quark model, chpt 9 in Thomson | She will post exercises that might be covered in tutorials

God given units (Natural units)

$\hbar = c = 1$, $\epsilon_0, \mu_0 = 1$

Electron mass: $m_e = 0.511$ MeV

Proton: $m_p = 938$ MeV

- Fine structure constant

$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$

Relativistic Kinematics

$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

$p^\mu = (E, \vec{p})^\mu = m\gamma(1,\vec{\beta})^\mu$

$\beta = \frac{v}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$E_p = (p + m^2)^{\frac{1}{2}}$ $\quad p^2 = m^2$

If $E = E_p \Rightarrow$ particle is on-shell

Decay of massive particle in its rest frame

$\hat{3}$ $\xleftarrow{1}$ (M) $\xrightarrow{2}$ $\quad p_M^\mu = (M, 0,0,0)$

Case 1: $\quad p_1^\mu = (\frac{M}{2}, 0, 0, \frac{M}{2})$

$m_1 = m_2 = 0 \Rightarrow p_1^2 = p_2^2 = 0 \Rightarrow p_2^\mu = (\frac{M}{2}, 0, 0, -\frac{M}{2})$

$+ \quad p_M^\mu = p_1^\mu + p_2^\mu$

Case 2:

$m_1 \neq 0, m_2 = 0 \quad p_1^\mu = (E_p, 0, 0, p)$

$p_2^\mu = (p, 0, 0, -p)$

where $E_p = \frac{M^2 + m_1^2}{2M}$, $p = \frac{M^2 - m^2}{2M}$

Case 3:

$m_1 \neq 0, m_2 \neq 0$

$p_1^\mu = (E_1, 0, 0, p)$

$p_2^\mu = (E_2, 0, 0, -p)$

where $E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$, $E_2 = \frac{M_1^2 - m_1^2 + m_2^2}{2M}$

$p = \sqrt{\frac{\lambda(M, m_1, m_2)}{2M}}$

$\lambda(a,b,c) = a^4 + b^4 + c^4 - 2cb - 2ac - 2bc$

Discrete groups

only really need $\mathbb{Z}_2$

- Unitary representation:

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

for particles this is related to parity and charge conjugation

QM system with 2 states

$|\pi^+\rangle \quad |\pi^-\rangle$

$C|\pi^+\rangle = |\pi^-\rangle$, $C|\pi^-\rangle = |\pi^+\rangle$

$C\begin{pmatrix} |\pi^+\rangle \\ |\pi^-\rangle \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} |\pi^+\rangle \\ |\pi^-\rangle \end{pmatrix}$

If $[C, H] = 0 \Rightarrow \pi^+$ and π^- have equal masses and decay rates

"H has $\mathbb{Z}_2$ symmetry"

Since $C^2 = 1 \Rightarrow \lambda = \pm 1$

$\Rightarrow \lambda = 1 \quad \frac{1}{2}(|\pi^+\rangle + |\pi^-\rangle)$

$\lambda = -1 \quad \frac{1}{2}(|\pi^+\rangle - |\pi^-\rangle)$

Discrete symmetries $\quad$ det $= -1$

Parity: $(x^0, \vec{x}) \longrightarrow (x^0, -\vec{x})$

Time reversal: $(x^0, \vec{x}) \rightarrow (-x^0, \vec{x})$

QM intrinsic Parity $\quad P|A(\vec{k})\rangle = \pm |A(-\vec{k})\rangle$ $\quad$ Parity ± 1

& intrinsic time reversal $\quad \Rightarrow$ antiparticles (conjugate fields)

C charge conjugation QFT $\quad$ Particle $\xleftrightarrow{C}$ antiparticle

Opposite charge, same mass

C, P, T are not required to be respected by the Hamiltonian. QFT has a theorem for conservation of CPT

Continuous groups

Spacetime: translations - $\mathbb{R}^4$ $\quad$ abelian

& Lorentz transformations $\quad SO(3,1)$ non-abelian

- For standard model (SM) we also want intrinsic $U(1)$, $SU(2)$, $SU(3)$

$U(n)$ = unitary $n \times n$ matrices

$U(1) = \{e^{i\alpha}, \alpha \in \mathbb{R}\}$ Rotations of a circle S^1

$SU(n)$ = Special unitary $n \times n$ matrices

$SU(n) = \{U_{n \times n} \mid U^\dagger U = 1 \text{ \& } \det U = 1\}$

$SO(3)$ rotation group of $\mathbb{R}^3$

$U(\vec{\alpha}) = \exp[-i\vec{\alpha}\cdot\vec{J}]$

$\vec{J}$ angular momentum

J_i are the generators of rotations

$[J^i, J^j] = i\epsilon^{ijk} J^k$

$SO(3)$ has a d-dimensional rep for every integer d

$d = 2j+1 \quad$ "spin j"

More generally generators of Lie groups satisfy a Lie algebra with commutation relation

$[t^a, t^b] = i \underbrace{f^{abc}}_{\text{structure constants}} t^c$

$U = \exp[-i\alpha^a t^a]$

Klein-Gordon Eq

Scalar field spin 0

EOM $\quad (\partial^2 + m^2)\phi(x) = 0$

solution $\quad \phi(x) = e^{-ip\cdot x}$

where $p^2 = m^2$

Both $E = \pm E_p$ are allowed

QFT! $\phi(x)$ is a field that creates and destroys particles

$|0\rangle$ Vacuum; $|\varphi(p)\rangle$ one particle

$\langle 0|\phi(x)|\varphi(p)\rangle = e^{-ip\cdot x}$ $\quad$ with $p^0 = E_p > 0$

$\Rightarrow$ positive energy sol

$\phi(x)$ is complex

$\Rightarrow \langle \varphi(p) | \phi^{\dagger}(x) | 0 \rangle = e^{ipx}$

creation of a particle

If $\phi^{\dagger}(x) \neq \phi(x)$, then we also have

$$\langle 0 | \phi^{\dagger}(x) | \varphi'(p) \rangle = e^{-ipx}$$

$\Rightarrow \varphi'(p)$ is the anti-particle

If $\phi^{\dagger}(x) = \phi(x) \Rightarrow \varphi$ can be its own anti-particle.

Maxwells eqs Spin 1

vector field $V^i(x)$ (vector representation of angular mom)

Particle destruction:

$$\langle 0 | V^i(x) | v(p,\epsilon) \rangle = \epsilon^i e^{-ipx}$$

- Need $\epsilon^i \to R_{ij} \epsilon^j$

(where $V^i \to R^i_j V^j$

Relativistic

$\langle 0 | V^{\mu}(x) | v(p,\epsilon) \rangle = \epsilon^{\mu} e^{ipx}$

Notice: Norm of this state is $\epsilon^{\mu}\epsilon_{\mu}$ can be <0

- This is solved by photons being massless

Variational principle

4-Vector potential $A^{\mu}(x)$

Define a tensor $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \underbrace{J^{\mu}}_{\text{external current}} A_{\mu}$$

with $\partial_{\mu} J^{\mu} = 0$

$\partial_{\mu} F^{\mu\nu} = -J^{\nu}$

Dirac Eq spin 1/2

Dirac Matricies (must have $d \geq 4$)

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$$

- We will use Dirac basis

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \to \text{for non-rel applications}$$

or Weyl basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \to \text{rel applications}$$

Dirac Eq

$$(i\gamma^{\mu}\partial_{\mu} - M)\psi = 0$$

- Solution of Dirac eq is solution of KG eq.

$\psi \sim e^{-ipx}$

- 2 positive energy and 2 negative energy sols

Given in terms of 2-compont Spinors: $s = 1, 2$

$$\psi = u^s(p) e^{-ipx}, \quad \psi = v^s(p) e^{ipx}$$

Matrix elements for destroying/creating one electron

$$\langle 0 | \psi(x) | e^-(p,s) \rangle = u^s(p) e^{-ipx}$$

$$\langle e^-(p,s) | \psi^{\dagger}(x) | 0 \rangle = u^{s\dagger}(p) e^{ipx}$$

or for positrons

$$\langle 0 | \psi^{\dagger}(x) | e^+(p,s) \rangle = v^{\dagger s} e^{ipx}$$

$$\langle e^+(p,s) | \psi(x) | 0 \rangle = v^s e^{-ipx}$$

- couple to Maxwells eqs

Electrons & photons $\to$ QED

- We can couple Dirac eq to EM field by $\partial_{\mu} \to D_{\mu} \equiv \partial_{\mu} - ieA_{\mu}$

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - M)\psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \quad \bar{\psi} = \gamma^0 \psi^{\dagger}$$

Relativistic Normalization

$$\langle p_1 | p_2 \rangle = 2E_{p_1} (2\pi)^3 \delta^{(3)}(\vec{p}_1 - \vec{p}_2)$$

$$\int \frac{d^4 p}{(2\pi)^4} (2\pi)\, \delta(p^2 - m^2) = \int \frac{d^3 p}{(2\pi)^3} \int dp^0\, \delta(|p^0|^2 - |p|^2 - m^2) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_0}$$

Spin-Statistics Thm of QFT

Fields w/ integer spin $\to$ particles with BE stats

Fields with half-integer spin $\to$ particles with FD stats

SM: fields of spin $0, \frac{1}{2}, 1$

Also composit particles

Hydrogen Atom

(Not to scale)

p e

- Non-rel:

$V(r) = \frac{\alpha}{r}$ $n = 1, 2, \dots$

Bound state energies $E = -\frac{1}{2}\alpha^2 m \frac{1}{n^2}$

. Orbital wave functions $Y_{lm}(\theta, \phi)$, $l = 0, 1, \dots, n-1$, $m = -l, \dots, l$

Parity:

$$P|nlm\rangle = (-1)^l |n\, lm\rangle$$

- Fine structure: electron spin 1/2

- Spin orbit interactions + relativistic effects

$\vec{L}$ = orbital angular mom

$\vec{S}$ = spin angular mom

$$\Delta H \sim \vec{L} \cdot \vec{S}$$

$$\Delta H = \frac{g-1}{2} \frac{\alpha}{M_e^2 r^3} \vec{L} \cdot \vec{S}$$

- We will diagonalize ΔH

Let $\vec{J} = \vec{L} + \vec{S}$ ← Tot angular mom ($\vec{L}$: orbital ang mom, $\vec{S}$: electron spin)

$$\vec{L} \cdot \vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$$

Each $(n, l) \longrightarrow \underset{\text{spin}}{2}\ \underset{m}{(2l+1)}$ qm states

- Fine structure splits degeneracy

1s orbital

$n = 1, l = 0, s = \frac{1}{2}, j = \frac{1}{2}$

$$\vec{L} \cdot \vec{S} = \frac{1}{2}\left[\frac{1}{2} \cdot \frac{3}{2} - 0 - \frac{1}{2} \frac{3}{2}\right]$$

2s $\quad n = 2 \quad l = 0 \quad s = \frac{1}{2} \quad j = \frac{1}{2} \quad \vec{L} \cdot \vec{S} = \frac{1}{2}$

