Problem Solving

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Sheet of equations useful for the Problem solving paper

"Mo money mo problems, gotta move carefully"

- Jay-Z

Contents

1 Mechanics

1.1 Constant acceleration

- For constant acceleration a the equations of motion can be written in three useful forms. The variables here are initial velocity u final velocity v, time t and displacement s:

$$\boxed{\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2as \\ s &= ut + \frac{1}{2}at^2 \end{aligned}}$$

1.2 Force

- The force due to a potential U is:

$$\boxed{\mathbf{F} = -\nabla U}$$

1.3 Friction

- Friction is given by the following expression, where μ is the *co-efficient of friction* and N is the normal force acted on the object by the surface it is sliding across.

$$\boxed{F_{\text{Frict}} = \mu N}$$

1.4 Work

- For constant Force work is just defined $W = \mathbf{F} \cdot \mathbf{s}$. (Force times displacement). If the force is not constant:

$$\boxed{W = \int_a^b \mathbf{F} \cdot d\mathbf{s}}$$

1.4.1 Power

- *Power* is defined as:

$$\boxed{P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}}$$

Where $\mathbf{v}$ is velocity.

1.5 Circular motion & Rotation

- In circular motion the force acting as the *centripetal* force (force pulling to wards the center, gravity, tension in rope, ect) is balanced by a *centrifugal* force equal in magnitude but opposite direction, given by:

$$\boxed{F_{\text{fugal}} = \frac{mv^2}{r} = mr\omega^2, \quad (\text{as } v = \omega r)}$$

Where here v is the tangential velocity, r is the radius, m is the mass of the object in motion and ω is the angular velocity.

1.5.1 Rotational Kinetic energy

- Usually kinetic energy is just $\frac{1}{2}mv^2$, but for rigid bodies rotation it is:

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Where I is the moment of inertia. A list of these can be found in the formula and tables booklet for different geometries.

1.5.2 Parallel axis Theorem

- This is the rule that tells us how to add the self rotation moment of inertia I_{self} to the orbital rotation of a mass M at radius R:

$$\boxed{I_{\text{total}} = I_{\text{self}} + MR^2}$$

1.5.3 Torque

- This is defined as:

$$\boxed{\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}}$$

We also have that:

$$\boxed{|\boldsymbol{\tau}| = \alpha I}$$

Where $\alpha = \frac{d\omega}{dt}$. We can then find that the work done by a source the torque is:

$$\boxed{W = \int_{\theta_0}^{\theta_1} \boldsymbol{\tau} d\theta, \qquad \Longrightarrow P = \tau\omega}$$

Where P is the power.

1.5.4 Angular momentum

- The standard definition is just:

$$\boxed{\begin{aligned} &\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad \text{for a particle} \\ &I\boldsymbol{\omega}, \quad \text{Ridged body motion} \end{aligned}}$$

We can also write the torque as $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$.

1.6 Center of Mass

- This is given by:

$$\boxed{\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}}$$

1.7 Stress/strain

- Shear modulus G is:

$$\boxed{G = \frac{\text{shear stress}}{\text{shear strain}}}$$

Where shear stress $= F/A$ (Force per Area) and shear strain $= \frac{\Delta s}{l}$. Length displaced Δs over the length of the object l. The same sort of equation holds for the other moduli. Youngs modulus is G with the word "shear" replaced with "Tensile". Same goes with Bulk and Elastic.

1.8 Gravitation

- The Gravitational potential energy of a mass m in a gravitational field produced by a body of mass M is:

$$\boxed{U = -\frac{GMm}{r} \quad \Longrightarrow \mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}}$$

1.8.1 Acceleration due to gravity

- This is just the gravitational force F_{grav} per unit mass:

$$\boxed{g = \frac{GM}{r^2}}$$

1.9 Orbits

- To find the escape velocity we just find the velocity that makes the total energy 0 when $r = R$, this results in:

$$\boxed{v_{\text{esc}} = \sqrt{\frac{2GM}{R}}}$$

The Schwartzchild radius is when this velocity v is the speed of light c.

