# Experiment 1
# Basic Tools of Digital Signal Processing

**Objective:**

Objective of this experiment is to introduce students with some basic mathematical operations frequently used in processing of discrete time signal. This experiment will cover plot of periodic and non periodic sequences, determination of period of a sinusoidal sequence, convolution of sequences, moving average filter and *z*-transform and its inverse operation.

1. Plot the sequence, {5, -2, 0, 3, 4, 6}.

```
y=[5 -2 0 3 4 6];
stem(y, 'k')
xlabel('n')
ylabel('x(n)')
grid on
```

The result of above code is shown in fig. 1.1.

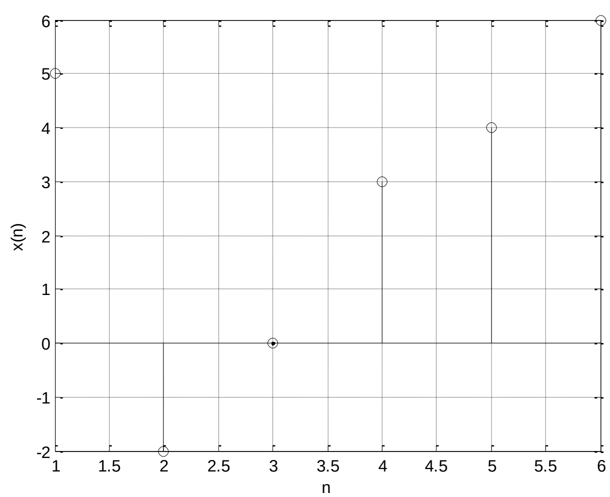
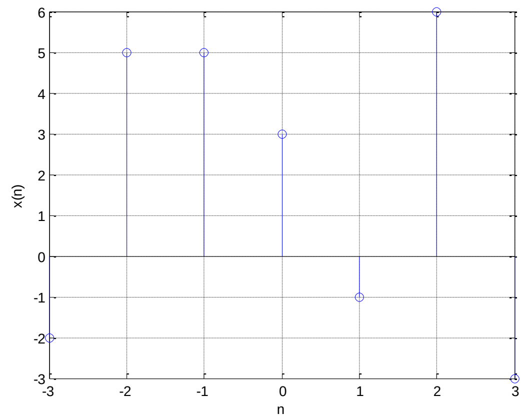



**Fig. 1.1** The sequence of section-1

Plot the sequence $x = \left\{-2, 5, 5, \underset{\uparrow}{3}, -1, 6, -3\right\}$

```
n=-3:3;
x=[-2 5 5 3 -1 6 -3];
```

```
stem(n,x)
grid on
xlabel('n')
ylabel('x(n)')
```




2. Plot of the exponential sequence, $y(n) = ar^n$ for $a = 2$, $r = 0.8$ and 1.2.

```
n=0:1:20;
subplot(2,1,1)
a=2;
r=0.8;
y=a*r.^n;
stem(n,y, 'k')
xlabel('n')
ylabel('y(n)')
title('y=ar^n; where r<1')
grid on
subplot(2,1,2)
a=2;
r=1.2;
y=a*r.^n;
stem(n,y, 'k')
xlabel('n')
ylabel('y(n)')
title('y=ar^n; where r>1')
grid on
```

The result of above code is shown in fig. 1.2.

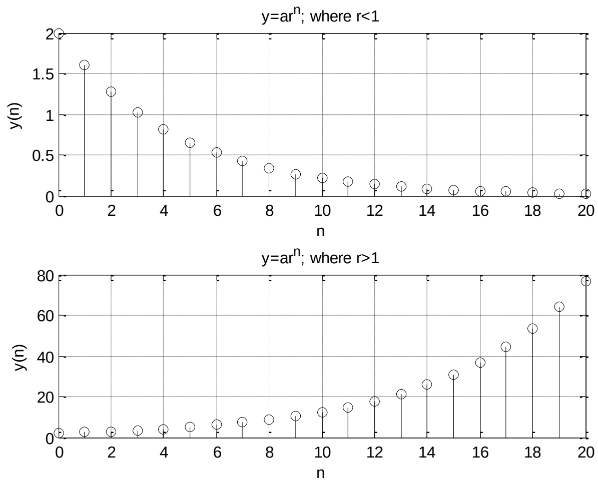



**Fig. 1.2** The sequences of section-2

3. Plot the periodic sequence, $x[n]$ = {….5,4,3,2,1, 5,4,3,2,1, 5,4,3,2,1, 5,4,3,2,1……..} for $n$ = -10 to 9.

```
% plot of periodic sequence
n=[-10:9];
x=[5,4,3,2,1];
p=4;
xy=x'*ones(1,p);
% xy indicates, p=4 column of vectors of  [ 5, 4 , 3,  2 , 1]
xy =(xy(:))'; %a long column vector will be converted
stem(n,xy, 'k')
xlabel('n')
ylabel('x(n)')
grid on
```

The result of above code is shown in fig. 1.3.