2p $\quad n = 2 \quad l = 1 \quad s = \pm\frac{1}{2} \quad j = \begin{cases} \frac{3}{2} & \text{4 states} \\ \frac{1}{2} & \text{2 states} \end{cases} \quad \vec{L} \cdot \vec{S} = -1$

Hyperfine Structure

Proton spin 1/2

$$\Delta H \sim \vec{S}_p \cdot \vec{S}_e = \frac{1}{2}\left[(\vec{S}_p + \vec{S}_e) - \vec{S}_p^2 - \vec{S}_e^2\right]$$

1s state is split

$\vec{S}_p + \vec{S}_e = 1 \qquad |\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \qquad |\downarrow\downarrow\rangle$

$\vec{S}_p + \vec{S}_e = 0 \qquad \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Positronium: Bound state of e^+, e^-

Label eigenstates by J^{PC} (J: ang mom, P: parity, C: charge)

Parity

Y_{lm} gets $(-1)^l$ under spatial inversion

electron and positron have opposit intrinsic parities $P = (-1)^{l+1}$

charge ① C exchanges $e^+ \longleftrightarrow e^-$

-1 is wavefunction from femionic exchange

② Co-ordinate reversal in Y_{lm} $(-1)^l$

③ exchange of spins $\vec{S}_{e^+}$ and $\vec{S}_{e^-}$

$$\begin{cases} +1 & S = 1 \\ -1 & S = 0 \end{cases}$$

$$\text{Total } C = (-1)^{l+1} \begin{cases} 1 & S = 1 \\ -1 & S = 0 \end{cases}$$

Spectrum

"s" is $l = 0$ $\Rightarrow P = -1$

2s: 1^{--} ($s = 1$), 0^{-+} ($s = 0$)

1s: 1^{--} ($s = 1$), 0^{-+} ($s = 0$)

2P: 1^{+-} ($s = 0$); $2^{++}, 1^{++}, 0^{++}$ ($s = 1$)

P is $l = 1$ $\Rightarrow P = 1$

QED interactions mediated by the photon

Couplings to e^+ and e^- are opposit $\Rightarrow$

$$C|\gamma(\epsilon, p)\rangle = -|\gamma(\epsilon, p)\rangle$$

Possible photon transitions

$2s(1^{--}) \to 2p(J^{++})$

$2p(J^{++}) \to 1s(1^{--})$

$2p(1^{++}) \to 1s(0^{-+})$

$1s(1^{--}) \to 1s(0^{-+})$

Ground states can decay to photons
- But decay to a single photon is not possible (momentum cons)

Decay to $\gamma\gamma$ possible only for $C=+$ (i.e. 0^{-+})

1s 1^{--} can decay to $\gamma\gamma\gamma$

The Quark model

- Hadrons are the particles observed in nuclei, they are bound states of quarks

Quarks appear in baryons or mesons, never alone

Pions: The lightest Hadrons are π-mesons

$\pi^\pm$ 139.57 MeV

π^0 134.98 MeV

Charmonium

Any particle + antiparticle $\to$ excited EM fields $\to$ any other particle + anti-particle

1974 Two groups found a sharp resonance at the same energy level (3.1 GeV).

- One collided e^+, e^- $\xrightarrow{\text{found}}$ "ψ" — Theory need a 4th quark, charm
- other $p+p \to e^+e^- + X$ "J"

Reasonance called J/ψ "gypsy"

Highest rate comes from annihilation by the EM current $\vec{J} = \bar{\psi}\vec{\gamma}\psi$

$\langle 0|\vec{J}(x)|e^+e^-\rangle$ — spin 1, $P=-1$, $C=-1$, $J^{PC}=1^{--}$

1s, 2S etc higher l-states also observed like positronium

$\Rightarrow$ conclude bound state $c\bar{c}$

- Potential is not $\frac{1}{r}$ but like $A\log r$ or $-\frac{A}{r}+Br$

$\to$ quarks are confined

Light mesons

Pions found to have $J^P = 0^-$

$\pi^0 \to 2\gamma \Rightarrow C(\pi^0) = +1$

Lightest hadrons all have $J^P = 0^-$ or 1^- 9 of each $L=0$

- Light psudo scalar Mesons.

$J^P = 0^-$
- η' 958 MeV
- η 548 MeV
- $K^- \bar{K}^0 K^0 K^+$ 498 MeV
- $\pi^- \pi^0 \pi^+$ 140 MeV

- Light vector Mesons ϕ 1020 MeV $\to \gamma\pi^0$

$J^P = 1^-$
- K^{*-} $\bar{K}^{*0}$ K^{*+} 892 MeV
- ρ^- ω ρ^0 ρ^+ 781, 770 MeV $\to \gamma\pi^0$

K, K^*, Λ^0 dont appear single only with another "strange particles"

(Λ^0: excited state proton)

Vaguely correct: the 9 1^- states are bound states of $(\bar{u},\bar{d},\bar{s})\cdot(u,d,s)$

$m_u \sim m_d$ 3-4 MeV $m_s \sim 95$ MeV

- Posit discrete symmetry $u \leftrightarrow d$ and approx symmetry $u \leftrightarrow s$, $d \leftrightarrow s$
- exstend to continuous symmetry SU(2) rotation "isospin"
- SU(3) rotation

Quark flavour Isospin (Thomson convention chp 9?)

$u = \begin{pmatrix}1\\0\end{pmatrix}$, $d = \begin{pmatrix}0\\1\end{pmatrix}$

- Useful if EM << strong interactions

Isospin:

$\vec{T} = \frac{1}{2}\vec{\sigma}$ Doublet $\begin{pmatrix}u\\d\end{pmatrix}$ Antiquark doublet $\begin{pmatrix}\bar{d}\\\bar{u}\end{pmatrix}$

$\underset{\text{spin }\frac{1}{2}}{2} \otimes \bar{2} = \underset{\text{spin 3}}{3} \oplus \underset{\text{spin 0}}{1}$

$q\bar{q}$ Isospin states

$\phi(1,1) = -u\bar{d} \sim \pi^+, \rho^+$

$\phi(1,0) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \sim \pi^0, \rho^0$

$\phi(1,-1) = d\bar{u} \sim \pi^-, \rho^-$

$\phi(0,0) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \sim \eta, \omega$

SU(3) flavour Symmetry

Triplet $u = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $d = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $s = \begin{pmatrix}0\\0\\1\end{pmatrix}$

8 generators of SU(3)

Gell mann matricies, prefer $u \leftrightarrow d$ isospin

$\lambda_1 = \begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$, $\lambda_2 = \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}$, $\lambda_3 = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$ SU(2) of u,d

$\lambda_4 = \begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}$, $\lambda_5 = \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}$ $u \leftrightarrow s$

$\lambda_6 = \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$ $\lambda_7 = \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}$ $d \leftrightarrow s$

$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$ equal treatment of u,d

Normalized so that $\mathrm{tr}(\lambda_i \lambda_j) = 2\delta_{ij}$

Isospin:

$\hat{T}_i = \frac{1}{2}\lambda_i$ eg $\hat{T}_3 u = \frac{1}{2}u$, $\hat{T}_3 d = -\frac{1}{2}d$, $\hat{T}_3 s = 0$

Total Isospin

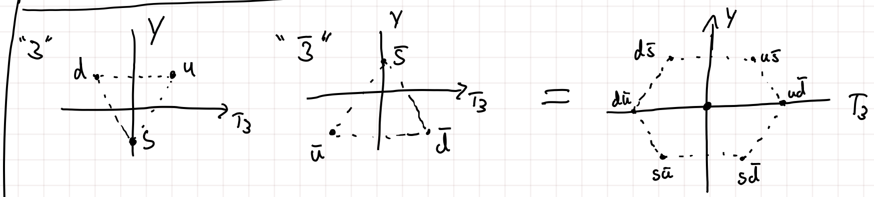
$\hat{T}^2 = \sum_i T_i^2 = \frac{1}{4}\sum_i \lambda_i^2 = \frac{4}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

2- commuting generators $\hat{T}_3$ $\hat{T}_8$

Quantum numbers $\hat{T}_3 = \frac{1}{2}\lambda_3 \leftarrow$ 3rd comp isospin

$\hat{Y} = \frac{1}{\sqrt{3}}\lambda_8 \leftarrow$ Hypercharge

$\hat{Y}u = \frac{1}{3}u$ $\hat{Y}d = \frac{1}{3}d$ $\hat{Y}s = -\frac{2}{3}s$



Combine remaining generators into ladder operators

$\hat{T}_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$ $\quad \hat{V}_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$ $\quad \hat{U}_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$

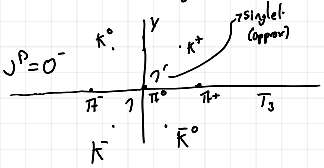
$u \uparrow -\bar{d}$, $d \downarrow \bar{u}$ $\quad$ $d \uparrow -\bar{s}$, $s \downarrow \bar{d}$ $\quad$ $u \uparrow -\bar{s}$, $s \downarrow \bar{u}$

3 states with $T_3 = Y = 0$

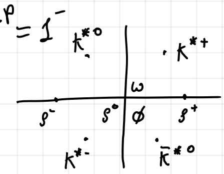
$T_-|u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle$
$V_-|u\bar{s}\rangle = |s\bar{s}\rangle - |u\bar{u}\rangle$ $\Big\}$ 2 lin indep
$U_-|d\bar{s}\rangle = |s\bar{s}\rangle - |d\bar{d}\rangle$

$3 \otimes \bar{3} = 8 \oplus 1$ (octet, singlet) $\quad$ Singlet $\frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$

$J^P = 0^-$



$J^P = 1^-$



$|\eta'\rangle \simeq \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$|\eta\rangle \simeq \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$

$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$|\omega\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$
$|\phi\rangle \simeq s\bar{s}$

G-Parity

Pions are not eigenstates of charge conjugation

$C|\pi^+\rangle = |\pi^-\rangle$

Define: $G = C e^{i\pi I_2}$ (rotation in isospin about 2 axis)

$T_1 \to T_1$
$T_2 \to T_2$
$T_3 \to -T_3$

$G|\pi^+\rangle = -|\pi^+\rangle$
$G|\pi^-\rangle = -|\pi^-\rangle$
$G|\pi^0\rangle = -|\pi^0\rangle$

$G = -1$ for pions $\to$ selection rules

Ex J/ψ has $C = -1$ and $I = 0$
$\Rightarrow G = -1$
$\Rightarrow$ can decay into an odd No of pions

Heavy mesons with b, c

$c\bar{s}$
$c\bar{u}$ $\quad c\bar{d}$ + anti-particles

$J^P = 0^-$ $\quad D_s^+$ 1968 MeV
D^0 D^+ 1869 MeV

$J^P = 1^-$ $\quad D_s^{*+}$ 2112 MeV
D^{*0} D^{*-} 2010 MeV

$b\bar{u}$ $\quad b\bar{s}$ $\quad b\bar{d}$ + anti

$J^P = 0^-$ $\quad B_s^0$ 5367 MeV
B^- B^0 5227 MeV

$J^P = 1^-$ $\quad B_s^{*0}$ 5415 MeV
B^{*-} B^{*0} 5325 MeV

. No mesons with top quark $M_t \sim 173$ GeV decays to rapidly

Baryons: Baryons are fermions

Baryon number B is conserved

$\to$ The lightest baryon is stable. Its the proton!

