# Problem Solving

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Sheet of equations useful for the Problem solving paper

"Mo money mo Problems"

- Samson

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# 1 Mechanics

## 1.1 Constant acceleration

- For constant acceleration $a$ the equations of motion can be written in three useful forms. The variables here are initial velocity $u$ final velocity $v$, time $t$ and displacement $s$:

$$\boxed{\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2as \\ s &= ut + \frac{1}{2}at^2 \end{aligned}}$$

## 1.2 Force

- The force due to a potential $U$ is:

$$\boxed{\mathbf{F} = -\nabla U}$$

## 1.3 Friction

- Friction is given by the following expression, where $\mu$ is the *co-efficient of friction* and $N$ is the normal force acted on the object by the surface it is sliding across.

$$\boxed{F_{\text{Frict}} = \mu N}$$

## 1.4 Work

- For constant Force work is just defined $W = \mathbf{F} \cdot \mathbf{s}$. (Force times displacement). If the force is not constant:

$$\boxed{W = \int_a^b \mathbf{F} \cdot d\mathbf{s}}$$

### 1.4.1 Power

- *Power* is defined as:

$$\boxed{P = \frac{dW}{dr} = \mathbf{F} \cdot \mathbf{v}}$$

Where $\mathbf{v}$ is velocity.

## 1.5 Circular motion & Rotation

- In circular motion the force acting as the *centripetal* force (force pulling to wards the center, gravity, tension in rope, ect) is balanced by a *centrifugal* force equal in magnitude but opposite direction, given by:

$$\boxed{F_{\text{fugal}} = \frac{mv^2}{r} = mr\omega^2, \quad (\text{as } v = \omega r)}$$

Where here $v$ is the tangential velocity, $r$ is the radius, $m$ is the mass of the object in motion and $\omega$ is the angular velocity.

### 1.5.1 Rotational Kinetic energy

- Usually kinetic energy is just $\frac{1}{2}mv^2$, but for rigid bodies rotation it is:

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Where $I$ is the moment of inertia. A list of these can be found in the formula and tables booklet for different geometries.

### 1.5.2 Parallel axis Theorem

- This is the rule that tells us how to add the self rotation moment of inertia $I_{\text{self}}$ to the orbital rotation of a mass $M$ at radius $R$:

$$\boxed{I_{\text{total}} = I_{\text{self}} + MR^2}$$

### 1.5.3 Torque

- This is defined as:

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}}$$

We also have that:

$$\boxed{|\boldsymbol{\tau}| = \alpha I}$$

Where $\alpha = \frac{d\omega}{dt}$. We can then find that the work done by a source the torque is:

$$\boxed{W = \int_{\theta_0}^{\theta_1} \boldsymbol{\tau} d\theta, \qquad \implies P = \tau\omega}$$

Where $P$ is the power.

### 1.5.4 Angular momentum

- The standard definition is just:

$$\boxed{\begin{array}{ll} \mathbf{L} = \mathbf{r} \times \mathbf{p}, & \text{for a particle} \\ I\boldsymbol{\omega}, & \text{Ridged body motion} \end{array}}$$

We can also write the torque as $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$.

## 1.6 Center of Mass

- This is given by:

$$\boxed{\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}}$$

## 1.7 Stress/strain

- Shear modulus $G$ is:

$$\boxed{G = \frac{\text{shear stress}}{\text{shear strain}}}$$

  Where shear stress $= F/A$ (Force per Area) and shear strain $= \frac{\Delta s}{l}$. Length displaced $\Delta s$ over the length of the object $l$. The same sort of equation holds for the other moduli. Youngs modulus is $G$ with the word "shear" replaced with "Tensile". Same goes with Bulk and Elastic.