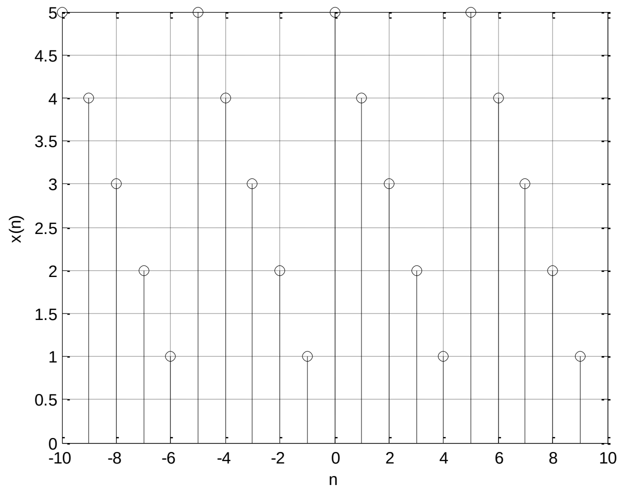



**Fig. 1.3** The sequence of section-3

4. Plot the sequence, $x(n)=\left\{0,\ -1,\ 2,\ 5,\ \underset{\uparrow}{7}\ -4,\ 3,\ -5,\ 6,\ -2\right\}$ , $y(n)$ = $x(n\text{-}2)$, and $z(n)$ = $x(n)$+$y(n)$.

```
n=-4:5;
x=[0 -1 2  5 7 -4  3 -5 6 -2];
stem(n,x,'r*')
hold on
k=2;
m=n+k;
y=x;
stem(m,y,'bd')
hold on
r=min(min(n),min(m)):max(max(n),max(m));%duration of z
z1=zeros(1,length(r));z2=z1;%initialization
z1(find((r>=min(n))&(r<=max(n))==1))=x;%x with duration of z
z2(find((r>=min(m))&(r<=max(m))==1))=y;%y with duration of z

z=z1+z2;
stem(r,z,'k>')
grid on
xlabel('n')
```

```
ylabel('x(n),y(n),z(n)')
legend('x(n)','y(n)=x(n-2)', 'z(n)=x(n)+y(n)')
```

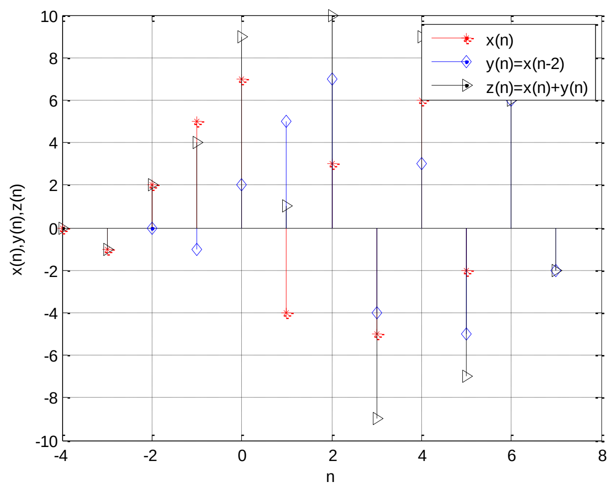



Fig.1.4 Plot of sequence *x(n), x(n-2)* and *x(n)+x(n-2)*

5. For discrete time sequence, a sinusoidal wave is periodic if its frequency is a rational number like, $f = M / K$. Period of the wave is the denominator i.e. $K$.

```
%periodic sinusoidal wave
n=0:1:50;
f=1/20; %is a rational number, therefore period is 20
y=sin(2*pi*f*n);
subplot(2,1,1)
stem(n,y, 'k')
xlabel('n')
ylabel('y(n)')
title('Periodic sine wave N=20')
grid on
f=sqrt(2); % is an irrational number, non-periodic sinusoidal wave
y=sin(2*pi*f*n);
subplot(2,1,2)
stem(n,y, 'k')
xlabel('n')
ylabel('y(n)')
```

```
title('Non-periodic sine wave')
grid on
```

The result of above code is shown in fig. 1.5.

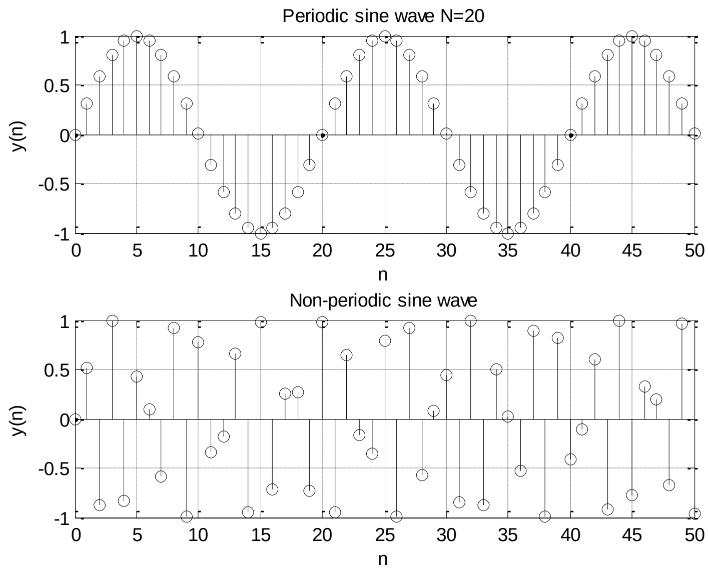



**Fig. 1.5** The periodic and non-periodic sinusoidal sequences of step-4

6. The sampling rate of $y(2n)$ is half the rate of $y(n)$ therefore $y(2n)$ is called the under sampled version of $y(n)$. Let us verify the phenomenon.

```
a=0.2;
r=0.8;
N=10;
for n=1:N,
s(n)=n;
y(n)=a*r.^n;
end

M=N/2;
for m=1:M,
m=2*m;
p(m)=m;
z(m)= y(m);
end
stem(s,y,'ks')%Original sequence y(n)
```

```
hold on
stem(p,z,'k*')%under sampled sequence y(2n)
xlabel('n')
ylabel('y(n) and y(2n)')
grid on
legend('Original sequence', 'Under sampled')
```

The result of above code is shown in fig. 1.6.