Light families

- Spin $\frac{1}{2}$ octet

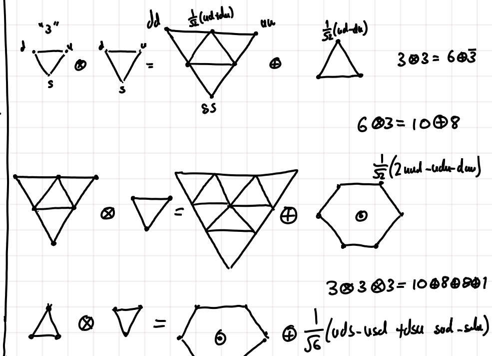
Ξ^- $\quad \Xi^0$ $\quad$ 1313 MeV
Σ^- $\quad \Lambda^0 \Sigma^0$ $\quad \Sigma^+$ $\quad$ 1192, 1176
n $\quad p$ $\quad$ 938 MeV

Spin $\frac{3}{2}$

Ω^- $\quad$ 1672
Ξ^{*-} $\quad \Xi^{*0}$ $\quad$ 1532
Σ^{*-} $\quad \Sigma^{*0}$ $\quad \Sigma^{*+}$ $\quad$ 1385
Δ^- $\quad \Delta^0$ $\quad \Delta^+$ $\quad \Delta^{++}$ $\quad$ 1232

- All states of $P = +1$ (by convention)

- Can be understood as qqq bound states



Sugested exercise:
- work through equations (9.8) – (9.15)
- Thomson exercise 9.3
- work through spin algebra

Total wave function for bound qqq state:

$\psi = \phi_{flavour} \chi_{spin} \xi_{color} \eta_{space}$

η_{space}: $L=0$ symmetric $\leftarrow$ what we look at

- ψ must be anti-symmetric
- χ turns out needs to be anti-symm (SU(3) invariants are δ^a_b, ϵ_{abc}, ϵ^{abc})

$\Rightarrow \phi \cdot \chi$ must be symm

Spin states

qqq spin $\frac{3}{2}$ quadruplet

$\chi(\frac{3}{2}, \frac{3}{2}) = |\uparrow\uparrow\uparrow\rangle$
$\chi(\frac{3}{2}, \frac{1}{2}) = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$
$\chi(\frac{3}{2}, -\frac{1}{2}) = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)$
$\chi(\frac{3}{2}, -\frac{3}{2}) = \downarrow\downarrow\downarrow$

Electric charges of quarks

It follows that

$Q_d = -\frac{1}{3}$, $Q_u = \frac{2}{3}$
$Q_s = -\frac{1}{3}$, $Q_c = \frac{2}{3}$
$Q_b = -\frac{1}{3}$ $\quad (Q_t = +\frac{2}{3})$

Detectors in a nutshell

2-types: - trackers $\to$ trajectory & momentum
- calorimeters $\to$ total energy (particle destroyed)

- Energy loss by ionisation
Em interactions that kick electrons

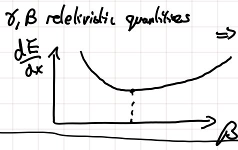
- Bethe-Bloch eq:

$$\frac{dE}{dx} \approx -4\pi\alpha^2 \frac{nZ}{m_e v^2}\left[\log\frac{2\gamma^2\beta^2 m_e}{\omega} - \beta^2\right]$$

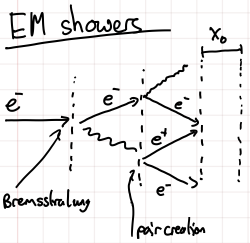
- n = number density
- Z = atomic number
- ω = atomic frequency
- γ, β relativistic quantities

- path depends on momentum
- $\frac{dE}{dx}$ depends on velocity

$\Rightarrow$ with measurement of both we can derive particle mass



EM showers



X_0 = radiation length

EM calorimeters are crystals containing these showers in compact regions

- Energy is measured from scintillation light

Hadronic showers

- far more irregular
Not all energy is detectable
Can go into the nucleus
Hadronic calorimeter are cruder by an order of mag

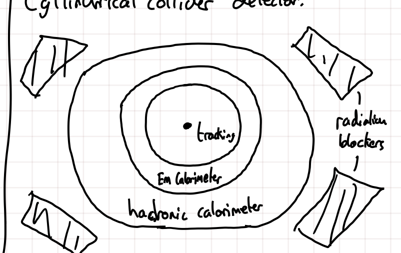
- Transition radiation
effect of crossing vacuum-medium boundary, Can distinguish electrons from pions

Cherenkov radiation

Superluminal velocity in medium
$\to$ shockwave

Discriminates particle velocity.

Cyllindrical collider detector:



Sujested exercises

Peskin 2.1, 2.5, 7.2, 8.1

Tools for calculation

two phenomena:
- scattering
- decay

- Unstable particle A

Constant rate of decay

Probability of existence

$dP = -P\Gamma_A dt$

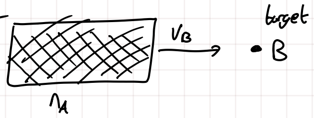
$\Rightarrow P(t) = e^{-t\Gamma_A}$

" total width Γ_A "

$\Gamma_A = \sum_F \Gamma(A\to f)$ for multible decay processes

Chapter 7 Peskin

Scattering



- Beam of particles with density n_A velocity v_A

Rate of scattering:

$$\frac{\text{events}}{\text{sec}} = n_A v_A \sigma$$ cross section, units cm^2

differential cross section

$$\frac{d\sigma}{d^3p_1 \cdots d^3p_n}$$ ← final state momenta

Total cross section:

$$\sigma = \int d^3p_1 \cdots d^3p_n \frac{d\sigma}{d^3p_1\cdots d^3p_n}$$

Master formula Fermi's Golden rule

QM transition matrix element

$$\langle 1\,2 \ldots n | T | A(p_A)\rangle = M(A\to|1\ldots n\rangle)(2\pi)^4\delta^{(4)}(p_A - \sum_j p_j)$$

Decay:

$$\Gamma(A\to F) = \frac{1}{2m_A}\int d\Pi_n |M(A\to f)|^2$$ - phase space integral

mass of A

where $d\Pi_n = \frac{d^3p_1}{(2\pi)^3 2E_1}\cdots\frac{d^3p_n}{(2\pi)^3 2E_n}(2\pi)^4\delta^{(4)}(p-\sum_i p_i)$

- Spin: sum $\sum$ over final spin states specify initial state if known, average if not.
- Cross section:

$$\sigma(A+B\to f) = \frac{1}{2E_A 2E_B|v_A - v_B|}\int d\Pi_n |M(A+B\to f)|^2$$

$$\int d\Pi_2 \int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta(P-p_1-p_2)$$ (2 particle collision)

CoM frame: $\vec{p}_1 = -\vec{p}_2 = \vec{p}$

carry out 3 integrals?

$$\int d\Pi_2 = \int \frac{d^3p\ (2\pi)}{(2\pi)^3 2E_1 2E_2}\delta(E_{cm} - E_1 - E_2)$$

$d^3p = p^2 dp \sin\theta\, d\theta\, d\varphi = p^2 dp\, d\Omega$

$$\int dp\, \delta(E_{cm} - E_1(p) - E_2(p)) = \frac{1}{\left|\frac{dE_1}{dp} + \frac{dE_2}{dp}\right|}$$

$E_{12}^2 = p^2 + m^2$

$\Rightarrow E_1 dE_1 = p\,dp$

$$= \frac{1}{\left|\frac{p}{E_1} + \frac{p}{E_2}\right|} = \frac{E_1 E_2}{\underbrace{(E_1+E_2)}_{E_{cm}} p}$$

$$\int d\Pi_2 = \int \frac{p^2 d\Omega}{16\pi^2 E_1 E_2}\frac{E_1 E_2}{E_{cm}\, p}$$

$$= \frac{1}{8\pi}\frac{2p}{E_{cm}}\int\frac{d\Omega}{4\pi}$$

Relativistic Limit

$E \gg m_1, m_2$

$E_{cm} \to 2p \qquad \int d\Pi_2 \to \frac{1}{8\pi}\int\frac{d\Omega}{4\pi}$

If two particles are identical

$\Rightarrow$ Cant distiguish final states

$\Rightarrow$ need extra factor of $\frac{1}{2}$ in sum over states and hence in $\int d\Pi_2$

- For 3 particles we find:

$$\int d\Pi_3 = \frac{E_{cm}^2}{128\pi^2}\int dx_1 dx_2$$

$\Rightarrow x_1 + x_2 + x_3 = 2$, where $x_i = \frac{2E_i}{E_{cm}}$

- Boundaries given implicitly

$-1 \leq \cos\theta_{ij} \leq 1$

θ_{ij} and between p_i, p_j

Resonance

Breit-Wigner formula for resonance

$$M \sim \frac{1}{E - E_R + \frac{i\Gamma}{2}}$$

E_R = Energy of resonant state

Γ = decay rate.

Fourier transform:

$$\psi(t) = \int\frac{dE}{2\pi}\frac{e^{-iEt}}{E - E_R + i\frac{\Gamma}{2}} = -i\, e^{-iE_R t}e^{-\Gamma t/2}$$

Probability of maintaining Resonance decays exponentially $|\psi(t)|^2 = e^{-\Gamma t}$

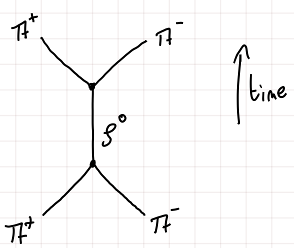
lifetime $\tau_R = \frac{1}{\Gamma}$

We need a relativistic version

Ex $\pi^-\pi^+ \to \rho^0 \to \pi^+\pi^-$ | $M_{\rho_0} = 770$ MeV

Feynman diagrams

- convention is time goes upwards



step ①: $\pi^+\pi^-\to\rho^0$
step ②: $\rho^0 \to \pi^+\pi^-$
matrix elements

③ ρ^0 resonance

Breit wigner + sum over polarizations

→ Lorentz invariance

$\to M(\pi^+\pi^-\to\rho^0)$

$= g\,\epsilon^*\cdot(p_A - p_B)$

const

spin-1 field eqs

- Can argue this is the only quantity that can be formed?