## 1.8 Gravitation

- The Gravitational potential energy of a mass $m$ in a gravitational field produced by a body of mass $M$ is:

$$\boxed{U = -\frac{GMm}{r} \quad \Longrightarrow \mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}}$$

### 1.8.1 Acceleration due to gravity

- This is just the gravitational force $F_{\text{grav}}$ per unit mass:

$$\boxed{g = \frac{GM}{r^2}}$$

## 1.9 Orbits

- To find the escape velocity we just find the velocity that makes the total energy 0 when $r = R$, this results in:

$$\boxed{v_{\text{esc}} = \sqrt{\frac{2GM}{R}}}$$

  The Schwartzchild radius is when this velocity $v$ is the speed of light $c$.

### 1.9.1 Kepler's Period

- This is the orbital period for a body in elliptical motion with semi-major axis $a$:

$$\boxed{T^2 = \frac{4\pi^2 a^3}{GM}}$$

## 1.10 Periodic motion

- *Simple Harmonic motion* is defined as motion where the force is:

$$\boxed{F = -kx}$$

- This motion will then have a angular frequency $\omega$, which is related to the period of the motion $T$, by: $T = 2\pi/\omega$.
- Frequency is defined as one over the period $f = 1/T = \omega/2\pi$.

### 1.10.1 Pendulum

- For a pendulum of length $L$ we have that $\omega = \sqrt{\frac{g}{L}}$. If this pendulum is a rigid body with moment of inertia $I$, then this takes the form: $\omega = \sqrt{\frac{mgL}{I}}$.

### 1.10.2 Damped Harmonic Motion

- The General solution to damped harmonic motion takes the form:

$$\boxed{x(t) = Ae^{-\gamma t}cos(\omega' t)}$$

Where $\omega' = \sqrt{\frac{k}{m} - \frac{k^2}{4m^2}}$.

## 1.11 Density and Pressure

- Density is defined as:

$$\boxed{\rho = \frac{m}{V}}$$

- Pressure is in its most general form perpendicular Force $F_\perp$ divided by area $A$:

$$\boxed{P = \frac{F_\perp}{A}}$$

### 1.11.1 Pressure in a fluid

- At a height $h$ below the surface is:

$$\boxed{P = P_0 + \rho g h}$$

### 1.11.2 Fluid Flow

- For a fluid flowing through a pipe of changing cross sectional area the conserved quantity is the area $A$ times the velocity of the fluid $v$. ($A_1 v_1 = A_2 v_2$).

### 1.11.3 Bernoulli's equation

- The relations ship between pressure and velocity in a fluid is given by:

$$\boxed{P + \frac{1}{2}\rho v^2 = \text{constant}}$$

### 1.11.4 Wave equation

- This takes the form:

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

Where $y - y(x,t)$ and $v = \lambda * f$ (wavelength by frequency) is the speed of the wave. The general form to the solutions of this equation are:

$$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

### 1.11.5 Wave Power

- Given the tension $T$ in a string with mass per length $\mu = m/L$ the wave power is:

$$\boxed{P = \frac{1}{2}\sqrt{\mu T}\omega^2 A^2}$$

### 1.11.6 Beat frequency

- When there is two sources of oscillatory motion the beat frequency is the difference between the two underlying frequencies $f_{\text{beat}} = |f_1 - f_2|$.

### 1.11.7 Doppler affect

- The observed frequency of a source emitting a frequency $f_0$ when the receiver is moving at velocity $v_r$ and the source at velocity $v_s$ is:

$$\boxed{f' = \left(\frac{c \pm v_r}{c \mp v_s}\right) f}$$

Where we have $+$ in the numerator if receiver is moving towards source and a $-$ for away. And we have a $+$ in the denominator if the source is moving towards the medium and a $-$ if away.

# 2 Thermodynamics

## 2.1 Heat

### 2.1.1 Heat Capacity

- Heat capacity is defined as the temperature derivative of the Energy:

$$\boxed{C_V = \left(\frac{\partial E}{\partial T}\right)_V}$$

  For $C_P$ just replace $V$ with $P$.

