

$$y = mx + c$$

slope

y-intercept

$$h(x) = w_0 + w_1 x + w_2$$

- $w_0$  and  $w_1$  are unknown coefficients
- $w_1$  has multiplicative or dividing effect on input  $x$ , whereas  $w_0$  is an additive term.
- We can also calculate these constants values from Inversion

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \theta_0 & \theta_1 \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \underbrace{\quad}_{\text{nx1}} \underbrace{\quad}_{\text{mx2}}$$

$$\vec{y} = A \vec{x}$$

$$A = \vec{y} \vec{x}^{-1}$$

Loss Function

$y \rightarrow$  est.-i     $y \rightarrow$  original value  
 $h(x) \rightarrow$  estimated value

$$y - (w_0 + w_1 x_1 + w_2 x_2) = 0$$

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$$y - h(x) = 0$$

Q.v

$$h(x) - y = 0$$

So

loss function

$$L(y, h(x)) = ((y - h(x))^2)$$

Other loss functions,

$$(y - h(x))$$
  
error

$$|y - h(x)|$$
  
absolute error

$$\sqrt{(y - h(x))^2}$$
 root squared error

For multiple values

total loss or total cost

$$\frac{1}{N} \sum_{j=1}^{N-1} (y_j - h(x_j))^2$$

Expanded form

$$\frac{1}{N} \sum_{j=1}^{N-1} (y_j - (w_1 x_j + w_0))^2$$

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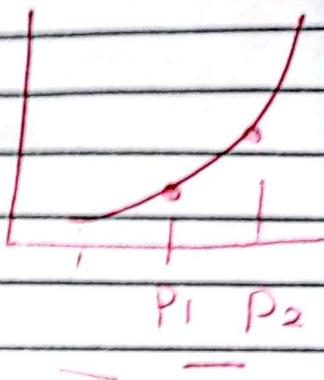


## Gradient Descent -

ignore  
Re lund  
Learn  
error

$$\frac{dp}{dt} = \lim_{\Delta t \rightarrow 0} \frac{P_2 - P_1}{t_2 - t_1} = r P_1$$

$$\frac{dP}{dt} = \frac{P_2 - P_1}{\Delta t} = \gamma_1 P_1$$



$$\frac{dp}{dt} = \frac{P_2 - P_1}{t_2 - t_1} = \gamma_1 P_1$$

$$dp_2 = \gamma_1 P_1 \Delta t$$

$$t_2 - t_1 = \Delta t$$

$$P_2 = \Delta t \times P_1 + P_1$$

$$P_2 = P_1 + \Delta t \times P_1$$

$$P_2 = P_1 + \Delta t \frac{dp}{dt}$$

new value = old value + learning rate (slope)

for  $\epsilon$

$$\text{for } \epsilon_j = \epsilon_{j-1} + \alpha \frac{d \epsilon}{d \text{steps}}$$

$$\epsilon_j = \epsilon_{j-1} + \alpha \frac{d \epsilon}{d w_1, w_2}$$

$$\epsilon_j = \epsilon_{j-1} + \alpha \frac{d \epsilon}{d w_1, w_2}$$

$$\frac{\partial}{\partial w_1} (y - h(x))^2$$

$$\frac{\partial}{\partial w_1} (y - (w_1 x + w_0))^2 = 2(y - (w_1 x + w_0))(-x)$$

$$= 2(y - (w_1 x + w_0))(-x)$$

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ate

$$\frac{\partial}{\partial w_0} (y - (w_1x + w_0))^2 =$$

$$2(y - (w_1x + w_0))$$

~~E~~

$$w_0 = w_0 + \alpha 2(y - (w_1x + w_0))$$

$$w_1 = w_1 + \alpha 2(y - (w_1x + w_0))(x)$$

## Regression with

For  $i$  in range (num\_iter):

$$m\text{-grad} = 0$$

$$b\text{-grad} = 0$$

$$N = \text{float}(\text{len(points)})$$

$$\text{total-cost} = 0$$

for  $j$  in range (0, len(points)):

$$x-j = \text{points}[j, 0] \quad y-j = \text{points}[j, 1]$$

$$\text{m-gradient} += \frac{1}{N} * 2 * (y-j - (m * x-j + b))$$

$$\text{b-gradient} += \frac{1}{N} * 2 * (y-j - (m * x-j + b))$$

$$\text{total cost} += (y-j - (m * x-j + b))^2$$

$$\text{cost} = \frac{\text{total-cost}}{N}$$

$$m = m + \text{learning rate} * \text{m-gradient}$$

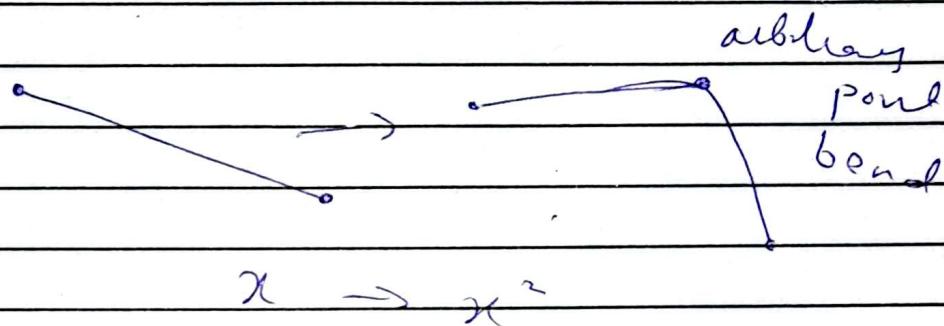
$$b = b + \text{learning rate} * \text{b-gradient}$$

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## Regression with polynomial Models.

linear equation  $\rightarrow$  change  $\rightarrow$  arbitrary points



arbitrary points depend on bend

polynomial equation

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \dots$$

Taylor series

$$\theta_0 + \theta_1 \frac{dx}{d\theta_1} + \theta_2 \frac{d^2x}{d\theta_2^2} + \theta_3 \frac{d^3x}{d\theta_3^3} + \dots$$

$$\frac{dp}{dt} = r_p$$

$$\frac{dp^2}{dt^2} = \gamma$$

$$\frac{d^3}{dt^3} = 0$$

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## Derivation

$$h(x) = w_0 + w_1 x + w_2 x^2$$

$$\frac{\partial}{\partial w_0} (y - (w_0 + w_1 x + w_2 x^2))^2 = \\ = \partial (y - (w_0 + w_1 x + w_2 x^2))$$

$$\frac{\partial}{\partial w_1} (y - (w_0 + w_1 x + w_2 x^2))^2 = \\ = \partial (y - (w_0 + w_1 x + w_2 x^2)) x$$

$$\frac{\partial}{\partial w_2} (y - (w_0 + w_1 x + w_2 x^2))^2 = \partial (y - (w_0 + w_1 x + w_2 x^2)) x^2$$

$$w_0 = w_0 + \alpha * \partial (y - (w_0 + w_1 x + w_2 x^2))$$

$$w_1 = w_1 + \alpha * \partial (y - (w_0 + w_1 x + w_2 x^2)) x$$

$$w_2 = w_2 + \alpha * \partial (y - (w_0 + w_1 x + w_2 x^2)) x^2$$



For  $i$  in range (num\_iterations),

$$w_0\text{-gradient} = 0$$

$$w_1\text{-gradient} = 0$$

$$w_2\text{-gradient} = 0$$

$$N = \text{float}(\text{len(points)})$$

$$\text{total\_cost} = 0$$

For  $j$  in range (0, len(points)):

$$x_j = \text{points}[j, 0]$$

$$y_j = \text{points}[j, 1]$$

$$w_0\text{-gradient} = \frac{1}{N} * 2 * (y_j - (w_0 + w_1 * x_j + w_2 * x_j^2))$$

$$w_1\text{-gradient} = \frac{1}{N} * 2 * (y_j - (w_0 + w_1 * x_j + w_2 * x_j^2)) * x_j$$

$$w_2\text{-gradient} = \frac{1}{N} * 2 * (y_j - (w_0 + w_1 * x_j + w_2 * x_j^2)) * x_j^2$$

~~cost~~

$$\text{total cost} = (y_j - (w_0 + w_1 * x_j + w_2 * x_j^2))^2$$

$$\text{cost} = \text{total cost} / N$$

$$w_0 = w_0 + \text{learning rate} * w_0\text{-gradient}$$

$$w_1 = w_1 + \text{learning rate} * w_1\text{-gradient}$$

$$w_2 = w_2 + \text{learning rate} * w_2\text{-gradient}$$

$$\begin{bmatrix} \sum x^0 & \sum x^1 & \sum x^2 \\ \sum x^1 & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \sum x^0 y \\ \sum x^1 y \\ \sum x^2 y \end{bmatrix}$$

$$\begin{bmatrix} \sum x^0 & \sum x^1 & \sum x^2 \\ \sum x^1 & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \sum x^0 y \\ \sum x^1 y \\ \sum x^2 y \end{bmatrix}$$

Univariant

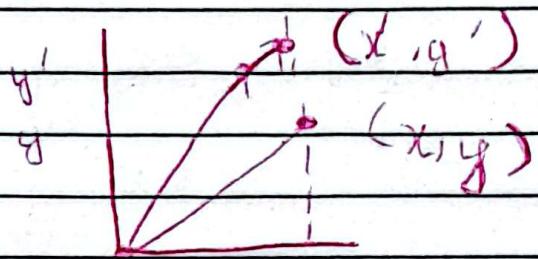
$$h(n) = w_0 + w_1 x$$

multivariant

$$h(n) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots$$

$$= w_0 + \sum_{i=1}^n w_i x_i$$

$$h(x) = \sum_{i=0}^n w_i x_i$$



$$\begin{bmatrix} x' \\ y' \end{bmatrix}, \begin{bmatrix} \quad \\ \quad \end{bmatrix} \begin{bmatrix} * \\ y_0 \end{bmatrix}$$

$$r' \rightarrow w_0, x + w_1, y$$

$$A \cdot B = |A||B| \cos \theta \quad \left. \begin{array}{l} \\ \cos(90^\circ) = 0 \end{array} \right\}$$

$$h(x) = w_0 + w_1 x \cos(0) \quad \left. \begin{array}{l} \\ \cos(0) = 1 \end{array} \right\}$$

$$h(x) = w_0 + w_1 x + w_2 x^2 \cos(0)$$

no effect.