CoM frame

$p_A = (E, \vec{p})$ $\quad E = \frac{1}{2}m_\rho \quad p^2 = \frac{m_\rho^2}{4} - m_\pi^2$

$p_B = (E, -\vec{p})$

$\epsilon^* = -\epsilon \Rightarrow \epsilon^*(p_A - p_B) = -2\vec{e}\cdot\vec{p}$

$$\sigma(\pi^+\pi^-\to\rho_0) = \frac{1}{\underbrace{2E_A E_B|v_A - v_B|}}\underbrace{\int\frac{d^3p_c}{(2\pi)^3}\frac{1}{2E_c}}|M|^2\delta(p_c - p_A - p_B)$$

$$\left|\frac{p_A}{E_A} - \frac{p_B}{E_B}\right| = 2\frac{p}{m_\rho/2}$$

$$\int\frac{d^4p}{(2\pi)^4}2\pi\,\delta(p_c^2 - m_\rho^2)$$

$$\Rightarrow \frac{1}{m_\rho^2\frac{4p}{m_\rho}}2\pi\,\delta\left((p_A+p_B)^2 - m_\rho^2\right)g_\rho^2\, 4|\vec{e}\cdot\vec{p}|^2$$

$\sum_\epsilon |\vec{e}\cdot\vec{p}|^2 = p_x^2 + p_y^2 + p_z^2 = p^2$

$$\Rightarrow \sigma = g_\rho^2\frac{p}{m_\rho}(2\pi)\,\delta\left((p_A+p_B)^2 - m_\rho^2\right)$$

① $M(\pi^+\pi^-\to\rho^0)$

$\pi^+(p_A)\pi^-(p_B)\to\rho^0(p_c)$

spin 1

$\Rightarrow$ polarization vectors ϵ^μ to describe the states, with $\epsilon^\mu\cdot p_c = 0$

Step 2

$\mathcal{M}(\rho_0 \to \pi^+\pi^-)$

$$\Gamma_\rho = \frac{1}{2M_\rho}\int d\Pi_2 |\mathcal{M}|^2$$

average instead of sum

$$= \frac{1}{2M_\rho}\frac{1}{8\pi}\frac{2p}{M_\rho} g_\rho^2 \underbrace{\langle 4|\vec{\epsilon}\cdot\vec{p}|^2\rangle}_{\frac{4}{3}p^2} = \frac{g_\rho^2 p^3}{6\pi M_\rho^2}$$

- Measure ρ^0 decay Find

$\frac{g_\rho^2}{4\pi} \simeq 2.9$ (reletivly large) process likes to occur.

Step 3

Resonance $\mathcal{M} \sim \frac{1}{p^2 - M_R^2 + iM_R\Gamma_R}$

CM frame: $P = (M_R + \Delta E, \vec{0})$

$\sigma(\pi^+(p_A)\pi^-(p_B) \to \rho_0 \to \pi^+(p_A')\pi^-(p_B'))$

$$= \frac{1}{2E_A 2E_B|V_A - V_B|}\int d\Pi_2 \left|\sum_{pol}\frac{\mathcal{M}(\pi^+\pi^- \to \rho^0(\epsilon))\mathcal{M}(\rho^0(\epsilon)\to\pi^+\pi^-)}{(p_A+p_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho}\right|^2$$

$$= \frac{1}{4M_\rho p\cdot 8\pi}\frac{2p}{M_\rho}\int\frac{d\Omega}{4\pi}\frac{1}{(E_{cm}^2 - M_\rho^2) + M_\rho^2\Gamma_\rho^2}\left|\sum_\epsilon (2g_\rho\vec{\epsilon}\cdot\vec{p})(2g_\rho\vec{\epsilon}\cdot\vec{p}')\right|^2$$

$$\boxed{\sigma = \frac{1}{4M_\rho^2}\frac{g_\rho^2}{(E_{cm}^2 - M_\rho^2) + M_\rho^2\Gamma_\rho^2}\int\frac{d\Omega}{4\pi}|\vec{p}\cdot\vec{p}'|^2}$$

---

check limit of long lived narrow resonance

$\sigma(\pi^+\pi^- \to \rho \to f)$

$$= \frac{1}{4M_\rho p}\int d\Pi \left|\sum_\epsilon \frac{(2g_\rho\vec{\epsilon}^*\cdot\vec{p})\mathcal{M}(\rho_\epsilon \to f)}{(p_A+p_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho}\right|^2$$

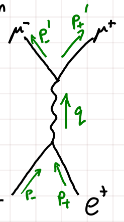
$$\sum_f \sigma(\pi^+\pi^- \to \rho \to f) - \frac{1}{4M_\rho p}2\pi g_\rho^2 p^2 \left[\frac{\Gamma_\rho/\pi}{(p_c^2 - M_\rho^2) + M_\rho\Gamma_\rho^2}\right] \to \delta(p_c^2 - M_\rho^2) \text{ as } \Gamma_\rho \to 0$$

$$\lim_{\Gamma_\rho\to 0}\sigma = \frac{g_\rho^2 p}{M_\rho}2\pi\,\delta((p_A+p_B)^2 - m_\rho)^2 \qquad \delta(x) = \frac{1}{\pi}\lim_{\epsilon\to 0}\frac{\epsilon}{x^2+\epsilon^2}$$

Strong interaction:

e^+e^- anhilation

$e^+e^- \to \mu\mu^-$



- Resonance = EM current Breit-Wigner at zero mass

$\mathcal{M} \sim \frac{1}{q^2}$ "virtual photon

Virtual particle $\Rightarrow$ off shell

$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = (-e)\langle\mu^+\mu^-|j^\mu|0\rangle\frac{1}{q^2}(-e)\langle 0|j_\mu|e^+e^-\rangle$$

Massless spin ½ particles

e, μ treated as massless (Also light quarks)

Massless Dirac Eq: $i\gamma^\mu\partial_\mu\psi = 0$

$\psi = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}$ Weyl spinors

- Dirac eq factorizes for $M=0$ into:

$i\bar{\sigma}\cdot\partial\psi_L = 0$, $i\sigma\cdot\partial\psi_R = 0$

- Couple to EM $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$

plane wave solutions, for $\vec{p} = p\hat{3}$

$\psi_R = \begin{pmatrix}1\\0\end{pmatrix}e^{-iEt + iEx^3}$ with $E>0$ distruction of right handed electron. $U_R(p)$

$\psi_L = \begin{pmatrix}0\\1\end{pmatrix}e^{+i|E|t + i|E|x^3}$ with $E<0$ Creation of left handed positron. $V_R(p)$

$\psi_L = \begin{pmatrix}0\\1\end{pmatrix}e^{-iEt + iEx^3}$ $E>0$ destruction of left-handed electron $U_L(p)$

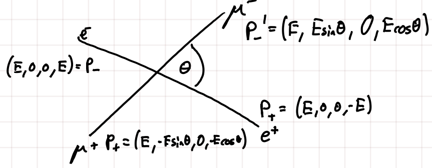
$\psi_L = \begin{pmatrix}1\\0\end{pmatrix}e^{i|E|t + i|E|x^3}$ $E<0$ creation of right handed positron $V_L(p)$

$$j^\mu = \bar{\psi}\gamma^\mu\psi = \psi_L^\dagger\bar{\sigma}\psi_L + \psi_R^\dagger\sigma^\mu\psi_R$$

Helicity conservation e_R^- can scatter into e_R^- or annihilate e_L^+, nothing else.

CM frame

$q = (E_{cm}, \vec{0})$



EM

$$\vec{\epsilon}_\pm = \frac{1}{\sqrt{2}}(1, \pm i, 0) \qquad \vec{\epsilon}_\pm' = \frac{1}{\sqrt{2}}(\cos\theta, \pm i, -\sin\theta)$$

$$|\mathcal{M}(e_R^-e_L^+ \to \mu_R^-\mu_L^+)|^2 = |\mathcal{M}(e_L^-e_R^+ \to \mu_L^-\mu_R^+)|^2 = e^4(1+\cos\theta)^2$$

$$|\mathcal{M}(e_R^-e_L^+ \to \mu_L^-\mu_R^+)|^2 = |\mathcal{M}(e_L^-e_R^+ \to \mu_R\mu_L^+)|^2 = e^4(1-\cos\theta)^2$$

cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2E_{cm}^2}(1+\cos^2\theta)$$

$$\Rightarrow \sigma = \frac{4\pi\alpha^2}{3E_{cm}^2}$$

peaks at $\cos^2\theta = 1$

we Average over 4 initial spin states including $e_R^+e_R^-$ and $e_L^+e_L^-$

Sum over 2 final spin states

Sujested exercises

PeskIn 8.1

$e^+e^- \to$ hadrons

For $E_{cm} \gg$ Light meson masses
quark model predicts the following modification

1. sum over light quark species uds (c, b)
2. $Q_\mu = -1 \to Q_f = \begin{cases} 2/3 & u, c \\ -1/3 & d\,s\,b \end{cases}$

$M \sim Q_f \Rightarrow \sigma \sim Q_f^2$

3. sum over final colour states

Result is:

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_f 3Q_f^2 \frac{4\pi\alpha}{3E_{cm}^2}$$

$$\frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f 3Q_f^2 = \begin{cases} 2 & u,d,s \\ 3\frac{1}{3} & u,d,s,c \\ 3\frac{2}{3} & u,d,s,c,b \end{cases}$$

- Strong interaction can be nearly neglected at this level

Deep inelastic Electron scattering

probing proton structure (quarks, gluons, anti-quarks)

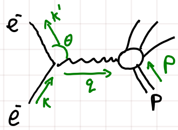
Elastic: $ep \to ep$

Inelastic: break proton, produce hadrons
High momentum transfer

Deep: Probe internal structure
Total mass of final hadrons $\gg m_p$

- Observation shows DIES is well approximated by $ep \to$ hadrons
- Also: p is not just a uud bound state

$\to$ Parton Distribution Functions (PDFs)



Define W = mass of final hadronic system

$$W^2 = (p+q)^2 = m_p^2 + 2p_q + q^2$$

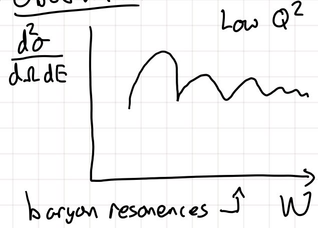
Assume large momentum transfer
$|q|^2 \gg m_p^2$ i.e. $m_p^2 \to 0$

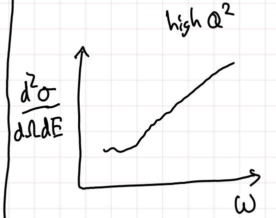
Define $Q^2 = -q^2$

Since q is space like $\Rightarrow Q^2 > 0$

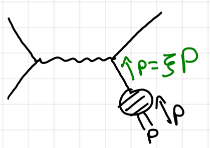
Large $Q^2 \Leftrightarrow$ large momentum transfer to proton

