Fig. 1.21 Impulse response of fig. 1.19

Experiment No. 02
Discrete Time Fourier Transform (DTFT)

Basic Theory:

A continuous time signal *f(t)* called analog signal has a finite amplitude at every instant *t*. A discrete time signal *x(n)* called sequence is defined at discrete instants *n* but *x(n)* is undefined at any instant between *n+k* and *n+k+1*. A digital signal is one for which both time and amplitude axis is discrete. Fourier transform is applicable for a discrete time sequence *x(n)* in a different form called Discrete Fourier Transform (DFT).

For a sequence *x(n)* in *n*-domain, its DFT will transform it in *m*-domain resembles to transformation of time to frequency domain. DFT of *x(n)* of infinite length is expressed like,

$X(m\omega) = X(m) = \sum_{n=-\infty}^{\infty} x(n)e^{-jm\omega n} = \sum_{n=-\infty}^{\infty} x(n)e^{-jm(2\pi/N)n}$; Which resembles to continuous time Fourier transform of, $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$.

A comparison between continuous and discrete time Fourier transform is shown in table-2.1.

Table-2.1

Continuous FT	Discrete FT
X(ω)	X(m)
$\int_{-\infty}^{\infty}$	$\sum_{n=-\infty}^{\infty}$

t	n
ω	mω or m
x(t)	x(n)
T	N

Now the inverse operation i.e. IDFT can be expressed like,

$$x(n) = Lt_{N\to\infty} \frac{1}{N} \sum_{m=-\infty}^{\infty} X(m) e^{jm\omega n} = Lt_{N\to\infty} \frac{1}{N} \sum_{n=-\infty}^{\infty} X(m) e^{jm(2\pi/N)n}$$

For a finite length sequence of length N over 0≤n≤N-1, DFT and IDFT is expressed like,

$$X(m\omega) = X(m) = \sum_{n=0}^{N-1} x(n) e^{-jm\omega n} = \sum_{n=0}^{N-1} x(n) e^{-jm(2\pi/N)n} \text{ and}$$

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) e^{jm\omega n} = \frac{1}{N} \sum_{n=0}^{N-1} X(m) e^{jm(2\pi/N)n}$$

1. Consider a continuous time signal, $x(t) = \sin(2\pi 1000t) + \frac{1}{2}\sin(2\pi 2000t + \frac{3\pi}{4})$.

Determine sampled signal *x(n.Ts)* = *x(n)*, using Matlab, taking *N* = 8 samples at sampling rate, *Fs* = 8000Hz (samples/sec).

The sampled signal,

$x_s(n) = x(nT_s) = \sin(2\pi 1000 nT_s) + \frac{1}{2}\sin(2\pi 2000 nT_s + \frac{3\pi}{4})$; where *Ts=1/Fs*=1/8000sec is the sampling period. Let us use Matlab code to determine sampled sequence *x(n)*.

```
n=0:1:7;
Ts=1/8000;
xn=sin(2*pi*1000*n*Ts)+0.5*sin(2*pi*2000*n*Ts+3*pi/4);
xn
```

```
xn =
   0.3536    0.3536    0.6464    1.0607    0.3536   -1.0607   -1.3536   -0.3536
```

Therefore,

x(0)= *0.3536*

x(1)= *0.3536*

x(2)= *0.6464*

x(3)= *1.0607*

x(4)= *0.3536*

x(5)= *-1.0607*

x(6)= *-1.3536*

x(7)= -0.3536

2. Determine DFT of above sequence using Matlab.

X=fft(xn)

X =

X(0)=0.0000

X(1)=-0.0000 - 4.0000i

X(2)=1.4142 + 1.4142i

X(3)=-0.0000 + 0.0000i

X(4)=-0.0000

X(5)=-0.0000 - 0.0000i

X(6)=1.4142 - 1.4142i

X(7)=-0.0000 + 4.0000i

IDFT of above sequence will retrieve the original sequence *x(n)* like,

Y=ifft(X)

Y =

0.3536 0.3536 0.6464 1.0607 0.3536 -1.0607 -1.3536 -0.3536

3. If $x(n) \leftrightarrow X(m)$; both *x(n)* and *X(m)* are the vector of length *N* then their relation can be expressed in a different way like,

$X = \mathbf{D}_N x$

Where $\mathbf{x} = [x(0)\ \ x(1)\ \ x(2)\ \ \ldots\ \ \ldots\ \ \ldots\ x(N\text{-}1)]^T$,

$X = [X(0)\ \ X(1)\ \ X(2)\ \ \ldots\ \ \ldots\ \ \ldots\ X(N\text{-}1)]^T$,

$$\mathbf{D}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots. & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \cdots & \cdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \vdots & W_N^{(N-1)(N-1)} \end{bmatrix} \text{ and } W_N = e^{-j2\pi/N}$$

Determine the $\mathbf{D}_N$ matrix of dimension of 4×4 using Matlab.

