Testing theories of gravity using pulsars

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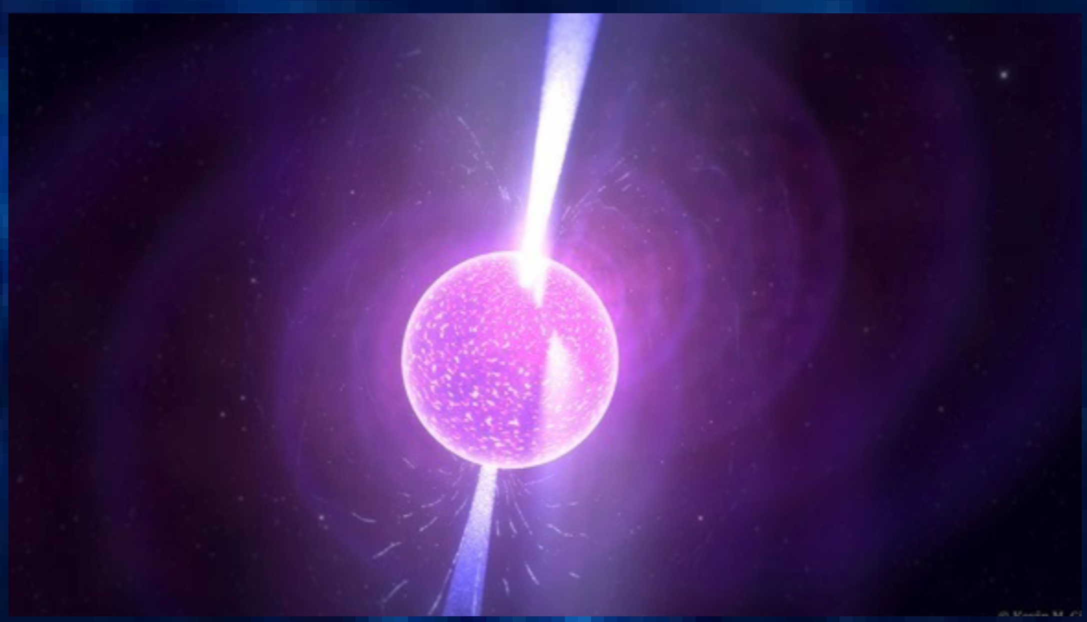
Introduction

Pulsars are rapidly-rotating and highly-magnetised neutron stars that emit radiation from their poles.



Why useful?, They provide tests of strong-field gravity in ways impossible on Earth-bound laboratories or even in our Solar system. The pulsars of interest are found in binary systems meaning they have a companion star. To fully describe a binary system 6 parameters known as Keplerian parameters are needed. Tests of the theories of gravity are carried out by examining deviations from the Keplerian description on a mass diagram.

Objectives

- The first objective of this project was to understand the orbital elements and post Keplerian (PK) parameterization in General relativity (GR), this meant working through the methods of derivations of the PK parameters in order to get a grasp of the process.
- Then once these methods were understood the next objective was to understand other theories of gravity and attempt to derive the same PK parametrizations.
- Finally the last objective was to develop code that can take in real pulsar data and produce mass-mass diagrams for a given theory of gravity to test whether its description of the PK parameters match reality. This involves understanding of the software that analyses the pulsar data (TEMPO).

Post Keplerian parameters

Due to various effects of gravity Keplerian parameters are not stable in time. This is very evident in pulsar binaries. Here the timing model of how the system changes is parameterized by post Keplerian (PK) parameters [1]. For example in General relativity:

- $\dot{\omega}$ the periastron advance
- $\dot{P}_b$ the orbital decay
- γ Einstein delay
- r Shapiro range
- $s = sin(i)$ Shapiro shape