Observation:





clue for interpretation: Parton model

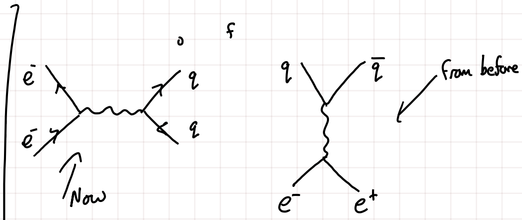


Assume that the parton is a spin $\frac{1}{2}$ quark carrying a flavour label f (or $\bar{f}$ for an anti-quark)

Def: $f_i(\xi)d\xi$ = probability of finding parton of type i carrying momentum fraction $\in (\xi, \xi + d\xi)$

- sum rule $\int_0^1 d\xi \sum_i f_i(\xi) = 1$ ← sum over $f, \bar{f}$

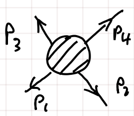
$$\sigma(e^-p \to e^-X) = \int_0^1 d\xi \sum \left[f_f(\xi) + f_{\bar{f}}(\xi)\right] \times \sigma(e^-q(\xi P) \to e^-q)$$



- These have the same M (invariant matrix element)

Crossing Symmetry

General $2 \to 2$ scattering



Orient all momenta outwards
$P_1 + P_2 + P_3 + P_4 = 0$

M only depend on Lorentz invariant

Mandelstam invariants

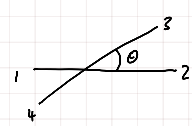
$P_i^2 = M_i^2$

Def: $(P_1+P_2)^2 = s$, $(P_1+P_3)^2 = t$, $(P_1+P_4)^2 = u$

These are not even independent due to momentum conservation

$s+t+u = M_1^2 + M_2^2 + M_3^2 + M_4^2$

CM frame



massless scattering

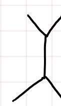
$P_1 = (-E, 0, 0, -E)$ $P_3 = (E, E\sin\theta, 0, E\cos\theta)$
$P_2 = (-E, 0, 0, E)$ $P_4 = (E, -E\sin\theta, 0, -E\cos\theta)$

$s = (2E)^2 = E_{cm}^2$ $s+t+u = 0$

$t = -2E^2(1-\cos\theta)$ $u = -2E^2(1+\cos\theta)$

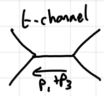
Intermediate state:

s-channel



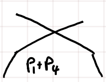
$P_1 + P_2 \sim \frac{1}{s - M_r^2 + iM_r\Gamma_R}$; $|M|^2 \sim \frac{1}{s^2} = \frac{1}{E_{cm}^4}$

t-channel



$|M|^2 \sim \frac{1}{t^2} = \frac{1}{E_{cm}^4 \sin^4(\frac{\theta}{2})}$

$\Rightarrow$ strong peak in forward dir $\theta = 0$



$|M|^2 \sim \frac{1}{u^2} = \frac{4}{E_{cm}^2(1+\cos\theta)^2}$

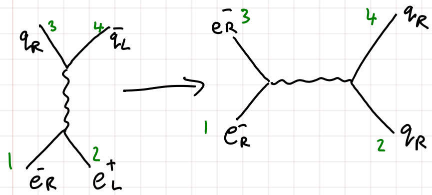
Strong peak in backwards dir $\theta = \pi$

$$|M(e_R^- e_L^+ \to q_R \bar{q}_L)|^2 = Q_f^2 e^4 (1+\cos\theta)^2 = 4Q_f^2 e^4 \frac{u^2}{s^2} \quad ①$$

$$|M(e_R^- e_L^+ \to q_L \bar{q}_R)|^2 = Q_f^2 e^4 (1-\cos\theta)^2 = 4Q_f^2 e^4 \frac{t^2}{s^2} \quad ②$$

Lorentz invariant

①



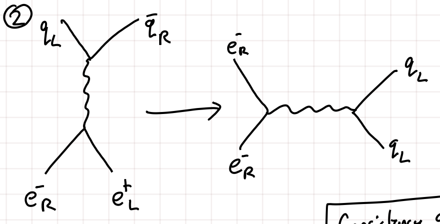
$P_1 \to P_1$
$P_2 \to P_3$
$P_3 \to P_4$
$P_4 \to P_2$

or:
$s \to t$
$t \to u$
$u \to s$

$\Rightarrow$ (for ①)

$$|M|^2 = 4Q_F^2 e^4 \frac{s^2}{t^2}$$

②



$$\Rightarrow |M|^2 = 4Q_F^2 e^4 \frac{u^2}{t^2}$$

Consistency check

$|M|^2 \sim u^2 \sim (1+\cos\theta)^2 \Rightarrow |M|^2 = 0$
for $\theta = \pi$ (no backwards scattering)
- Makes sense with spin conservation

Total cross section:

$$\sigma(e q \to e q) = \frac{1}{2E}\frac{1}{2E}\frac{1}{8\pi}\int \frac{d\cos\theta}{2}\,\frac{1}{4}\sum_{\text{spins}} |M(\bar{e} q \to \bar{e} q)|^2$$

$$\frac{d\sigma}{d\cos\theta} = \pi \frac{Q_f^2 \alpha^2}{s}\frac{s^2+u^2}{t^2}$$

- can use $t = -2E^2(1-\cos\theta)$

$\Rightarrow dt = \frac{1}{2} s \, d\cos\theta$

$$\frac{d\sigma}{dt}(eq \to eq) = \frac{2\pi Q_f^2 \alpha^2}{s^2}\frac{s^2+u^2}{t^2}$$

Back to D.I.S   Hats ^ refer to partonic kinimatics

$$\sigma(e^- p \to e^- X) = \int d\xi \int d\hat{t} \sum_f \left[f_f(\xi) + f_{\bar{f}}(\xi)\right] \frac{2\pi Q_f \alpha^2}{\hat{s}^2}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Kinematics of DIS

introduce Q^2, x, y

$\hat{t} = q^2 = -Q^2$ momentum transfer

$\hat{s} = (k+p)^2 = 2k\cdot p = 2k\xi P$ (recall we neglect proton mass)

$= \xi S$

define $y = \frac{2P\cdot q}{2P\cdot K}$ - In proton rest frame $y = \frac{q_0}{K_0}$ = fraction of initial e^- energy transfered to proton

$$y = \frac{2\xi P\cdot q}{2\xi P\cdot K} = \frac{2p(k-k')}{2pk} = \frac{\hat{s}+\hat{u}}{\hat{s}} = 1 + \frac{\hat{u}}{\hat{s}}$$

$$\Rightarrow \frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2} = 1 + (1-y)^2$$

Masslessness of the final state quark means $0 = (p+q)^2$

$= 2p\cdot q + q^2 = 2\xi P\cdot q - Q^2$

define:

$x = \frac{Q^2}{2P\cdot q}$ Final quark massless $\Rightarrow \xi = x$

Can measure x to find ξ

Relation

$Q^2 = xys \Rightarrow dQ^2 = xs\,dy$

$$\frac{d^2\sigma}{d\xi\, d\hat{t}} = \sum_f \left[f_f(\xi) + f_{\bar{f}}(\xi)\right] 2\pi Q_f \alpha^2 \frac{(1+(1-y)^2)}{Q^4}$$

$d\hat{t} = dQ^2 = xs\,dy$

$$\Rightarrow \frac{d^2\sigma}{dy\,dx} = \sum_f x Q_f^2 \left[f_f(\xi) + f_{\bar{f}}(\xi)\right]\frac{2\pi\alpha^2}{Q^4}(1+(1-y)^2)$$ with $0<x<1$, $0<y<1$

Bjorken scaling

On general grounds, we expected the elementary QED cross section

$$\frac{d^2\sigma}{dx\,dy}(e^- p \to e^- X) = \underbrace{f_2(x,Q)}_{\text{unknown form factor.}} \frac{2\pi\alpha^2 s}{Q^4}(1+(1-y)^2)$$

Parton model $\Rightarrow f_2(x,Q) = F_2(x)$

$$= \sum_f Q_f^2 x \left[f_f(x) + f_{\bar{f}}(x)\right]$$

- independence of F_2 on Q^2 is called Bjorken scaling

Q^2 dependence is not fully constant but slow logarithmic

The Gluon

$f_f(x)$, $f_{\bar{f}}(x)$ are parton distribution functions (pdfs) of the proton | (or any other hadron)

- Measurment of pdfs: Quark model

$$F_2(x) = \sum_f Q_f^2 x \left[f_f(x) + f_{\bar{f}}(x)\right]$$

$$= \frac{4}{9} x f_u(x) + \frac{1}{9} x f_d(x)$$

- preserving charge and Isospin, any extra quark must appear in $q\bar{q}$ pairs and satisfy spin rules

$$\int_0^1 dx\left[f_u(x) - f_{\bar{u}}(x)\right] = 2$$

$$\int_0^1 dx\left[f_d(x) - f_{\bar{d}}(x)\right] = 1$$

$$\int_0^1 dx\left[f_s(x) - f_{\bar{s}}(x)\right] = 0$$

- These cannot be derived from first princibles, only experiment
- DIS of $e^- p \to e^- X$ gives only one compination
- Get an independent combination from scattering on a deuterium target

Isospin Relation:

Proton and nutron pdfs are related:

$f_u^{(n)}(x) = f_d(x)$ $f_d^{(n)}(x) = f_u(x)$

$f_{\bar{u}}^{(n)} = f_{(\bar{d})}(x)$ $f_{\bar{d}}^{(n)}(x) = f_{\bar{u}}(x)$

$\Rightarrow F_n^{(2)}(x) = \frac{4}{9} x f_d(x) + \frac{1}{9} x f_u(x)$

$\Rightarrow$ can fix $f_u(x)$ and $f_d(x)$

More info from DIS by nutrinos. (Weak interactions)

Sum rules: do not require

$f_f(x) = f_{\bar{f}}(x)$

Ex. quantum fluxuation $p \leftrightarrow \Lambda^0 + K^+$
uud   uds $u\bar{s}$

- relativly strong f_s at higher x compared to $f_{\bar{s}}$

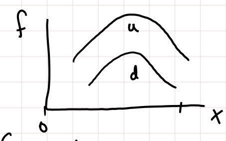
$p \leftrightarrow n + \pi^+$
uud   udd $u\bar{d}$

- relativly higher $f_{\bar{d}}$ compared to $f_{\bar{u}}$

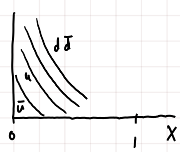
ud can form a low energy $I=0$ $s=0$ leaves the other u with higher momentum

$\to$ relativly higher f_u compared to f_d

Valence quark pdfs



Sea quarks:



Found to diverge as $x \to 0$

- divergences cancle between any $f, \bar{f}$ for the sum rules to hold.

Total sum rule: $\int_0^1 dx\, x \sum_{i=f,\bar{f}} f_i(x) = 1$

- We observe that:

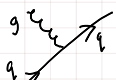
$$\frac{P_{q+\bar{q}}}{P} = \int_0^1 dx\, x \sum_f \left[f_f(x) + f_{\bar{f}}(x)\right] \approx 0.5$$

What is missing? Gluons!