angle value adjusted in the weight we assume that the angle is

slope  $\rightarrow$  coefficient

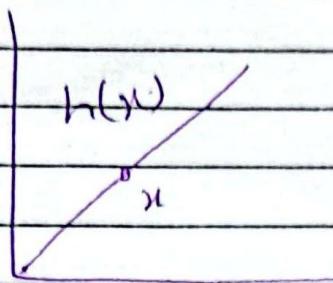
Value b/w 0, 1

$w \in [0, 1]$

because only

# Classification

if else



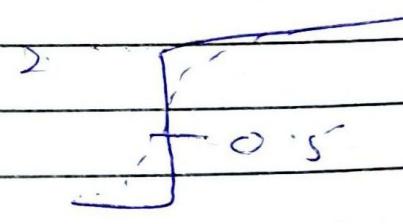
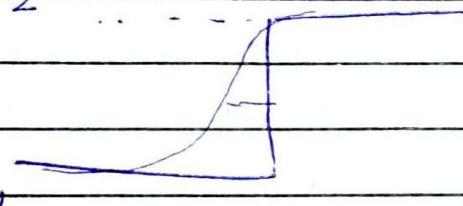
$$h(x) = \begin{cases} 0 & \text{if } x < h(x) \\ 1 & \text{if } x = h(x) \\ 2 & \text{otherwise} \end{cases}$$

# MonteCarlo Method

## Activation Functions.

Non-linearity

choose based on smooth boundary



Softmax

$$z_i = \frac{1}{\sum_j e^{z_j}}$$

~~max(z)~~

$$z' = \frac{e^{z_i}}{\sum_{j=1}^n e^{z_j}}$$

Sigmoid

$$z' = \frac{1}{1 + e^{-z}}$$

tanh

$$z' = \frac{e^{z+2} - e^{-z}}{e^{z+2} + e^{-z}}$$

ReLU  $z' = \sigma_i \max(0, z_i)$

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activation function in  
linear scenario

$$h(x) = mx + c$$

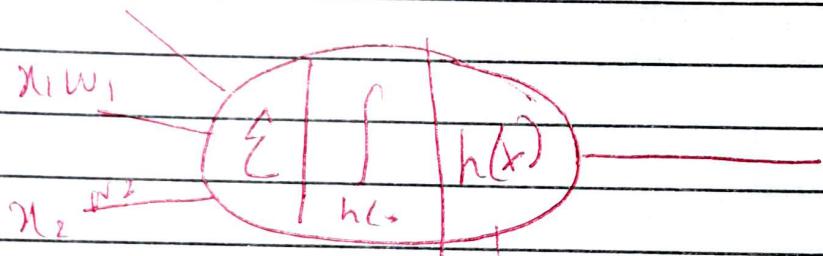
$$= \frac{1}{1 + e^{-h(x)}}$$

$$= \frac{1}{1 + e^{-(mx + c)}}$$

$$\frac{1}{1 + e^{-(w_0 + w_1 x_1 + w_2 x_2 + \dots)}}$$

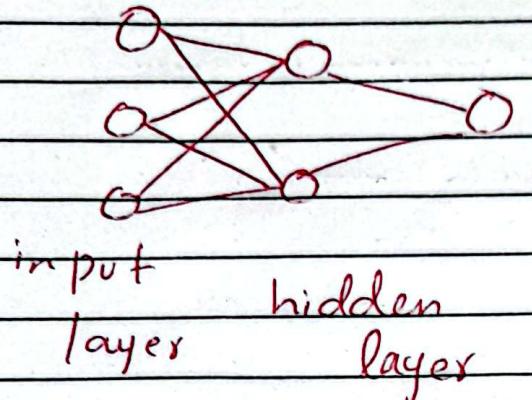
w<sub>0</sub> w<sub>1</sub>

Perception



activation

function

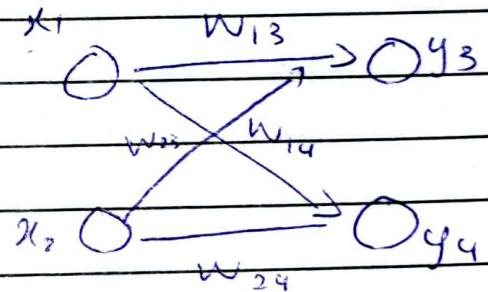


Reval  
 architeclure  
 → adjacency matrx  
 → incident matrx  
 linked matrx  
 tree list (slow  
 (matrix complex)  
 multiplication)

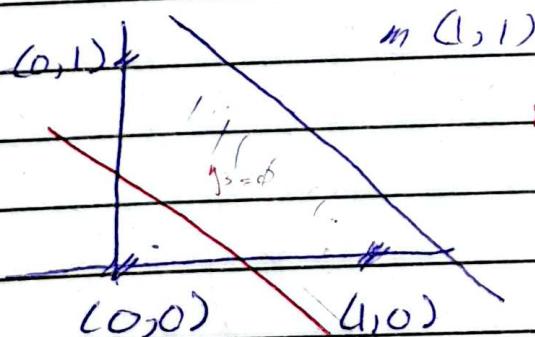
NN architecture → Feed forward  
 ↗ Feed back ward (learning)

Example

$x_1$	$x_2$	$y_3$	$y_4$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



$$\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} w_{13} & w_{23} \\ w_{14} & w_{24} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$m(1,1)$$

$$y_3 = w_{13}x_1 + w_{23}x_2$$

$$y_4 = w_{14}x_1 + w_{24}x_2$$

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$$\begin{array}{c} x_1 \xrightarrow{y_3} y_5 \\ x_2 \xrightarrow{y_3} y_5 \end{array}$$

$$\begin{bmatrix} y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} w_{35} & w_{45} \\ w_{36} & w_{46} \end{bmatrix} \begin{bmatrix} w_{13} & w_{23} \\ w_{14} & w_{24} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\begin{bmatrix} w_{1+2, 3+2} & w_{2+2, 3+2} \\ w_{1+2, 4+2} & w_{2+2, 4+2} \end{bmatrix}$$

$$= w_{35} [w_{13}x_1 + w_{23}x_2]$$

$$+ w_{45} [w_{14}x_1 + w_{24}x_2]$$

$$\begin{bmatrix} w_{13}x_1 & w_{23}x_2 \\ w_{14}x_1 & w_{24}x_2 \end{bmatrix}$$

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Required				Obtained	
$x_1$	$x_2$	$y_3'$	$y_4'$	$y_3$	$y_4$
0	0	0	0	0	0
0	1	0	1	0.5	0.5
1	0	0	1	0.5	0.5
1	1	1	0	1	1

$$E = \frac{1}{4} [(0-0.5)^2 + (1-0.5)^2]$$

$$y_3 = W_{13}x_1 + W_{23}x_2$$

$$y_4 = W_{14}x_1 + W_{24}x_2$$

$$W = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

$$E = \frac{1}{2} (y_3 - y_s)^2$$

$$E = \frac{1}{2} [(y_3 - y_s)^2 + (y_4 - y_s)^2]$$

$$y_3 \Rightarrow E = \frac{1}{4} [0-0]^2 + (0-0.5)^2 + (0-0.5)^2 + (0-1)^2$$

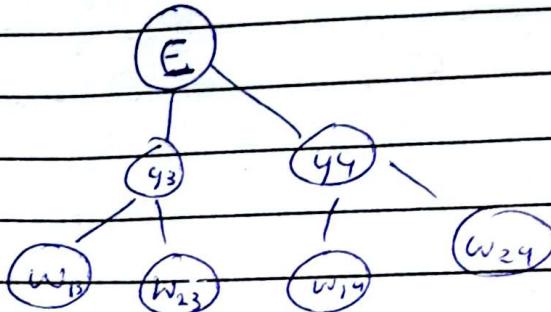
$$y_4 \Rightarrow E = \frac{1}{4} [(-0)^2 + (-0.5)^2 + (1-0.5)^2 + (1-1)^2]$$

$$\frac{1}{2} (0.25 + 0.25 + 0) = \frac{0.5}{2} = 0.25$$

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## Gradient Descent



$$\frac{\partial E}{\partial y_3}$$

$$\frac{\partial E}{\partial y_4}$$

$$\frac{\partial y_3}{\partial w_{13}}$$

$$\frac{\partial y_3}{\partial w_{23}}$$

$$\frac{\partial y_4}{\partial w_{14}}, \frac{\partial y_4}{\partial w_{24}}$$

$$\frac{\partial E}{\partial y_3} = \frac{1}{2} (y_3' - y_3)^2 + (y_4' - y_4)^2$$

$$\frac{\partial E}{\partial y_3} = \frac{\partial}{\partial y_3} (y_3' - y_3)(-1) \quad \frac{\partial E}{\partial y_4} = (y_4' - y_4)(-1)$$

$$\frac{\partial E}{\partial y_3} = -(y_3' - y_3)$$

$$\frac{\partial E}{\partial y_4} = -(y_4' - y_4)$$

$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial y_3} \cdot \frac{\partial y_3}{\partial w_{13}}$$

$$\frac{\partial E}{\partial w_{14}} = \frac{\partial E}{\partial y_4} \cdot \frac{\partial y_4}{\partial w_{14}}$$

$$\frac{\partial E}{\partial w_{23}} = \frac{\partial E}{\partial y_3} \cdot \frac{\partial y_3}{\partial w_{23}}$$

$$\frac{\partial E}{\partial w_{24}} = \frac{\partial E}{\partial y_4} \cdot \frac{\partial y_4}{\partial w_{24}}$$