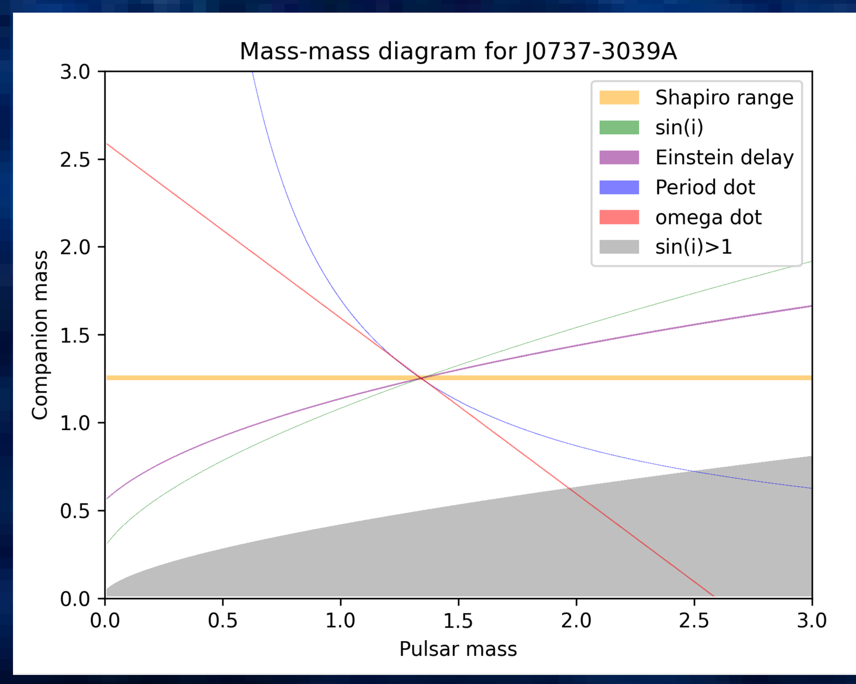
These parameters can be expressed purely as functions of the mass of the pulsar m_p and its companion m_c [2]:

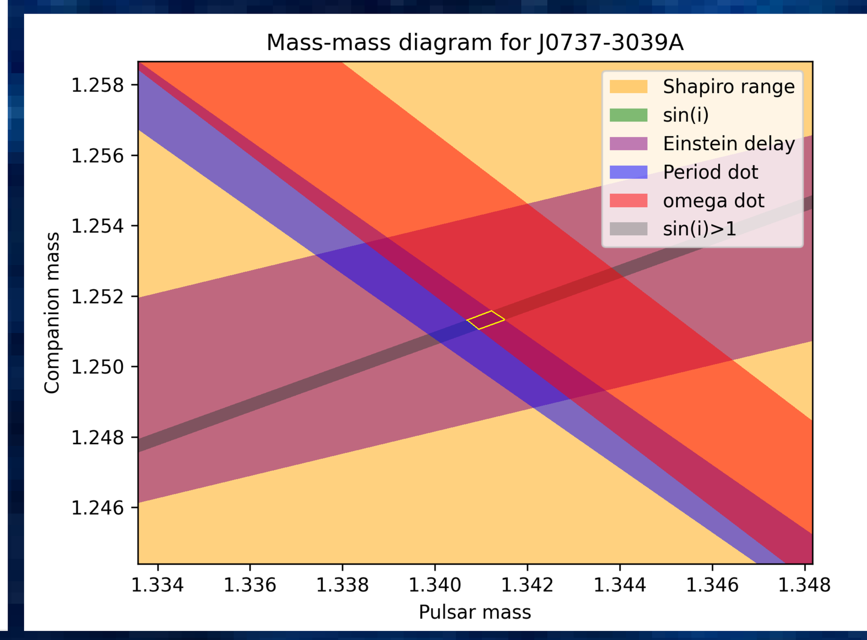
- $\dot{\omega} = 3T_{\odot}^{2/3}\left(\frac{2\pi}{P_b}\right)^{\frac{5}{3}}\frac{(m_p+m_c)^{\frac{2}{3}}}{(1-e^2)}$,
- $\gamma = T_{\odot}^{2/3}\left(\frac{2\pi}{P_b}\right)^{-\frac{1}{3}} e\,\frac{m_c(m_p+2m_c)}{(m_p+m_c)^{\frac{4}{3}}}$
- $r = T_{\odot} m_c$
- $s = sin(i) = T_{\odot}^{-\frac{1}{3}}\left(\frac{2\pi}{P_b}\right)^{-\frac{2}{3}}\frac{x(m_p+m_c)^{\frac{2}{3}}}{m_c}$
- $\dot{P}_b = -\frac{192}{5}T_{\odot}^{\frac{5}{3}}\left(\frac{2\pi}{P_b}\right)^{\frac{5}{3}}\frac{(1+(\frac{73}{24})e^2+(\frac{37}{96})e^4)}{(1-e^2)^{\frac{7}{2}}}\frac{m_p m_c}{(m_p+m_c)^{\frac{1}{3}}}$