- Heat energy change due to a temperature change $\Delta T$ for a substance with heat specific capacity $c \equiv C/m$ and mass $m$:

$$\boxed{\Delta Q = mc\Delta T}$$

- The two heat capacities are related by:

$$\boxed{C_p = C_v + nR, \quad \gamma = \frac{C_p}{C_v}}$$

- At constant volume we have from the first law since $dV = 0$:

$$\boxed{C_v = \frac{dQ}{dT} = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)}$$

### 2.1.2 Latent Heat

- For a mass $m$ and specific latent heat $l$:

$$\boxed{\Delta E = ml}$$

### 2.1.3 Heat Flux

- This is the heat flux radiated by a source of temperature $T$ with emissivity $\varepsilon$:

$$\boxed{F = \varepsilon\sigma T^4}$$

  Where $\sigma$ is the Stephan Boltzmann constant. Flux is also related to power $P$ and area $A$ via:

$$\boxed{F = P/A}$$

## 2.2 Ideal Gas

- The ideal Gas Law is:

$$\boxed{PV = NkT}$$

N is the number of particles and $k$ the Boltzmann factor.

### 2.2.1 Energy of a Gas

- The relation for energy of a Gas is:

$$\boxed{E = \frac{d}{2}NkT}$$

Where $d$ is the degrees of freedom of the gas molecules, for ideal Gas this is just $d = 3$. Note that this means the heat capacity is $C_v = \frac{d}{2}Nk$

- This can also be expressed as:

$$\boxed{E = \frac{1}{2}mN\bar{v}^2}$$

Where $\bar{v}$ is the average velocity given by $\bar{v} = \sqrt{\frac{dkT}{m}}$

## 2.3 Mean free Path

- This is just defined as the velocity of the particles times the mean free time $t_{\text{mean}}$. Which can be expressed as:

$$\boxed{\lambda = \frac{1}{\sigma n}}$$

Where $\sigma$ is the cross section and $n$ the number of particles per unit volume. This is assuming stationary particles, or that the incoming particle has a much higher velocity then the targets. If this is not the case, i.e. thee incoming particle is at thermal equilibrium with the targets then the mean free path changes as the number of collisions becomes $\sqrt{2}$ times that of the stationary case. $l = 1/(\sqrt{2}n\sigma)$.

- The intensity of particles in to a medium falls off exponentially, depending on the mean free path, $I = I_0 e^{x/l}$. The probability can then be calculated from this:

$$\boxed{dP(x) = \frac{I(x) - I(x + dx)}{dx} = \frac{1}{l}e^{-x/l}dx}$$

## 2.4 First Law

- The first law of thermodynamics is:

$$\boxed{dU = dW + dQ}$$

Where $U$ is internal energy, $W$ the work (which is not always exact) and $Q$ the heat. Often we will have the expression that the infinitesimal work done is $dW = PdV$ with the sign convention that work done on the system is positive.

## 2.5 Processes

- The following processes mean the following things:
  - Adiabatic: $\Delta Q = 0$, no heat transfer.
  - Isochoric/mechanically isolated: $\Delta W = 0$ (constant volume).
  - Isobaric: (constant pressure)
  - Isothermal: $\Delta T = 0$. (constant temperature).

### 2.5.1 Adiabatic Process

- In this case we have the following expression where $\gamma$ is the ratio of heat capacities:

$$\boxed{TV^{\gamma-1} = const, \quad PV^{\gamma-1} = const}$$

- $dW = dU \implies W = C_v NkT$.

## 2.6 Efficacy of a heat engine

- This is defined as the ratio of output work to input heat:

$$\boxed{\varepsilon = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_1} = 1 - \frac{Q_1}{Q_2}}$$

### 2.6.1 Carnot cycle

- In the case of a Carnot cycle we can find a relationship between the heat and the temperatures such that:

$$\boxed{\eta = 1 - \frac{T_1}{T_2}}$$

In the case of a refrigerator the cycle is performed in the opposite direction so we have a *coefficient of performance* defined as:

$$\boxed{c = \frac{Q_1}{Q_2 - Q_1}}$$

## 2.7 Entropy

- From the definition, for a reversible process, Entropy is defined as:

$$\boxed{dS = \frac{dQ}{T}}$$

We have from the Clausius Inequality that $dS \geq 0$. If we have an isobaric process then the heat is just $dQ = C_p dT$

## 2.8 Thermal conductivity

- This is a reformulation of Fourier's law:

  $k$ is the *thermal conductivity*.