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[M] [T] [W] [T] [F] [S]

$$\frac{\partial y^3}{\partial w_{13}} = \frac{\partial}{\partial w_{13}} (w_{13}x_1 + w_{23}x_2) \\ = x_1$$

$$\frac{\partial y^3}{\partial w_{14}} = \frac{\partial}{\partial w_{14}} (w_{14}x_1 + w_{24}x_2) \\ = x_1$$

$$\frac{\partial y^3}{\partial w_{13}} = x_2$$

$$\frac{\partial y^3}{\partial w_{24}} = x_2$$

$$\frac{\partial E}{\partial w_{13}} = -(y^3' - y_3) \cdot x_1$$

$$\frac{\partial E}{\partial w_{14}} = -(y^3' - y_3) \cdot x_1$$

$$\frac{\partial E}{\partial w_{23}} = -(y^3' - y_3) \cdot x_2$$

$$\frac{\partial E}{\partial w_{24}} = -(y^3' - y_3) \cdot x_2$$

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case (0,0)	case (0,1)	case (1,0)	case (1,1)
------------	------------	------------	------------

$\frac{\partial E}{\partial w_{13}}$	0	0	0.5	0
$\frac{\partial E}{\partial w_{23}}$	0	0.5	0	0
$\frac{\partial E}{\partial w_{14}}$	0	0	-0.5	1
$\frac{\partial E}{\partial w_{24}}$	0	-0.5	0	1

$$n = 1$$

When s Hold  $\frac{\partial E}{\partial w_{13}}$

$$\rightarrow W = \begin{bmatrix} w_{13} & w_{23} \\ 0.5 & 0.5 \\ 0.5 & 0.5 \\ w_{14} & w_{24} \end{bmatrix}$$

for case (0,1)

for case (1,0)

for case (1,1)

$$W_{1,1} = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1.0 & 1.0 \\ 0.5 & 0 \end{bmatrix} \rightarrow W_s = \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 1.0 \end{bmatrix}$$

$0.5 + 1(0.5)$



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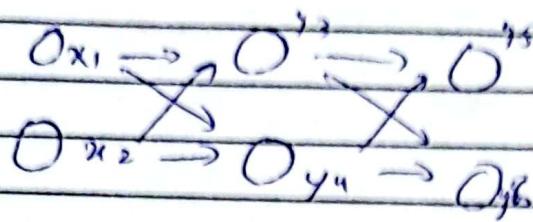
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$$\begin{bmatrix} y_1^0 & y_1^1 & y_1^2 & y_1^3 \\ y_2^0 & y_2^1 & y_2^2 & y_2^3 \end{bmatrix} = \begin{bmatrix} w_{13} & w_{23} \\ w_{11} & w_{21} \end{bmatrix} \begin{bmatrix} 1, 1, 1, 1 \\ 1, 2, 3, 4 \end{bmatrix}$$

$$= [W] + n \begin{bmatrix} \frac{\partial E}{\partial w_{13}} & \frac{\partial E}{\partial w_{23}} \\ \frac{\partial E}{\partial w_{11}} & \frac{\partial E}{\partial w_{21}} \end{bmatrix}$$

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$$\begin{bmatrix} y_3^- \\ y_4 \end{bmatrix} = \begin{bmatrix} w_{13} & w_{23} \\ w_{14} & w_{24} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} y_5^- \\ y_6 \end{bmatrix} = \begin{bmatrix} w_{35} & w_{45}^- \\ w_{36} & w_{46} \end{bmatrix} \begin{bmatrix} y_3^- \\ y_4 \end{bmatrix}$$

$$y_5^- = w_{35} y_3^- + w_{45}^- y_4$$

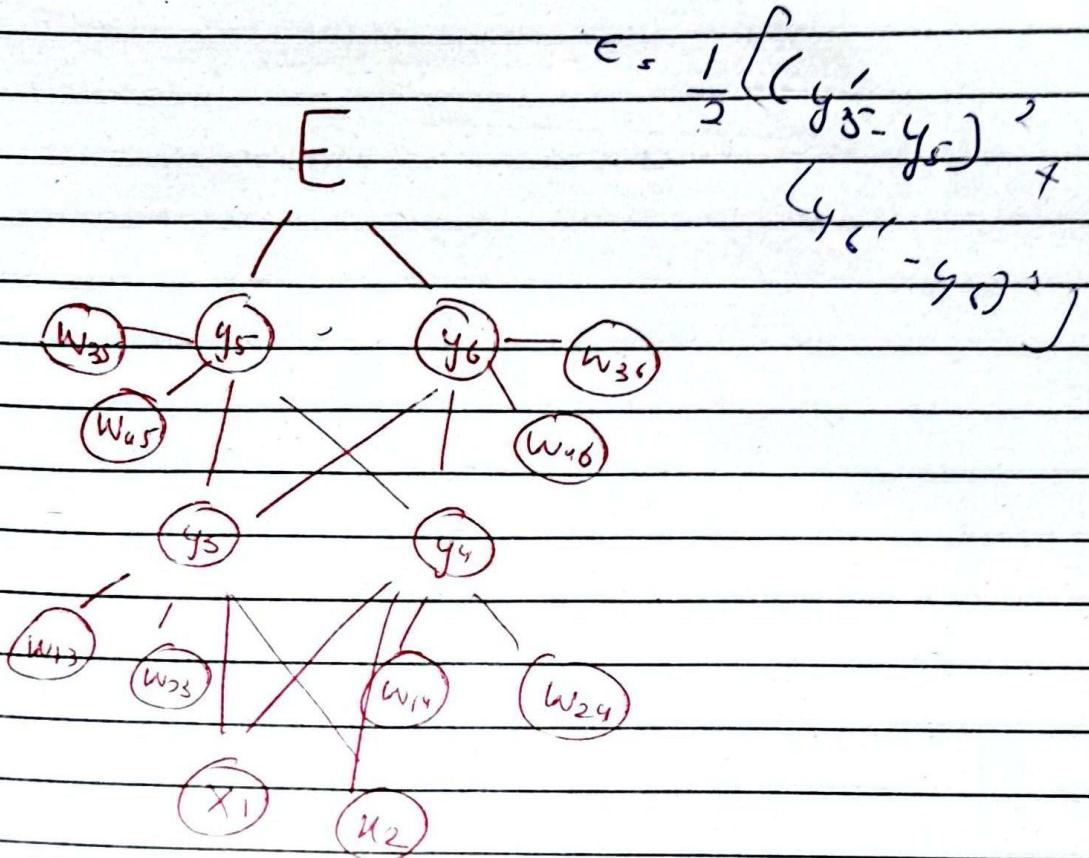
$$y_6 = w_{36} y_3^- + w_{46} y_4$$

$$\begin{aligned} y_5^- &= w_{35} (w_{13} x_1 + w_{23} x_2) + \\ &\quad w_{45}^- (w_{14} x_1 + w_{24} x_2) \end{aligned}$$

$$\begin{aligned} y_6 &= w_{36} (w_{13} x_1 + w_{23} x_2) + \\ &\quad w_{46} (w_{14} x_1 + w_{24} x_2) \end{aligned}$$

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$$\frac{\partial E}{\partial y_5}, \frac{\partial E}{\partial y_6}, \frac{\partial y_5}{\partial w_{35}}, \frac{\partial y_5}{\partial w_{45}}, \frac{\partial y_6}{\partial w_{36}}, \frac{\partial y_6}{\partial w_{46}}$$

$$\frac{\partial y_5}{\partial y_3}, \frac{\partial y_5}{\partial y_4}, \frac{\partial y_6}{\partial y_3}, \frac{\partial y_6}{\partial y_4}, \frac{\partial y_3}{\partial w_{13}}, \frac{\partial y_3}{\partial w_{23}}, \frac{\partial y_4}{\partial w_{14}}, \frac{\partial y_4}{\partial w_{24}}$$

$$\frac{\partial E}{\partial y_5} = \frac{\partial y_5}{\partial w_{35}} \cdot \frac{\partial y_5}{\partial y_3} + \frac{\partial y_5}{\partial w_{45}} \cdot \frac{\partial y_5}{\partial y_4}$$

$$\frac{\partial E}{\partial y_6} = -(\frac{\partial y_5}{\partial y_3} - \frac{\partial y_5}{\partial y_4}) \quad \frac{\partial y_5}{\partial w_{35}} = y_3 \quad \frac{\partial y_5}{\partial w_{45}} = y_4$$

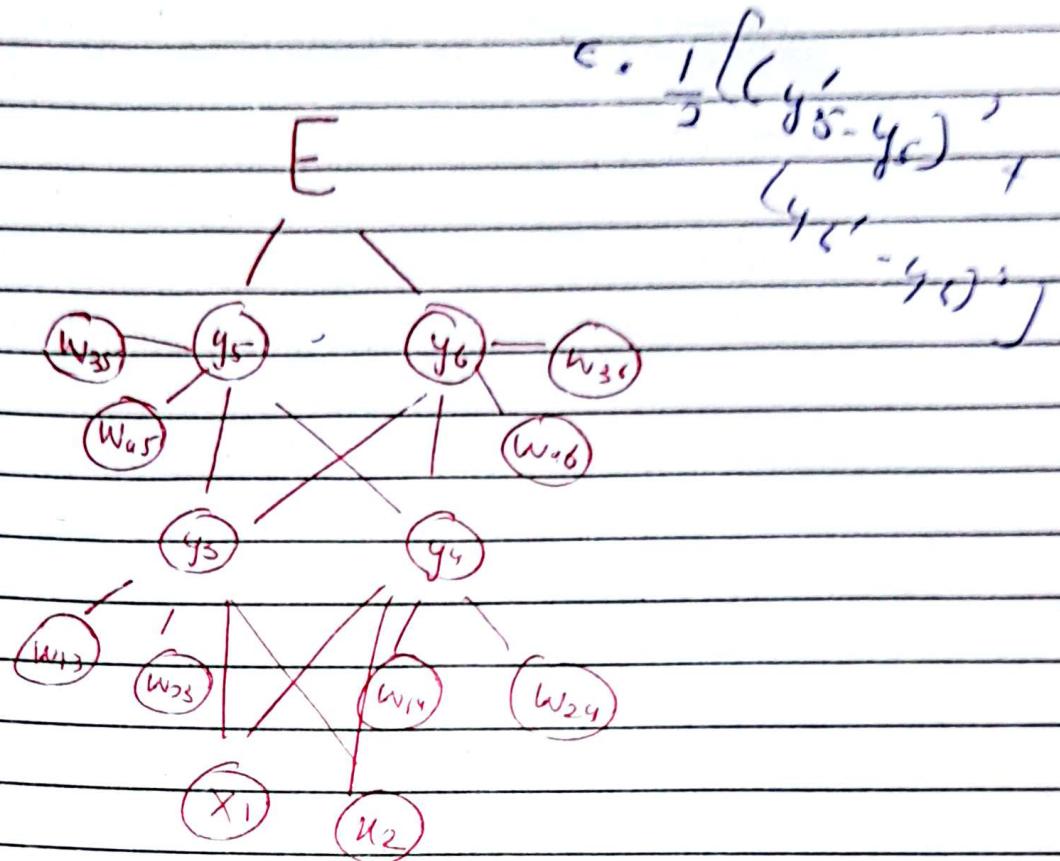
$$\frac{\partial y_5}{\partial w_{35}} = y_3 \quad \frac{\partial y_6}{\partial w_{46}} = y_4 \quad \frac{\partial y_5}{\partial y_3} = w_{35} \quad \frac{\partial y_5}{\partial w_{45}} = w_{45}$$

$$\frac{\partial y_5}{\partial y_3} = w_{35} \quad \frac{\partial y_6}{\partial y_4} = w_{46} \quad \frac{\partial y_3}{\partial w_{13}} = x_1, \quad \frac{\partial y_3}{\partial w_{23}} = x_2, \quad \frac{\partial y_4}{\partial w_{14}} = x_1, \quad \frac{\partial y_4}{\partial w_{24}} = x_2$$



