Problem Solving

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Sheet of equations useful for the Problem solving paper

"Mo money mo Problems"

- Samson

Contents

1 Mechanics

1.1 Constant acceleration

- For constant acceleration a the equations of motion can be written in three useful forms. The variables here are initial velocity u final velocity v, time t and displacement s:

$$\boxed{\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2as \\ s &= ut + \frac{1}{2}at^2 \end{aligned}}$$

1.2 Force

- The force due to a potential U is:

$$\boxed{\mathbf{F} = -\nabla U}$$

1.3 Friction

- Friction is given by the following expression, where μ is the *co-efficient of friction* and N is the normal force acted on the object by the surface it is sliding across.

$$\boxed{F_{\text{Frict}} = \mu N}$$

1.4 Work

- For constant Force work is just defined $W = \mathbf{F} \cdot \mathbf{s}$. (Force times displacement). If the force is not constant:

$$\boxed{W = \int_a^b \mathbf{F} \cdot d\mathbf{s}}$$

1.4.1 Power

- *Power* is defined as:

$$\boxed{P = \frac{dW}{dr} = \mathbf{F} \cdot \mathbf{v}}$$

  Where $\mathbf{v}$ is velocity.

1.5 Circular motion & Rotation

- In circular motion the force acting as the *centripetal* force (force pulling to wards the center, gravity, tension in rope, ect) is balanced by a *centrifugal* force equal in magnitude but opposite direction, given by:

$$\boxed{F_{\text{fugal}} = \frac{mv^2}{r} = mr\omega^2, \quad (\text{as } v = \omega r)}$$

Where here v is the tangential velocity, r is the radius, m is the mass of the object in motion and ω is the angular velocity.

1.5.1 Rotational Kinetic energy

- Usually kinetic energy is just $\frac{1}{2}mv^2$, but for rigid bodies rotation it is:

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Where I is the moment of inertia. A list of these can be found in the formula and tables booklet for different geometries.

1.5.2 Parallel axis Theorem

- This is the rule that tells us how to add the self rotation moment of inertia I_{self} to the orbital rotation of a mass M at radius R:

$$\boxed{I_{\text{total}} = I_{\text{self}} + MR^2}$$

1.5.3 Torque

- This is defined as:

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}}$$

We also have that:

$$\boxed{|\boldsymbol{\tau}| = \alpha I}$$

Where $\alpha = \frac{d\omega}{dt}$. We can then find that the work done by a source the torque is:

$$\boxed{W = \int_{\theta_0}^{\theta_1} \boldsymbol{\tau} d\theta, \qquad \implies P = \tau\omega}$$

Where P is the power.

1.5.4 Angular momentum

- The standard definition is just:

$$\boxed{\begin{aligned} &\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad \text{for a particle} \\ &I\boldsymbol{\omega}, \quad \text{Ridged body motion} \end{aligned}}$$

We can also write the torque as $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$.

1.6 Center of Mass

- This is given by:

$$\boxed{\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}}$$

1.7 Stress/strain

- Shear modulus G is:

$$\boxed{G = \frac{\text{shear stress}}{\text{shear strain}}}$$

Where shear stress $= F/A$ (Force per Area) and shear strain $= \frac{\Delta s}{l}$. Length displaced Δs over the length of the object l. The same sort of equation holds for the other moduli. Youngs modulus is G with the word "shear" replaced with "Tensile". Same goes with Bulk and Elastic.

1.8 Gravitation

- The Gravitational potential energy of a mass m in a gravitational field produced by a body of mass M is:

$$\boxed{U = -\frac{GMm}{r} \qquad \Longrightarrow \mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}}$$

1.8.1 Acceleration due to gravity

- This is just the gravitational force F_{grav} per unit mass:

$$\boxed{g = \frac{GM}{r^2}}$$

1.9 Orbits

- To find the escape velocity we just find the velocity that makes the total energy 0 when $r = R$, this results in:

$$\boxed{v_{\text{esc}} = \sqrt{\frac{2GM}{R}}}$$

The Schwartzchild radius is when this velocity v is the speed of light c.

1.9.1 Kepler's Period

- This is the orbital period for a body in elliptical motion with semi-major axis a:

$$\boxed{T^2 = \frac{4\pi^2 a^3}{GM}}$$

1.10 Periodic motion

- *Simple Harmonic motion* is defined as motion where the force is:

$$\boxed{F = -kx}$$

- This motion will then have a angular frequency ω, which is related to the period of the motion T, by: $T = 2\pi/\omega$.
- Frequency is defined as one over the period $f = 1/T = \omega/2\pi$.