- Posit that their is a field responsible for strong interaction (with particles!)

If gluons exist $\Rightarrow$

$e^+e^- \to$ hadrons should also produce gluons radiated from outgoing $q\bar{q}$



- Prediction of observed 3-Jet events

- Found at $E_{cm} = 30$ GeV

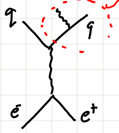
4 jet events observed at 91 GeV (at Desy)

Model Gluons as photons (Spin-1) could try spin-0/2 only this works

- coupling to q as γ couples to e

For Jets, assume colinear splitting i.e. small momentum transfer.

$e^+e^- \to q\bar{q}\gamma$



can be slightly off mass shell?

$P \approx (E, 0, 0, E)$

let z be momentum fraction transfered to γ

$$q = \left(zE, q_\perp, 0, zE - \frac{q_\perp^2}{2zE}\right)$$

so that $q^2 = 0 + \mathcal{O}\left(\left(\frac{q_\perp}{E}\right)^4\right)$

$$K = \left((1-z)E, -q_\perp, 0, (1-z)E - \frac{q_\perp^2}{2(1-z)E}\right)$$

$\Rightarrow k^2 = 0 + \mathcal{O}(q_\perp^4)$

$$P = \left(E, 0, 0, E - \frac{q_\perp^2}{2z(1-z)E}\right)$$

$$P^2 = \frac{q_\perp^2}{z(1-z)} + \mathcal{O}(q_\perp^4)$$

$$M(q_R \to \gamma q_R) = Q_f e \langle q_R(k) | j^\mu | q_R(p) \rangle \epsilon^*_\mu(q)$$

For Right handed only:

$$j^\mu = \psi_R^\dagger \sigma^\mu \psi_R$$

$$= Q_R e\, u_R^\dagger(k) \sigma^\mu u_R(p)\, \epsilon^*_\mu(q)$$

- Spinors $U_R(p) = \sqrt{2E}\begin{pmatrix}1\\0\end{pmatrix}$

$$U_R(k) = \sqrt{2(1-z)E}\begin{pmatrix}1\\ -\frac{q_\perp}{2(1-z)E}\end{pmatrix}$$

To $\mathcal{O}(q_\perp)$:
photon polarizations

So $\epsilon \cdot q = 0$

$$\epsilon_R = \frac{1}{\sqrt2}\left(0,1,i,-\frac{q_\perp}{2E}\right),\quad \epsilon_L = \frac{1}{\sqrt3}\left(0,1,-1,-\frac{q_\perp}{2E}\right)$$

$$u_R^\dagger \sigma\cdot\epsilon_L^* U_R = -\frac{2E}{\sqrt{1-z}}\left(1,-\frac{q_\perp}{2(1-z)E}\right)\frac{1}{\sqrt2}\left[\sigma^1 + i\sigma^2 - \frac{q_\perp}{2E}\sigma^3\right]\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \sqrt2\, q_\perp \frac{\sqrt{1-z}}{z}$$

$$u_R^\dagger \sigma\cdot\epsilon_R^* U_R = \cdots = \sqrt2\, q_\perp \frac{\sqrt{1-z}}{2(1-z)}$$

$$\sum_\epsilon |M|^2 = 2Q_f^2 e^2 q_\perp^2 \frac{1}{z^2(1-z)}\left(1+(1-z)^2\right)$$

$$\sigma(e^+e^- \to q\bar q\gamma) = \frac{1}{2E_A 2E_B 2}\int d\Pi_3 \left|M(e^+e^- \to q\bar q\gamma)\right|^2$$

$$\int \Pi_3 = \int \frac{d^3\bar p\, d^3k\, d^3q}{(2\pi)^9 2\bar p\, 2k\, 2q}(2\pi)^4\delta^{(4)}(Q-\bar p-q-k)$$

from q

$K = p - q \Rightarrow d^3k = d^3p$

$k = (1-z)p + \mathcal{O}(q_\perp)$

$q = zp + \mathcal{O}(p_\perp)$

$d^3q = dq^z d^2q_\perp = p\,dz\,\pi\,dq_\perp$

$$\sigma(e^+e^- \to q\bar q\gamma) = \frac{1}{2E_A 2E_B 2}\int d\Pi_2 \left|M(e^+e^- \to q\bar q)\right|^2$$

$$\times \int \frac{dz\,\pi\,dq_\perp}{(2\pi)^2 2z(1-z)}\left|\frac{1}{p}\right|^2 \left|M(q\to\gamma q)\right|^2$$

$$= \sigma(e^+e^- \to q\bar q)\int \frac{dz\,dq_\perp^2}{16\pi^2 z(1-z)}\left(\frac{z(1-z)}{q_\perp}\right) 2Q_f^2 e^2 q_\perp^2 \frac{1}{z^2(1-z)}\left(1+(1-z)^2\right)$$

- remaining integrals diverge as z or $q_\perp \to 0$ - soft and co-linear divergences.

Weizsacker-Williams distribution

$$\sigma(A\to B+f+\gamma) \approx \sigma(A\to B+f)\int dz \int \frac{dq_\perp}{q_\perp}\frac{Q_f^2\alpha}{\pi}\frac{1+(1-z)^2}{z}$$

$$\sigma(A+f\to B+\gamma) \approx \sigma(A+f\to B)\ \text{"} \qquad \text{"}$$

For f = electrons, cut of these integrals at the scale of $m_e \le q_\perp \le E_{cm}$

$$\frac{m_e}{E_{cm}} \le z \le 1$$

- we get $\frac{2\alpha}{\pi}\log^2\frac{E_{cm}}{m_e}$ for $E_{cm} >> m_e$

$e^+e^- \to 3$ jets

Let g_s = strong interaction coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

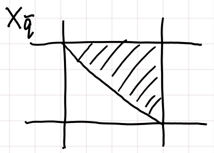
$$\sigma(A\to B+q+g) \approx \sigma(A\to B+f)\int dz\int\frac{dq_\perp}{q_\perp}\overbrace{\frac{4}{3}}^{\text{added colour}}\frac{\alpha_s}{\pi}\frac{(1+(1-z)^2)}{z}$$

Relax co-linear assumption

$E_q,\ E_{\bar q},\ E_g \qquad E_{cm} = Q = E_q + E_{\bar q} + E_g$

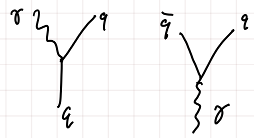
Define fractions

$$x_q = \frac{2E_q}{Q},\quad x_{\bar q} = \frac{2E_{\bar q}}{Q},\quad x_g = \frac{2E_g}{Q}$$



Co-linear at boundaries (are favourite)

$$\sigma(e^+e^- \to q\bar q g) = \sigma(e^+e^- \to q\bar q)\int dx_q\, dx_{\bar q}\frac{2\alpha_s}{3\pi}\frac{x_q^2 + x_{\bar q}^2}{(1-x_q)(1-x_{\bar q})}$$



Noethers thm $\to$ conservation law

$$\delta\Psi = i\delta\alpha\Psi,\quad \delta\bar\Psi = -i\delta\alpha\bar\Psi$$

$$\delta\mathcal{L} = (\delta\bar\Psi)(i\gamma^\mu D_\mu - m)\Psi + \bar\Psi(i\gamma^\mu D_\mu - m)(\delta\Psi)$$

$$= 0$$

- If we allow local variation:

$$\delta\Psi(x) = i(\delta\alpha(x))\Psi(x),\quad \delta\bar\Psi = -i(\delta\alpha(x))\bar\Psi(x)$$

$$\Rightarrow \delta\mathcal{L} = \bar\Psi i\gamma^\mu\partial_\mu(i\delta\alpha(x)\bar\Psi(x)) = \bar\Psi i\gamma^\mu\Psi(i\partial_\mu\alpha(x))$$

$$\delta S = \int d^4x\,(\delta\alpha(x))\,\partial_\mu(\bar\Psi\gamma^\mu\Psi)$$

$\Rightarrow$ Conservation of $j^\mu = \bar\Psi\gamma^\mu\Psi$

$\partial_\mu j^\mu = 0$

Quantum Chromodynamics (QCD)

- Quarks as spin $\frac12$ fermions
- Gluons as massless spin-1 bosons
- Maxwell like equations
- "Colour"
- Consistency in neglecting α_s
- to lowest order in $e^+e^- \to$ Hadrons and DIS

Gauge Invariance, modled on QED

$$\mathcal{L}_{QED} = -\frac14 F^{\mu\nu}F_{\mu\nu} + \bar\Psi(i\gamma^\mu D_\mu - m)\Psi$$

- $D_\mu = \partial_\mu + ieA_\mu$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Global Guage invariance

$\mathcal{L}_{QED}$ is invariant under

$$\Psi(x) \to e^{i\alpha}\Psi(x)$$

$$\bar\Psi(x) \to e^{-i\alpha}\bar\Psi(x)$$

$\mathcal{L}$ is more generally invariant under a local gauge transformation

- use also $A_\mu(x) = -\frac1e\partial_\mu\delta\alpha(x)$

Then $\delta\mathcal{L} = 0$

$$\delta\mathcal{L} = \bar\Psi(i\gamma^\mu\Psi)(i\partial_\mu\delta\alpha)$$

$$+ \bar\Psi(i\gamma^\mu)(+ie)(-\tfrac1e\partial_\mu\delta\alpha)\Psi = 0$$

Lie groups

Group operators are hermitian operators

$T^a,\ a = 1,\dots,d_G$

$$[T^a, T^b] = if^{abc}T^c$$

structure constants

- Generators of d_R-dim unitary representation are $d_R\times d_R$ Hermitian matricies t^a_R

$$[t^a_R, t^b_R] = if^{abc}t^c_R$$

acting on complex vectors Φ

Infitesimal group transformations

$\phi \to (1+i\alpha^a t^a_R)\Phi$

Unitary $U(\alpha) = \exp[i\alpha^a t^a_R]$

$\Phi \to U(\alpha)\Phi$

Representations of $SU(3)$

- 1 trivial
- Smallest non-trivial rep, 3 & $\bar{3}$, complex conjugates

fundementa ; anti-fundemental $\sim$ anti-symm 2-din tensor

Define a normalization factor for each rep

$tr[t^a_R t^b_R] = C(R)\,\delta^{ab}$

for $R=N$ of $SU(N)$ $C(R) = \frac{1}{2}$

- $SU(N)$ also has an adjoint representation

Define matricies $(t^b_G)_{ac} = if^{abc}$

$tr[t^a_G t^b_G] = f^{acd}f^{bcd} = C(G)\,\delta^{ab}$

for $SU(N)$, $C(G) = N$

Dim of adjoint of $SU(N)$ is N^2-1

$N=3 \to 8$ (octet) of $SU(3)$

Non-abelian gauge symmetry

$\mathcal{L}_0 = \bar{\Psi}_j\, i\gamma^\mu \partial_\mu \psi_j$

- where Ψ_j transforms in Rep R of gauge group G

Infitesimal local gauge transformation

$\Psi_j(x) \to \Psi'_j(x) = (1+i\alpha^a(x)t^a_R)_{jk}\,\psi_k$

we will drop the label of representation now

$\delta\mathcal{L}_0 = \bar{\Psi}_j\, i\gamma^\mu(i\partial_\mu\alpha^a(x)t^a_{jk})\Psi_k$

- $D_\mu = \partial_\mu - igA^a_\mu t^a$ ← trying to get rid of

one spin-1 field for each generator

- To do this we need:

$A^a_\mu(x) \to A^a_\mu(x) + \frac{1}{g}\partial_\mu\alpha^a(x) + A^b_\mu f^{abc}\alpha^c(x)$

In adjoint rep,

$D_\mu\alpha^a(x) = \partial_\mu\alpha^a(x) + gA^b_\mu f^{abc}\alpha^c(x)$

(from $(t^b)_{ac} = if^{abc}$

So we can write:

$A^a_\mu(x) \to A^a_\mu(x) + \frac{1}{g}D_\mu\alpha^a(x)$

- Finite local transformations

$\Psi \to \exp[i\alpha^a t^a]\Psi$

$D_\mu \to \exp[i\alpha^a t^a]D_\mu\exp[-i\alpha^a t^a]$

Infitesimally

$D_\mu \to (1+i\alpha^b t^b)D_\mu(1-i\alpha^a t^a) + O(\alpha^2)$

$= D_\mu - i\partial_\mu(\alpha^a t^a) + [i\alpha^b t^b, -igA^a_\mu t^a]$

$= D_\mu - it^a\partial_\mu\alpha^a + g\alpha^b A^a_\mu \underbrace{[t^a, t^b]}_{= -if^{abc}t^c})$?

$= D_\mu - it^a\partial_\mu\alpha^a - igf^{abc}t^a A^b_\mu\alpha^c$

$D_\mu\Psi \to \exp[i\alpha^a t^a]D_\mu\Psi$

$\mathcal{L} = \bar{\Psi}i\gamma^\mu D_\mu\Psi$ is invariant

Need a gauge invariant kinetic term:

$\mathcal{L} = -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu}$

$[D_\mu, D_\nu] \to \exp[i\alpha^a t^a][D_\mu, D_\nu]\exp[-i\alpha^a t^a]$

$= [\partial_\mu - igA^a_\mu t^a, \partial_\nu - igA^b_\nu t^b]$

$= ig\partial_\mu A^a_\nu t^a + igA^a_\mu t^a + (-i)^2[A^a_\mu t^a, A^b_\nu t^b]$

- This motivates field strength

$[D_\mu, D_\nu] = -igF^a_{\mu\nu}t^a$

i.e. $\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu}$

(indep of rep R)

We have made it so that

$\mathcal{L} = \bar{\Psi}i\gamma^\mu D_\mu\Psi$ is locally gauge invariant

Defined F by $[D_\mu, D_\nu] = -igF^a_{\mu\nu}t^a$

$F^a_{\mu\nu} = \partial_\mu A^a - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$

$F^a_{\mu\nu}t^a \to \exp[i\alpha^b t^b]F^a_{\mu\nu}t^a\exp[-i\alpha^c t^c]$

so that $tr[(F^a_{\mu\nu}t^a)(F^{\mu\nu b}t^b)]$ is invariant

$\sim F^a_{\mu\nu}F^{\mu\nu b}\delta_{ab}$

$\Rightarrow \mathcal{L} = -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu}$ is Lorentz invariant and gauge invariant

- Complete Lagrangian with vector bosons and Dirac fermions

- Field eq are Dirac eq for Ψ and non linear for A

$\mathcal{L} = -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + \bar{\Psi}[i\gamma^\mu D_\mu - m]\Psi$

QCD Lagrangian

$\mathcal{L} = -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + \bar{\Psi}_f(i\gamma^\mu D_\mu - m)\Psi_f$

- f sums over flavours of quarks
- $G = SU(3)$ $a = 1, \dots, 8$
- $(\Psi_f)_i$ in fundemental rep 3 $i = 1,2,3$

$D_\mu = \partial_\mu - ig_s A^a_\mu t^a$ we define $\alpha_s = \frac{g_s^2}{4\pi}$

- Gluon emmission :

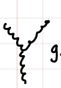
$q \to qg$ 

Recall

$M(q_{f_i}(p) \to g^a(\varepsilon)q_{f_j}(k)) = g_s\bar{u}(k)\gamma^\mu t^a_{ji}\varepsilon_\mu(q)$

$|M|^2$ summed over final colours, avg over initial colours

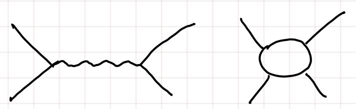
$\frac{1}{3}\sum_{i,j,a} g_s^2|t^a_{ji}|^2 = g_s^2 tr[t^a t^a] = \frac{4}{3}g_s^2$

 $g \to gg$ replace $R \to 8$ adj

$\frac{1}{8}\sum_{a,b,c} g_s^2|(t^b_G)_{ac}|^2 = \frac{g_s^2}{8}tr[t^a_G t^a_G] = \sum_a \frac{g_s^2}{8}3\delta^{aa} = 3g_s^2$

Vacuum Polarization

In QED, electric charge is screened, charge we measure is smaller then the coupling in $\mathcal{L}$



Includes 

- $e_0 \to e$
- Renormalization group eq as $e = e(Q)$

$$\frac{d}{d\log Q}e(Q) = \beta(e(Q))$$

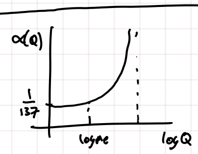
Q = momentum transfere

β = "The Beta function"

for $Q \gg m_e$ $\beta(e) = \frac{e^3}{12\pi^2}$ ⇒ $e^2(Q) = \frac{e_0^2}{1 - \frac{e_0^2}{6\pi^2}\log\frac{Q}{Q_0}}$ where $e_0 = e(Q_0)$

Also

$\alpha(Q) = \frac{\alpha_0}{1 - \frac{2\alpha_0}{3\pi}\log\frac{Q}{Q_0}}$



LEP $\sqrt{s} = 193$ GeV

$\alpha \sim \frac{1}{127}$

Asymmtopic freedom

There are gluon self interactions

bosonic loops ⇒ changes Renormalization eq

$$\frac{d}{d\log Q}g_s(Q) = \beta(g_s(Q)) \; ; \; \beta(g_s) = -\left[\frac{11}{3}C(G) - \frac{4}{3}n_f C(R)\right]\frac{g_s^3}{16\pi^2}$$

SU(N) ; Number of flavours

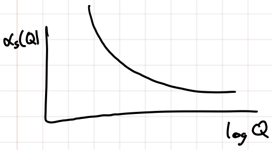
In the case of $SU(3)$ $= -\left[11 - \frac{2}{3}n_f\right]\frac{g_s^3}{16\pi^2} \equiv b_0$

$n_f = 5$ at high energies

no t loops for most purposes

$$\alpha_s(Q) \simeq \frac{\alpha_s(Q_0)}{1 + b_0 \frac{\alpha_s(Q_0)}{2\pi} \log\frac{Q}{Q_0}} = \frac{2\pi / b}{\log\frac{Q}{\Lambda}}$$

$$\Lambda \sim Q_0 \exp\left[-\frac{2\pi}{b_0 \alpha_s(Q_0)}\right]$$



- Assymtopic freedom
- strongly coupled bound states use high $\alpha_s(Q)$

Hard scattering uses low $\alpha_s(Q)$

At short distances, gluon exchange produces a Coulomb potential between heavy quarks

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} \qquad \left(\begin{matrix} \Psi \sim c\bar{c} \\ \Upsilon \sim b\bar{b} \end{matrix}\right)$$

Next order in $e^+e^- \to$ hadrons

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_f Q_f^2 \left(1 + \frac{\alpha_s(s)}{\pi} \cdots\right) \qquad s = \sqrt{s}_{cm}$$

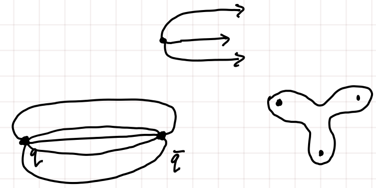
$$\alpha_s(91\,\text{GeV}) \approx \frac{1}{8.5}$$

Need next-to-leading order corrections and more

Confinement of quarks in hadrons

Is not fully explained by asymtotic freedom

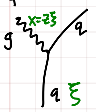
Colour "flux tubes" carry infinite energy



Lattice QCD calculations interpolate from strong to weak coupling.

We Omit chpt 13.1, 13.3

Evolution of pdfs with Q^2



$$\sigma(A \to B + q + g) \simeq \sigma(A \to B + q)\int dz \int \frac{dq_\perp}{q_\perp}\frac{4}{3}\frac{\alpha_s}{\pi}\frac{1+(1-z)^2}{z}$$

$$\text{Prob}(q \to gq) = \int dz \int \frac{dq_\perp}{q_\perp}\Big(\qquad \Big)$$

With the proton the probability of emmitting gluon off a quark parton is:

$$\int dx\, f_g(x) = \int d\xi \int dz \int^Q \frac{dq_\perp}{q_\perp}\frac{4}{3}\frac{\alpha_s(q_\perp)}{\pi}\frac{1+(1-z)^2}{z} f_f\left(\frac{x}{z}\right)$$

flavour of parton

$x = z\xi$ ← parton momentum fraction

$f_g(x)$ satisfies an ODE

$$\frac{d}{d\log Q} f_g(x) = \frac{4}{3}\frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\frac{1+(1-z)^2}{z} f_f\left(\frac{x}{z}\right)$$

Limits: $0 < z < 1$, and $\frac{z}{x} = \frac{1}{\xi} > 1$

Similar analysis for other collinear splitting

$q\ \bar{q}\ \xi \qquad g\ g\ g$

Total set of DEs for pdfs are called Altarelli-Parisi or DGLAP eqs

$$\frac{d}{d\log Q} f_g(x) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{g\leftarrow g}(z) f_g\left(\frac{x}{z}\right) + \sum_f P_{g\leftarrow q}(z)\left[f\left(\frac{x}{z}\right) + f_{\bar{f}}\left(\frac{x}{z}\right)\right]\right\}$$

$$\frac{d}{d\log Q} f_f(x) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{q\leftarrow q}(z) f_f\left(\frac{x}{z}\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z}\right)\right\}$$

$$\frac{d}{d\log Q} f_{\bar{f}}(x) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{q\leftarrow q}(z) f_{\bar{f}}\left(\frac{x}{z}\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z}\right)\right\}$$

- $P_{g\leftarrow g}(z)$ ect are called DGLAP splitting functions
- $P_{g\leftarrow q}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$

$g\ q\ \xi$ related by $z \to 1-z$ $\quad P_{q\leftarrow q} = \frac{4}{3}\left[\frac{1+z^2}{1-z} + A\,\delta(z-1)\right]$

- Conservation of quark number:

$$\int_0^1 dz\, P_{q\leftarrow q}(z) = 0$$

still need a cutoff

say $z < 1-\epsilon$

$$A = -\left[2\log\frac{1}{\epsilon} - \frac{3}{2}\right]$$

$$P_{q\to g}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right] \qquad \text{Ex 12.1}$$

$$P_{g\leftarrow g}(z) = 3\left[\frac{1+z^4+(1-z)^4}{z(1-z)} + B\,\delta(z-1)\right]$$

from considering colour

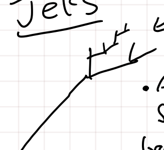
As $z \to 0$, the singular behaviour is:

$$P_{g\leftarrow q}(z) \sim \frac{4}{3}\frac{2}{z} \qquad P_{g\leftarrow g}(z) \sim 3\frac{2}{z}$$

in the ratio $\frac{9}{4}$

- Hence gluon jets are wider than quark jets

Jets



- After multible splitting momentum transfer decreases
- To where $\alpha_s(q_\perp)$ is large and strong interactions take over: Hadronisation.
- Quark gluon interactions → divergences for soft and collinear emmisions

Low energy.

