CS 3120 - Module 1 HW 1

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1 Cardinality of Σ^n

$|\Sigma^n| = m^n$ where $m = |\Sigma|$

The alphabet Σ^n means the set of all elements such that each element is a string of size n composed of the elements in Σ which means that each individual character of a string in alphabet Σ^n can be any element in Σ. Since there are m alphabets in Σ, this means that there are m possible choices for each letter of any string in Σ^n and this can be represented as $m * m * ... * m$ for n times leading to the equation $|\Sigma^n| = m^n$.

2 Cardinality of $A \times B$

If the $|A| = a$ and $|B| = b$, then the cardinality of $A \times B$ or $|A \times B| = a \cdot b$ because each element in A needs to be paired with each individual element in B meaning for each element in A, there are b pairs associated with each element in A. Since you can have b sets for each element in A, this means that the cardinality of $A \times B = a \cdot b$.

3 Set Notation with $\Sigma = 0, 1$

- $\{x \in (\{011\} \times \{0,1\}^*) \cup (\{100\} \times \{0,1\}^*)\}$
- $\{0,1\}^* \times \{1111\} \times \{0,1\}^*$
- $(\{0,1\}^* \times \{11\} \times \{0,1\}^*) \cap \neg((\{11\} \times \{0,1\}^*) \cup (\{0,1\}^* \times \{11\}))$

4 *Flogrammable Device*

We cannot program 10,000,000 functions because there are only $5^{10} = 9,765,625$ possible permutations which means there is not even enough possible strings to even have 10,000,000 functions. If there are less than 10,000,000 possible permutations with the given alphabet and conditions, there is a stronger case that there are certainly less than 10,000,000 functions that can be programmed in the Flogrammable Device. Therefore, it is not possible.

5 Proof for a graph

The assumptions I will make in this proof are that $G = (V, E)$ are undirected graphs with no self loops.

Proof:

Assume that there exists a graph with $|V| \geq 2$ such that the graph does not contain two or more nodes with equal degrees. This means that each vertex must have a unique degree. However, each vertex can make at most $|V| - 1$ edges. If there are $|V|$ nodes and $|V| - 1$ possible edges for each node, this means that there must exist at least two nodes that have the same number of edges, or degrees, by the pigeonhole principle. This contradicts the assumption that there exists a graph with $|V| \geq 2$ such that the graph does not contain two or more nodes with equal degrees. Therefore, we can conclude that for every graph $G = (V, E)$ such that $|V| \geq 2$, there exists two or more nodes with equal degrees.