1.10.1 Pendulum

- For a pendulum of length L we have that $\omega = \sqrt{\frac{g}{L}}$. If this pendulum is a rigid body with moment of inertia I, then this takes the form: $\omega = \sqrt{\frac{mgL}{I}}$.

1.10.2 Damped Harmonic Motion

- The General solution to damped harmonic motion takes the form:

$$\boxed{x(t) = Ae^{-\gamma t}cos(\omega' t)}$$

Where $\omega' = \sqrt{\frac{k}{m} - \frac{k^2}{4m^2}}$.

1.11 Density and Pressure

- Density is defined as:

$$\boxed{\rho = \frac{m}{V}}$$

- Pressure is in its most general form perpendicular Force $F_\perp$ divided by area A:

$$\boxed{P = \frac{F_\perp}{A}}$$

1.11.1 Pressure in a fluid

- At a height h below the surface is:

$$\boxed{P = P_0 + \rho gh}$$

1.11.2 Fluid Flow

- For a fluid flowing through a pipe of changing cross sectional area the conserved quantity is the area A times the velocity of the fluid v. ($A_1v_1 = A_2v_2$).

1.11.3 Bernoulli's equation

- The relations ship between pressure and velocity in a fluid is given by:

$$\boxed{P + \frac{1}{2}\rho v^2 = \text{constant}}$$

1.11.4 Wave equation

- This takes the form:

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

Where $y - y(x,t)$ and $v = \lambda * f$ (wavelength by frequency) is the speed of the wave. The general form to the solutions of this equation are:

$$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

1.11.5 Wave Power

- Given the tension T in a string with mass per length $\mu = m/L$ the wave power is:

$$\boxed{P = \frac{1}{2}\sqrt{\mu T}\omega^2 A^2}$$

1.11.6 Beat frequency

- When there is two sources of oscillatory motion the beat frequency is the difference between the two underlying frequencies $f_{\text{beat}} = |f_1 - f_2|$.

1.11.7 Doppler affect

- The observed frequency of a source emitting a frequency f_0 when the receiver is moving at velocity v_r and the source at velocity v_s is:

$$\boxed{f = \left(\frac{c \pm v_r}{c \mp v_s}\right)}$$

Where we have $+$ in the numerator if receiver is moving towards source and a $-$ for away. And we have a $+$ in the denominator if the source is moving towards the medium and a $-$ if away.

2 Thermodynamics

2.1 Heat

2.1.1 Heat Capacity

- Heat capacity is defined as the temperature derivative of the Energy:

$$\boxed{C_V = \left(\frac{\partial E}{\partial T}\right)_V}$$

  For C_P just replace V with P.

- Heat energy change due to a temperature change ΔT for a substance with heat specific capacity $c \equiv C/m$ and mass m:

$$\boxed{\Delta Q = mc\Delta T}$$

2.1.2 Latent Heat

- For a mass m and specific latent heat l:

$$\boxed{\Delta E = ml}$$

2.1.3 Heat Flux

- This is the heat flux radiated by a source of temperature T with emissivity ε:

$$\boxed{F = \varepsilon\sigma T^4}$$

  Where σ is the Stephan Boltzmann constant. Flux is also related to power P and area A via:

$$\boxed{F = P/A}$$

2.2 Ideal Gas

- The ideal Gas Law is:

$$\boxed{PV = NkT}$$

  N is the number of particles and k the Boltzmann factor.

2.2.1 Energy of a Gas

- The relation for energy of a Gas is:

$$\boxed{E = \frac{d}{2}NkT}$$

Where d is the degrees of freedom of the gas molecules, for ideal Gas this is just $d = 3$.

- This can also be expressed as:

$$\boxed{E = \frac{1}{2}mN\bar{v}^2}$$

Where $\bar{v}$ is the average velocity given by $\bar{v} = \sqrt{\frac{dkT}{m}}$

2.3 Mean free Path

- This is just defined as the velocity of the particles times the mean free time t_{mean}. Which can be expressed as:

$$\boxed{\lambda = \frac{1}{\sigma n}}$$

Where σ is the cross section and n the number of particles per unit volume. This is assuming stationary