

# Problem Solving

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Sheet of equations useful for the Problem solving paper

”Mo money mo Problems”  
- Samson

# Contents

<b>1</b>	<b>Mechanics</b>	<b>4</b>
1.1	Constant acceleration . . . . .	4
1.2	Force . . . . .	4
1.3	Friction . . . . .	4
1.4	Work . . . . .	4
1.4.1	Power . . . . .	4
1.5	Circular motion & Rotation . . . . .	4
1.5.1	Rotational Kinetic energy . . . . .	5
1.5.2	Parallel axis Theorem . . . . .	5
1.5.3	Torque . . . . .	5
1.5.4	Angular momentum . . . . .	5
1.6	Center of Mass . . . . .	6
1.7	Stress/strain . . . . .	6
1.8	Gravitation . . . . .	6
1.8.1	Acceleration due to gravity . . . . .	6
1.9	Orbits . . . . .	6
1.9.1	Kepler's Period . . . . .	6
1.10	Periodic motion . . . . .	7
1.10.1	Pendulum . . . . .	7
1.10.2	Damped Harmonic Motion . . . . .	7
1.11	Density and Pressure . . . . .	7
1.11.1	Pressure in a fluid . . . . .	7
1.11.2	Fluid Flow . . . . .	7
1.11.3	Bernoulli's equation . . . . .	8
1.11.4	Wave equation . . . . .	8
1.11.5	Wave Power . . . . .	8
1.11.6	Beat frequency . . . . .	8
1.11.7	Doppler affect . . . . .	8
<b>2</b>	<b>Thermodynamics</b>	<b>9</b>
2.1	Heat . . . . .	9
2.1.1	Heat Capacity . . . . .	9
2.1.2	Latent Heat . . . . .	9
2.1.3	Heat Flux . . . . .	9
2.2	Ideal Gas . . . . .	9
2.2.1	Energy of a Gas . . . . .	10

# 1 Mechanics

## 1.1 Constant acceleration

- For constant acceleration  $a$  the equations of motion can be written in three useful forms. The variables here are initial velocity  $u$  final velocity  $v$ , time  $t$  and displacement  $s$ :

$$\begin{aligned}v &= u + at \\v^2 &= u^2 + 2as \\s &= ut + \frac{1}{2}at^2\end{aligned}$$

## 1.2 Force

- The force due to a potential  $U$  is:

$$\mathbf{F} = -\nabla U$$

## 1.3 Friction

- Friction is given by the following expression, where  $\mu$  is the *co-efficient of friction* and  $N$  is the normal force acted on the object by the surface it is sliding across.

$$F_{\text{Frict}} = \mu N$$

## 1.4 Work

- For constant Force work is just defined  $W = \mathbf{F} \cdot \mathbf{s}$ . (Force times displacement). If the force is not constant:

$$W = \int_a^b \mathbf{F} \cdot d\mathbf{s}$$

### 1.4.1 Power

- Power* is defined as:

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

Where  $\mathbf{v}$  is velocity.

## 1.5 Circular motion & Rotation

- In circular motion the force acting as the *centripetal* force (force pulling towards the center, gravity, tension in rope, ect) is balanced by a *centrifugal* force equal in magnitude but opposite direction, given by:

$$F_{\text{fugal}} = \frac{mv^2}{r} = mr\omega^2, \quad (\text{as } v = \omega r)$$

Where here  $v$  is the tangential velocity,  $r$  is the radius,  $m$  is the mass of the object in motion and  $\omega$  is the angular velocity.

### 1.5.1 Rotational Kinetic energy

- Usually kinetic energy is just  $\frac{1}{2}mv^2$ , but for rigid bodies rotation it is:

$$K = \frac{1}{2}I\omega^2$$

Where  $I$  is the moment of inertia. A list of these can be found in the formula and tables booklet for different geometries.

### 1.5.2 Parallel axis Theorem

- This is the rule that tells us how to add the self rotation moment of inertia  $I_{\text{self}}$  to the orbital rotation of a mass  $M$  at radius  $R$ :

$$I_{\text{total}} = I_{\text{self}} + MR^2$$

### 1.5.3 Torque

- This is defined as:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

We also have that:

$$|\boldsymbol{\tau}| = \alpha I$$

Where  $\alpha = \frac{d\omega}{dt}$ . We can then find that the work done by a source the torque is:

$$W = \int_{\theta_0}^{\theta_1} \tau d\theta, \quad \implies P = \tau\omega$$

Where  $P$  is the power.

### 1.5.4 Angular momentum

- The standard definition is just:

$$\begin{aligned} \mathbf{L} &= \mathbf{r} \times \mathbf{p}, & \text{for a particle} \\ I\boldsymbol{\omega}, & & \text{Ridged body motion} \end{aligned}$$

We can also write the torque as  $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$ .

## 1.6 Center of Mass

- This is given by:

$$\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

## 1.7 Stress/strain

- Shear modulus  $G$  is:

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

Where shear stress =  $F/A$  (Force per Area) and shear strain =  $\frac{\Delta s}{l}$ . Length displaced  $\Delta s$  over the length of the object  $l$ . The same sort of equation holds for the other moduli. Youngs modulus is  $G$  with the word “shear” replaced with “Tensile”. Same goes with Bulk and Elastic.