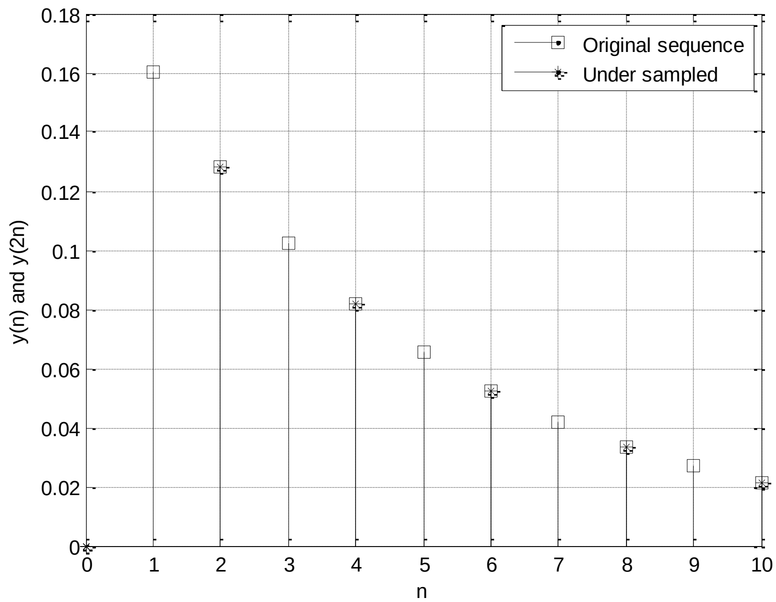



**Fig. 1.6** plot of y(n) and y(2n)

7. Determine convolution of two sequences $A$ = {6, 2, 0, 5, 6} and $B$ = {-4, -2, 1, 0, 7} and find the length of resultant sequence.

```
A=[6 2 0 5 6];
B=[-4 -2 1 0 7];
Y=conv(A,B);
stem(A,'bs')
hold on
stem(B, 'r>')
hold on
stem(Y,'k*')
legend('Sequence A', ' Sequence B', 'Convolution of sequences A*B')
xlabel('n')
ylabel('Amplitude')
length(Y)
```

**grid on**

Result of above code is shown in fig. 1.7

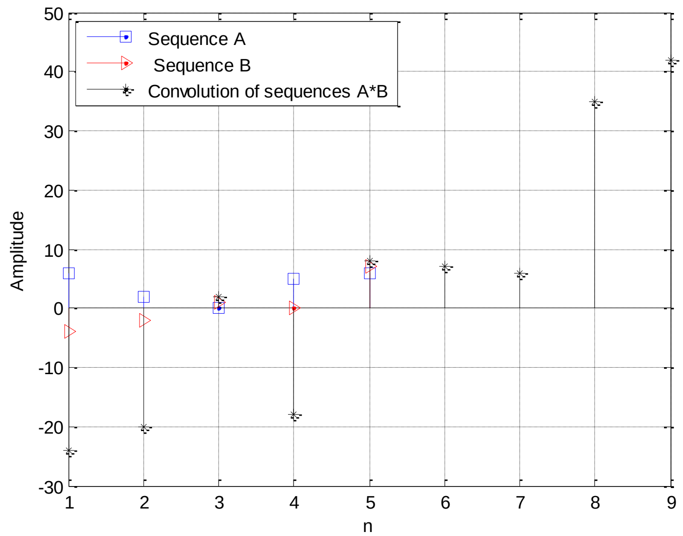



**Fig. 1.7** Sequences A and B and their convolution

8. Generate 100 random numbers between 0 and 1. Add them with an exponential sequence as a random noise. Plot both noisy and noiseless signals.

```
a=0.5;
r=0.96;
N=100; %number of samples
for n=1:N,
e(n)=n;
y(n)=a*r.^n;
yn(n)=y(n)+0.04*rand();%noisy sequence
end
stem(e,y,'k*') %values of y(n)
hold on
stem(e,yn,'kd') %plot of noisy sequence
xlabel('n')
ylabel('y(n)')
legend('Original sequence', 'Noisy Sequence')
grid on
```

The result of above code is shown in fig. 1.8.

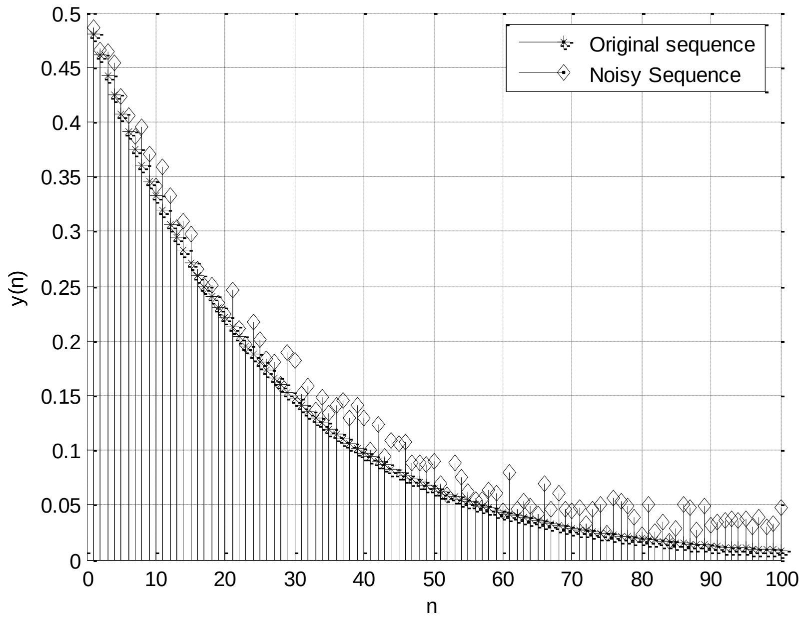



**Fig. 1.8** The result of section-7.

9. Generate a unit step sequence for $n$ = [-6, 6].

```
n =[-6 -5 -4 -3 -2 -1 0 1 2 3  4 5 6];
x=[0   0  0  0  0  0 1 1 1  1  1  1  1];
stem(n,x, 'k')
xlabel('n')
ylabel('x(n)')
title('Unit step sequence')
grid on
```