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$$\frac{\partial E}{\partial y_5}, \frac{\partial E}{\partial y_6}, \frac{\partial y_5}{\partial w_{35}}, \frac{\partial y_5}{\partial w_{45}}, \frac{\partial y_6}{\partial w_{36}}, \frac{\partial y_6}{\partial w_{46}}$$

$$\frac{\partial y_5}{\partial y_3}, \frac{\partial y_5}{\partial y_4}, \frac{\partial y_6}{\partial y_3}, \frac{\partial y_6}{\partial y_4}, \frac{\partial y_3}{\partial w_{13}}, \frac{\partial y_3}{\partial w_{23}}, \frac{\partial y_4}{\partial w_{14}}, \frac{\partial y_4}{\partial w_{24}}$$

$$\frac{\partial E}{\partial y_5} = \cancel{\frac{\partial E}{\partial y_5}} \cdot \cancel{\frac{\partial y_5}{\partial w_{35}}} \cdot \cancel{\frac{\partial y_5}{\partial y_3}} \cdot \cancel{\frac{\partial y_3}{\partial w_{13}}} \cdot \cancel{\frac{\partial y_3}{\partial y_1}} \cdot \cancel{\frac{\partial y_1}{\partial w_{13}}}$$

$$\frac{\partial E}{\partial y_5} = -(y_5' - y_5) \quad \frac{\partial E}{\partial y_6} = -(y_6' - y_6) \quad \frac{\partial y_5}{\partial w_{35}} = y_3 \quad \frac{\partial y_5}{\partial w_{45}} = y_4$$

$$\frac{\partial y_5}{\partial w_{36}} = y_3 \quad \frac{\partial y_6}{\partial w_{46}} = y_4 \quad \frac{\partial y_5}{\partial y_3} = w_{35} \quad \frac{\partial y_5}{\partial y_4} = w_{45}$$

$$\frac{\partial y_6}{\partial y_3} = w_{36} \quad \frac{\partial y_6}{\partial y_4} = w_{46} \quad \frac{\partial y_3}{\partial w_{13}} = x_1, \quad \frac{\partial y_3}{\partial w_{23}} = x_2, \quad \frac{\partial y_4}{\partial w_{14}} = x_1, \quad \frac{\partial y_4}{\partial w_{24}} = x_2$$

Date \_\_\_\_\_

(M|T|W|T|F|S)



$$\frac{\partial E}{\partial w_{35}} \rightarrow \frac{\partial y_5}{\partial w_{35}} \rightarrow \alpha$$

$$\frac{\partial E}{\partial w_{35}} = \frac{\partial E}{\partial y_5} \cdot \frac{\partial y_5}{\partial w_{35}}$$

$$\frac{\partial E}{\partial w_{45}} = \frac{\partial E}{\partial y_5} \cdot \frac{\partial y_5}{\partial w_{45}}$$

$$\frac{\partial E}{\partial w_{36}} = \frac{\partial E}{\partial y_6} \cdot \frac{\partial y_6}{\partial w_{36}}$$

$$\frac{\partial E}{\partial w_{46}} = \frac{\partial E}{\partial y_6} \cdot \frac{\partial y_6}{\partial w_{46}}$$

$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial y_5} \cdot \frac{\partial y_5}{\partial y_3} \cdot \frac{\partial y_3}{\partial w_{13}} +$$

$$\frac{\partial E}{\partial y_6} \cdot \frac{\partial y_6}{\partial y_3} \cdot \frac{\partial y_3}{\partial w_{13}}$$

We ignore this approach (tree traversal  
make calculation explicit)

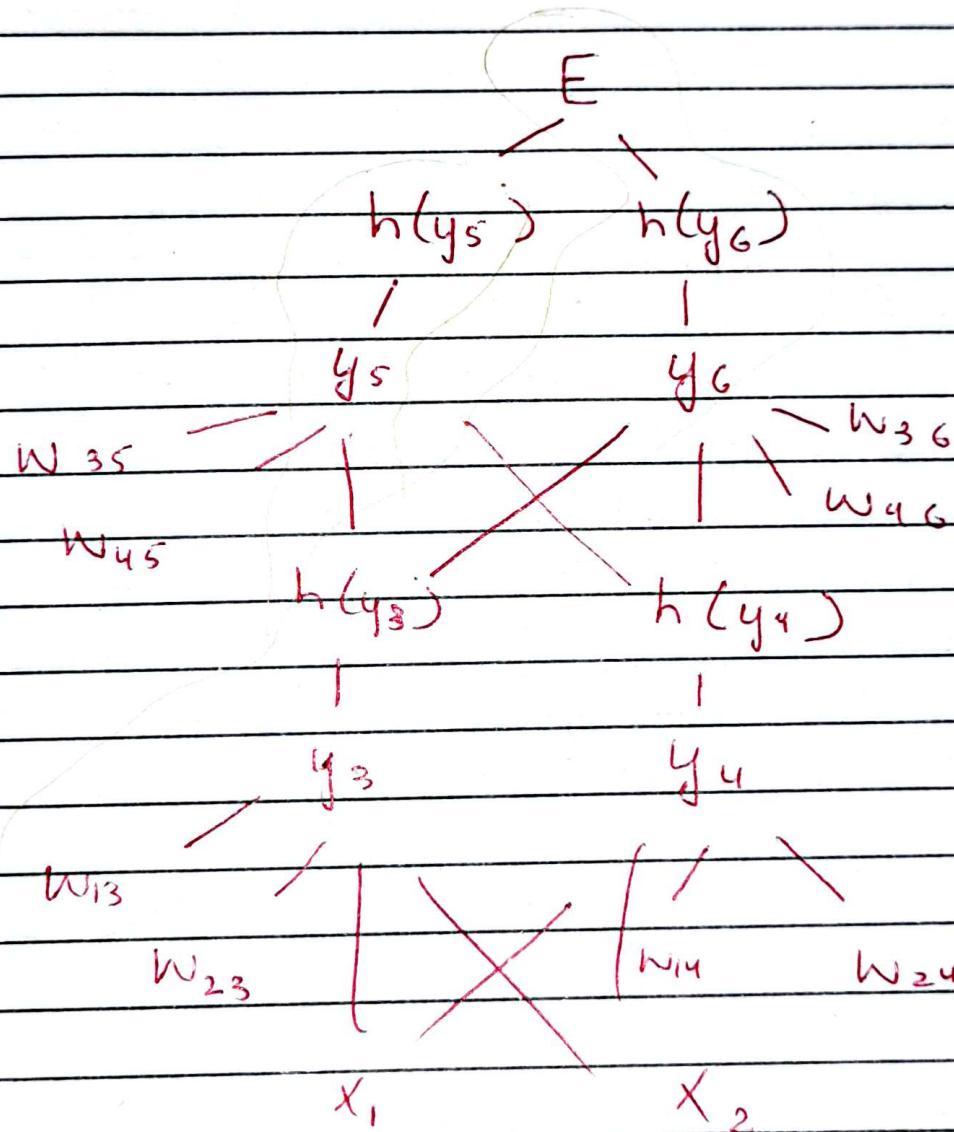


Date \_\_\_\_\_

(M T W T F S)

$$h(y) = \frac{1}{1+e^{-y}}$$

$$\epsilon = \frac{1}{2} (h(y)' - h(y))^2$$



Date \_\_\_\_\_

MTWTFSS

$$\frac{\partial E}{\partial h(y_5)} = -(h(y_5)' - h(y_5^-))$$

$$\frac{\partial E}{\partial h(y_6)} = -(h(y_6)' - h(y_6^-))$$

 $\partial H_{ij}$ 

$$\frac{\partial h(y_5)}{y_5} = \frac{e^{-y_5}}{(1+e^{-y_5})^2}$$

$$\frac{\partial h(y_6)}{y_6} = \frac{e^{-y_6}}{(1+e^{-y_6})^2}$$

$$y_5 = W_{51}h_1 +$$

$$W_{52}h_2$$

$$h(y_k) = \frac{1}{1+e^{-y_k}}$$

$$= (1+e^{-y_k})^{-1}$$

$$= -1 (1+e^{-y_k})^{-2} (e^{-y_k}) (-1)$$

$$= \frac{(e^{-y_k})}{(1+e^{-y_k})^2}$$

$$- \frac{e^{-y_k}}{(1+e^{-y_k})} \cdot \frac{1}{(1+e^{-y_k})} \quad [P - R(y_k)]$$