D=dftmtx(4)

D =

1.0000	1.0000	1.0000	1.0000
1.0000	0 - 1.0000i	-1.0000	0 + 1.0000i
1.0000	-1.0000	1.0000	-1.0000
1.0000	0 + 1.0000i	-1.0000	0 - 1.0000i

4. Write Matlab code to determine DFT of $x(n) = \{1, 0, 0, 1\}$ hence show the plot of $X(m)$.

```
x=[1 0 0 1];
y=fft(x);
subplot(2,1,1)
stem(abs(y), 'k')
xlabel('m')
ylabel('X(m)')
title('Absolute value of DFT sequence')
subplot(2,1,2)
stem(angle(y), 'k')
xlabel('m')
ylabel('Angle(X(m))')
title('Angle of DFT sequence')
```

The result of above code is shown in fig. 2.1.

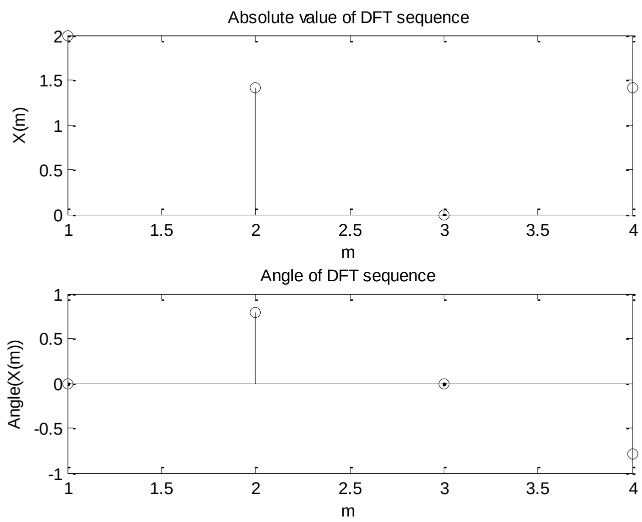
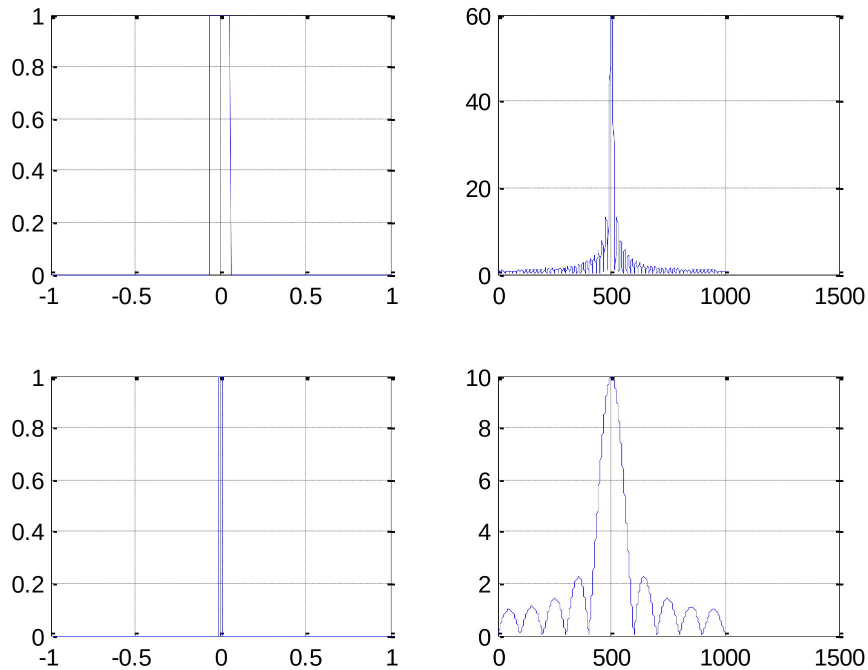


Fig. 2.1 Plot of $|X(m)|$ and $\angle X(m)$

DTFT on a rectangular pulse

```
fs = 500;
t = -1:1/fs:1;
x = rectpuls(t,0.12);
subplot(2,2,1)
plot(t, x)
grid on
y = fft(x);
y = fftshift(y);
subplot(2,2,2)
plot( abs(y))
grid on
x = rectpuls(t,0.02);
subplot(2,2,3)
plot(t, x)
grid on
y = fft(x);
y = fftshift(y);
subplot(2,2,4)
plot( abs(y))
grid on
```



DTFT on a 3D rectangular pulse

```
x=zeros(32);
x(12:17)=ones(6,1);
subplot(2,2,1)
title('2D rectangular pulse')
mesh(x)
x=fft(x);
x=fftshift(x);%Dc avlue is at the corner of the array
%let us move it at the middle
subplot(2,2,2)
mesh(abs(x))
title('2D sinc in frequency domain')
```

```
x=zeros(32);
x(12:17,12:17)=ones(6);
subplot(2,2,3)
title('3D rectangular pulse')
mesh(x)
x=fft2(x);
x=fftshift(x);%Dc avlue is at the corner of the array
%let us move it at the middle
subplot(2,2,4)
mesh(abs(x))
title('3D sinc in frequency domain')
```