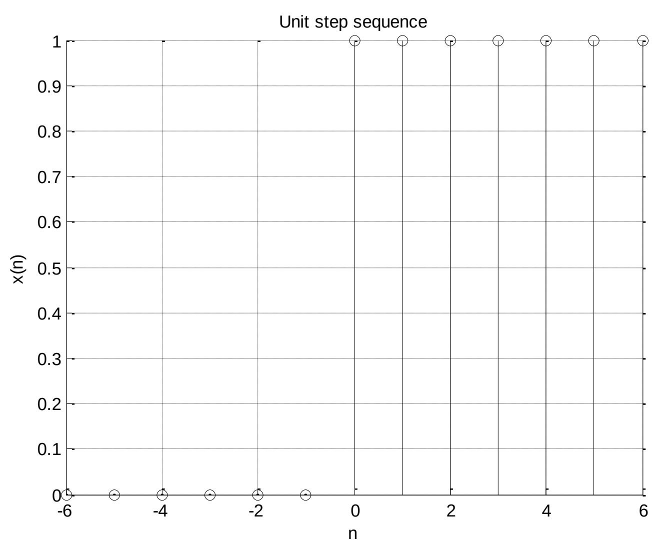



**Fig. 1.9** Unit step sequence.

For impulse sequence above code will be modified like,

```
n =[-6 -5 -4 -3 -2 -1 0 1 2 3  4 5 6];
x=[0   0  0  0  0  0 1 0 0  0  0  0  0];
stem(n,x, 'k')
xlabel('n')
ylabel('x(n)')
title('Impulse sequence')
grid on
```

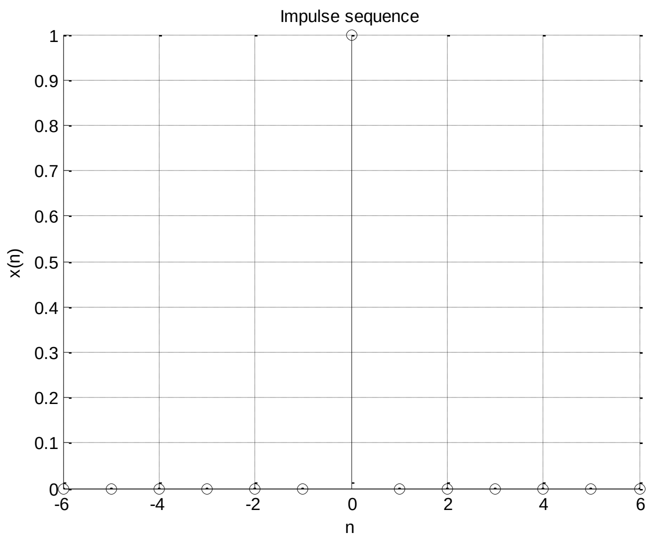



**Fig. 1.10** Impulse sequence

10. Computes the two-dimensional convolution of matrices, $A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -11 \\ 6 & -5 & 2 \end{bmatrix}$ and

$$B = \begin{bmatrix} -3 & 2 & -8 \\ 2 & -7 & 0 \\ -1 & 4 & -1 \end{bmatrix}.$$

```
A = [1 -2  0; 3  4 -11; 6  -5  2];
B = [-3  2  -8; 2  -7 0; -1  4  -1];
H = conv2(A, B);
mesh(H)
```

Result of above code is shown in fig. 1.11.

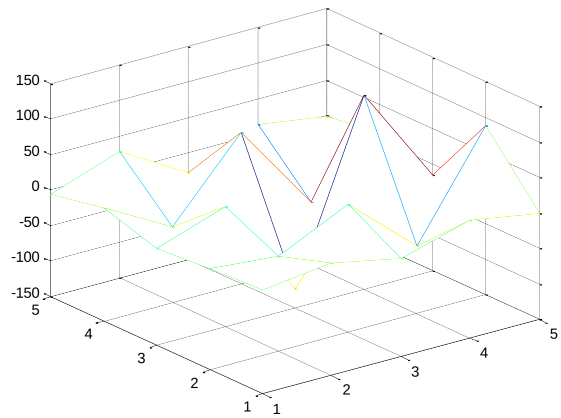



**Fig. 1.11** The result of section-8

11. Generate 50 random numbers in the range [1,-1] and add it with the sequence *x(n)* $=2n(0.9)^n$. Pass the noisy signal through a moving average filter and observe the signal before and after filtering.

Basic Theory:
Moving average system is defined as,

$$y(n)=\frac{1}{M_1+M_2+1}\sum_{n=-M_1}^{M_2}x(n-k)$$

For example if $M_1$=1 and $M_2$=2 then,

$$y(n)=\frac{1}{1+2+1}\sum_{n=-1}^{2}x(n-k)$$

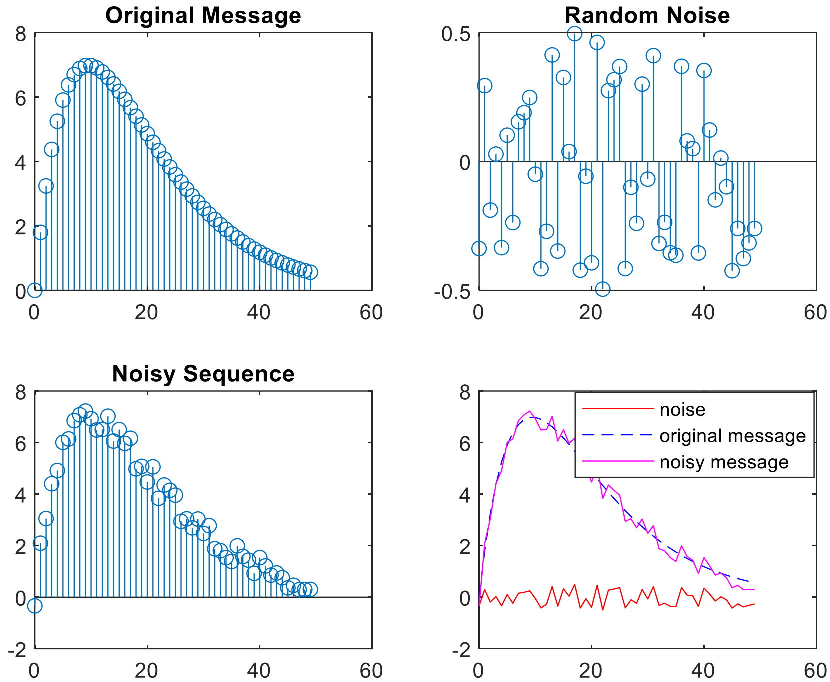
$$=\frac{1}{4}\{x(n+1)+x(n)+x(n-1)+x(n-2)\}$$

Therefore,
y(0)=0.25{x(1)+x(0)+x(-1)+x(-2)}
y(1)=0.25{x(2)+x(1)+x(0)+x(-1)}
y(-1)=0.25{x(0)+x(-1)+x(-2)+x(-3)} etc.

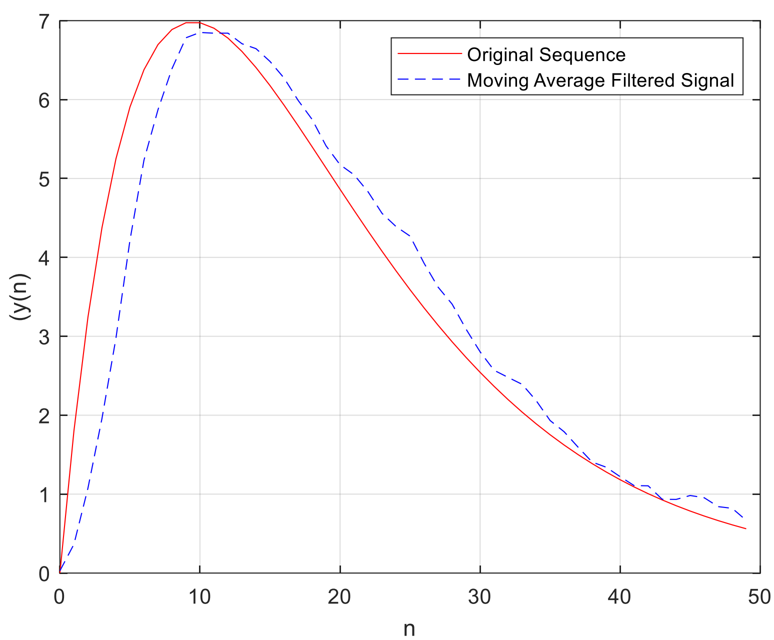
```
N=50;%number of samples
d=rand(N,1)-0.5;%noise with mean 0 and lies between -0.5 to 0.5
m=0:1:N-1;
s=2*m.*(0.9.^m);%original Sequence
x=s+d'; %noisy sequence
subplot(2,2,1)
stem(m,s)
title('Original Message')
subplot(2,2,2)
```

```
stem(m,d)
title('Random Noise')
subplot(2,2,3)
stem(m, x)
title('Noisy Sequence')
subplot(2,2,4)
plot(m, d, 'r-', m, s, 'b--', m, x, 'm-')
legend('noise','original message', 'noisy message')
figure
M=input('Value of M=');% Value of M from key board
b=ones(M, 1)/M;
y=filter(b,1,x);
plot(m,s,'r-',m,y,'b--')
legend('Original Sequence', 'Moving Average Filtered Signal')
grid on
xlabel('n')
ylabel('(y(n)')
```

The result of above code is shown in fig. 1.12 for $M = 4$ which is equivalent to curve smoothing technique of regression.




(a) Original and noisy sequences




(b) Original and filtered signal

**Fig. 1.12** Curve smoothing using moving average filter

12. Now we will listen to the speech signal with noise removal.

```
load handel %original music signal
u=y(1:20000);
sound(u);
d=0.5*rand(length(u),1)-0.5;%noise with mean 0 and lies between -0.25 to 0.25
x=u+d; %noisy sequence
sound(x)
M=5;% Value of M
b=ones(M,1)/M;
z=2*filter(b,1,x);
sound(z)
subplot(3,1,1)
plot(u)
subplot(3,1,2)
plot(x)
subplot(3,1,3)
plot(z)
```

13. In this section we will deal with elimination of noise by moving average method. Let us first load a voice/music signal and add some noise with it. Finally we will filter the signal and observe the signals and corresponding spectrograms.

```
load handel %original signal
u=y(1:16000);
[num,den]=ellip(4,3,40,0.75,'high');
noise=filter(num,den,randn(length(u),1));
x=u+noise;
x=x/max(max(x));
M=5;% 5 sample will be averaged
b=ones(M,1)/M;
z=2*filter(b,1,x);
figure(1)
subplot(1,3,1)
```

```
specgram(u,[],Fs)
title('Original wave')
subplot(1,3,2)
specgram(x,[],Fs)
title('Noisy wave')
subplot(1,3,3)
specgram(z,[],Fs)
title('Filtered wave')
figure(2)
subplot(3,1,1)
plot(u)
title('Original wave')
subplot(3,1,2)
plot(x)
title('Noisy wave')
subplot(3,1,3)
plot(z)
title('Filtered wave')
```

Result of above code is shown in fig. 1.13.

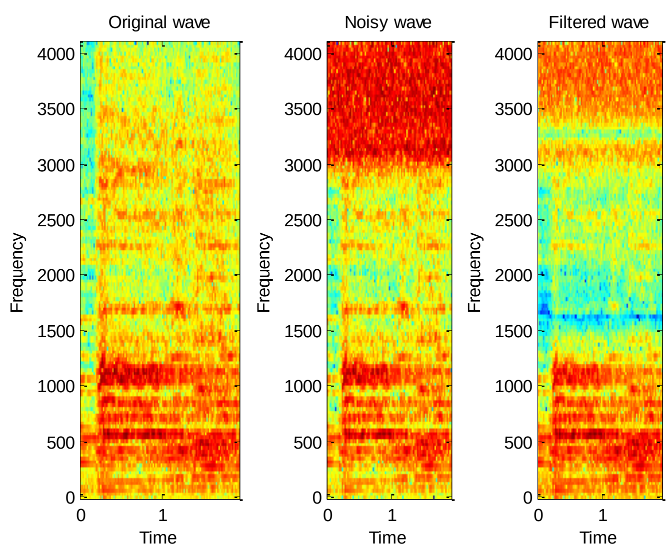



**Fig.1.13 (a)** Spectrogram of original, noisy and recovered signal

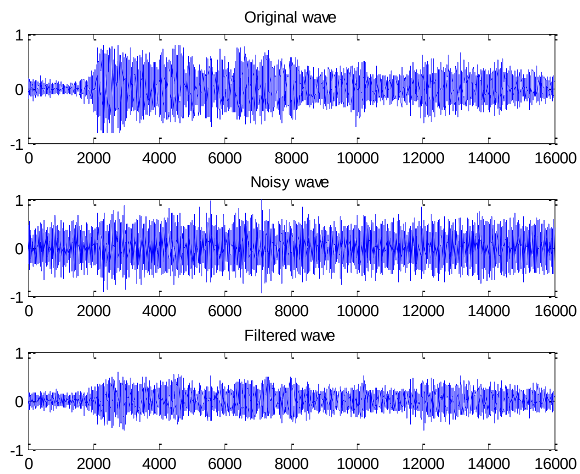



**Fig. 1.13 (b)** Plot of original, noisy and recovered signal

If your PC contains multimedia service, you can listen to them using,

```
sound(u,Fs); % original sound
sound(x,Fs); % Noisy sound
sound(z,Fs); % filtered sound
```

14. Determine *Z*-transform of $x(n) = kn$ and $x(n) = cos(kn)$

```
syms k n
ztrans(k*n)
ans =
k*z/(z-1)^2
```

```
syms k n
ztrans(cos(k*n))
ans =
 (z-cos(k))*z/(z^2-2*z*cos(k)+1)
```

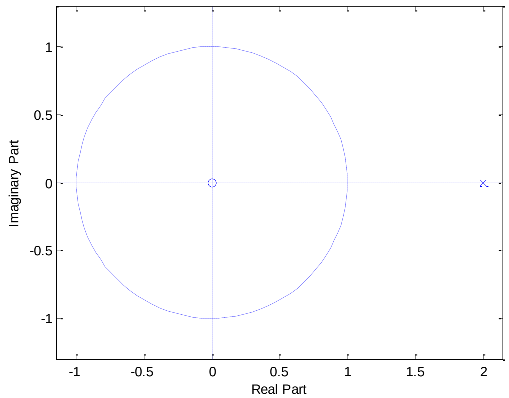
Find ztransform of $2^n U(n)$ and ROC

```
syms n
ztrans(2^n)

ans =

z/(z - 2)
```

```
[z,p,k]=tf2zp([1 0],[1 -2])
zplane(z, p);
```




**syms z n**
**iztrans(z/(z-2))**

ans =

2^n

15. Find inverse z-transform of $X(z) = \dfrac{kz}{(z-1)^2}$

**syms k z**
**iztrans(k*z/(z-1)^2)**

ans =
k*n

Example-1
Given, $X_1(z) = 1 + 2z^{-1} + 5z^{-2}$ and $X_1(z) = 1 + 3z^{-1} + 6z^{-2} + 8z^{-3}$. Determine $Y(z) = X_1(z).X_2(z)$
Ans.
$x_1(n) = \left\{ \underset{\uparrow}{1}, 2, 5 \right\}$ and $x_2(n) = \left\{ \underset{\uparrow}{1}, 3, 6, 8 \right\}$

```
x1=[1 2 5];
x2=[1 3 6 8];
y=conv(x1,x2)

y =

     1     5    17    35    46    40
```

$\therefore X_1(z) = 1 + 5z^{-1} + 17z^{-2} + 35z^{-3} + 46z^{-4} + 40z^{-5}$

Find inverse z transform of $X_1(z) = 1 + 3z^{-1} + 6z^{-2} + 8z^{-3}$

**x = filter([1 3 6 8], [1] ,[1 0 0 0])**

```
x =
    1     3     6     8
```

Find inverse z transform of

$$X(z)\frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+0.3561z^{-2}}$$

**b=[1  2  1];**
**a=[1  -1  0.3561];**
**z=filter(b,a,[1 0 0 0 0 0 0 0 0 0])**

z =

1.0000    3.0000    3.6439    2.5756    1.2780    0.3608   -0.0943   -0.2228   -0.1892   -0.1099

16. Plot the transfer function, $F(z) = 1/(z^4+2z^3+5z^2+3z+2)$ in dB scale; where $z = x+jy$

```
clear
clf
x = -3:0.1:0;
y = -4:0.1:4.0;
[X,Y] = meshgrid(x,y);
p = X + j*Y;
den = [ 1 2 5 3 2];
s = polyval(den, p); %value of polynomial at p
h = surf(x, y, 20*log10(1./abs(s)));
xlabel('real')
ylabel('imag')
zlabel('magnitude (dB)')
```

Result of above code is shown in fig. 1.14

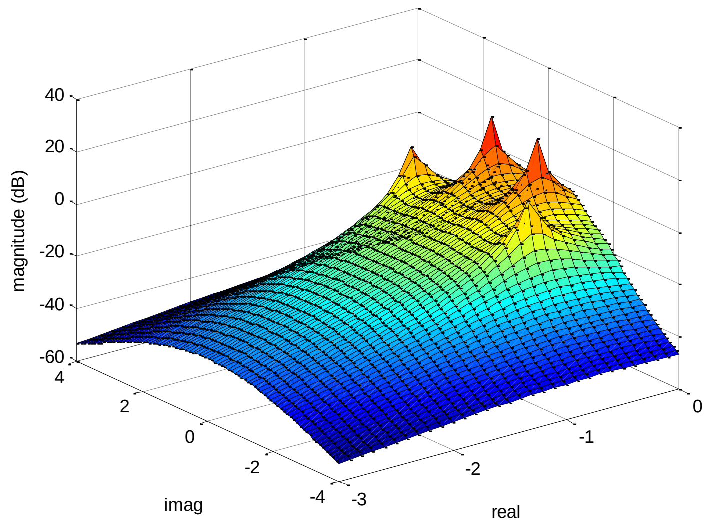



**Fig.1.14** Plot of $F(z) = 1/(z^4+2z^3+5z^2+3z+2)$ in dB scale

Example-1

Verify the relation with first 10 terms.

