Problem Solving

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Sheet of equations useful for the Problem solving paper

"Mo money mo Problems"

- Samson

Contents

1 Mechanics

1.1 Constant acceleration

- For constant acceleration a the equations of motion can be written in three useful forms. The variables here are initial velocity u final velocity v, time t and displacement s:

$$\boxed{\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2as \\ s &= ut + \frac{1}{2}at^2 \end{aligned}}$$

1.2 Force

- The force due to a potential U is:

$$\boxed{\mathbf{F} = -\nabla U}$$

1.3 Friction

- Friction is given by the following expression, where μ is the *co-efficient of friction* and N is the normal force acted on the object by the surface it is sliding across.

$$\boxed{F_{\text{Frict}} = \mu N}$$

1.4 Work

- For constant Force work is just defined $W = \mathbf{F} \cdot \mathbf{s}$. (Force times displacement). If the force is not constant:

$$\boxed{W = \int_a^b \mathbf{F} \cdot d\mathbf{s}}$$

1.4.1 Power

- *Power* is defined as:

$$\boxed{P = \frac{dW}{dr} = \mathbf{F} \cdot \mathbf{v}}$$

  Where $\mathbf{v}$ is velocity.

1.5 Circular motion & Rotation

- In circular motion the force acting as the *centripetal* force (force pulling to wards the center, gravity, tension in rope, ect) is balanced by a *centrifugal* force equal in magnitude but opposite direction, given by:

$$\boxed{F_{\text{fugal}} = \frac{mv^2}{r} = mr\omega^2, \quad (\text{as } v = \omega r)}$$

Where here v is the tangential velocity, r is the radius, m is the mass of the object in motion and ω is the angular velocity.

1.5.1 Rotational Kinetic energy

- Usually kinetic energy is just $\frac{1}{2}mv^2$, but for rigid bodies rotation it is:

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Where I is the moment of inertia. A list of these can be found in the formula and tables booklet for different geometries.

1.5.2 Parallel axis Theorem

- This is the rule that tells us how to add the self rotation moment of inertia I_{self} to the orbital rotation of a mass M at radius R:

$$I_{\text{total}} = I_{\text{self}} + MR^2$$

1.5.3 Torque

- This is defined as:

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}}$$

We also have that:

$$\boxed{|\boldsymbol{\tau}| = \alpha I}$$

Where $\alpha = \frac{d\omega}{dt}$. We can then find that the work done by a source the torque is:

$$\boxed{W = \int_{\theta_0}^{\theta_1} \boldsymbol{\tau} d\theta, \qquad \implies P = \tau\omega}$$

Where P is the power.

1.5.4 Angular momentum

- The standard definition is just:

$$\boxed{\begin{array}{ll} \mathbf{L} = \mathbf{r} \times \mathbf{p}, & \text{for a particle} \\ I\boldsymbol{\omega}, & \text{Ridged body motion} \end{array}}$$

We can also write the torque as $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$.

1.6 Center of Mass

- This is given by:

$$\boxed{\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}}$$

1.7 Stress/strain

- Shear modulus G is:

$$\boxed{G = \frac{\text{shear stress}}{\text{shear strain}}}$$

Where shear stress = F/A (Force per Area) and shear strain = $\frac{\Delta s}{l}$. Length displaced Δs over the length of the object l. The same sort of equation holds for the other moduli. Youngs modulus is G with the word "shear" replaced with "Tensile". Same goes with Bulk and Elastic.

1.7.1 Pressure

- Pressure is in its most general form perpendicular Force $F_\perp$ divided by area A:

$$\boxed{P = \frac{F_\perp}{A}}$$

1.8 Gravitation

- The Gravitational potential energy of a mass m in a gravitational field produced by a body of mass M is:

$$\boxed{U = -\frac{GMm}{r} \quad \implies \mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}}$$