## 2.9 Thermodynamic potentials

### 2.9.1 Helmholtz Free

- This is $\mathcal{F}(T, V, N_i)$ and is given by:

$$\boxed{\mathcal{F} = E - TS} \tag{2.1}$$

We use $\mathcal{F}$ in the case of an isothermal process along with there being chemical and mechanical isolation. $\mathcal{F}$ also has:

$$\boxed{-P = \left(\frac{\partial F}{\partial V}\right), \quad -S = \left(\frac{\partial F}{\partial T}\right), \quad \mu_i = \left(\frac{\partial F}{\partial N_i}\right)} \tag{2.2}$$

### 2.9.2 Gibbs free

- This is $\mathcal{G}(T, P, N_i)$ and is given by:

$$\boxed{\mathcal{G} = F + PV} \tag{2.3}$$

We use $\mathcal{G}$ in the case of an isothermal and Isobaric system along with chemical and mechanical isolation. $\mathcal{G}$ also has:

$$\boxed{V = \left(\frac{\partial G}{\partial P}\right), \quad -S = \left(\frac{\partial G}{\partial T}\right), \quad N_i = \left(\frac{\partial G}{\partial \mu_i}\right)} \tag{2.4}$$

### 2.9.3 Enthalpy

- This is $\mathcal{H}(S, P, N_i)$ and is given by:

$$\boxed{\mathcal{H} = E + PV} \tag{2.5}$$

We use $\mathcal{H}$ in the case of an Isobaric process along with thermal and chemical isolation. $\mathcal{H}$ also has:

$$\boxed{P = \left(\frac{\partial H}{\partial V}\right), \quad S = \left(\frac{\partial H}{\partial T}\right), \quad \mu_i = \left(\frac{\partial H}{\partial N_i}\right)} \tag{2.6}$$

# 3 Electromagnetism

## 3.1 Coulombs Law

- The force of between two charged particles $q_1$ and $q_2$

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}$$

## 3.2 Electric Field

- The electric field is related to the force of coulombs law via:

$$\boxed{\boldsymbol{E} = \frac{\boldsymbol{F}}{q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

## 3.3 Electric Potential

- This is defined as, $\phi$ such that $\boldsymbol{E} = -\nabla\phi$. This means the potential difference between two points along some curve is:

$$\boxed{\phi = \int_\gamma \mathbf{E}\cdot d\mathbf{l}}$$

### 3.3.1 Electric Energy Density

- This is:

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$$

## 3.4 Dipoles

- Dipoles have a quantity called the dipole moment $\boldsymbol{\mu}$, this has magnitude $qd$, where $q$ is the charge and $d$ the separation. he potential energy due to a dipole in an electric field is:

$$\boxed{U = -\boldsymbol{E}\cdot\boldsymbol{\mu}}$$

The torque on a dipole is $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{E}$

## 3.5 Gauss' Law

- This says the total electric flux (integral of electric field perpendicular to a surface) is the same as the charge enclosed by that surface:

$$\boxed{\frac{Q_{\text{encl}}}{\epsilon_0} = \int_S \mathbf{E}\cdot d\mathbf{A}}$$

## 3.6 Capacitors

- Capacitance is defined as:

$$\boxed{C = \frac{Q}{V}}$$

  Where $V$ is the potential or voltage. For two plate capacitor of area $A$ and separation $d$, this is $C = \epsilon_0 \frac{A}{d}$.

- Capacitors add in series via:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

  And in parallel via:

$$\boxed{C = C_1 + C_2}$$

### 3.6.1 Energy in a Capacitor

- This is:

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

## 3.7