$$(n-2)(0.5)^{n-2}\cos\left\{\frac{\pi}{3}(n-2)\right\}u(n-2) \leftrightarrow \frac{0.25z^{-3}-0.5z^{-4}+0.0625z^{-5}}{1-z^{-1}+0.75z^{-2}-0.25z^{-3}+0.0625z^{-4}}$$

```
n=0:9;
u2=[0 0 1 1 1 1 1 1 1 1];
xn=(n-2).*((0.5).^(n-2)).*cos((pi/3)*(n-2)).*u2; %LHS
b=[0,0,0,0.25,-0.5,0.0625];
a=[1, -1, 0.75,-0.25,0.0625];
z=filter(b,a,[1 0 0 0 0 0 0 0 0 0]); %RHS
```

```
>> xn
xn =
     0      0      0   0.2500   -0.2500   -0.3750   -0.1250    0.0781    0.0938    0.0273
>> z
z =
     0      0      0   0.2500   -0.2500   -0.3750   -0.1250    0.0781    0.0938    0.0273
```

Example-2

The ratio of two polynomial of z, can be expressed in terms of residues $R_k$, poles, $p_k$ and coefficients of direct terns $C_k$ like,

$$X(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}} = \sum_{k=1}^{N} \frac{R_k}{1 - p_k z^{-1}} + \sum_{\substack{k=0 \\ When \quad M \geq N}}^{M-n} C_k z^{-k}$$

Determine residues, poles and coefficients of direct terms of,

$$X(z) = \frac{1 + \sqrt{3} z^{-1}}{1 - 0.5 z^{-1} + 0.75 z^{-2} + 0.25 z^{-3}}$$

b=[1,sqrt(3)];

a=[1, -0.5, 0.75, 0.25];

[R,P,C] = residuez (b,a); R is residues, P is the poles and C is coefficients of direct terms.

```
R =
   0.6583 - 0.9175i
   0.6583 + 0.9175i
  -0.3166
P =
   0.3815 + 0.8973i
   0.3815 - 0.8973i
  -0.2630
C =
     []
```

Example-2

Determine inverse z-transform of,

$X(z) = \frac{1}{(1 - 0.9 z^{-1})^2 (1 + 0.9 z^{-1})}$ and verify the result.

b = 1; a = poly([0.9, 0.9, -0.9]);

[R,P,C] = residuez (b,a);

```
R =

   0.2500
   0.2500 + 0.0000i
   0.5000 - 0.0000i
P =
```

-0.9000

0.9000 + 0.0000i

0.9000 - 0.0000i

C =

[]

$$\therefore X(z) = \frac{1}{\left(1-0.9z^{-1}\right)^2\left(1+0.9z^{-1}\right)} = \frac{0.25}{\left(1-0.9z^{-1}\right)} + \frac{0.5}{\left(1-0.9z^{-1}\right)^2} + \frac{0.25}{\left(1+0.9z^{-1}\right)}$$

$$= \frac{0.25}{\left(1-0.9z^{-1}\right)} + \frac{0.5z}{0.9}\frac{\left(0.5z^{-1}\right)}{\left(1-0.9z^{-1}\right)^2} + \frac{0.25}{\left(1+0.9z^{-1}\right)} \leftrightarrow 0.25(0.9)^n u(n) + \frac{5}{9}(n+1)(0.9)^{n+1}u(n+1) + 0.25(-0.9)^n u(n)$$

```
b = 1; a = poly([0.9, 0.9,  -0.9]);
xz=filter(b,a,[1 0 0 0 0 0 0 0 0 0]); %RHS
n=0:9;
u=[1 1 1 1 1 1 1 1 1 1];
xn=0.25*((0.9).^n).*u+(5/9)*(n+1).*((0.9).^(n+1)).*u+0.25*((-0.9).^n).*u;
%srart from n = 0 gives the LHS
```

xz =

1.0000 0.9000 1.6200 1.4580 1.9683 1.7715 2.1258 1.9132 2.1523 1.9371

xn =

1.0000 0.9000 1.6200 1.4580 1.9683 1.7715 2.1258 1.9132 2.1523 1.9371

17. Determine pole-zero diagram of a system with zeros, $Z$ = 1 + 2j, -j and poles $P$ = 1, 1-j, 5 + 5j;

```
% vector for zeros
z = [1+2j ; -j];
% vector for poles
p = [1 ; 1-j ; 5+5j];
zplane(z, p);
title('Pole/Zero Plot ');
```

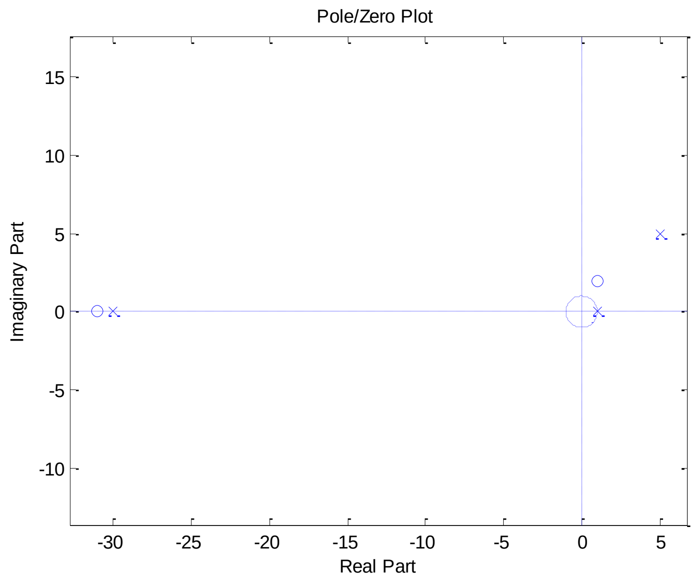



**Fig.1.15** Pole zero plot

18. Let us now determine root locus, bode diagram, step response and impulse response of open or closed loop systems in 's' domain of Laplace transform. Such analysis is done in determination of system stability. This part of the experiment is taken based on concept of, A. J. Chipperfield, P. J. Fleming and C. M. Fonseca, 'MATLAB toolboxes an application for control,' Peter Peregrinus Ltd.