1.9.1 Kepler's Period

- This is the orbital period for a body in elliptical motion with semi-major axis a:

$$\boxed{T^2 = \frac{4\pi^2 a^3}{GM}}$$

1.10 Periodic motion

- *Simple Harmonic motion* is defined as motion where the force is:

$$\boxed{F = -kx}$$

- This motion will then have a angular frequency ω, which is related to the period of the motion T, by: $T = 2\pi/\omega$.
- Frequency is defined as one over the period $f = 1/T = \omega/2\pi$.

1.10.1 Pendulum

- For a pendulum of length L we have that $\omega = \sqrt{\frac{g}{L}}$. If this pendulum is a rigid body with moment of inertia I, then this takes the form: $\omega = \sqrt{\frac{mgL}{I}}$.

1.10.2 Damped Harmonic Motion

- The General solution to damped harmonic motion takes the form:

$$\boxed{x(t) = Ae^{-\gamma t}cos(\omega' t)}$$

Where $\omega' = \sqrt{\frac{k}{m} - \frac{k^2}{4m^2}}$.

1.11 Density and Pressure

- Density is defined as:

$$\boxed{\rho = \frac{m}{V}}$$

- Pressure is in its most general form perpendicular Force $F_\perp$ divided by area A:

$$\boxed{P = \frac{F_\perp}{A}}$$

1.11.1 Pressure in a fluid

- At a height h below the surface is:

$$\boxed{P = P_0 + \rho g h}$$

1.11.2 Fluid Flow

- For a fluid flowing through a pipe of changing cross sectional area the conserved quantity is the area A times the velocity of the fluid v. ($A_1 v_1 = A_2 v_2$).

1.11.3 Bernoulli's equation

- The relations ship between pressure and velocity in a fluid is given by:

$$\boxed{P + \frac{1}{2}\rho v^2 = \text{constant}}$$

1.12 Drag Force

- The force sue to air resistance depends on the area A of a body, its speed v, the density of the medium ρ , and some drag co-efficient C_D:

$$\boxed{F_D = \frac{1}{2} C_D A \rho v^2}$$

1.13 Archimedes Principle

- This says that the buoyancy force action on an object in a liquid is the same as the weight of the fluid being displaced by that object.

1.13.1 Wave equation

- This takes the form:

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

Where $y - y(x,t)$ and $v = \lambda * f$ (wavelength by frequency) is the speed of the wave. The general form to the solutions of this equation are:

$$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

1.13.2 Wave Power

- Given the tension T in a string with mass per length $\mu = m/L$ the wave power is:

$$\boxed{P = \frac{1}{2}\sqrt{\mu T}\omega^2 A^2}$$

1.13.3 Beat frequency

- When there is two sources of oscillatory motion the beat frequency is the difference between the two underlying frequencies $f_{\text{beat}} = |f_1 - f_2|$.

1.13.4 Doppler affect

- The observed frequency of a source emitting a frequency f_0 when the receiver is moving at velocity v_r and the source at velocity v_s is:

$$\boxed{f' = \left(\frac{c \pm v_r}{c \mp v_s}\right) f_0}$$

  Where we have $+$ in the numerator if receiver is moving towards source and a $-$ for away. And we have a $+$ in the denominator if the source is moving towards the medium and a $-$ if away.

2 Thermodynamics

2.1 Heat

2.1.1 Heat Capacity

- Heat capacity is defined as the temperature derivative of the Energy:

$$\boxed{C_V = \left(\frac{\partial E}{\partial T}\right)_V}$$

For C_P just replace V with P.

- Heat energy change due to a temperature change ΔT for a substance with heat specific capacity $c \equiv C/m$ and mass m:

$$\boxed{\Delta Q = mc\Delta T}$$

- The two heat capacities are related by:

$$\boxed{C_p = C_v + nR, \quad \gamma = \frac{C_p}{C_v}}$$

- At constant volume we have from the first law since $dV = 0$:

$$\boxed{C_v = \frac{dQ}{dT} = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)}$$

2.1.2 Latent Heat

- For a mass m and specific latent heat l:

$$\boxed{\Delta E = ml}$$

2.1.3 Heat Flux

- This is the heat flux radiated by a source of temperature T with emissivity ε:

$$\boxed{F = \varepsilon\sigma T^4}$$

Where σ is the Stephan Boltzmann constant. Flux is also related to power P and area A via:

$$\boxed{F = P/A}$$

2.2 Ideal Gas

- The ideal Gas Law is:

$$\boxed{PV = NkT}$$

N is the number of particles and k the Boltzmann factor.

2.2.1 Energy of a Gas

- The relation for energy of a Gas is:

$$\boxed{E = \frac{d}{2}NkT}$$

Where d is the degrees of freedom of the gas molecules, for ideal Gas this is just $d = 3$. Note that this means the heat capacity is $C_v = \frac{d}{2}Nk = \frac{d}{2}nR$ (Dulong Petit Law).

- This can also be expressed as:

$$\boxed{E = \frac{1}{2}mN\bar{v}^2}$$

Where $\bar{v}$ is the average velocity given by $\bar{v} = \sqrt{\frac{dkT}{m}}$

2.3 Mean free Path

- This is just defined as the velocity of the particles times the mean free time t_{mean}. Which can be expressed as:

$$\boxed{\lambda = \frac{1}{\sigma n}}$$

Where σ is the cross section and n the number of particles per unit volume. This is assuming stationary particles, or that the incoming particle has a much higher velocity then the targets. If this is not the case, i.e. thee incoming particle is at thermal equilibrium with the targets then the mean free path changes as the number of collisions becomes $\sqrt{2}$ times that of the stationary case. $l = 1/(\sqrt{2}n\sigma)$.

- The intensity of particles in to a medium falls off exponentially, depending on the mean free path, $I = I_0 e^{x/l}$. The probability can then be calculated from this:

$$\boxed{dP(x) = \frac{I(x) - I(x+dx)}{dx} = \frac{1}{l}e^{-x/l}dx}$$

2.4 First Law

- The first law of thermodynamics is:

$$\boxed{dU = dW + dQ}$$

Where U is internal energy, W the work (which is not always exact) and Q the heat. Often we will have the expression that the infinitesimal work done is $dW = PdV$ with the sign convention that work done on the system is positive.

2.5 Processes

- The following processes mean the following things:
  - Adiabatic: $\Delta Q = 0$, no heat transfer.
  - Isochoric/mechanically isolated: $\Delta W = 0$ (constant volume).
  - Isobaric: (constant pressure)
  - Isothermal: $\Delta T = 0$. (constant temperature).

2.5.1 Adiabatic Process

- In this case we have the following expression where γ is the ratio of heat capacities:

$$\boxed{TV^{\gamma-1} = const, \quad PV^{\gamma-1} = const}$$

- $dW = dU \implies W = C_v NkT$.

2.6 Efficacy of a heat engine

- This is defined as the ratio of output work to input heat:

$$\boxed{\varepsilon = \frac{W}{Q_2} = \frac{Q_2 - Q_1}{Q_1} = 1 - \frac{Q_1}{Q_2}}$$

2.6.1 Carnot cycle

- In the case of a Carnot cycle we can find a relationship between the heat and the temperatures such that:

$$\boxed{\eta = 1 - \frac{T_1}{T_2}}$$

  In the case of a refrigerator the cycle is performed in the opposite direction so we have a *coefficient of performance* defined as:

$$\boxed{c = \frac{Q_1}{Q_2 - Q_1}}$$

2.7 Entropy

- From the definition, for a reversible process, Entropy is defined as:

$$\boxed{dS = \frac{dQ}{T}}$$

We have from the Clausius Inequality that $dS \geq 0$. If we have an isobaric process then the heat is just $dQ = C_p dT$

2.8 Thermal conductivity

- This is a reformulation of Fourier's law:

$$\boxed{\frac{dQ}{dt} = kA\frac{\Delta T}{\Delta x}}$$

k is the *thermal conductivity*.

2.9 Measurable Quantities

- Here are a list of definitions of quantities ascribed to materials:

2.9.1 Bulk Modulus

- This is a measure of the resistance of a substance to compression:

$$\boxed{\kappa = -V\left(\frac{\partial P}{\partial V}\right)_T}$$

2.9.2 Thermal Expansivity

- This is a measure of the tendency for a body to increase in size upon rising in temperature:

$$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}$$

2.9.3 Compressibilities

- These are the measure of the instantaneous relative volume change in response to some change. There is the *isothermal compressibility*:

$$\boxed{\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T}$$

And *adiabatic or isentropic* compressibility:

$$\boxed{\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S}$$

2.10 Maxwell relations

- These are the following 4 relations:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V, \quad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P, \tag{2.1}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T, \quad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T. \tag{2.2}$$

These are the *Maxwell relations.* These can be remembered using the following:

$$\begin{array}{ccc} & -S & \\ P & & V \\ & T & \end{array}$$

To construct each Maxwell relation, start at some point and go clockwise around three letters to obtain the left-hand side. If you have both P and S, include a minus sign. Then move on one letter and count back anti-clockwise three letters, inserting a minus sign if you have both P and S.

2.11 Thermodynamic potentials

2.11.1 Helmholtz Free

- This is $\mathcal{F}(T, V, N_i)$ and is given by:

$$\mathcal{F} = E - TS \tag{2.3}$$

We use $\mathcal{F}$ in the case of an isothermal process along with there being chemical and mechanical isolation. $\mathcal{F}$ also has:

$$-P = \left(\frac{\partial F}{\partial V}\right), \quad -S = \left(\frac{\partial F}{\partial T}\right), \quad \mu_i = \left(\frac{\partial F}{\partial N_i}\right) \tag{2.4}$$

2.11.2 Gibbs free

- This is $\mathcal{G}(T, P, N_i)$ and is given by:

$$\mathcal{G} = F + PV \tag{2.5}$$

We use $\mathcal{G}$ in the case of an isothermal and Isobaric system along with chemical and mechanical isolation. $\mathcal{G}$ also has:

$$V = \left(\frac{\partial G}{\partial P}\right), \quad -S = \left(\frac{\partial G}{\partial T}\right), \quad N_i = \left(\frac{\partial G}{\partial \mu_i}\right) \tag{2.6}$$

2.11.3 Enthalpy

- This is $\mathcal{H}(S, P, N_i)$ and is given by:

$$\boxed{\mathcal{H} = E + PV} \tag{2.7}$$

We use $\mathcal{H}$ in the case of an Isobaric process along with thermal and chemical isolation. $\mathcal{H}$ also has:

$$\boxed{P = \left(\frac{\partial H}{\partial V}\right), \quad S = \left(\frac{\partial H}{\partial T}\right), \quad \mu_i = \left(\frac{\partial H}{\partial N_i}\right)} \tag{2.8}$$

2.12 Partition function

- This is defined to normalize the probability distribution of a particle being in a certain energy state $P(E_n) = \frac{1}{Z}e^{-\beta E_n}$, where $\beta = 1/kT$. This is imposed so that the total probabilities sum to 1:

$$\boxed{Z = \sum_n e^{-\beta E_n}}$$

Note this is for a single particle.

- Note also that there is a connection between statistical mechanics and thermodynamics that allows us to write the Helmholtz free energy as:

$$\boxed{F = -kT \ln Z}$$

- Average quantities can then be calculated:

$$\boxed{\langle A \rangle = \frac{1}{Z}\sum_n A e^{-\beta E_n}}$$

For certain quantities we can express these sums in terms of the partition function, for example the average energy is:

$$\boxed{\langle E \rangle = -\frac{\partial \ln Z}{\partial \beta}}$$

3 Electromagnetism

3.1 Coulombs Law

- The force of between two charged particles q_1 and q_2

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}$$

3.2 Electric Field

- The electric field is related to the force of coulombs law via:

$$\boxed{\boldsymbol{E} = \frac{\boldsymbol{F}}{q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

3.3 Electric Potential

- This is defined as, ϕ such that $\boldsymbol{E} = -\nabla\phi$. This means the potential difference between two points along some curve is:

$$\boxed{\phi = \int_\gamma \mathbf{E}\cdot d\mathbf{l}}$$

3.3.1 Electric Energy Density

- This is:

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$$

3.4 Dipoles

- Dipoles have a quantity called the dipole moment $\boldsymbol{\mu}$, this has magnitude qd, where q is the charge and d the separation. he potential energy due to a dipole in an electric field is:

$$\boxed{U = -\boldsymbol{E}\cdot\boldsymbol{\mu}}$$

  The torque on a dipole is $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{E}$

3.5 Gauss' Law

- This says the total electric flux (integral of electric field perpendicular to a surface) is the same as the charge enclosed by that surface:

$$\boxed{\frac{Q_{\text{encl}}}{\epsilon_0} = \int_S \mathbf{E}\cdot d\mathbf{A}}$$

3.6 Capacitors

- Capacitance is defined as:

$$\boxed{C = \frac{Q}{V}}$$

Where V is the potential or voltage. For two plate capacitor of area A and separation d, this is $C = \epsilon_0 \frac{A}{d}$.

- Capacitors add in series via:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

And in parallel via:

$$\boxed{C = C_1 + C_2}$$

3.6.1 Charging a capacitor

- The charge collected on a capacitor due to an induced emf $\mathcal{E}$ is:

$$\boxed{q = C\mathcal{E}(1 - e^{-t/\tau})}$$

Where $\tau = RC$ (for an RC circuit) is the charging time. $C\mathcal{E}$ is the final charge.

3.6.2 Energy in a Capacitor

- This is:

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

- When discharging the charge on the capacitor is:

$$\boxed{q = Q_0 e^{-t/\tau}}$$

3.7 Di-electric

- In a dielectric material ϵ_0 changes to $\epsilon = K\epsilon_0$ where K is the dielectric constant.

3.8 Current

- This is defined as:

$$\boxed{I = \frac{dQ}{dt} = n|q|vA}$$

Here n is the number of charge carriers per unit volume, A is the area, v the velocity and q the charge.

3.8.1 Current density

- This is defined as:

$$\boxed{\mathbf{J} = n|q|\mathbf{v}}$$

3.9 Resistivity

- This is the ratio of electric field to the current density:

$$\boxed{\rho = \frac{E}{J}}$$

- The resistivity depends on the temperature via:

$$\boxed{\rho(T) = \rho_0(1 + \alpha(T - T_0))}$$

  Where α is the *temperature coefficient of resistivity*.

3.10 Resistance

- Ohms law is that the potential is related linearly to the current via the resistance:

$$\boxed{V = IR}$$

  If we have a resistor of length of length L and cross section A:

$$\boxed{R = \frac{\rho L}{A}}$$

- Resistors in series:

$$\boxed{R = R_1 + R_2}$$

3.11 Power loss

- The power lost due to heat (Joules law) is:

$$\boxed{P = IV = I^2 R = \frac{V^2}{R}}$$

- Resistors in parallel:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

3.12 Kirchhoff's laws

- These are, for a junction:

$$\boxed{\sum I = 0}$$

- For a loop:

$$\boxed{\sum V = 0}$$

3.13 Gauss's Law for Magnetism

- This is also known as Ampere's Law:

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I}$$

Where I is the current.

3.14 Force on Charged particle

- Due to the presence of a magnetic and Electric field is:

$$\boxed{\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})}$$

3.15 Magnetic Field due to a Wire

- This is:

$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{r}}{r^3}}$$

The Biot Savar Law for a straight wire is:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

3.16 Faraday's law

- This is that the emf in a wire is related to the magnetic flux $\phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ via:

$$\boxed{\mathcal{E} = -\frac{d\phi_B}{dt}}$$

- Lenz's law tells us that this sign should be a minus.

3.16.1 Magnetic Energy Density

- This is:

$$\boxed{u = \frac{1}{2\mu}B^2}$$

Where μ is the magnetic permeability (usually μ_0?).

3.17 RCL Circuit

- Here the voltage is $V = IZ$, where Z is the impedance given by:

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

- Where $X_L = \omega L$ is the *inductive reactance* depending on the inductance L. And $X_C = \frac{1}{\omega C}$ is the *Capacitive Reactance* depending on the capacitance C. The ω is the frequency of the AC source.

3.18 Energy stored in a Inductor

- This is:

$$\boxed{U = \frac{1}{2}LI^2}$$

3.19 Poyting Vector

- This is:

$$\boxed{\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}}$$

4 Special Relativity

- The postulates of special relativity are:
  - The law of physics are invariant under transformations between inertial frames.
  - The speed of light in a vacuum is measured to be the same by all the observers

4.1 Lorentz Transformation

- t and x transform as follows:

$$\boxed{\begin{aligned} t' &= \gamma(t - \frac{vx}{c^2}) \\ x' &= \gamma(x - vt) \end{aligned}}$$

Where v is the velocity of the S' frame relative to the rest frame. γ is the *Lorentz factor.*:

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Note that $\gamma > 1$

4.1.1 Length Contraction

- This depends on the Lorentz factor, just need to remember that $\gamma > 1$ and that the faster you go the shorter things look:

$$\boxed{L = \frac{L_0}{\gamma}}$$

4.1.2 Time Dilation

- Same as length contraction, but here remember that clocks tick slower for faster moving observers:

$$\boxed{\Delta t' = \gamma \Delta t}$$

4.1.3 Proper time

- τ is proper time, i.e. the time as observed by an observer in his own rest frame.

$$\boxed{\tau = \frac{t}{\gamma}}$$

4.2 Velocity transformation

- If a particle has a velocity u in one frame, then in the frame moving with velocity v with respect to the rest frame, its velocity u' will be:

$$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

4.3 Relativistic Doppler Effect

- Here the observed frequency due to Lorentz contraction effects is:

$$\boxed{f = \sqrt{\frac{c \pm u}{c \mp u}} f_0}$$

Where the $+$ is for an object moving towards the observer and the $-$ is for an object moving away.

4.4 Relativistic Energy

- This is:

$$\boxed{E^2 = p^2c^2 + (mc^2)^2}$$

4.5 Relativistic momentum

- Note that in special relativity momentum changes from $p = mv$ to $p = \gamma mv$.

5 Waves and Optics

5.1 Photon Relations

- For a photon with frequency f its energy is:

$$\boxed{E = hf}$$

Where h is planks constant. Its frequency is related to wavelength via:

$$\boxed{c = f\lambda}$$

5.2 Radiation Pressure

- Radiation pressure is the pressure felt due to photons. It is given by the intensity I (power per unit area), divided by the speed of light c:

$$\boxed{P = \frac{I}{c}}$$

5.3 Refractive index

- The refractive index between media 1 and 2 is defined as the ratio of the speed of light in material 1 over the speed of light in material 2 In general denser the material the higher the refractive index.

$$\boxed{n_{12} = \frac{v_1}{v_2}}$$

One of these media is usually chosen the be the vacuum.

5.4 Snell's law

- The angle of incidence θ_i and angle of refraction θ_r are related via:

$$\boxed{\frac{n_i}{n_r} = \frac{\sin\theta_r}{\sin\theta_i}}$$

5.5 Critical Angle

- Incident angle greater then the critical angle will result in *total internal reflection*:

$$\boxed{\theta_c = \arcsin(\frac{n_2}{n_1})}$$

5.6 Polarization

- A source of intensity I_0 when incident on a polarization slit at an angle ϕ to the polarization of the wave, will have an intensity that passes through of:

$$\boxed{I = I_0 \cos^2 \phi}$$

5.7 Interference

- In the double slit experiment with slits of width d, we get interference in the following two ways, for light incident with a wavelength of λ
  - Constructive interference for $m\lambda = d \sin\theta$
  - Destructive interference for $(m + \frac{1}{2})\lambda = d \sin\theta$

5.8 Rayleigh Criterion

- This is the smallest angle at which an image can be resolved through a circular aperture of diameter D:

$$\boxed{\sin\theta = \frac{1.22\lambda}{D}}$$

6 Quantum Mechanics

6.1 Time-Independent Schrödinger Equation

- This is:

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi}$$

6.2 Time-Dependent Schrödinger Equation

- This is:

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi = (-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x,t))\psi}$$

6.3 Infinite potential well

- For the infinite potential well, i.e. when the potential is 0 for some interval 0 to L and ∞ elsewhere. The general solution is:

$$\boxed{\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)}$$

- And the energy levels are:

$$\boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

6.4 DeBroglie Wavelength

- This is the wavelength associated with massive particles:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

6.5 Uncertainty relation

- There are two versions of this:

$$\boxed{\begin{aligned}\Delta x\Delta p &\geq \frac{\hbar}{2}\\ \Delta E\Delta t &\geq \frac{\hbar}{2}\end{aligned}}$$

6.6 Harmonic oscillator

- This is when the potential takes the form $V(x) = \frac{1}{2}kx^2$, then the energy levels are given by:

$$\boxed{E_n = (n + \frac{1}{2})\hbar\omega}$$

Where $\omega = \sqrt{\frac{k}{m}}$

6.7 Zeemann effect

- Here the potential energy is $V = -\boldsymbol{\mu} \cdot \mathbf{B}$, where $\boldsymbol{\mu}$ is given by:

$$\boxed{\boldsymbol{\mu} = \frac{\mu_B}{\hbar}\left(g_l\hat{L} + g_s\hat{S}\right)}$$

Where g_l and g_s are the gyro-magnetic ratios, (usually just 2).

6.8 Hydrogen Atom

- Here the potential is given by:

$$\boxed{V(r) = -\frac{e^2}{4\pi\epsilon_0 r}}$$

- The energy levels are:

$$\boxed{E_n = -\frac{E_1}{n^2}}$$

Where $E_1 = 13.6\text{eV}$.

6.9 Rydberg formula

- This is the formula for the wavelength of light emitted from the transition from the nth energy level to the n'th energy level:

$$\boxed{\frac{1}{\lambda} = R_\infty(\frac{1}{n'^2} - \frac{1}{n^2})}$$

7 Condensed Matter

7.1 Bragg's Law

- This is similar to 2 slit interference, but for the angle at which light is incident on a lattice with separation d and wavelength λ:

$$\boxed{n\lambda = 2d\sin\theta}$$

- For a cubic lattice with miller indices (hkl) the spacing:

$$\boxed{d_{hkl} = \frac{h^2+k^2+l^2}{a^2}}$$

Where a is the atomic spacing. In general the d is related to the wave-vector $\mathbf{k} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3$, which is related to d via $|\mathbf{d}| = \frac{2\pi}{\mathbf{k}}$

8 Nuclear

8.1 Radius of Nucleus

- This is related to the atomic number A via:

$$\boxed{R = R_0 A^{1/3}}$$

Where R_0 is 1.2×10^{-18}

8.2 Radioactive Decay

- The number of nuclei as a function of time $N(t)$ is:

$$\boxed{N(t) = N_0 e^{-\lambda t}}$$

Where λ is the *decay constant*, which is related to the *half life* via:

$$\boxed{T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}}$$

$1/\lambda$ is often called the mean time T_{mean}.

8.3 Activity

- This is defined as:

$$\boxed{A(t) = \lambda N(t)}$$

8.4 Bateman Equation

- This describes the time evolution of two isotopes A and B is a radioactive decay chain. If The parent nucleus A decays via:

$$\boxed{\frac{dN_A}{dt} = -\lambda_A N_A}$$

Then its daughter isotope B will decay via:

$$\boxed{\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B}$$

9 Taylor Expansions

- The only Taylor expansion you need to know is the following, for $x << 1$:

$$\boxed{(1+x)^\alpha = 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2 + \mathcal{O}(x^3)}$$