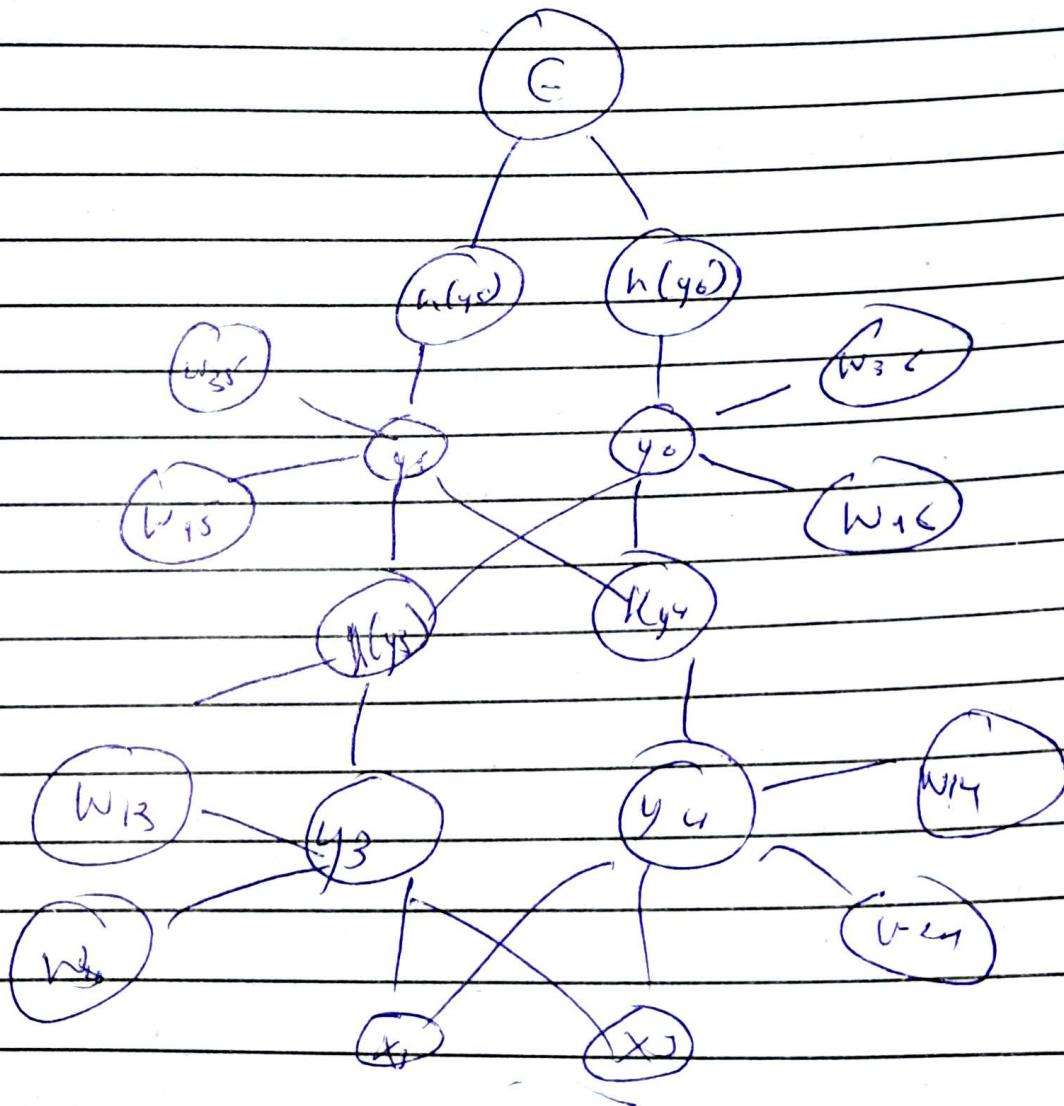
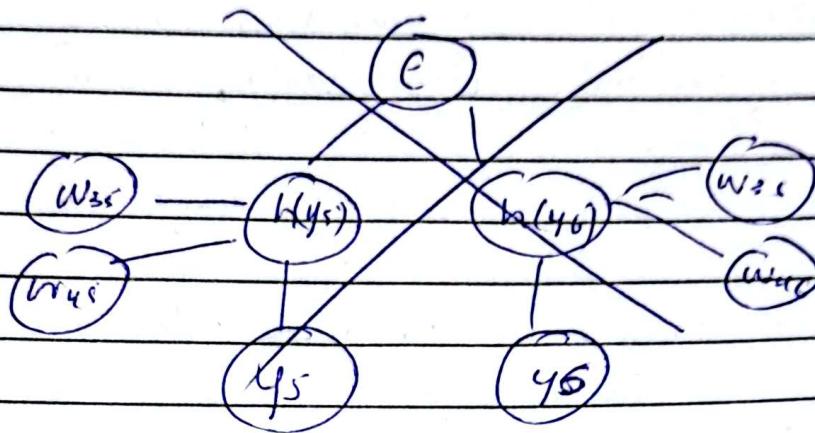
$$(1-y_k)(1+y_k)]$$

## Subject:



Date \_\_\_\_\_

(M|T|W|T|F|S)





Date \_\_\_\_\_

(M|T|W|T|F|S)

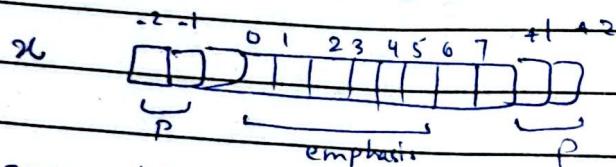
CNN

 $x_0, \dots, x_7$   
 $w_0, w_1, w_2$ 

Convolve

$$\text{convolve}(x, w) = \left[ \begin{array}{l} x_0 w_0 + x_1 w_1 + x_2 w_2 \\ x_1 w_0 + x_2 w_1 + x_3 w_2 \\ x_2 w_0 + x_3 w_1 + x_4 w_2 \\ \vdots \\ x_5 w_0 + x_6 w_1 + x_7 w_2 \end{array} \right]$$

Use padding to give importance to corners



② P

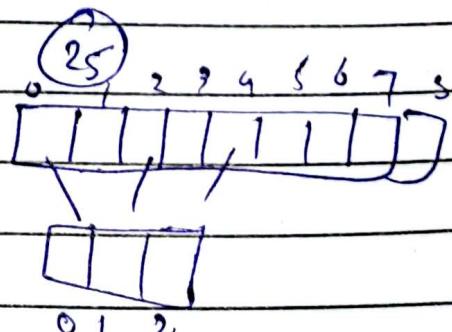
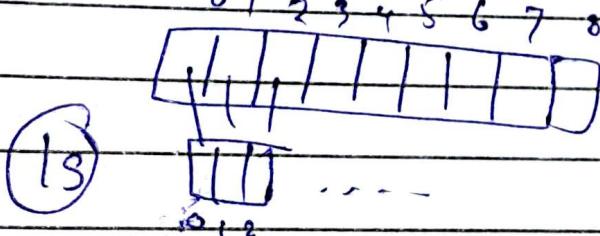
zeros add on both side  
 $P^l$  left side  
 $P^r$  right side

Formulate to calculate new size after padding and convolve operation

$$\text{size} \left( \text{size}(x) - \text{size}(k) + 2p \right) + 1$$

↑ original matrix      ↑ kernel matrix  
 ↓ stride                ↓ padding

stride





Date \_\_\_\_\_

M T W T F S

Convolution:  $I^{m \times n} \rightarrow O^{m' \times n'}$ 

Without padding and stride

$$S_I = S_O + 1$$

With padding and stride

$$\left[ \frac{S_I - S_O + 2P}{S_K} \right] + 1$$

different kernels and make more examples

Kernel matrix  $\rightarrow$  just like operator

$$x_i - x_{i+1}$$

$$x_i [0 \ 1]$$

↖  
tells position

$$x_i + x_{i+1}$$

$$x_i [0 \ 1]$$

Required

Output matrix  $\rightarrow$   $O^{256 \times 256}$  $I^{1600 \times 920} \quad k^{3 \times 3} \quad O^{256 \times 256}$ 

Calculation



Date \_\_\_\_\_

M T W T F S

$$\frac{1600 - 3}{2} + 1$$

$$\frac{920 - 3}{2} + 1$$

adjust padding  
and stride to  
end on even  
number  
for equally  
divisible.

$$\left[ \frac{1600 - 3 + 2P}{S} \right] + 1 = 600$$

$$\frac{1600 - 3 + 2P}{S} = 600 - 1$$

$$\frac{1600 - 3 + 2P}{S} = 599$$

$$\frac{1597 + 2P}{S} = 599$$

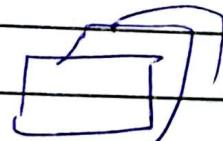
$$1597 + 2P = 599S$$

$$2P = 599S - 1597$$

$$P = \frac{599S - 1597}{2}$$

$$\left( \frac{S_I - S_{k+1} + 2P}{S} \right) + 1$$

$$\left( \frac{16 - 3 + 2(1)}{2} \right) + 1$$



$$H_k \times W_k \times C_k \times N_k$$

Date \_\_\_\_\_

[M T W T F S]



if we want to adjust it according  
LeNet / AlexNet

which take input image as  $32 \times 32 \times 1$

$1600 \times 920$

3

$$\begin{array}{r} 1600 \\ 32 \end{array} \quad \begin{array}{r} \times 920 \\ 32 \end{array}$$

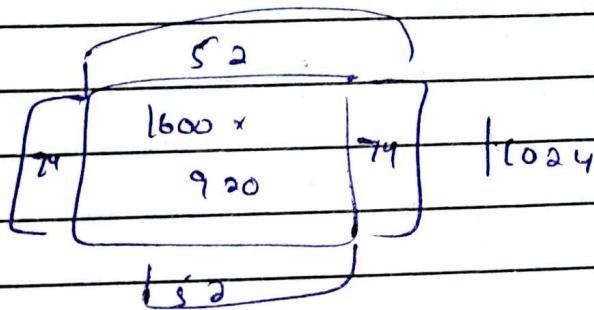
$32 \times 32 \times 50 \times$

3

$28.75^{\circ}$

50  $28.75^{\circ}$

or if we add padding



2048

$$1600 \times 920 \rightarrow 32 \times 32 \times 2048$$

$1600 \times 920$  resizing

$$\begin{array}{r} 1600 \\ 32 \end{array} \quad \begin{array}{r} 920 \\ 32 \end{array}$$

