Diferential Geometry

Thomas Brosnan

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"hokay" -Sergey Frolov

Contents

1 Definition of a Manifold

1.1 Regions

- A *region* ("open set") is a set of D points in $\mathbb{R}^n$ such that together with each point p_0, D also contains all points sufficiently closer to p_0, i.e.:

$$\forall \; p_0 = (x_0^1, \ldots, x_0^n) \in D \; \exists \; \epsilon > 0,$$
$$st : p = (x^1, \ldots, x^n) \in D, \; \text{iff} \; |x^i - x_0^i| < \epsilon.$$

- A *region with out a boundary* is obtained fro ma region D by adjoining all boundary points to D. The *boundary* of a region is the set of all boundary points.

1.2 Differentiable Manifold

- A differentiable n-dimensional manifold is a set M together with the following structure on it. The set M is the union of a finite or countably infinite collection of subsets U_q with the following properties:

  - Each subset U_q has defined on it co-ords $x_q^\alpha, \alpha = 1, \ldots, n$ called local co-ords by virtue of which U_q is identifiable with a region of Euclidean