a b
c d

FIGURE 12.26 (a) and (b) Images of unordered and ordered objects. (c) and (d) Corresponding spectra.

12.4.3 Moment Invariants

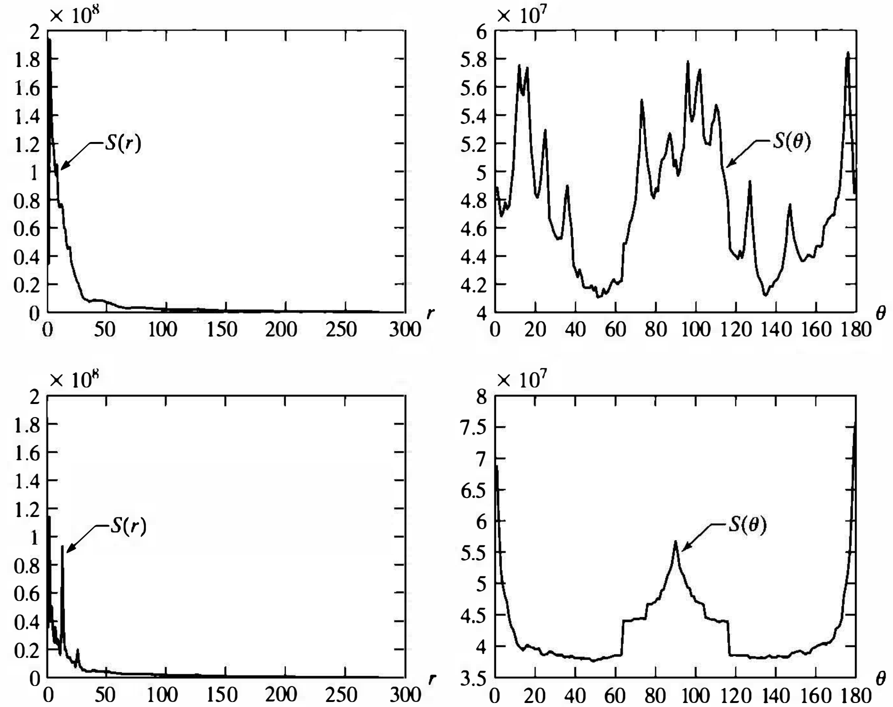
The 2-D *moment* of order $(p+q)$ of a digital image $f(x, y)$ of size $M \times N$ is defined as

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q f(x, y)$$

where $p = 0, 1, 2, \ldots$ and $q = 0, 1, 2, \ldots$ are integers. The corresponding *central moment* of order $(p+q)$ is defined as

$$\mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \bar{x})^p (y - \bar{y})^q f(x, y)$$

for $p = 0, 1, 2, \ldots$ and $q = 0, 1, 2, \ldots$, where



a b
c d

FIGURE 12.27 (a) and (b) Plots of $S(r)$ and $S(\theta)$ for the random image in Fig. 12.26(a). (c) and (d) Plots of $S(r)$ and $S(\theta)$ for the ordered image.

$$\bar{x} = \frac{m_{10}}{m_{00}} \quad \text{and} \quad \bar{y} = \frac{m_{01}}{m_{00}}$$

The *normalized central moment* of order $(p + q)$ is defined as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}$$

where

$$\gamma = \frac{p + q}{2} + 1$$

for $p + q = 2, 3, \ldots$.

A set of seven 2-D *moment invariants* that are insensitive to translation, scale change, mirroring (to within a minus sign), and rotation can be derived from these equations.[†] They are listed in Table 12.6.

[†]Derivation of these results involves concepts that are beyond the scope of this discussion. The book by Bell [1965] and a paper by Hu [1962] contain detailed discussions of these concepts. For generating moment invariants of order higher than seven, see Flusser [2000]. Moment invariants can be generalized to n dimensions (see Mamistvalov [1998]).

TABLE 12.6
A set of seven moment invariants.

Moment order	Expression
1	$\phi_1 = \eta_{20} + \eta_{02}$
2	$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$
3	$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$
4	$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$
5	$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$
6	$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$
7	$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{30})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$

Custom M-function `invmoments` implements these seven equations. The syntax is as follows (see Appendix C for the code):

`invmoments`

```
phi = invmoments(f)
```

where `f` is the input image and `phi` is a seven-element row vector containing the moment invariants just defined.

EXAMPLE 12.13: Moment invariants.

■ The image in Fig. 12.28(a) was obtained from an original of size 400 × 400 pixels using the following commands:

```
>> f = imread('Fig1228(a).tif');
>> fp = padarray(f, [84 84], 'both'); % Padded for display.
```

This image was created using zero padding to make all displayed images consistent in size with the image occupying the largest area (568 × 568) which, as explained below, is the image rotated by 45°. The padding is for display purposes only, and was not used in moment computations. A translated image was created using the following commands: