

quark model, chpt 9 in Thomson | she will post exercises that night
be covered in tutorials

Good given units (Natural units)

$$\hbar = c = 1, \epsilon_0, \mu_0 = 1$$

Electron mass: $m_e = 0.511 \text{ MeV}$

Proton: $m_p = 938 \text{ MeV}$

- Fine structure constant

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

Relativistic kinematics

$$\gamma_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

$$p^M = (E, \vec{p}) \gamma^\mu = m \gamma(1, \vec{\beta})^M$$

$$\beta = \frac{\vec{v}}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$E_p = (p + m^2)^{\frac{1}{2}} \quad p^2 = m^2$$

If $E = E_p \Rightarrow$ particle is on-shell

Decay of massive particle
in its rest frame

$$\hat{s} \xleftarrow{=} \cancel{m}^2 \rightarrow p_m^M = (m, 0, 0, 0)$$

$$\text{Case 1: } p_1^M = \left(\frac{m}{2}, 0, 0, \frac{m}{2}\right)$$

$$\begin{aligned} m_1 = m_2 = 0 \Rightarrow p_2^M &= \left(\frac{m}{2}, 0, 0, \frac{m}{2}\right) \\ \Rightarrow p_1^2 = p_2^2 &= 0 \end{aligned}$$

$$+ p_m^M = p_1^M + p_2^M$$

Case 2:

$$m_1 \neq 0, m_2 \neq 0 \quad p_1^M = (E_p, 0, 0, P) \\ p_2^M = (P, 0, 0, -P)$$

$$\text{where } E_p = \frac{m_1^2 + m_2^2}{2M}, P = \frac{M^2 - m_1^2 - m_2^2}{2M}$$

Case 3:

$$m_1 \neq 0, m_2 \neq 0$$

$$p_1^M = (E, 0, 0, P)$$

$$p_2^M = (E_2, 0, 0, -P)$$

$$\text{where } E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}, E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

$$P = \sqrt{\frac{\lambda(M_1 M_2)}{2M}}$$

$$\lambda(a, b, c) = a^4 + b^4 + c^4 - 2ab - 2ac - 2bc$$

Discrete groups

only really need \mathbb{Z}_2

- Unitary representation:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

for particles this is
related to parity and charge
conjugation

QM system with 2 states

$$|T^+\rangle, |T^-\rangle$$

$$C|T^+\rangle = |T^-\rangle, C|T^-\rangle = |T^+\rangle$$

$$C \begin{pmatrix} |T^+\rangle \\ |T^-\rangle \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} |T^+\rangle \\ |T^-\rangle \end{pmatrix}$$

If $[C, H] = 0 \Rightarrow T^+$ and T^-

have equal masses and decay rates
"H has \mathbb{Z}_2 symmetry"

Since $C^2 = I \Rightarrow \lambda = \pm 1$

$$\Rightarrow \lambda = 1 \quad \frac{1}{2} (|T^+\rangle + |T^-\rangle)$$

$$\lambda = -1 \quad \frac{1}{2} (|T^+\rangle - |T^-\rangle)$$

Discrete symmetries

$\det = -1$

Parity: $(x^0, \vec{x}) \rightarrow (x^0, -\vec{x})$

Time reversal: $(x^0, \vec{x}) \rightarrow (-x^0, \vec{x})$

QM intrinsic Parity $P/A(\vec{R}) = \pm A(\vec{R})$

& intrinsic time reversal \Rightarrow anti-particles (conjugate fields)

C charge conjugation QFT \Rightarrow anti-particles (opposite charge, same mass)

Particle \leftrightarrow antiparticle

opposite charge, same mass

Continuous groups

Spacetime: translations - \mathbb{R}^4 abelian

& Lorentz transformations $SO(3, 1)$
non-abelian

$SU(n)$ = special unitary $n \times n$ matrices

$$SU(n) = \{ U_{n \times n} \mid U^\dagger U = I \text{ & } \det U \neq 0 \}$$

- For Standard model (SM) we also
want intrinsic $U(1)$, $SU(2)$, $SU(3)$

$U(n)$ = unitary $n \times n$ matrices

Klein-Gordan Eq.

Scalar field spin 0

$$\text{EOM } (\partial^2 + m^2) \phi(x) = 0$$

solution $\phi(x) = e^{ip \cdot x}$

$$\text{where } p^2 = m^2$$

Both $E = E_p$ are allowed

QFT! $\phi(x)$ is a field that
creates and destroys particles

$|0\rangle$ Vacuum: $|\psi(p)\rangle$ one particle

$$\langle 0 | \phi(x) | \psi(p) \rangle = e^{ip \cdot x} \quad \text{with } p^0 = E_p, p^1 = \dots$$

\Rightarrow positive energy sol.

$$[J^i, J^j] = i \epsilon^{ijk} J^k$$

$SO(3)$ has a d-dimensional rep
for every integer d

$$d = 2j + 1 \quad \text{"spin j"}$$

More generally

generators of Lie groups

satisfy a Lie algebra

with commutation relation

$$[t^a, t^b] = i f^{abc} t^c$$

structure constants

$$U = \exp[-i \alpha^a t^a]$$

<p>$\phi(x)$ is complex $\Rightarrow \langle \phi(p) \phi^*(x) \rangle = e^{ipx}$ creation of a particle If $\phi^*(x) \neq \phi(x)$, then we also have $\langle 0 \phi(x) \phi^*(p) \rangle = e^{-ipx}$ $\Rightarrow \phi^*(p)$ is the anti-particle If $\phi^*(\omega) = \phi(\omega) \Rightarrow \varphi$ can be its own anti-particle.</p>	<p><u>Maxwells eqs</u> spin 1 vector field $V^i(x)$ (vector representation) of angular mom</p> <p>Particle destruction: $\langle 0 V^i(x) v(p, \epsilon) \rangle = e^i e^{-ipx}$</p> <p>- Need $e^i \rightarrow R_{ij} e^j$ (where $V^i \rightarrow R_{ij} V^j$)</p>	<p><u>Relativistic</u> $\langle 0 V^i(x) v(p, \epsilon) \rangle = e^i e^{-ipx}$ Notice: Norm of this state is $e^m \epsilon_m$ can be ∞ - This is solved by photons being massless</p>	$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu$ extonal current with $\partial_\mu J^\mu = 0$ $\partial_\mu F^{\mu\nu} = -J^\nu$ Define a tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$
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<p><u>Piral Eq</u> spin $\frac{1}{2}$ <u>Dirac Matrices</u> must have $d \geq 4$ $\{ \gamma^\mu, \gamma^\nu \} = 2 \gamma^{\mu\nu}$</p> <p>- We will use Dirac basis $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \rightarrow$ for non-rel applications or Weyl basis $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \rightarrow$ rel applications</p> <p>- Couple to Maxwell's eqs Electrons & photons \rightarrow QED - We can couple Dirac eq to EM field by $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ieA_\mu$</p>	<p><u>Dirac Eq</u> $(i \gamma^\mu \partial_\mu - m) \psi = 0$</p> <p>- Solution of Dirac eq is solution of KG eq. $\psi \sim e^{-ipx}$</p> <p>- 2 positive energy sols 2 negative energy sols Given in terms of 2-component Spinors: $s=1, 2$ $\psi = u^s(p) e^{-ipx}, \bar{\psi} = v^s(p) e^{-ipx}$</p>	<p><u>Matrix elements for destroying/creating one electron</u> $\langle 0 \psi(x) \bar{e}(p, s) \rangle = u^s(p) \bar{e}^{-ipx}$ $\langle \bar{e}(p, s) \psi^+(x) 0 \rangle = u^{s\dagger}(p) e^{ipx}$ or for positions $\langle 0 \psi^+(x) e^+(p, s) \rangle = v^{s\dagger}(p) e^{ipx}$ $\langle e^+(p, s) \psi(x) 0 \rangle = v^s(p) e^{-ipx}$</p>
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<p>- Fine structure: electron spin $\frac{1}{2}$ - Spin orbit interactions + relativistic effects $\vec{L} = \text{orbital angular mom}$ $\Delta H \sim \vec{L} \cdot \vec{s}$ $\vec{s} = \text{spin angular mom}$ $\Delta H = \frac{g-1}{2} \frac{\alpha}{M_e^2 r^3} \vec{L} \cdot \vec{s}$</p> <p>- We will diagonalize ΔH Let $\vec{j} = \vec{l} + \vec{s}$ \leftarrow Tot angular mom orbital \uparrow Electron spin any mom $\vec{l} \cdot \vec{s} = \frac{1}{2} (\vec{j}^2 - \vec{l}^2 - \vec{s}^2)$</p> <p>Each $(n, l) \rightarrow 2 (2l+1) q_m$ states $\uparrow \quad \uparrow$ $\text{spin } m$</p> <p>- Fine structure splits degeneracy</p>	<p><u>Relativistic normalization</u> $\langle k_1 k_2 \rangle = \frac{1}{(2\pi)^3} \delta^3(\vec{p}_1 - \vec{p}_2)$</p> $\int \frac{d^4 p}{(2\pi)^4} (2\pi) \delta(p^2 - m^2)$ $= \int \frac{d^3 p}{(2\pi)^3} \int d^3 p' \delta((p')^2 - (p^2 - m^2))$ $= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_0}$	<p><u>Spin-statistics Thm of QFT</u> Fields w/ integer spin \rightarrow particles with BE stats Fields with half-integer spin \rightarrow particles with FD stats SM: fields of spin $0, \frac{1}{2}, 1, \dots$ Also composite particles</p>	<p><u>Hydrogen Atom</u> (Not to scale) - Non-rel: $V(r) = \frac{e^2}{r}$ Bound state energies $E = -\frac{1}{2} \alpha^2 n^2 \frac{1}{r^2}$ Orbital wave functions $Y_m(\theta, \phi), l=0, 1, 2, \dots$ m=-l, ..., l Parity: $P lm\rangle = (-1)^l lnlm\rangle$</p>
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<p><u>Positronium</u>: Bound state of e^+, e^- Label eigen states by J^{PC} charge parity Any mom</p> <p>Parity χ_m gets $(-1)^l$ under spatial inversion electron and position have opposite intrinsic parities $P = (-1)^{l+1}$</p> <p>charge DC exchanges $e^+ \leftrightarrow e^-$</p> <p>- 1 is wavefunction from fermionic exchange</p> <p>① Co-ordinate reversal in χ_m $(-1)^l$</p> <p>② Exchange of spins $\vec{S}_{e^+}^2$ and $\vec{S}_{e^-}^2$</p>
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<p>Total $C = (-1)^{l+1} \sum_{s=1}^l 1$</p> <p><u>Spectrum</u> $2s \longrightarrow 1^- \quad s=1$ $0^+ \quad s=0$</p>	<p><u>1s</u> $-\quad i^{--} \quad s=1$ $0^+ \quad s=0$</p> <p><u>2P</u> $s=0 \quad 1^{+-} \quad s=1$ $s=1 \quad 1^{++} \quad s=0$ $0^{++} \quad s=0$</p>	<p><u>QED interactions</u> mediated by the photon</p> <p><u>Couplings to e^+ and e^-</u> are opposite \Rightarrow</p> $C \gamma(\epsilon, p)\rangle = - \gamma(\epsilon, p)\rangle$	<p><u>Possible photon transitions</u></p> $2s (1^-) \rightarrow 2p (J^{++})$ $2p (J^{++}) \rightarrow 1s (1^-)$ $2p (1^{++}) \rightarrow 1s (0^{-+})$ $ s (1^-) \rightarrow s (0^{-+})$
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Ground states can decay to photons

Decay to $\gamma\gamma$ possible only for $C=+$ (i.e. 0^+)

But decay to a single photon is not possible (momentum cons)

1s 1^- can decay to $\gamma\gamma\gamma$

The Quark Model

- Hadrons are the particles observed in nuclei, they are bound states of quarks

Quarks appear in baryons or mesons, never alone

Pions: The lightest Hadrons are π -mesons

$\pi^\pm 139.57 \text{ MeV}$

$\pi^0 134.98 \text{ MeV}$

Charmonium

Any particle + antiparticle

→ excited EM fields

→ any other particle+anti-particle

1974 Two groups found a sharp resonance at the same energy level (3.1 GeV). $\xrightarrow{\text{found}}$

- One collided $e^+ e^-$ "ψ" need a 4th quark, charm

- other $p+p \rightarrow e^+ e^- + X$ "J"

Reasonance called J/ψ "gypsy"

Highest rate comes from annihilation by the EM current $J^\mu = \bar{q} \gamma^\mu q$

$$\langle 0 | \bar{J}(x) | e^+ e^- \rangle$$

$$\uparrow \text{spin } 1 \quad p=1, C=-1 \quad J^\mu = 1^-$$

1s, 2s etc higher l-states also observed like positronium

⇒ conclude bound state $c \bar{c}$

- Potential is not $\frac{1}{r}$

but like Alogr or $-\frac{A}{r} + Br$
→ quarks are confined

Light mesons

Pions found to have $J^P = 0^-$
 $\pi^0 \rightarrow 2\gamma \Rightarrow C(\pi^0) = +1$

Lightest hadrons all have $J^P = 0^-$

or 1^- 9 of each $l=0$

- Light pseudo scalar Mesons.

$$J^P = 0^- \quad \pi^0 958 \text{ MeV}$$

$$\pi^- 548 \text{ MeV}$$

$$K^0 \bar{K}^0 \pi^0 \pi^+ 498 \text{ MeV}$$

$$\pi^- \pi^0 \pi^+ 140 \text{ MeV}$$

- Light vector Mesons $\phi 1020 \text{ MeV} \rightarrow \gamma \eta^0$

$$J^P = 1^- \quad K^{*-} \bar{K}^{*0} \pi^{*+} 892 \text{ MeV}$$

$$\begin{matrix} \bar{s}^- & \omega & s^+ \\ s^0 & \sigma & s^+ \\ \bar{s}^0 & \rho^0 & 770 \text{ MeV} \end{matrix} \rightarrow \pi \eta^0$$

K, K^*, Λ^0 don't appear singly only with another "Strange Particles"
excited state proton

Vaguely correct: the 9 1^- states are bound states of $(\bar{u}, \bar{d}, \bar{s}) \cdot (u, d, s)$

- Posit discrete symmetry $u \leftrightarrow d$ and approx symmetry $u \leftrightarrow s, d \leftrightarrow s$
- extend to continuous symmetry SU(2) rotation "isospin"
- SU(3) rotation

Quark flavour Isospin (Thomson convention chp 9?)

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Useful if EM << strong interactions

Isospin:

$$\vec{T} = \frac{1}{2} \vec{\sigma}^2 \quad \text{Doublet } (\begin{pmatrix} u \\ d \end{pmatrix}) \quad \text{Antiquark doublet } (\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix})$$

$$2 \otimes 2 = 3 \oplus 1 \quad \begin{array}{ll} \phi(1,1) = -ud & \sim \pi^+, \pi^0 \\ \phi(1,0) = \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) & \sim \pi^0, \pi^- \\ \phi(1,-1) = d\bar{u} & \sim \pi^-, \pi^0 \\ \phi(0,0) = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) & \sim \gamma, \omega \end{array}$$

$q\bar{q}$ Isospin states

SU(3) Flavour Symmetry

$$\text{Triplet } u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

8 generators of SU(3)

Gellmann matrices, prefer $u \leftrightarrow d$ isospin

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ SU(2) of } u, d$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad u \leftrightarrow s$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad d \leftrightarrow s$$

Normalized so that $tr(\lambda_i \lambda_j) = 2 \delta_{ij}$

Isospin:

$$\hat{T}_i = \frac{1}{2} \lambda_i \quad \text{e.g. } \hat{T}_3 u = \frac{1}{2} u$$

$$\hat{T}_3 d = -\frac{1}{2} d$$

$$\hat{T}_3 s = 0$$

$$\text{Total Isospin } \hat{T}^2 = \sum_i T_i^2 = \frac{1}{4} \sum_i \lambda_i^2 = \frac{4}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ of } u, d$$

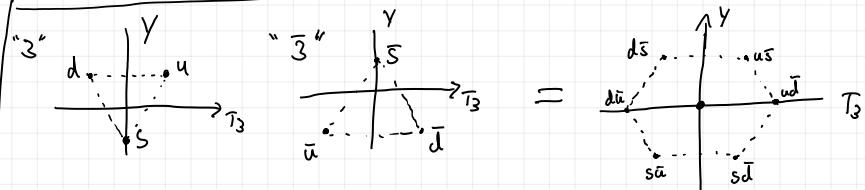
$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \text{ of } u, d$$

2-commuting generators $\hat{T}_3 \hat{T}_8$

Quantum numbers $\hat{T}_3 = \frac{1}{2} \lambda_3 \leftarrow 3rd \text{ comp isospin}$

$$\hat{Y} = \frac{1}{\sqrt{3}} \lambda_8 \leftarrow \text{Hypercharge}$$

$$\hat{Y}u = \frac{1}{3}u \quad \hat{Y}d = \frac{1}{3}d \quad \hat{Y}s = -\frac{2}{3}s$$



Combine remaining generators
into ladder operators

$$\hat{T}_{\pm} = \frac{1}{2}(\lambda_1 \pm i\lambda_2) \quad \hat{V}_{\pm} = \frac{1}{2}(\lambda_4 \pm i\lambda_5) \quad \hat{U}_{\pm} = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

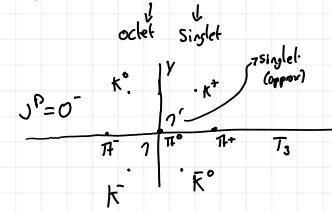
$\begin{matrix} u & \uparrow -d \\ d & \downarrow \bar{u} \end{matrix}$
 $\begin{matrix} d & \uparrow -s \\ s & \downarrow \bar{d} \end{matrix}$
 $\begin{matrix} u & \uparrow -s \\ s & \downarrow \bar{u} \end{matrix}$

3 states with $T_3 = Y=0$

$$\begin{aligned} T_- |u\bar{d}\rangle &= |d\bar{d}\rangle - |u\bar{s}\rangle \\ V_- |u\bar{s}\rangle &= |s\bar{s}\rangle - |u\bar{d}\rangle \\ U_- |d\bar{s}\rangle &= |s\bar{s}\rangle - |d\bar{d}\rangle \end{aligned}$$

2 lin
index

$$3 \otimes \bar{3} = 8 \oplus 1 \quad \text{Singlet } \frac{1}{3}(u\bar{u}) + (d\bar{d}) + (s\bar{s})$$



$$J^P = \begin{cases} S^- & K^+ \\ K^- & \bar{K}^0 \end{cases} \quad \begin{matrix} \omega & \\ s^- & s^0 & \phi & s^+ \\ K^+ & \bar{K}^0 & \bar{K}^0 & \bar{K}^{*-} \end{matrix}$$

$$\begin{aligned} |1\rangle &\simeq \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \\ |1T^0\rangle &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ |1\rangle &\simeq \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \end{aligned}$$

$$\begin{aligned} |1S^0\rangle &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ |1W\rangle &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\ |1\phi\rangle &\approx s\bar{s} \end{aligned}$$

G - Parity

Pions are not eigenstates
of charge conjugation

$$G|T^+\rangle = -|T^+\rangle$$

$$G|T^-\rangle = -|T^-\rangle$$

$$G|T^0\rangle = -|T^0\rangle$$

$G = -1$ for pions \rightarrow selection rules

Ex J/ψ has $C=-1$ and $I=0$

$$\Rightarrow G=-1$$

\Rightarrow can decay into an odd No of
pions

$$C|T^+\rangle = |T^-\rangle$$

$$\text{Define: } G = C e^{iT_1 I_2}$$

$$T_1 \rightarrow T_1$$

$$T_2 \rightarrow T_2$$

$$T_3 \rightarrow T_3$$

rotation in
isospin about
 \hat{z} axis

Heavy mesons with b, c

$$c\bar{s} \quad \text{+ anti-particles}$$

$$c\bar{u} \quad c\bar{d}$$

$$J^P = 0^- \quad D_s^+ \quad 1968 \text{ MeV}$$

$$0^0 \quad D^+ \quad 1989 \text{ MeV}$$

$$b\bar{u} \quad b\bar{s} \quad b\bar{d} \quad \text{+ anti}$$

$$J^P = 1^- \quad B_s^0 \quad 5367 \text{ MeV}$$

$$J^P = 0^- \quad B^0 \quad 5227 \text{ MeV}$$

$$D_s^{*+} \quad 2112 \text{ MeV}$$

$$D^* \bar{D}^* \quad 2010 \text{ MeV}$$

$$J^P = 1^- \quad B_s^{*-} \quad 5415 \text{ MeV}$$

$$B^* - \quad B^* \bar{0} \quad 5325 \text{ MeV}$$

No mesons with top quark decays to rapidly

Baryons: Baryons are fermions

Baryon number B is conserved

\rightarrow The lightest baryon is stable. Its the proton!

Light families

$$\Sigma \quad \Xi \quad \Xi^0 \quad 1313 \text{ MeV}$$

- Spin $\frac{1}{2}$ octet

$$\sum \quad \Lambda^0 \Sigma^0 \quad \Sigma^+ \quad 1192 \quad 1176$$

$$n \quad p \quad 938 \text{ MeV}$$

Spin $\frac{3}{2}$

$$\begin{matrix} \Sigma^- & \Sigma^{*-} & \Sigma^{*0} \\ \Sigma^+ & \Sigma^{\circ} & \Sigma^{*+} \\ \Delta^- & \Delta^0 & \Delta^+ \\ \Delta^+ & \Delta^{*+} & \Delta^{**0} \end{matrix}$$

- All states of $P=+$

(by convention)

- Can be understood as $q\bar{q}q$ bound states

$$\begin{matrix} "3" & "3" & "3" \\ \nabla \otimes \nabla = & \Delta & \Delta \end{matrix} \quad \begin{matrix} dd & uu \\ ss & ss \end{matrix} \quad 3 \otimes 3 = 6 \otimes 8$$

$$\begin{matrix} \Delta \otimes \Delta = & \Delta & \Delta \\ \Delta \otimes \Delta = & \Delta & \Delta \end{matrix} \quad \begin{matrix} \frac{1}{2}(ud+ud) \\ 3 \otimes 3 \otimes 3 = 10 \otimes 8 \otimes 8 \end{matrix}$$

$$\Delta \otimes \Delta = \Delta \oplus \frac{1}{2}(uds-usd+uds-sd-sd)$$

Suggested exercise:
- work through equations (9.8) - (9.15)



- Thomson exercise 9.3



Spin states

9 \otimes 9 Spin $\frac{3}{2}$ quadruplet

$$\chi\left(\frac{3}{2}, \frac{3}{2}\right) = |1111\rangle$$

$$\chi\left(\frac{3}{2}, \frac{1}{2}\right) = \frac{1}{\sqrt{3}}(|111\rangle + |11\downarrow\rangle + |1\downarrow\downarrow\rangle)$$

$$\chi\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{\sqrt{3}}(|1\uparrow\uparrow\rangle + |1\uparrow\downarrow\rangle + |1\downarrow\downarrow\rangle)$$

$$\chi\left(\frac{3}{2}, -\frac{3}{2}\right) = |\downarrow\downarrow\rangle$$

Electric charges of quarks

It follows that

$$Q_d = -\frac{1}{3}, \quad Q_u = \frac{2}{3}$$

$$Q_s = -\frac{1}{3}, \quad Q_c = \frac{2}{3}$$

$$Q_b = -\frac{1}{3}, \quad Q_t = +\frac{2}{3}$$

Detectors in a nutshell

2-types: - trackers \rightarrow trajectory & momentum
- calorimeters \rightarrow total energy

- Energy loss by ionisation
(particle destroyed)

- Em interactions that tick electrons

Bethe-Bloch eq:

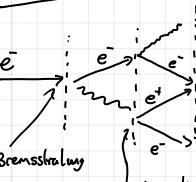
$$\frac{dE}{dx} \approx -4\pi Z^2 \frac{N}{Z} \log \frac{2\pi^2 \rho^2 m_e}{w} - \beta^2$$

- N = number density
- Z = atomic number
- w = atomic frequency

- $\frac{dE}{dx}$ depends on velocity
 \Rightarrow with measurement of both we can derive particle mass



EM showers



EM calorimeters are crystals containing these showers in compact regions

- Energy is measured from scintillation light

$$X_0 = \text{radiation length}$$

Hadronic showers

- far more irregular
Not all energy is detectable
Can go into the nucleus
Hadronic calorimeter are cruder by an order of mag
- Transition radiation
effect of crossing vacuum
medium boundary, can distinguish electrons from pions

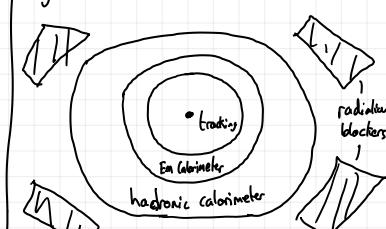
Cherenkov radiation

Superluminal velocity in medium

\rightarrow shockwave

Discriminates particle velocit.

Cylindrical collider detector:

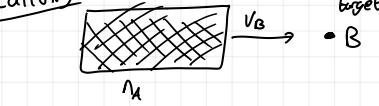


Suggested Exercises

Pestin 2.1, 2.5, 7.2, 8.1

Chapter 7 Pestkin

Scattering



- Beam of particles with density n_A velocity v_A

Rate of scattering:

$$\frac{\text{events}}{\text{sec}} = n_A V_A \sigma \quad \text{cross section, units cm}^2$$

differential cross section

$$\frac{d\sigma}{d^3 p_1 \dots d^3 p_n} \quad \text{final state momenta}$$

Total cross section:

$$\sigma = \int d^3 p_1 \dots d^3 p_n \frac{d\sigma}{d^3 p_1 \dots d^3 p_n}$$

Tools for calculation

two phenomena:

- scattering
- decay

- Unstable particle A

Constant rate of decay

Probability of existence

$$dP = -P \Gamma_A dt$$

$$\Rightarrow P(t) = e^{-t \Gamma_A}$$

"total width Γ_A "

$$\Gamma_A = \sum_F \Gamma(A \rightarrow f)$$

for multiple decay processes

$$\int dT_2 \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta(P - p_1 - p_2) \quad (2 \text{ particle collision})$$

CoM frame:

$$\vec{p}_i = \vec{p}_1 = \vec{p}_2$$

carry out 3 integrals?

$$\int dT_2 = \int \frac{d^3 p}{(2\pi)^3 2E_1 2E_2} S(E_{cm} - E_1 - E_2)$$

$$d^3 p = p^2 d\theta \sin\theta d\phi dP = p^3 dP d\Omega$$

$$\int dP S(E_{cm} - E_1(p) - E_2(p)) = \frac{1}{\left| \frac{dE_1}{dp} + \frac{dE_2}{dp} \right|}$$

$$\stackrel{E_n^2 = p^2 + m^2}{\Rightarrow E_1 dE_1 = p dp} \Rightarrow \frac{1}{\left| \frac{p}{E_1} + \frac{p}{E_2} \right|} = \frac{E_1 E_2}{(E_1 + E_2)^2}$$

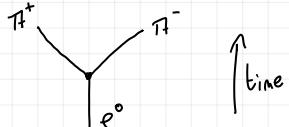
$$\int dT_2 = \int \frac{p^2 d\Omega}{8\pi^2 E_1 E_2} \frac{E_1 E_2}{E_{cm} P}$$

$$= \frac{1}{8\pi^2} \frac{2p}{E_{cm}} \int \frac{d\Omega}{4\pi^2}$$

$$Ex: \bar{T}\bar{T} T^+ \rightarrow g^0 \rightarrow T^+ T^- \quad |M_{g^0}| = 770 \text{ MeV}$$

Feynman diagrams

- convention is time goes upwards



$$\textcircled{1} \quad M(T^+ T^- \rightarrow g^0)$$

$$T^+(p_A) T^-(p_B) \rightarrow g^0(p_c)$$

Step ①: $T^+ T^- \rightarrow g^0$
Step ②: $g^0 \rightarrow T^+ T^-$

③ g^0 resonance
Breit-Wigner + sum over polarizations

spin 1
→ polarization vectors
 ϵ^μ to describe
the states, with
 $\epsilon^\mu \cdot p_c = 0$

Master Formula Fermi's Golden rule

QM transition matrix element

$$\langle 1 2 \dots n | T | A(p_A) \rangle = M(A \rightarrow |1 \dots n\rangle) (2\pi)^4 \delta^{(4)}(p_A - \sum_j p_j)$$

Decay:

$$\Gamma(A \rightarrow f) = \frac{1}{2M_A} \int dT_n |M(A \rightarrow f)|^2 \quad \text{- phase space integral}$$

$$\text{mass } \uparrow \quad \text{where } dT_n = \frac{d^3 p_1}{(2\pi)^3 2E_1} \dots \frac{d^3 p_n}{(2\pi)^3 2E_n}$$

- Spin: sum \sum over final spin states

specify initial state if known, average if not.

- Cross section:

$$\sigma(A + B \rightarrow f) = \frac{1}{2E_A 2E_B |v_A - v_B|} \int dT_n |M(A+B \rightarrow f)|^2$$

Resonance

Breit-Wigner formula for resonance

$$M \sim \frac{1}{E - E_R + i\frac{\Gamma}{2}}$$

E_R = Energy of resonant state

Γ = decay rate.

Fourier transform:

$$\psi(t) = \int \frac{dE}{2\pi} \frac{e^{iEt}}{E - E_R + i\frac{\Gamma}{2}} = -i e^{iE_R t} e^{-\Gamma t/2}$$

Probability of maintaining Resonance

decays exponentially $|\psi(t)|^2 = e^{-\Gamma t}$

$$\text{lifetime } \tau_R = \frac{1}{\Gamma}$$

We need a relativistic version

→ Lorentz invariance

$$\rightarrow M(\bar{T}\bar{T} \rightarrow g^0)$$

$$= g^* \cdot (p_A - p_B)$$

↑ spin-1 field eqs
const

- Can argue this is
the only quantity that
can be formed?

CoM frame

$$p_A = (E, \vec{p}) \quad E = \frac{1}{2} m_g \quad p^2 = \frac{m_g^2 - m_\pi^2}{4}$$

$$p_B = (E, -\vec{p})$$

$$\epsilon^* = -\epsilon \Rightarrow \epsilon^* (p_A - p_B) = -2 \vec{e} \cdot \vec{p}$$

$$\sigma(\bar{T}\bar{T} \rightarrow g^0) = \frac{1}{2E_A E_B |v_A - v_B|} \int \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{1}{2E_c} |M|^2 S(p_c^2 - m_g^2)$$

$$\left| \frac{p_A - p_B}{E_A - E_B} \right| = 2 \frac{p}{m_g/2}$$

$$\int \frac{d^3 p_c}{(2\pi)^3 2E_c} S(p_c^2 - m_g^2)$$

$$= \frac{1}{m_g^2 \frac{4p}{m_g}} 2\pi \delta((p_A + p_B)^2 - m_g^2) g_s^2 4 |\vec{e} \cdot \vec{p}|^2$$

$$\sum_{\epsilon} |\vec{e} \cdot \vec{p}|^2 = p_A^2 + p_B^2 + p_c^2 = P^2 \Rightarrow \sigma = g_s^2 \frac{P}{m_g} (2\pi) \delta((p_A + p_B)^2 - m_g^2)$$

Step 2

$$M(\gamma_0 \rightarrow \pi^+ \pi^-)$$

$$\Gamma_\gamma = \frac{1}{2m_\gamma} \int d\Omega_2 |M|^2$$

$$= \frac{1}{2m_\gamma} \frac{1}{8\pi} \frac{2P}{m_\gamma} g_s^2 \underbrace{\langle 4|\vec{e}^2 \vec{p}|^2 \rangle}_{\frac{4}{3}P^2} = \frac{g_s^2 P^3}{6\pi m_\gamma^2}$$

- Measure γ° decay. Find

$$\frac{g_s^2}{4\pi} \approx 2.9 \quad (\text{relatively large})$$

process likes to occur.

Step 3
Resonance $M \sim \frac{1}{P^2 - M_R^2 + iM_R\Gamma_R}$

CM frame: $P = (M_R + \Delta E, \vec{0})$

$$\sigma(\pi^+(\rho_A)\pi^-(\rho_B) \rightarrow \gamma_0 \rightarrow \pi^+(\rho_A)\pi^-(\rho_B))$$

$$= \frac{1}{2E_A 2E_B (V_A - V_B)} \int d\Omega_2 \left| \sum_{P\bar{P}} \frac{M(\pi^+\pi^- \rightarrow \rho^0 \epsilon) M(\rho^0 \epsilon \rightarrow \pi^+\pi^-)}{(P_A + P_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho} \right|^2$$

$$= \frac{1}{4m_\rho P \cdot 8\pi} \frac{-2P}{m_\rho^3} \int d\Omega \frac{1}{(E_{cm}^2 - m_\rho^2) + M_\rho^2 \Gamma_\rho^2} \left| \sum_{\epsilon} (2g_\rho \vec{e} \cdot \vec{p})(2g_\rho \vec{e}' \cdot \vec{p}') \right|^2$$

$$\boxed{\sigma = \frac{1}{4m_\rho^2} \frac{g_s^2}{(E_{cm}^2 - m_\rho^2) + M_\rho^2 \Gamma_\rho^2} \int d\Omega | \vec{p} \cdot \vec{p}' |^2}$$

check limit of long lived narrow resonance

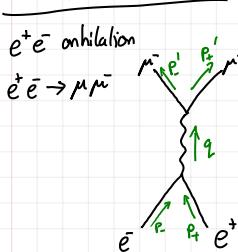
$$\sigma(\pi^+\pi^- \rightarrow \rho \rightarrow f)$$

$$= \frac{1}{4m_\rho P} \int d\Omega \left| \sum_{\epsilon} \frac{12g_\rho \vec{e}^* \cdot \vec{p}}{(P_A + P_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho} M(\rho \rightarrow f) \right|^2$$

$$\sum_f \sigma(\pi^+\pi^- \rightarrow \rho \rightarrow f) = \frac{1}{4m_\rho P} 2\pi g_\rho^2 \frac{P^2 \Gamma_\rho^2 / 1\pi}{(P_\rho^2 - M_\rho^2) + M_\rho^2 \Gamma_\rho^2} \xrightarrow{\rho \rightarrow 0} \frac{8(P_\rho^2 - M_\rho^2)}{M_\rho^2 \Gamma_\rho^2} \rightarrow 0$$

$$\lim_{\Gamma_\rho \rightarrow 0} \sigma = \frac{g_\rho^2 P}{m_\rho^3} 2\pi \delta((P_A + P_B)^2 - M_\rho^2) : \quad \delta(x) = \frac{1}{\pi} \lim_{G \rightarrow 0} \frac{G}{x^2 + G^2}$$

Strong interaction:



$$M(e^+e^- \rightarrow \mu^+\mu^-) = (-e) \langle \mu^+ \mu^- | j^\mu | 0 \rangle \frac{1}{q^2} (e) \langle 0 | j_\mu | e^+ e^- \rangle$$

Massless spin $\frac{1}{2}$ particles

e, μ treated as massless (Also light quarks)

$$\text{Massless Dirac Eq: } i \gamma^\mu \partial_\mu \psi = 0$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \text{Weyl spinors}$$

- Resonance = EM current

Breit-Wigner at zero mass

$$M \sim \frac{1}{q^2} \text{ "virtual photon"}$$

Virtual particle \Rightarrow off shell

- Dirac eq factorizes for $M=0$ into:

$$i \bar{\sigma} \cdot \vec{j} \psi_L = 0, i \sigma \cdot \vec{j} \psi_R = 0$$

- Couple to EM $\partial_\mu \rightarrow D_\mu = \partial_\mu - i e A_\mu$

plane wave solutions, for $\vec{p} = \vec{p}_3$

$$\psi_R = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-iEt + iEx^3} \quad \text{with} \quad E > 0 \quad \text{distinction of right handed electron.} \quad u_R(p)$$

$$\psi_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{+iEt + iEx^3} \quad \text{with} \quad E > 0 \quad \text{creation of left handed positron.} \quad v_L(p)$$

$$\psi_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-iEt + iEx^3} \quad E > 0 \quad \text{destruction of left-handed electron} \quad u_L(p)$$

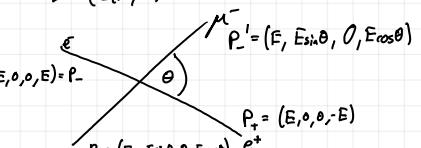
$$\psi_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iEt + iEx^3} \quad E < 0 \quad \text{creation of right-handed positron} \quad v_L(p)$$

$$j^\mu = \overline{\psi} \gamma^\mu \psi = \psi_L^+ \bar{\sigma} \psi_L + \psi_R^+ \sigma \psi_R$$

Helicity conservation e_R^- can scatter into e_R^- or annihilate e_L^+ , nothing else.

CM frame

$$q = (E_m, \vec{0})$$



$$\vec{E}_\pm = \frac{1}{S_2} (1, \pm i, 0) \quad \vec{Z}_\pm' = \frac{1}{S_2} (\cos\theta, \pm i, -\sin\theta)$$

$$|M(e_R^- e_L^+ \rightarrow \mu_R^- \mu_L^+)|^2 = |M(e_L^- e_R^+ \rightarrow \mu_L^+ \mu_R^-)|^2 = e^4 (1 + \cos\theta)^2$$

Cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi \alpha^2}{2E_m^2} (1 + \cos^2\theta)$$

$$\Rightarrow \sigma = \frac{4\pi \alpha^2}{3E_m^2}$$

Suggested exercises

PostN 8.1

peaks at $\cos^2\theta = 1$ { we Average over 4 initial spin states including $e_R^- e_R^-$ and $e_L^+ e_L^+$. Sum over 2 final spin states }

$$e^+ e^- \rightarrow \text{hadrons}$$

For $E_{cm} \gg$ light meson masses
quark model predicts the following modification:

1. sum over light quark species uds (c,b)

$$2. Q_f = -1 \rightarrow Q_f = \int_{-1/3}^{2/3} u, c \quad d s b$$

$$M \sim Q_f \Rightarrow \sigma \sim Q_f^2$$

3. sum over final colour states

Result is:

$$\sigma(e^+ e^- \rightarrow \text{hadrons}) = \sum_f 3Q_f^2 \frac{4\pi\alpha}{3E_{cm}^2}$$

$$\frac{\sigma(e^+ e^- \rightarrow q\bar{q})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = \sum_f 3Q_f^2 = \begin{cases} 2 & u,d,s \\ 3\frac{1}{3} & u,d,s,c \\ 3\frac{2}{3} & u,d,s,c,b \end{cases}$$

- Strong interaction can be nearly neglected at this level

Deep inelastic Electron scattering

probing proton structure (quarks, gluons, anti-quarks)

Elastic: $e p \rightarrow e p$

Inelastic: break proton, produce hadrons

High momentum transfer

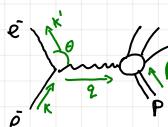
Deep: probe internal structure

Total mass of final hadrons $\gg m_p$

- Observation shows DIS is well approximated by $e p \rightarrow \text{hadrons}$

- Also: p is not just a uud bound state

\rightarrow Parton Distribution Functions (PDFs)



Define $W = \text{mass of final hadronic system}$

$$W^2 = (p + q)^2 = M_p^2 + 2 P_q + q^2$$

Assume large momentum transfer

$$|q|^2 \gg M_p^2 \quad \text{i.e. } M_p^2 \rightarrow 0$$

$$\text{Define } Q^2 = -q^2$$

Since q is space-like $\Rightarrow Q^2 > 0$

Large $Q^2 \Leftrightarrow$ large momentum transfer to proton

Observation:

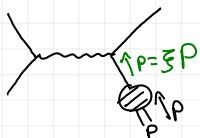


baryon resonances \uparrow W
high Q^2



clue for interpretation: parton model

$\xi = \text{momentum fraction of parton}$

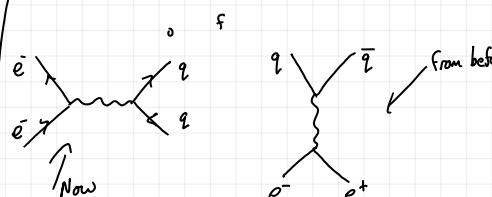


Assume that the parton is a spin $\frac{1}{2}$ quark carrying a flavour label f (or \bar{f} for an anti-quark)

Def: $f_i(\xi) d\xi = \text{probability of finding parton of type } i \text{ carrying momentum fraction } \xi (\xi, \xi+d\xi)$

- Sum rule $\int_0^1 d\xi \sum_i f_i(\xi) = 1$
 \downarrow sum over f, \bar{f}

$$\sigma(\bar{e} p \rightarrow \bar{e} X) = \int d\xi \sum [f_f(\xi) + f_{\bar{f}}(\xi)] \times \sigma(\bar{e} q (\bar{e} p) \rightarrow \bar{e} q)$$



- These have the same M (invariant matrix element)

Crossing Symmetry

General $2 \rightarrow 2$ scattering

Orient all momenta outwards

$$P_3 \rightarrow P_4 \quad P_4 \rightarrow P_1 \quad P_1 + P_2 + P_3 + P_4 = 0$$

M only depend on Lorentz invariant

Mandelstam invariants

$$P_i^2 = M_i^2 \quad (P_1 + P_2)^2 = S$$

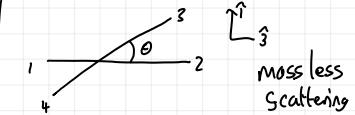
$$(P_1 + P_3)^2 = t \quad (P_1 + P_4)^2 = u$$

$$S+t+u = M_1^2 + M_2^2 + M_3^2 + M_4^2$$

These are no even independent due to momentum conservation

$$S+t+u = M_1^2 + M_2^2 + M_3^2 + M_4^2$$

CM frame



$$P_1 = (E, 0, 0, -E) \quad P_3 = (\bar{E}, E \sin\theta, 0, E \cos\theta)$$

$$P_2 = (-E, 0, 0, E) \quad P_4 = (E, -E \sin\theta, 0, -E \cos\theta)$$

$$S = (2E)^2 = E_{cm}^2 \quad S+t+u=0$$

$$t = -2E^2(1-\cos\theta) \quad u = -2E^2(1+\cos\theta)$$

Intermediate state:

S-channel

$$P_1 + P_2 \sim \frac{1}{S - M^2 + iM_F \Gamma_R} \quad ; \quad |M|^2 \sim \frac{1}{S} = \frac{1}{E_{cm}^2}$$

t-channel

$$|M|^2 \sim \frac{1}{t} = \frac{1}{E_{cm}^2 \sin^4(\frac{\theta}{2})} \quad \Rightarrow \text{strong peak in forward dir } \theta=0$$

u-channel

$$|M|^2 \sim \frac{1}{u} = \frac{4}{E_{cm}^2 (1+\cos\theta)^2} \quad \text{strong peak in backwards dir } \theta=0$$

$$|M(e^- R e_L^+ \rightarrow q_L \bar{q}_R)|^2 = Q_f^2 e^4 (1+\cos\theta)^2 = 4 Q_f^2 e^4 \frac{u^2}{S^2} \quad (1)$$

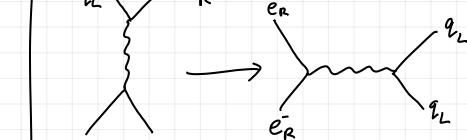
$$|M(e^- R e_L^+ \rightarrow q_L \bar{q}_R)|^2 = Q_f^2 e^4 (1-\cos\theta)^2 = 4 Q_f^2 e^4 \frac{t^2}{S^2} \quad (2)$$

Lorentz invariant

\Rightarrow (for 1)

$$|M|^2 = 4 Q_f^2 e^4 \frac{s^2}{t^2}$$

(2)



$$\Rightarrow |M|^2 = 4 Q_f^2 e^4 \frac{u^2}{t^2}$$

Consistency check

$$|M|^2 \sim u^2 \sim (1+\cos\theta)^2 \Rightarrow |M|^2 = 0$$

for $\theta = \pi$ (no backwards scattering)

Makes sense with Spin conservation $\rightarrow \bar{e}_R \leftarrow \bar{e}_L$

Total cross section:

$$\sigma(e\bar{e} \rightarrow e\bar{e}) = \frac{1}{2E} \frac{1}{2E} \frac{1}{8\pi} \int \frac{d\cos\theta}{2} \frac{1}{4} \sum_{f \in s} |M(e\bar{e} \rightarrow e\bar{e})|^2$$

Back to D.I.S. Hats ^ refer to partonic kinematics

$$\sigma(e\bar{p} \rightarrow e\bar{X}) = \int d\xi d\hat{t} \sum_f [f_f(\xi) + f_{\bar{f}}(\xi)] \frac{2\pi Q_g \alpha^2}{\xi^2} \frac{\hat{s}^2 \hat{t}^2}{\hat{t}^2}$$

kinematics of DIS

introduce Q^2, x, y

$\hat{t} = q^2 = -Q^2$ momentum transfer

$\hat{s} = (k+p)^2 = 2k \cdot p = 2k \xi P$ (we neglect proton mass)

recall $\hat{s} = \xi S$

define $y = \frac{2P \cdot \bar{q}}{2P \cdot k}$ In proton rest frame

$y = \frac{q_0}{k_0}$ fraction of initial e^- energy transferred to proton

$\hat{t} = \xi S$

$\hat{s}^2 + \hat{t}^2 = 1 + (1-y)^2$

Masslessness of the final state quark means $O = (P+q)^2 = 2P \cdot q + q^2 = 2S P \cdot q - Q^2$

define: $X = \frac{Q^2}{2P \cdot q}$ final quark massless $\Rightarrow \hat{t} = X$

(Can measure x to find S)

Relation

$$Q^2 = x y S \Rightarrow dQ^2 = x S dy$$

$$\frac{d^2\sigma}{dS d\hat{t}} = \sum_f [f_f(\xi) + f_{\bar{f}}(\xi)] 2\pi Q_g \alpha^2 \frac{(1+(1-y)^2)}{Q^4}$$

$d\hat{t} = dQ^2 = x S dy$

$$\Rightarrow \frac{d^2\sigma}{dy dx} = \sum_f x Q_g^2 [f_f(\xi) + f_{\bar{f}}(\xi)] \frac{2\pi \alpha^2}{Q^4} (1+(1-y)^2) \quad \text{with } 0 < x < 1 \quad 0 < y < 1$$

Bjorken scaling

On general grounds, we expected the elementary QED cross section

$$\frac{d\sigma}{dx dy} (e\bar{p} \rightarrow e\bar{X}) = f_e(x, \bar{q}) \frac{2\pi \alpha^2 S}{Q^4} (1+(1-y)^2)$$

un known form factor.

- independence of F_2 on Q^2 is called Bjorken Scaling

Q^2 dependence is not fully constant but slow logarithmic

Parton model $\Rightarrow f_f(x, Q) = F_f(x)$

$$= \sum_f Q_g^2 x [f_f(x) + f_{\bar{f}}(x)]$$

The Gluon

$f_f(x), f_{\bar{f}}(x)$ are parton distribution functions (pdfs) of the proton (or any other hadron)

- Measurement of pdfs: Quark model

$$F_2(x) = \sum_f Q_g^2 x [f_f(x) + f_{\bar{f}}(x)]$$

$$= \frac{4}{9} x f_u(x) + \frac{1}{9} x f_d(x)$$

- preserving charge and isospin, any extra quark must appear in $q\bar{q}$ pairs and satisfy spin rules

- These cannot be derived from first principles, only experiment

- DIS of $e\bar{p} \rightarrow e\bar{X}$ gives only one combination

- Get an independent combination from scattering on a deuterium target

Isospin Rotation:

proton and neutron pdfs are related:

$$f_u^{(n)}(x) = f_d(x) \quad f_d^{(n)}(x) = f_u(x)$$

$$f_{\bar{u}}^{(n)}(x) = f_{\bar{d}}(x) \quad f_{\bar{d}}^{(n)}(x) = f_{\bar{u}}(x)$$

$$\Rightarrow F_n^{(2)}(x) = \frac{4}{9} x f_d(x) + \frac{1}{9} x f_u(x)$$

$$\Rightarrow \text{can fix } f_u(x) \text{ and } f_d(x)$$

More info from DIS by neutrinos. (Weak interactions)

Sum rules: do not require $f_f(x) = f_{\bar{f}}(x)$

Ex. quantum fluxuation $p \leftarrow \Lambda^0 + K^+$

ud can form a low energy I=0 sea

leaves the other u with higher momentum \rightarrow relatively higher f_u compared to f_d

- relatively strong fs at higher x compared to $f_{\bar{s}}$

$p \leftarrow n + \bar{n}^+$

udd uds u \bar{u}

- relatively higher $f_{\bar{s}}$ compared to f_s

Valence quark pdf's

Sea quarks:

found to diverge as $x \rightarrow 0$

- divergences cancel between any $f_f, f_{\bar{f}}$ for the sum rules to hold.

Total sum rule: $\int dx x \sum_{f \in f} f_f(x) = 1$

We observe that:

$$\frac{p_g + \bar{q}}{p} = \int dx x \sum_f [f_f(x) + f_{\bar{f}}(x)] \approx 0.5$$

What is missing? Gluons!

- Posit that there is a field responsible for strong interaction (with particles)

If gluons exist \Rightarrow $e^+ e^- \rightarrow \text{hadrons}$ should also produce gluons radiated from outgoing $q\bar{q}$

- Prediction of observed 3-jet events

- Found at $E_{cm} = 30 \text{ GeV}$ 4 jet events observed at 91 GeV (at DESY)

Model gluons as photons (spin-1) could try spin-0/2 only this works

- coupling to q as q couples to e

For jets, assume collinear splitting i.e. small momentum transfer.

$e^+ e^- \rightarrow g\bar{q} q$

$P \approx (E, 0, 0, E)$

let z be momentum fraction transferred to q

$q = (zE, z\perp, 0, zE - \frac{q\perp^2}{2zE})$

so that $q^2 = 0 + O((\frac{q\perp}{E})^4)$

$k = ((1-z)E, -z\perp, 0, (1-z)E - \frac{q\perp^2}{2(1-z)E})$

$k^2 = 0 + O((\frac{q\perp}{E})^4)$

$P = (E, 0, 0, E - \frac{q\perp^2}{2z(1-z)})$

$P^2 = \frac{q\perp^2}{z(1-z)} + O((\frac{q\perp}{E})^4)$

$$M(q_R \rightarrow \tau q_R) = Q_F e \langle q_R(k) | j^\mu | q_R(p) \rangle \epsilon_\mu^*(\tau)$$

For Right handed only:

$$j^\mu = \psi_R^\dagger \sigma^\mu \psi_R$$

$$= Q_F e U_R^\dagger(k) \sigma^\mu U_R(p) \epsilon_\mu^*(\tau)$$

$$\text{- Spinors } U_R(p) = \sqrt{2E} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$U_R(k) = \sqrt{2(1-z)} E \begin{pmatrix} 1 \\ -q_\perp \end{pmatrix}$$

To $\partial(q_\perp)$:

photon polarizations

$$\text{so } \epsilon \cdot q = 0$$

$$E_R = \frac{1}{\sqrt{2}} (0, 1, i, -\frac{q_\perp}{2E}), \quad E_L = \frac{1}{\sqrt{3}} (0, 1, -1, -\frac{q_\perp}{2E})$$

$$U_R^\dagger \sigma \cdot \epsilon_L^* U_R = -\frac{2E}{\sqrt{2}} \left(1, -\frac{q_\perp}{2(1-z)E} \right) \frac{1}{\sqrt{2}} \left[\sigma^1 + i\sigma^2 - \frac{q_\perp}{2E} \sigma^3 \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \sqrt{2} q_\perp \frac{\sqrt{1-z}}{2(1-z)}$$

$$U_R^\dagger \sigma \cdot \epsilon_L^* U_R = \dots = \sqrt{2} q_\perp \frac{\sqrt{1-z}}{2(1-z)}$$

$$\sum_{\epsilon} |M|^2 = 2 Q_F^2 \epsilon^2 q_\perp^2 \frac{1}{z^2(1-z)} (1 + (1-z)^2)$$

Weizsäcker-Williams distribution

$$\sigma(A \rightarrow B + f + \gamma) \approx \sigma(A \rightarrow B + f) \int dz \int \frac{d\epsilon_\perp}{q_\perp} \frac{\alpha_s^2 \alpha}{4\pi} \frac{1 + (1-z)^2}{z}$$

$$\sigma(A + f \rightarrow B + \gamma) \approx \sigma(A + f \rightarrow B) \quad \dots$$

For $f = \text{electrons}$, cut off these

integrals at the scale of $M_e \leq q_\perp \leq E_m$

$$\frac{M_e}{E_m} \leq z \leq 1$$

$$\text{- we get } \frac{2\alpha}{\pi} \log^2 \frac{E_m}{M_e} \quad \text{for } E_m \gg M_e$$

$$e^+ e^- \rightarrow 3 \text{ jets}$$

Let $q_s = \text{strong interaction coupling}$

$$\text{constant } \alpha_s = \frac{g_s^2}{4\pi}$$

$$\sigma(A \rightarrow B + f + \gamma) \approx \sigma(A \rightarrow B + f) \int dz \int \frac{d\epsilon_\perp}{q_\perp} \frac{4}{3} \frac{\alpha_s}{\pi} \frac{(1 + (1-z)^2)}{z}$$

Relax co-linear assumption

$$E_q, E_{\bar{q}}, E_g \quad E_m = Q = E_q + E_{\bar{q}} + E_g$$

Define fractions

$$x_q = \frac{2E_q}{Q}, \quad x_{\bar{q}} = \frac{2E_{\bar{q}}}{Q}, \quad x_g = \frac{2E_g}{Q}$$

Nothers then \rightarrow conservation law

$$\delta \overline{\Psi} = i \delta a \overline{\Psi}, \quad \delta \overline{\Psi} = -i \delta a \overline{\Psi}$$

$$\delta \overline{\Psi} = (\delta \overline{\Psi}) (i \gamma^\mu \partial_\mu - m) \overline{\Psi} + \overline{\Psi} (i \gamma^\mu \partial_\mu - m) (\delta \overline{\Psi})$$

$$= 0$$

- { } we allow local variation:

$$\delta \overline{\Psi}(x) = i(\delta a(x)) \overline{\Psi}(x), \quad \delta \overline{\Psi} = -i(\delta a(x)) \overline{\Psi}(x)$$

$$\Rightarrow \delta \overline{\Psi} = \overline{\Psi} i \partial^\mu \partial_\mu (i \delta a(x)) \overline{\Psi}(x) = \overline{\Psi} i \gamma^\mu \overline{\Psi} (i \partial_\mu \delta a(x))$$

$$\delta S = \int d^4x (S(x)) \partial_\mu (\overline{\Psi} \gamma^\mu \overline{\Psi})$$

\Rightarrow Conservation of $j^\mu = \overline{\Psi} \gamma^\mu \overline{\Psi}$

$$\partial_\mu j^\mu = 0$$

$$\sigma(e^+ e^- \rightarrow q \bar{q} \gamma) = \frac{1}{2E_A 2E_B 2} \int dT \prod_{i=1}^3 |M(e^+ e^- \rightarrow q_i \bar{q}_i \gamma)|^2$$

$$\prod_{i=1}^3 = \int \frac{d^3 p_i d^3 k_i d^3 q_i}{(2\pi)^3 2\bar{p}_i 2k_i 2q_i} (2\pi)^4 \delta^{(4)}(Q - \bar{p} - q - k)$$

$$k = p - q \Rightarrow d^3 k = d^3 p$$

$$k = (1-z)p + O(q_\perp)$$

$$q = zp + O(p_\perp)$$

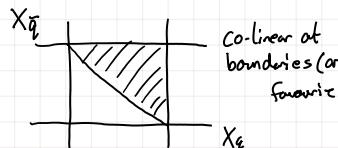
$$d^3 q = d\vec{q}^2 d^2 q_\perp = pdz \pi d\vec{q}$$

$$\sigma(e^+ e^- \rightarrow q \bar{q} \gamma) = \frac{1}{2E_A 2E_B 2} \int dT \prod_{i=1}^3 |M(e^+ e^- \rightarrow q_i \bar{q}_i \gamma)|^2$$

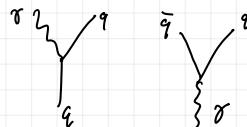
$$\times \int \frac{dz \pi d\vec{q}}{(2\pi)^2 2z(1-z)} \left| \frac{1}{p} \right|^2 |M(q \rightarrow \gamma \gamma)|^2$$

$$= \sigma(e^+ e^- \rightarrow q \bar{q}) \int \frac{dz d\vec{q}}{(6\pi)^2 2(1-z)} \left(\frac{z(1-z)}{q_\perp} \right) 2Q_F^2 e^2 q_\perp^2 \frac{1}{2^2 (1-z)} (1 + (1-z)^2)$$

- remaining integrals diverge as z or $q_\perp \rightarrow 0$ - soft and co-linear divergences.



$$\sigma(e^+ e^- \rightarrow q \bar{q} \gamma) = \sigma(e^+ e^- \rightarrow q \bar{q}) \int dx_q dx_{\bar{q}} \frac{2\alpha_s}{3\pi} \frac{x_q^2 + x_{\bar{q}}^2}{((1-x_q)(1-x_{\bar{q}}))}$$



Quantum Chromodynamics (QCD)

- Quarks as spin $\frac{1}{2}$ fermions
- Gluons as massless spin-1 bosons
- Maxwell-like equations
- "Colour"
- Consistency in neglecting α_s
- to lowest order in $e^+ e^- \rightarrow$ Hadrons and DIS

Gauge Invariance, modeled on QED

$$\delta_{\text{QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\Psi} (i \gamma^\mu \partial_\mu - m) \Psi$$

$$\bullet D_\mu = \partial_\mu + ie A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Global Gauge Invariance

δ_{QED} is invariant under

$$\overline{\Psi}(x) \rightarrow e^{i\alpha} \overline{\Psi}(x)$$

$$\overline{\Psi}(x) \rightarrow \bar{e}^{-i\alpha} \overline{\Psi}(x)$$

Lie groups

Group operators are hermitian operators

$$T^a, a=1, \dots, d_G$$

$$[T^a, T^b] = if^{abc} T^c$$

structure constants

- Generators of d_R -dim unitary representation are $d \times d$ Hermitian matrices t_R^a

$$[t_R^a, t_R^b] = if^{abc} t_R^c$$

acting on complex vectors Φ

Infinitesimal group transformations

$$\phi \rightarrow (1 + i\alpha^a t_R^a) \Phi$$

$$\text{Unitary } U(\omega) = \exp[i\alpha^a t_R^a]$$

$$\Phi \rightarrow U(\omega) \Phi$$

Representations of $SU(3)$

- 1 trivial

- Smallest non-trivial rep, $3 \otimes \bar{3}$, complex conjugates
fundamental anti-fundamental
~ anti-symm 2d tensor

Define a normalization factor for each rep

$$b_R [t_R^a | t_R^b] = C(R) \delta^{ab}$$

$$\text{for } R=N \text{ of } SU(N) \quad C(R) = \frac{1}{2}$$

- $SU(N)$ also has an adjoint representation

$$\text{Define matrices } (t_G^b)_{ac} = i f^{abc}$$

$$t_R [t_G^a | t_G^b] = f^{acd} f^{bcd} = C(G) \delta^{ab}$$

$$\text{for } SU(N), \quad C(G) = N$$

Dim of adjoint of $SU(N)$ is $N^2 - 1$

$$N=3 \rightarrow 8 \text{ (octet) of } SU(3)$$

Non-abelian gauge symmetry

$$\varPhi_0 = \bar{\Psi}_j i \gamma^\mu \partial_\mu \Psi_j$$

- where $\bar{\Psi}_j$ transforms in Rep R
of gauge group G

Infinitesimal local gauge transformation

$$\bar{\Psi}_j(x) \rightarrow \bar{\Psi}'_j(x) = (1 + i\alpha^a(x) t_R^a) \bar{\Psi}_j(x)$$

we will drop the
label of representation now

$$\delta \bar{\Psi}_0 = \bar{\Psi}_j i \gamma^\mu (i \partial_\mu \alpha^a(x) t_R^a) \bar{\Psi}_k$$

- $D_\mu = \partial_\mu - i g A_\mu^a t^a$ \leftarrow trying to get rid of
one spin-1 field for each generator

- To do this we need:

$$A_\mu^a(x) \rightarrow A_\mu^a(x) + \frac{1}{g} \partial_\mu \alpha^a(x) + A_{\mu b}^b f^{abc} \alpha^c(x)$$

Need a gauge invariant

Kinetic term:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a$$

$$[D_\mu, D_\nu] \rightarrow \exp[i\alpha^a t^a] [D_\mu, D_\nu] \exp[-i\alpha^a t^a]$$

$$= [\partial_\mu - i g A_\mu^a t^a, \partial_\nu - i g A_\nu^b t^b]$$

$$= i g \partial_\mu A_\nu^a t^a + i g A_\mu^a t^a + (i g)^2 [A_\mu^a t^a, A_\nu^b t^b]$$

- This motivates field strength

$$[D_\mu, D_\nu] = -i g F_{\mu\nu}^a t^a$$

$$\text{i.e. } F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

(index of rep R)

Gluon emission:

$$g \rightarrow g g$$

Recall

$$M(p_i(p) \rightarrow g^a(k) p_f(k)) = g_s \bar{u}(k) \gamma^\mu t_{ji} \epsilon_\mu(g)$$

$|M|^2$ summed over final colours, avg over initial colours

$$\frac{1}{2} \sum_{i,j,a} g_s^2 |t_{ji}^a|^2 = g_s^2 \text{tr}[t^a t^a] = \frac{4}{3} g_s^2$$

$g \rightarrow gg$ replace $R \rightarrow 8$ add

$$\frac{1}{8} \sum_{a,b,c} g_s^2 |t_{abc}^b|^2 = \frac{g_s^2}{8} \text{tr}[t^a t^a] = \sum_a g_s^2 8 \delta^{aa} = 3 g_s^2$$

In adjoint rep,

$$D_\mu \alpha^a(x) = \partial_\mu \alpha^a(x) + g A_\mu^b f^{abc} \alpha^c(x)$$

$$(\text{from } (t_G^b)_{ac} = i f^{abc})$$

so we can write:

$$A_\mu^a(x) \rightarrow A_\mu^a(x) + \frac{1}{g} D_\mu \alpha^a(x)$$

- Finite local transformations

$$\bar{\Psi} \rightarrow \exp[i\alpha^a t^a] \bar{\Psi}$$

$$D_\mu \rightarrow \exp[i\alpha^a t^a] D_\mu \exp[-i\alpha^a t^a]$$

We have made it so that

$$\mathcal{L} = \bar{\Psi} i \gamma^\mu D_\mu \bar{\Psi}$$

is locally gauge invariant

$$\text{Defined } F \text{ by } [D_\mu, D_\nu] = -i g F_{\mu\nu}^a t^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$F_{\mu\nu}^a t^a \rightarrow \exp[i\alpha^b t^b] F_{\mu\nu}^a t^a \exp[-i\alpha^c t^c]$$

so that $\text{tr}[F_{\mu\nu}^a t^a (F_{\mu\nu}^b t^b)]$ is invariant

$$\sim F_{\mu\nu}^a F^{\mu\nu}{}^b S_{ab}$$

$\Rightarrow \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a$ is Lorentz invariant
and gauge invariant

- Complete Lagrangian with vector bosons
and Dirac fermions

- Field eq. are Dirac
eq. for Ψ and nonlinear
for A

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\Psi} i \gamma^\mu D_\mu \Psi$$

Infinitesimally

$$D_\mu \rightarrow (1 + i\alpha^b t^b) D_\mu (1 - i\alpha^a t^a) + \partial(x^2)$$

$$= D_\mu - i \partial_\mu (\alpha^a t^a) + [i\alpha^b t^b, -i g A_\mu^a t^a]$$

$$= D_\mu - i t^a \partial_\mu \alpha^a + g \alpha^b A_\mu^a [t^a, t^b]$$

$$= D_\mu - i t^a \partial_\mu \alpha^a - i g f^{abc} t^a A_\mu^b \alpha^c$$

$$D_\mu \bar{\Psi} \rightarrow \exp[i\alpha^a t^a] D_\mu \bar{\Psi}$$

$$\mathcal{L} = \bar{\Psi} i \gamma^\mu D_\mu \bar{\Psi}$$

is invariant

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\Psi}_f (i \gamma^\mu D_\mu - m) \Psi_f$$

- f sums over flavours of quarks

- G = $SU(3)$ $a=1, \dots, 8$

- (q_s): in fundamental rep 3 $i=1, 2, 3$

$$D_\mu = \partial_\mu - i g_s A_\mu^a t^a \text{ we define } \alpha_S = \frac{g_s^2}{4\pi^2}$$

Vacuum polarization

In QED, electric charge is
screened, charge we measure
is smaller than the coupling in \mathcal{L}



Includes



- $e_0 \rightarrow e$

- Renormalization group eq. as

$$e = e(Q)$$

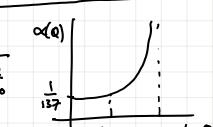
$$\frac{d}{d \log Q} e(Q) = \beta(e(Q))$$

Q = momentum transfer

$\beta = \text{The beta function}$

$$\text{for } Q \gg M_e \quad \frac{e^2}{e_0^2} = \frac{1}{1 - \frac{e_0^2}{\pi^2} \log \frac{Q}{Q_0}}$$

$$\text{Also } \alpha(Q) = \frac{\alpha_0}{1 - \frac{2\alpha_0}{3\pi^2} \log \frac{Q}{Q_0}}$$



$$\text{LEP } \sqrt{s} = 193 \text{ GeV}$$

$$\alpha \approx \frac{1}{127}$$

Asymptotic freedom

There are gluon self interactions
bosonic loops \Rightarrow changes Renormalization eq.

$$\frac{d}{d \log Q} g_s(Q) = \beta(g_s(Q)) ; \beta(Q) = \left[\frac{11}{3} C_F - \frac{4}{3} n_F(C_F) \right] \frac{g_s^2}{16\pi^2}$$

SU(3)
Number of
flavours

$$\text{In the case of } SU(3) = - \left[11 - \frac{2}{3} n_F \right] \frac{g_s^2}{16\pi^2} \equiv b_0$$

$n_F = 6$ at
high energies
no t loops for
most purposes

$$\alpha_s(Q) \approx \frac{\alpha_s(Q_0)}{1 + b_0 \frac{\alpha_s(Q_0)}{2\pi} \log \frac{Q}{Q_0}} = \frac{2\pi}{b_0} \left(\alpha_s(Q_0) \right)$$

$$1 - Q_0 \exp \left[- \frac{2\pi}{b_0 \alpha_s(Q_0)} \right]$$

At short distances, gluon exchange produces a Coulomb potential between heavy quarks

$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} \quad (\text{Y} \sim c\bar{c}) \quad (\text{X} \sim bb)$$

Next order in $e^+e^- \rightarrow \text{hadrons}$

$$\frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_f Q_f^2 \left(1 + \frac{\alpha_s(Q)}{\pi} \dots \right)$$

$$\alpha_s(91 \text{ GeV}) \approx \frac{1}{8.5} \quad \text{Near next-to-leading order corrections and more}$$

Confinement of quarks in hadrons
Is not fully explained by asymptotic freedom

Colour "flux tubes" carry infinite energy



Lattice QCD calculations interpolate from strong to weak coupling.

- Asymptotic freedom

- Strongly coupled bound states use high $\alpha_s(Q)$

Hard scattering uses low $\alpha_s(Q)$

We omit chpt 13.1, 13.3

Evolution of pdfs with Q^2



$$\sigma(A \rightarrow B + g + g) \approx \sigma(A \rightarrow B + g) \int dz \int \frac{d\epsilon_L}{zL^3} \frac{4}{\pi} \frac{\alpha_s}{z} \frac{1+(1-z)^2}{z}$$

$$P_{g\leftarrow q}(q \rightarrow gg) = \int dz \int \frac{d\epsilon_L}{zL^3} \left(\dots \right)$$

With the proton the probability of emitting gluon off a quark parton is:

$$\int dx f_g(x) = \int d\zeta \int dz \int \frac{d\epsilon_L}{zL^3} \frac{4}{\pi} \frac{\alpha_s(q_0)}{z} \frac{1+(1-z)^2}{z} f_q\left(\frac{z}{x}\right)$$

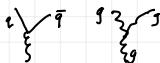
$x = z\zeta$ ~ parton momentum fraction flavour of parton

$f_g(x)$ satisfies an ODE

$$\frac{d}{d \log Q} f_g(x) = \frac{4}{3} \frac{\alpha_s(Q)}{\pi} \int_x^1 \frac{dz}{z} \frac{1+(1-z)^2}{z} f_q\left(\frac{z}{x}\right)$$

Limits: $0 < z < 1$, and $\frac{z}{x} = \frac{1}{\zeta} \geq 1$

Similar analysis for other collinear splitting



Total set of DEs for pdfs are called Altarelli-Parisi or DGLAP eqs

$$\frac{d}{d \log Q} f_g(x) = \frac{\alpha_s(Q)}{\pi} \int_x^1 \frac{dz}{z} \left\{ P_{g \leftarrow g}(z) f_g\left(\frac{z}{x}\right) + \sum_f P_{g \leftarrow f}(z) \left[f_q\left(\frac{z}{x}\right) + f_{q\bar{q}}\left(\frac{z}{x}\right) \right] \right\}$$

$$\frac{d}{d \log Q} f_f(x) = \frac{\alpha_s(Q)}{\pi} \int_x^1 \frac{dz}{z} \left\{ P_{f \leftarrow f}(z) f_f\left(\frac{z}{x}\right) + P_{q \leftarrow f}(z) f_q\left(\frac{z}{x}\right) \right\}$$

$$\frac{d}{d \log Q} f_{q\bar{q}}(x) = \frac{\alpha_s(Q)}{\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q\bar{q} \leftarrow q\bar{q}}(z) f_{q\bar{q}}\left(\frac{z}{x}\right) + P_{q \leftarrow q\bar{q}}(z) f_q\left(\frac{z}{x}\right) \right\}$$

- $P_{g \leftarrow g}(z)$ etc are called DGLAP splitting functions

- $P_{g \leftarrow q}(z) = \frac{4}{3} \frac{1+(1-z)^2}{z}$

related by $z \rightarrow 1-z$ $P_{q \leftarrow q} = \frac{4}{3} \left[\frac{1+z^2}{1-z} + A \delta(z-1) \right]$

- Conservation of quark number:

$$\int dz P_{q \leftarrow q}(z) = 0$$

still need a cutoff

say $z < 1-\epsilon$

$$A = -[2 \log \frac{1}{\epsilon} - \frac{3}{2}]$$

$$P_{q \leftarrow g}(z) = \frac{1}{2} [z^2 + (1-z)^2]$$

$$P_{g \leftarrow g}(z) = 3 \left[\frac{1+z^4 + (1-z)^4}{z(1-z)} + \Theta(z-1) \right]$$

↑
ignoring colour

As $z \rightarrow 0$, the singular behaviour is:

$$P_{g \leftarrow q}(z) \sim \frac{4}{3} \frac{2}{z} \quad P_{g \leftarrow g}(z) \sim 3 \frac{2}{z}$$

in the ratio $\frac{9}{4}$

Hence gluon jets are wider than quark jets

Ex 12.1

Jets



- After multiple splitting momentum transfer decreases

- To where $\alpha_s(Q_L)$ is large and strong interactions take over: Hadronisation.

- Quark-gluon interactions \rightarrow divergences for soft and collinear emissions at low energy.

- Jet structure is analysed in terms of IR-safe observables

- One such observable is thrust

$$T = \max_{\vec{n}} \left| \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i^2|} \right|$$

\vec{n} is a direction \sim initial \vec{q} dir

i runs over all detectable particles

Collinear splitting

$$q \approx zP, \bar{q} \approx (1-z)P$$

$$P = q + k$$

$$|\vec{p}^2| \approx |k^2| + |\vec{P}|^2$$

$$|\vec{P} \cdot \vec{p}| \approx |\vec{k} \cdot \vec{q}| + |\vec{P} \cdot \vec{R}|$$

\rightarrow same T

- Lowest order

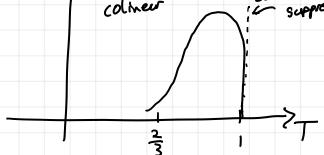
$$\leftarrow \rightarrow T = 1$$

$$\rightarrow T = \frac{2}{3}$$

\rightarrow any hadronisation is slightly non-collinear

distribution of Thrust values

Sudakov suppression



"shape probes deviation from basic, 2 jet structure"

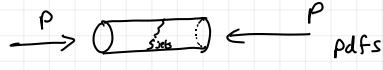
Alternative clustering.

- Pairs of particles with low value of $y_{ii} = \frac{(p_i + p_j)^2}{s}$

combine these $p_i + p_j = p_K$

Jets are composite particles obtained up to some cutoff y_{cut}

Hadron Colliders



QCD background swamps signals of interest

- $P = \begin{pmatrix} u \\ u \\ d \end{pmatrix}$
- Soft collisions \rightarrow many final state particles of small transfer momentum
 - Hard collisions \rightarrow particle scattering, jets with large transverse momentum