## 1.8 Gravitation

- The Gravitational potential energy of a mass  $m$  in a gravitational field produced by a body of mass  $M$  is:

$$U = -\frac{GMm}{r} \implies \mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2} \hat{r}$$

### 1.8.1 Acceleration due to gravity

- This is just the gravitational force  $F_{\text{grav}}$  per unit mass:

$$g = \frac{GM}{r^2}$$

## 1.9 Orbits

- To find the escape velocity we just find the velocity that makes the total energy 0 when  $r = R$ , this results in:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

The Schwartzchild radius is when this velocity  $v$  is the speed of light  $c$ .

### 1.9.1 Kepler's Period

- This is the orbital period for a body in elliptical motion with semi-major axis  $a$ :

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

### 1.10 Periodic motion

- *Simple Harmonic motion* is defined as motion where the force is:

$$F = -kx$$

- This motion will then have a angular frequency  $\omega$ , which is related to the period of the motion  $T$ , by:  $T = 2\pi/\omega$ .
- Frequency is defined as one over the period  $f = 1/T = \omega/2\pi$ .

#### 1.10.1 Pendulum

- For a pendulum of length  $L$  we have that  $\omega = \sqrt{\frac{g}{L}}$ . If this pendulum is a rigid body with moment of inertia  $I$ , then this takes the form:  $\omega = \sqrt{\frac{mgL}{I}}$ .

#### 1.10.2 Damped Harmonic Motion

- The General solution to damped harmonic motion takes the form:

$$x(t) = Ae^{-\gamma t} \cos(\omega' t)$$

Where  $\omega' = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}}$ .

### 1.11 Density and Pressure

- Density is defined as:

$$\rho = \frac{m}{V}$$

- Pressure is in its most general form perpendicular Force  $F_{\perp}$  divided by area  $A$ :

$$P = \frac{F_{\perp}}{A}$$

#### 1.11.1 Pressure in a fluid

- At a height  $h$  below the surface is:

$$P = P_0 + \rho gh$$

#### 1.11.2 Fluid Flow

- For a fluid flowing through a pipe of changing cross sectional area the conserved quantity is the area  $A$  times the velocity of the fluid  $v$ . ( $A_1 v_1 = A_2 v_2$ ).

**1.11.3 Bernoulli's equation**

- The relationship between pressure and velocity in a fluid is given by:

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

**1.11.4 Wave equation**

- This takes the form:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Where  $y = y(x, t)$  and  $v = \lambda * f$  (wavelength by frequency) is the speed of the wave. The general form to the solutions of this equation are:

$$y(x, t) = A \cos(kx - \omega t)$$

**1.11.5 Wave Power**

- Given the tension  $T$  in a string with mass per length  $\mu = m/L$  the wave power is:

$$P = \frac{1}{2} \sqrt{\mu T} \omega^2 A^2$$

**1.11.6 Beat frequency**

- When there are two sources of oscillatory motion the beat frequency is the difference between the two underlying frequencies  $f_{\text{beat}} = |f_1 - f_2|$ .

**1.11.7 Doppler effect**

- The observed frequency of a source emitting a frequency  $f_0$  when the receiver is moving at velocity  $v_r$  and the source at velocity  $v_s$  is:

$$f = \left( \frac{c \pm v_r}{c \mp v_s} \right) f_0$$

Where we have + in the numerator if receiver is moving towards source and a - for away. And we have a + in the denominator if the source is moving towards the medium and a - if away.



## 2 Thermodynamics

### 2.1 Heat

#### 2.1.1 Heat Capacity

- Heat capacity is defined as the temperature derivative of the Energy:

$$C_V = \left( \frac{\partial E}{\partial T} \right)_V$$

For  $C_P$  just replace  $V$  with  $P$ .

- Heat energy change due to a temperature change  $\Delta T$  for a substance with heat specific capacity  $c \equiv C/m$  and mass  $m$ :

$$\Delta Q = mc\Delta T$$

#### 2.1.2 Latent Heat

- For a mass  $m$  and specific latent heat  $l$ :

$$\Delta E = ml$$

#### 2.1.3 Heat Flux

- This is the heat flux radiated by a source of temperature  $T$  with emissivity  $\varepsilon$ :

$$F = \varepsilon \sigma T^4$$

Where  $\sigma$  is the Stephan Boltzmann constant. Flux is also related to power  $P$  and area  $A$  via:

$$F = P/A$$

### 2.2 Ideal Gas

- The ideal Gas Law is:

$$PV = NkT$$

$N$  is the number of particles and  $k$  the Boltzmann factor.

#### 2.2.1 Energy of a Gas

- The relation for energy of a Gas is:

$$E = \frac{d}{2}NkT$$

Where  $d$  is the degrees of freedom of the gas molecules, for ideal Gas this is just  $d = 3$ .