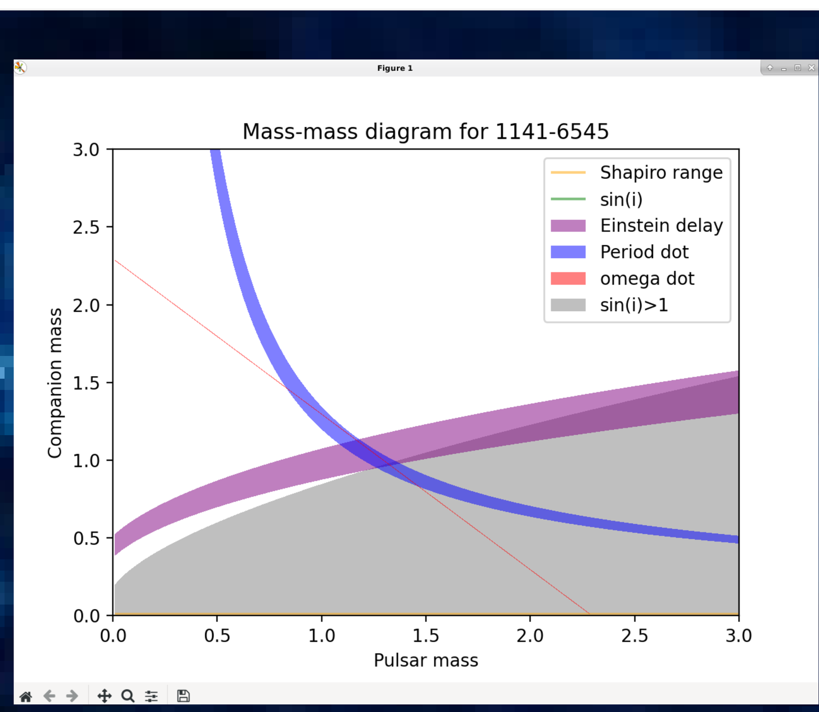
Derivations of most of these parameters were not easily available in any publication I could find. So, time was spent carefully deriving the above expressions and compiling them all in one place to avoid such an issue for anyone else.