$50$   $28.75^{\circ}$

$(32 \times 32 \times 50 \times 28.75^{\circ})$

Date \_\_\_\_\_

(M T W T F S)

ker

$$H_k \times W_k \times C_k \times N_k$$

$$H_s = \left\lceil \frac{H - H_k + 2p}{s} \right\rceil + 1$$

$$W_s = \left\lceil \frac{W - W_k + 2p}{s} \right\rceil + 1$$

$$\begin{array}{c} [3 \times 3] \quad \textcircled{X} \quad [3 \times 3] \\ \text{dot} \\ \text{product} \end{array}$$

One value

$$3 \times 3 \times 3 \times 8$$

$$\frac{3^2 - 3 \times 8}{1} + 1 = 90$$

$$\left\lceil \frac{1600 - 3 + 2p}{s} \right\rceil + 1 = 600$$

Date \_\_\_\_\_

M T W T F S



Image

$3 \times 3 \times 3 \times 8$

$30 \times 30 \times 1 \times 8$

$15 \times 15 \times 1 \times 8$

Conv

(2)

pooling

Avg

RELU

Softmax

back propagation

$$\frac{\partial E}{\partial k_1} + \frac{\partial E}{\partial k_2} + \frac{\partial E}{\partial k_3} + \dots \frac{\partial E}{\partial k_{19}}$$

Pooling

Max

Avg

Min

1/8

$$15 \times 15 \times 1 \times 8 \rightarrow 10 \times 10 \times 4$$

$$10 = \left[ \frac{15 - 1 + 2(0)}{1} \right] + 1 = 6 \times 6$$

~~$6 \times 6 \times 1 \times 8$~~

pooling

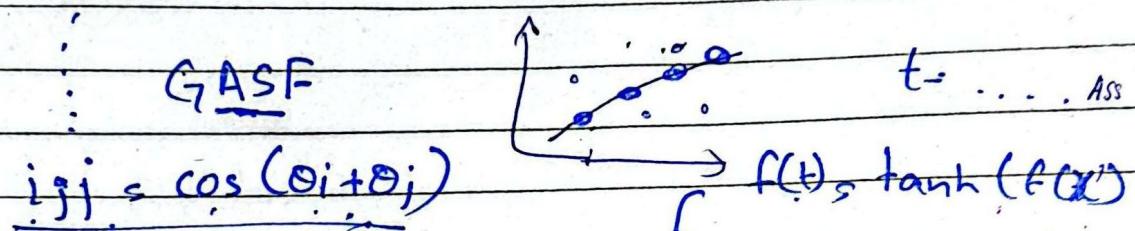
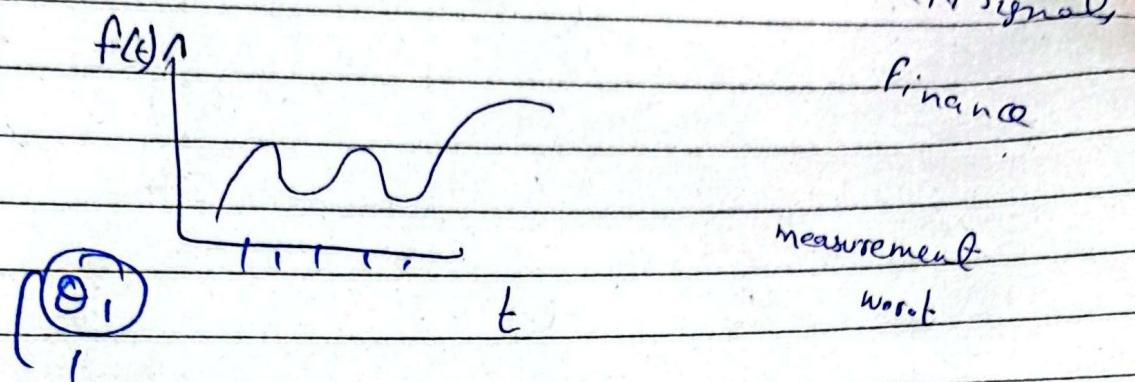
$6 \times 6 \times 1 \times 8$   $\xrightarrow{2}$   $3 \times 3 \times 1 \times 8$

$6 \times 6 \times 1 \times 4$   $\xrightarrow{2}$   $3 \times 3 \times 1 \times 4$

Day: \_\_\_\_\_

Date: \_\_\_\_\_

## Time Series Plots



GADF Transformation  $\Rightarrow$  Image  $\Theta_i = \cos'(f(t))$

$$i,j = \sin(\Theta_i + \Theta_j) \quad i,j = \cos(\Theta_i + \Theta_j)$$

NN

SIN

## 2 techniques

- Grammian Angular Summation Field
- h
- Difference Field  $\overset{(GADF)}{\sim}$

$$f(t), f(t) = \tanh(f(t))$$

Angle  $\rightarrow \cos$

$$\Theta = \cos^{-1}(f(t))$$

① Normalized Form  $(-1, +1) \tanh$

$$g(t) = \tanh(f(t))$$

② Angle  $(0, 2\pi)$

$$\theta = \cos^{-1}(g(t))$$

noise free  
data

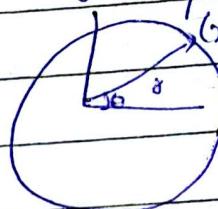
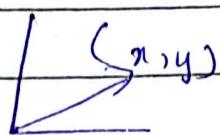
polar  
coordinates

are superior.

accuracy in angle

Transformation

Cartesian space to a polar space

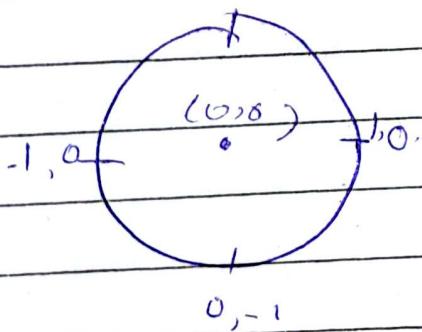


$(x, y) \sim (r\theta)$

Polar coordinates

background

$0, 1$



$$x = \cos(\theta)$$

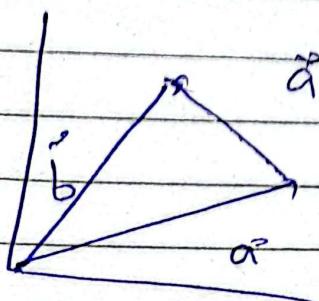
$$y = -\sin(\theta)$$

$$t = \cos(\theta)$$

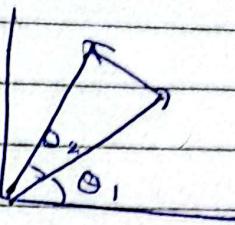
$$f(t) = -\sin(\theta)$$

Day: \_\_\_\_\_

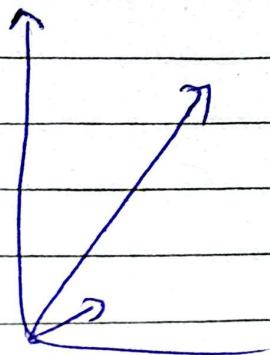
Date: \_\_\_\_\_



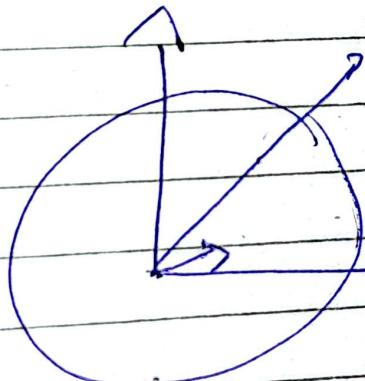
$$a - b \rightarrow a + b$$



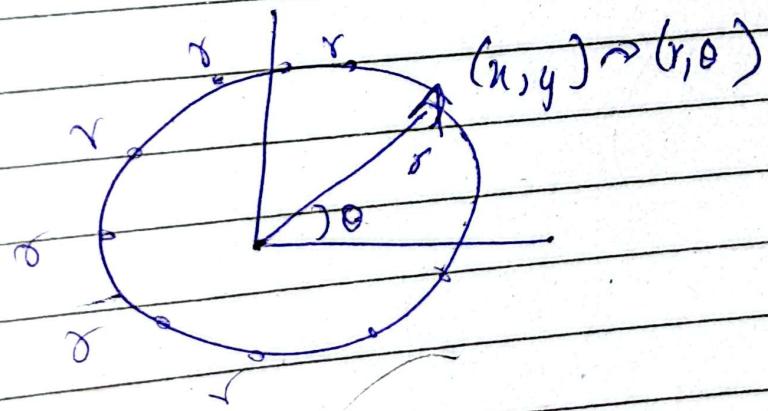
$$O_2 - O_1$$



angle difference less,  
r diff difference more



In  
Unit circle  
representation



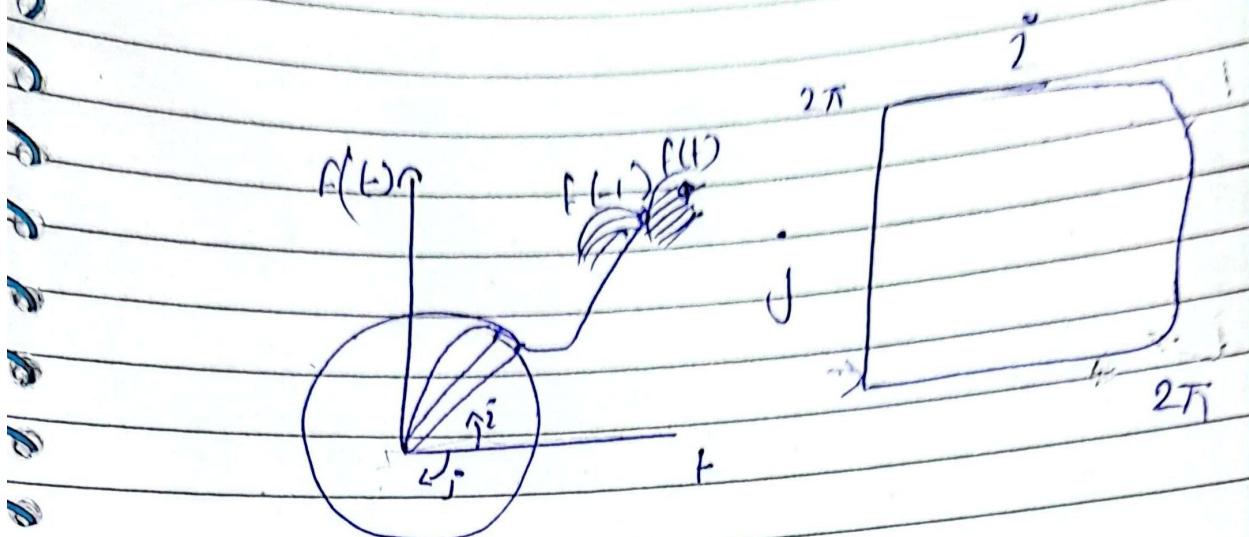
$$(x, y) \rightarrow (r, \theta)$$

Day:

Date:

$$\begin{array}{c} f(t) \\ \curvearrowleft \curvearrowright \\ f(t+1) \end{array}$$

$$f(t), f(t+1) + \text{cloud}$$



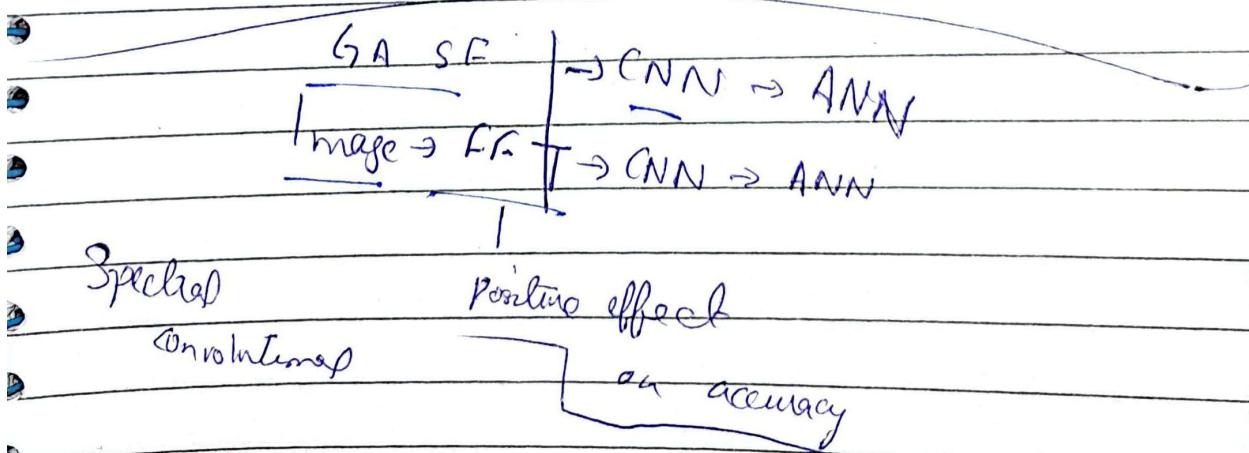
③ Construct Field (GASF)

$$\text{for loop } \rightarrow (i, j) = \cos(\theta_i + \theta_j) \quad \begin{matrix} \text{base level} \\ k \\ \text{pattern} \end{matrix}$$

global GADF

$$(i, j) = \sin(\theta_i - \theta_j) \quad \begin{matrix} \text{close by} \\ k \\ \text{pattern} \end{matrix}$$

minutes

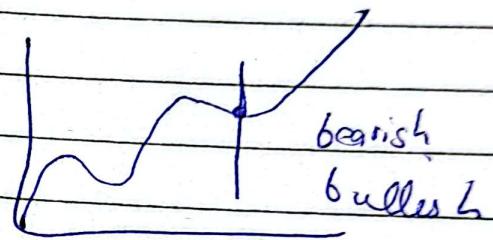


Day:

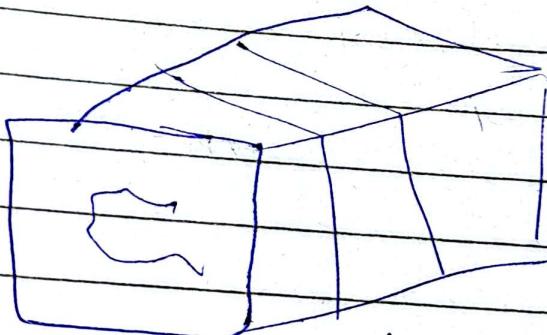
Date:

obtained required

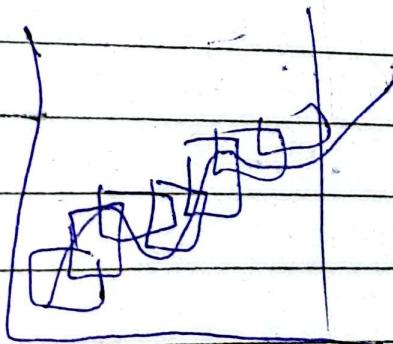
O O - H  
O O - H  
O O - H



a+a  
a+b

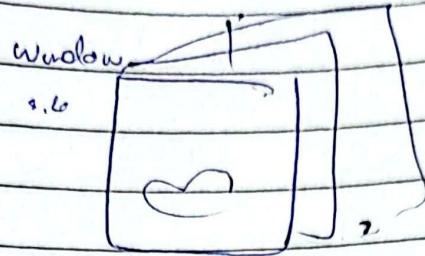


bearish bullsh. O ?  
O I

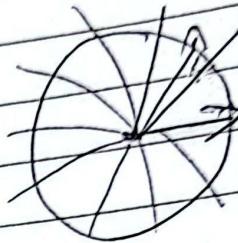


Day:

Date:



Window  
size,



① data  $f(n)$ .

②  $f(t) = \tanh(f(x))$   
 $[a, b, c, d]$

$\theta = \cos^{-1}(f(t))$

$\theta_1, \theta_2$

$E_{ij} = \cos(\theta_i + \theta_j)$

Date \_\_\_\_\_

GASF

MTWTFSS



G ADF

(1) Normalized form  $[-1, 1]$

(2)  $\theta = \tanh^{-1}(f(t))$

(3) Angle  $(0, 2\pi)$

$$\theta = \cos^{-1}(G(t))$$

(4) Construct field (GASF)

for loop

$$(i, j) = \cos(\theta_i + \theta_j)$$

$\sum_{i=0}^n$

$(0, i)$

$-3x - 1 \times 3$

$$= 0 + 9 + 9 + 9$$

$3x \quad 3x \quad +$   
 $-3x \times 1 \times 3$

$$= 27$$

Day: \_\_\_\_\_

Date: \_\_\_\_\_

RNN

Hopfield Neural Networks

2024 Nobel Prize Physics → AI

Unsupervised

Energy  $\leftrightarrow$  Weights

(Ising Model)



spin effect magnetic spin

both energy are in same direction

positive effect

no need to spin



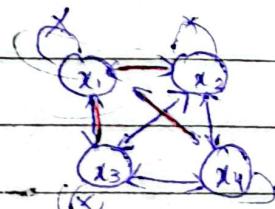
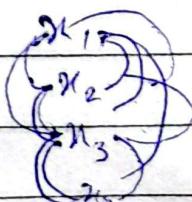
→ need energy to align them

Energy represented as -1 and +1

① 1 layer

② Ac: Sign ( $\sim$ )  $\{-1, +1\}$  Bipolar $\{0, 1\}$  P- binary

$$\delta(z) = \begin{cases} +1 & z \geq 0 \\ -1 & z < 0 \end{cases}$$

2x2 Image  $\rightarrow$  $\hookrightarrow w^{4 \times 4}$ 

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \\ w_{41} & w_{42} \end{bmatrix}$$

Day: Images  
Date:

## Outer product

Date:

(3)  $W = \sum_{i=1}^k x_i^k (x_i^k)^T$  if  $i \neq j$   
 $0$  if  $i = j$

$$= \{ x_i^k (x_i^k)^T \}$$

Diagonal

Input  $x$   $\in \mathbb{R}^n$   
 $x = [x_1 \ x_2 \ x_3 \ x_4]$   
 $x \in \mathbb{R}^4$

$$W \in \mathbb{R}^{4 \times 4}$$

$2 \times 2$

$2 \times 2$ ,  
so

$$x^T W x = \sum w_{ij} x_{ij}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

This could be an image

$$W = \sum w_{ij} x_{ij}$$

neural network  
one directional

Here,

bidirectional

(4) Stability Test

$$x_i^k (x_i^k)^T$$

$$S = 8 (\sum w_{ij} x_{ij}) + \beta I$$

$$S = \begin{bmatrix} -3 & 3 & 3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & -1 & 1 \\ -1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

-1 -1 -1

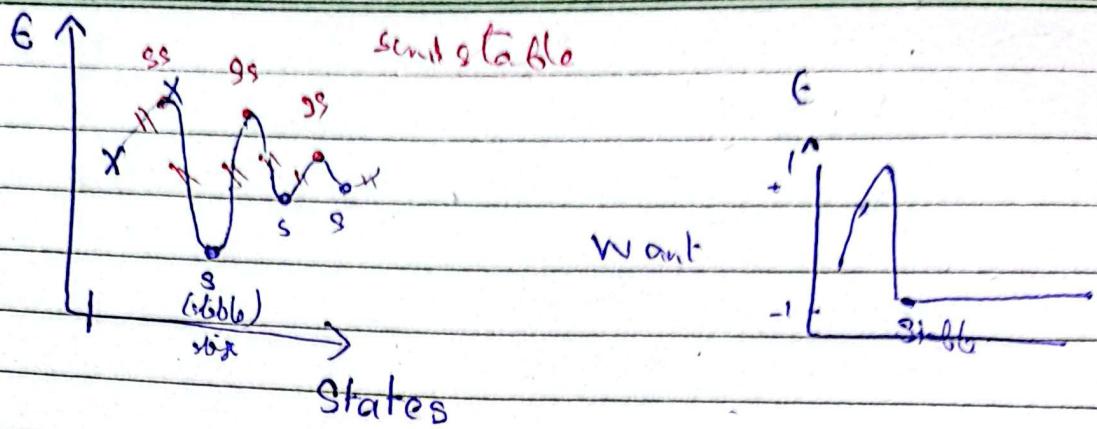
(5) Energy

$$E = \sum_{i,j} w_{ij} s_i s_j$$

$$= \sum_i \sum_j w_{ij} s_i s_j \text{ (using equation)}$$

Day: \_\_\_\_\_

Date: \_\_\_\_\_



Energy is increasing in a system, more distortion in the system (between the images)