    (a) Find root locus of the feedback system of fig. 1.16 for $K = 2$.

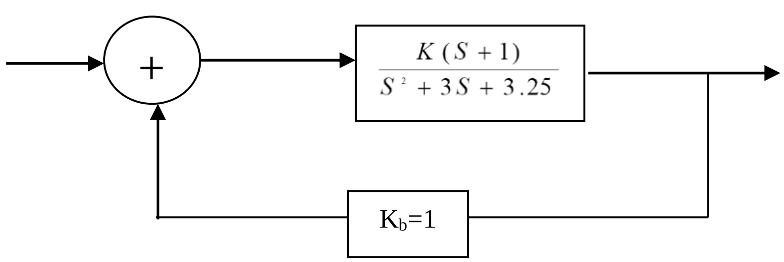



**Fig. 1.16** A feed back system in S domain

```
%Source code
K=2;
h1=tf(K, [1.0  3.0   3.25]);
h2=tf([1  1], 1);
```

**dcm=feedback(h1*h2,1,1);** % h1*h2 is system-1, Kb=1 system-2 and +1 is addition

**rlocus(dcm,'k')**

The result of the code is shown in fig. 1.17

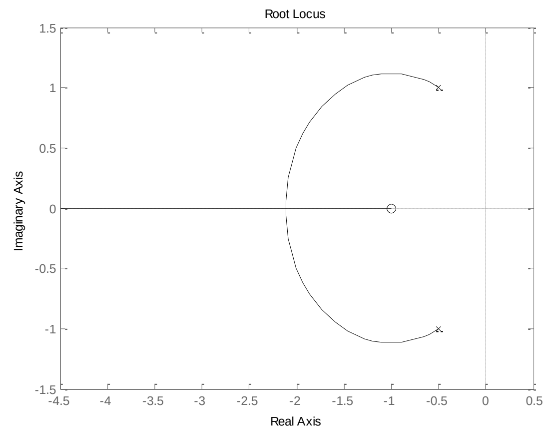



**Fig. 1.17** Root locus of fig. 1.16

(b) Find bode diagram of a system of transfer function of, $G(s) = \dfrac{4}{S(S+1)(S+2)}$

**num=4;**
**den=conv([1  0],conv([1,1],[1,2]));**
**Bode(num,den)**

Result of above code is shown in fig. 1.18

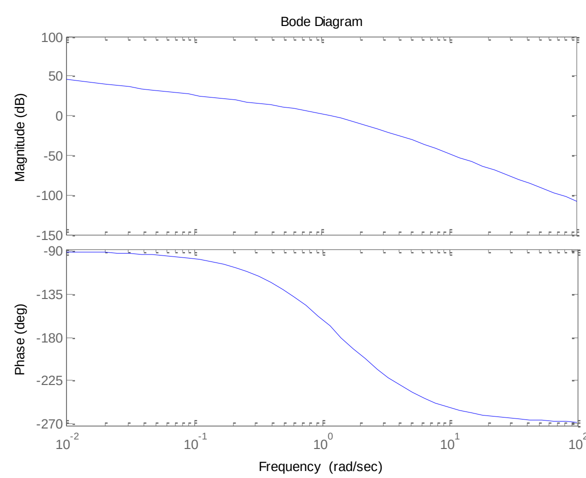



**Fig. 1.18**Bode diagram of above transfer function

(c) Determine step response of the following feedback system of fig. 1.19.

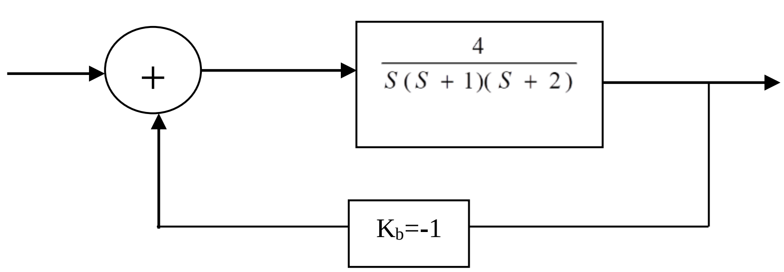


**Fig. 1.19** A feedback system

```
num=4;
den=conv([1  0],conv([1,1],[1,2]));
[N, D]=cloop(num, den, -1);
t=0:0.005:25;
step(N, D, t)
```

Result of above code is shown in fig. 1.20.

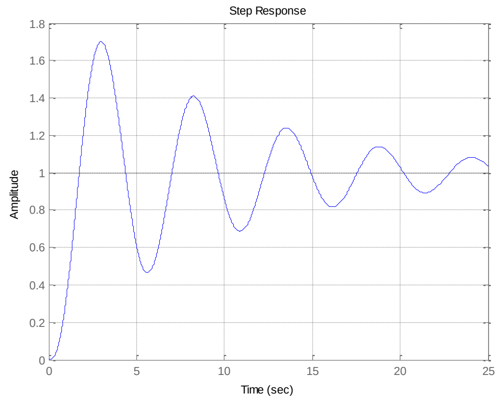



**Fig. 1.20** Step response of fig. 1.19

(d) Determine impulse response of the system.

```
impulse(N, D, t)
```

Result of above code is shown in fig. 1.21.