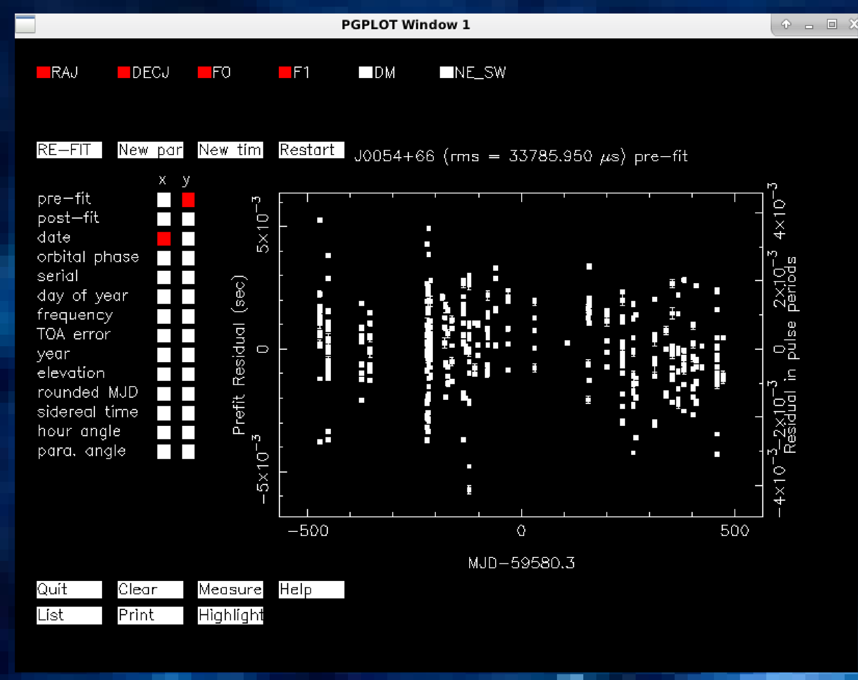
These parameters allowed me to create mass-mass diagrams for several pulsars, providing a method of testing a given theory based on the fact that all the parameter curves must cross at the same point. For N measured parameters the mass-mass diagram provides $N-2$ tests of the given theory of gravity. To no surprise GR passes these tests with flying colours as can be seen in the graphs above. These mass-mass diagrams were developed with code that can be found at: https://github.com/Tbrosnan12/Hamilton-trust-2023.

Mass-mass diagrams









The bottom right image shows the pulsar timing residuals in the tempo2 software. Here time of arrival of pulsars can be analysed and the parameters describing them like the Keplerian and post Keplerian parameters can be fitted for.

Yukawa like gravity and $f(R)$ theories

The simplest way to modify GR is to generalize the Einstein-Hilbert action by changing from just the Ricci scalar to an arbitrary function of the Ricci scalar $f(R)$, with the simple condition that in the weak field limit the theory tends towards GR. Modifying the Einstein-Hilbert action leads to:

$$S = \int \sqrt{-g}f(R)d^4x + S_m$$

This then leads to a slightly changed Schwarzschild metric that takes the following form:

$$g = -\left(1 - \frac{2GM}{rc^2}\frac{(1+\delta e^{-\frac{r}{\lambda}})}{1+\delta}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\frac{(1+\delta e^{-\frac{r}{\lambda}})}{1+\delta}\right)^{-1}dr^2 + r^2d\Omega^2$$

Using this metric it was found that derivations of the PK parameters for this theory resulted in the original GR PK parameters with added corrections. However since through methods like gravitational wave detection, a lower bound of $\lambda > 1.6\times10^{16}$ m has been set [3]. This leads to the extra contributions to the PK parameters being smaller than the terms that are all ready ignored in the calculations under GR. This means that the simplest f(R) theory cannot be tested without also including the next order corrections in GR.

Conclusion

In conclusion a thorough investigation into the GR PK parameters and their derivations was carried out. Effective code was developed to produce mass-mass diagrams for given pulsar data and confirm GR's description of binary pulsars. This was aided by tempo2's fitting of the PK parameters. Analysis of the PK equations of Yukawa-like gravity revealed extra contributions to the parameters, though it was concluded it would be difficult to accurately test this theory of gravity with binary pulsars as the contributions were of smaller order than the order set out to calculate the GR parameters. Further analysis of GR PK equations and far more accurate measurements of pulsars would be needed to come to a more concrete conclusion on Yukawa gravity.

References

[1] M Kramer et al. "Strong-field gravity tests with the double pulsar". In: *Physical Review X* 11.4 (2021), p. 041050.

[2] Duncan Ross Lorimer and Michael Kramer. *Handbook of pulsar astronomy*. Vol. 4. Cambridge university press, 2005.

[3] Clifford M Will. "Solar system versus gravitational-wave bounds on the graviton mass". In: *Classical and Quantum Gravity* 35.17 (2018).