$\Rightarrow$  Min amount of variation

Min amount will always at stable points

- local minima

$$E = \sum_i \sum_j W_{ij} S_i S_j \quad \begin{bmatrix} \dots \\ \vdots \\ \dots \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$i=0$   
 $j=0, 1, 2, 3$

$$\begin{aligned} &= 0 + (-1) \times (-3) \times (3) + \dots \\ &= 0 + 9 + 9 + 9 \end{aligned}$$

$$= 27$$

+1 Answer

Some case positive and negative

Day:

Date:

$$i = 1$$

$$j = 0, 1, 2, 3$$

$$9 + 0 + 9 + \dots$$

$$\begin{bmatrix} i \\ -i \end{bmatrix}^n \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

add the weight  
of both images;

$$\begin{bmatrix} 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & -2 \\ 2 & 0 & -2 & 0 \end{bmatrix}$$

getting small,  
increasing  
depth  
size  
High  
acc  
accuracy

Stability

$$S_s \begin{bmatrix} 4 & 0 & -4 & 4 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$0 + 0 + 32 + 32$$

High  
acc  
accuracy  
Energy Based  
Machine Learning

Quadratic unconstrained  
binary

QUBO

We want to find

$$\min \left( E = \sum_{ij} N_{ij} s_i s_j \right)$$

$$(-2)(4)(-4) + (2)(4)(4)$$

Quantum

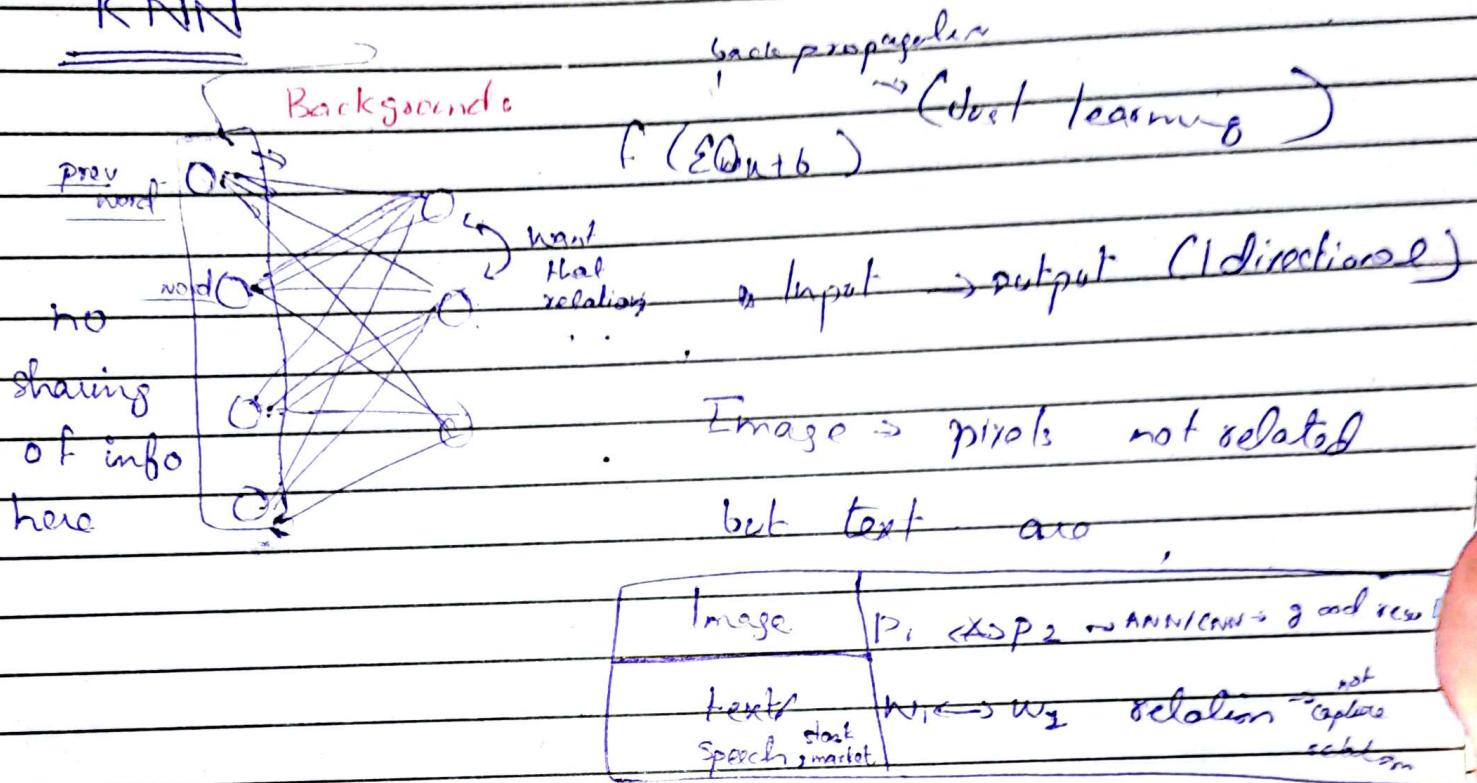


Castelli

Date \_\_\_\_\_

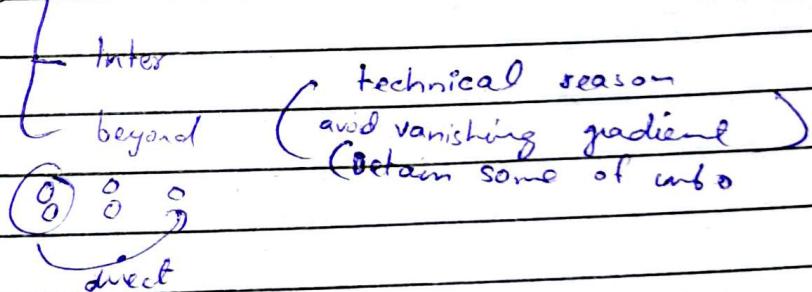
MTWTF

## RNN



Recurrent  $\Rightarrow$  recall  
→ previous

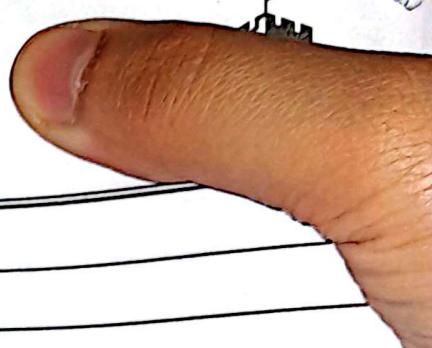
Two approaches



attention: take something from a new older layer and take that info relate to a further layer

Date \_\_\_\_\_

MTWTF



hope field



connected

Everybody with everybody

RNN

(immediate neighbour)

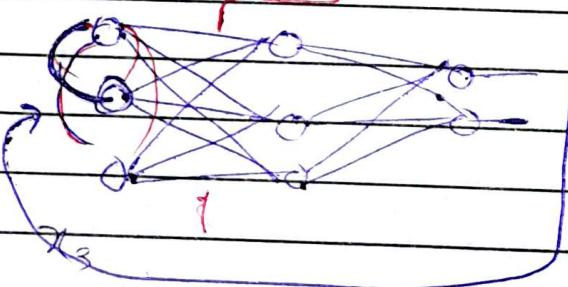
$$\sigma(Wx + b)$$

(bidirectional)

RNN

$$\sigma(Wx + W_h h)$$

$$\sigma(Wx + Ux_{t-1} + b)$$



→ (Just select the current previous element neighbor)

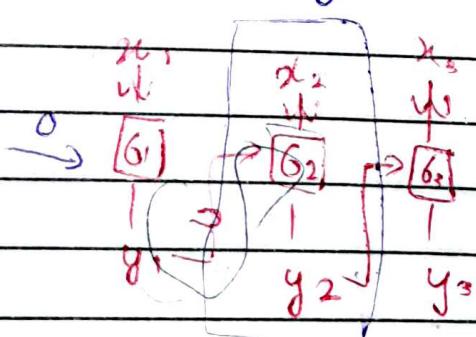
$$h_1 \quad h_2$$

$$x_1 \quad x_2$$

$$x \rightarrow \text{forget}$$

$$Wx + Ux_{t-1} + b$$

$$Wx + Ux_{t-1} + b$$



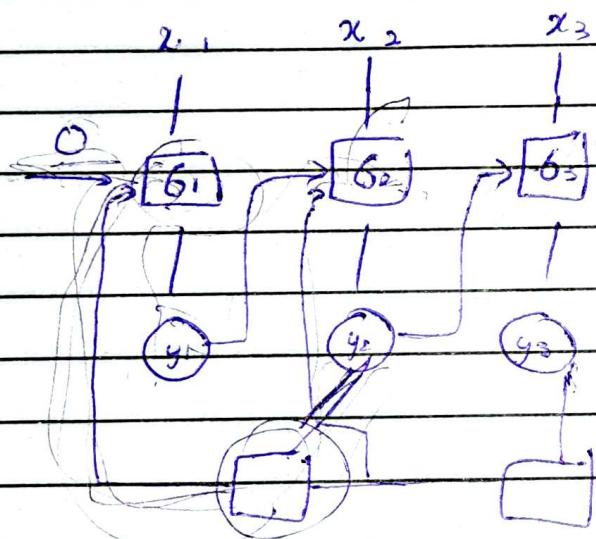
→ Dependency

↳ slow

RNN

## Bidirectional RNN

$$\delta \left( Wx_t + U^B x_{t-1} + U^F x_{t+1} + b \right)$$



The cat ate a mouse.

S-O relation

The previous cat infuture ate next.

O-S relation (positioning depend on previous and future objects)

Bidirectional adding more context.