Quantum Field Theory I

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Notes taken in Professor Samson Shatashvili class, Michaelmas Term 2024

"We will work in "God-given" units, where $\hbar = 1 = c$"

-Peskin & Schroeder

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1 QFT trailer

- We wish to see if we can perform a calculation in quantum field theory, just by elementary means, i.e. via dimensional analysis ect.

  Consider the head on collision of an electron e^- and e^+ that results in the production of a muon μ^- anti muon μ^+ pair, shown below:

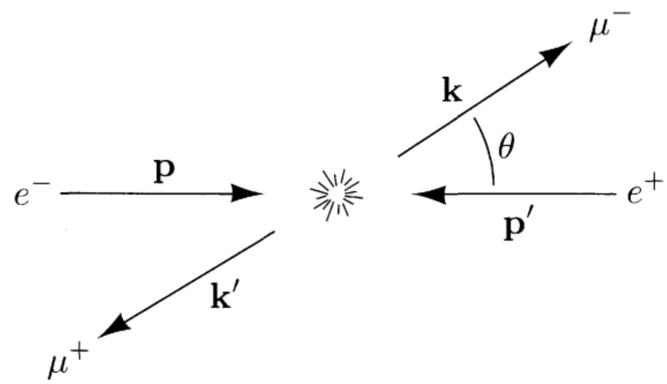

Figure 1: *electron positron annihilation*

- The calculation we would like to perform is the differential cross section, that is the derivative of the cross section σ with respect to the solid angle Ω, $\frac{d\sigma}{d\Omega}$. This is a useful quantity as it is easily experimentally observed. In a particle collider, electrons and positrons are prepared in batches of length l_A and l_B and densities ρ_A and ρ_B respectively. When the two batches collide if the over lapping area of the head on collision is A, then the cross section is given by:

$$\sigma = \frac{\text{Number of events}}{\rho_A \rho_B l_A l_B A}$$

- We now look at the dimensions of our quantities. Conveniently from our use of God given units, i.e. $\hbar = c = 1$ we have that momentum, mass and energy have the same units as the energy mass equivalence becomes $E^2 = p^2 + m^2$. We also have from the Heisenberg's uncertainty principle that $\Delta p \Delta x \sim 1$. Thus the dimensions of mass and length are inversely related.

  We can from this easily see that the dimensions of the quantity $[\rho_A \rho_B l_A l_B A]$ is $[m^2]$, which makes the dimensions of $\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{1}{m^2}\right]$ as angles are unitless. With this we can say that this quantity is also inversely proportional to the energy squared times some positive quantity that depends on the angle:

$$\boxed{\frac{d\sigma}{d\Omega} \propto \frac{1}{E^2}|\mathcal{M}(\theta)|^2} \tag{1.1}$$