Statistical Thermodynamics

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" Put a cool quote here or your lame "

-Thomas Brosnan

Contents

1 Concepts and Terminology

1.1 System macrostate

- Properties at large scale of the system when we know constraining thermodynamic parameters such as P,V and T.

1.2 Quantum microstate

- Each quantum state is a separate and distinct microstate of the system.

1.3 Weakly coupled systems

- We assume weakly coupled systems. These are isolated systems such that the total macro-parameters E, V, N, are constant. Weak coupling implies that the energy levels of a single particle are effectively unchanged by particle interactions, but the interaction is sufficiently large to allow energy exchange. This means that at equilibrium the system must have a common temperature.

  This assumption makes the quantum mechanics involved easier, as coupled systems in quantum mechanics can be quite complex where as simple systems like a gas of H_2 can be solved analytically (eigenvalues of vibration,rotation and translation).

1.4 Locality

- Localised particles are particles that are restricted to a certain place like a lattice for example. This way each particle of the lattice is *distinguishable*, i.e. it can be told apart from the other particles in the gas due to its position. If particles in a system are non-localised they are *indistinguishable*.

1.5 Bosons and Fermions

- In the case of indistinguishable particles we have two cases. Consider a two state system with two single particle states (orbitals) a and b and a wave function Ψ describing the system. We assume (weakly coupled systems) that the particles are non-interacting so that we can write the wavefunction as a product of the two states:

$$\boxed{\Psi(1,2) = \psi_a(1)\psi_b(2) \quad \mathit{or} \quad \Psi(2,1) = \psi_a(1)\psi_b(2)} \tag{1.1}$$

  Where here there are two possibilities for the states as the particles are indistinguishable. Quantum mechanics tells us that the wavefunction must be a linear combination of all the (equally likely thus same amplitude) possible states , so we write:

$$\boxed{\Psi(1,2) \propto \psi_a(1)\psi_b(2) \pm \psi_a(1)\psi_b(2)} \tag{1.2}$$

  The $\pm$ here indicates the two possibilities with respect to particle exchange. If we swap two particles and the system is symmetric (i.e. we get a $+$) then these particles are *Bosons*. And we can have any number of them in a single state, as if the states $a = b$ 1.2 is not 0.

However if we swap two particles and the system is anti-symmetric (now we get a $-$ sign) then the particles are *Fermions* and we can only have one fermion per state, as 1.2 is 0 if $a = b$. This is the Pauli exclusion principle.