- Jet structure is analysed in terms of IR-safe observables
- One such observable is thrust

$$T = \max_{\hat{n}}\left|\frac{\sum_i |\hat{n}\cdot\vec{p}_i|}{\sum_i |\vec{p}_i|}\right|$$

$\hat{n}$ is a direction ~ initial q dir

- i runs over all detectable particles

Collinear splitting

$q \simeq zP$, $k \simeq (1-z)P$

$P = q + k$

$|\vec{P}| \simeq |\vec{q}| + |\vec{k}|$

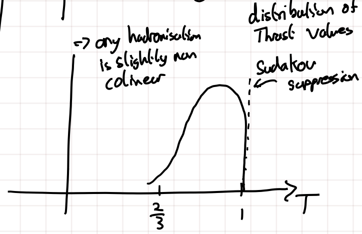
$|\hat{n}\cdot\vec{P}| \simeq |\hat{n}\cdot q| + |\hat{n}\cdot\vec{k}|$

→ Same T

- Lowest order

$T = 1$

$\to T = \frac{2}{3}$



"shape probes deviation from basic, 2 jet structure"

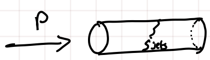
Alternative clustering.

- Pairs of particles with low value of $y_{ij} = \frac{(p_i+p_j)^2}{S}$

combine these $p_i+p_j = p_K$

Jets are composite particles obtained up to some cutoff y_{cut}

Hadron Colliders



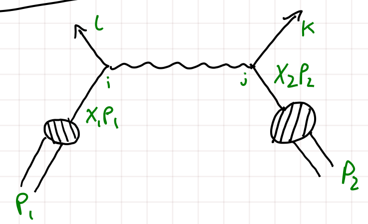
pdfs

QCD background swamps signals of interest

$p = \begin{pmatrix} u \\ u\ d \end{pmatrix}$

- Soft collisions → many final state particles of small transfer momentum
- Hard collisions → particle scattering, jets with large transverse momentum

2-jet events



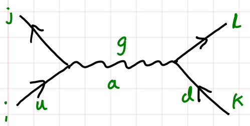
$$\sigma(pp \to 2jets) = \sum_{i,j,K,L} \int dx_1 f_i(x_1) \int dx_2 f_j(x_2) \int d\cos\theta_* \frac{d\sigma(ij\to KL)}{d\cos\theta_*}$$

recall $\hat{s} = (p_i+p_j)^2 = 2p_ip_j = 2x_1x_2P_1P_2 = x_1x_2S$

From $eq\to eq$ to $ud\to ud$, insert color factors

Recall: $\frac{d\sigma}{d\cos\theta}(eq\to eq) = \frac{\pi Q_f^2 \alpha^2}{s}\frac{s^2+u^2}{t^2}$

taking colour into account:



$$\frac{1}{3}\frac{1}{3}\sum_{i,k}\sum_{j,l}\sum_a |t^a_{ji} t^a_{lk}|^2$$

$$= \frac{1}{9} t^a_{ji} t^a_{lk} (t^b_{ij})(t^b_{kl})$$

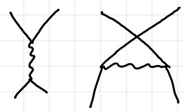
$$= \frac{1}{9} tr(t^a t^b) tr(t^a t^b)$$

$$= \frac{1}{9}\frac{1}{2}\delta^{ab}\frac{1}{2}\delta^{ab} = \frac{2}{9}$$

With this:

$$\frac{d\sigma}{\cos\theta_*}(ud\to ud) = \frac{2}{9}\frac{\pi\alpha_s^2}{\hat{s}}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

For $uu\to uu$ need more diagrams



t and u channel diagrams

results in:

$$\frac{d\sigma}{d\cos\theta_*}(qq\to qq) \sim \frac{2}{9}\frac{\pi\alpha_s^2}{\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2} - \frac{2}{3}\frac{\hat{s}^2}{\hat{u}\hat{t}}\right]$$

1st diagram², second diagram², cross term

Check the forward limit.

$\theta_* \to 0$ $\hat{t}\to 0$, $\hat{u}\to -\hat{s}$

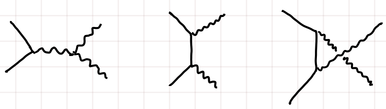
$$\lim \frac{d\sigma}{d\cos\theta}(qq\to qq) = \frac{4}{9}\frac{\pi\alpha_s^2\hat{s}^2}{3\hat{t}^2}$$

$$\sim \frac{4}{9}\frac{\pi\alpha_s^2}{\hat{s}}\frac{1}{\sin^4\frac{\theta}{2}}$$

⇒ Recover familiar coulomb scattering from QCD potential

We can use crossing symmetry to figure out $u\bar{u}\to d\bar{d}$, $u\bar{u}\to u\bar{u}$ and $u\bar{d}\to u\bar{d}$

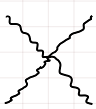
- With gluons, $u\bar{u}\to gg$ or $ug\to ug$; Diagrams:



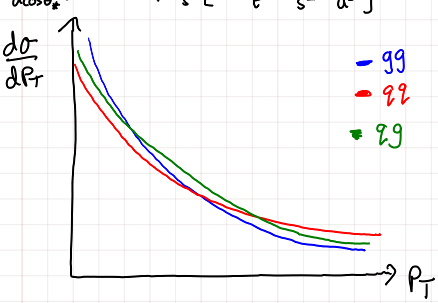
- Result is:

$$\frac{d\sigma}{d\cos\theta_*}(qg\to qg) = \frac{\pi\alpha_s^2}{2\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{4}{9}\left(\frac{\hat{u}}{\hat{s}}+\frac{\hat{s}}{\hat{u}}\right)\right]$$

Gluons → Gluons include



$$\frac{d\sigma}{d\cos\theta_*}(gg\to gg) = \frac{9}{4}\frac{\pi\alpha_s^2}{s}\left[3 - \frac{su}{t^2} - \frac{ut}{s^2} - \frac{st}{u^2}\right]$$



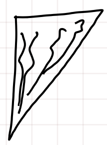
Low P_T gg dominates, larger intrinsic cross section

High P_T valence quarks dominate

In between qg dominate

Skipping 13.3

Jet width



Two observable effects

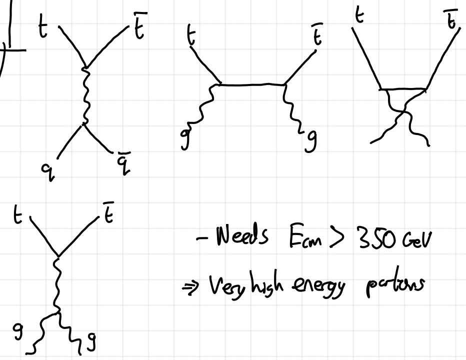
1. increased $P_\perp$ → decreased α_s → narrower jet
2. Gluon jets wider then quark jets, due to splitting function.

   Also leads to narrower jets at higher $P_\perp$

Top Quark production

u 2.3 MeV d 4.8 MeV 95 MeV

c 1.3 GeV b 4.2 GeV t 173 GeV



- Needs $E_{cm} > 350$ GeV

⇒ Very high energy partons

- First only valence quarks can produce t

⇒ First diagram only

- As we increase E_{cm} include other partons

Gluons give intrinsically larger σ then quarks

(Gluons become dominant process)

- Discovery 1995, Tevatron. Big boost in production once we got LHC

Chiral Symmetry & Spontaneous Symmetry Breaking

Light quark masses are non-zero and governed by chiral symmetry

Chiral symmetry is a symmetry of QCD in the limit of zero quark masses

- It is spontaneously broken

QCD with non zero quark masses

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu a} F^{a}_{\mu\nu} + \bar{\Psi}_f \left(i\gamma^\mu D_\mu - m\right)\Psi_f$$

- Symmetries: Lorentz P, C, T
- Global charge conservation (quark/baryon number)
  SU(3) color
- If $m_u = m_d \Rightarrow$ also isospin (in SU(2))

$$\Psi \to \left[\exp i\vec{\alpha}\cdot\frac{\vec{\sigma}}{2}\right]_{ij} \Psi \qquad i,j = u,d$$

Iso spin rotation of $\mathcal{L}$

$$\Psi = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix} \qquad m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

$$\bar{\Psi}\left(i\gamma^\mu D_\mu - m\right)\Psi$$

$$\bar{\Psi}\left(i\gamma^\mu D_\mu - \exp\left[-i\vec{\alpha}\cdot\frac{\vec{\sigma}}{2}\right] M \exp\left[i\vec{\alpha}\cdot\frac{\vec{\sigma}}{2}\right]\right)\Psi$$

$\mathcal{L}$ invariant if $[m, \sigma^i] = 0 \iff m_u = m_d$.

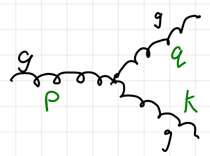
- Further if $m_u = m_d = 0$ then there is an extra SU(2) Chiral symmetry.

$$\int_0^{1-\varepsilon} \frac{1+z^2}{1-z}\,dz = \frac{1}{2}\left(-z^2 - 12z + 3 - 4\ln(1-z)\right)$$

$z = 0,\ \frac{3}{2}$

$z = 1-\epsilon = \frac{1}{2}\left(4 \ln\frac{1}{\varepsilon}\right)$

OOPS Tutorial Here



$$M\left(g^a(p, \epsilon_p) \to g^b(q, \epsilon_q) + g^c(k, \epsilon_k)\right)$$

$$= g_s f^{abc}\left[(k+p)\cdot\epsilon_q^* \;\epsilon_k^*\cdot\epsilon_p - (p+q)\cdot\epsilon_k^* \;\epsilon_p\cdot\epsilon_q^* + (q-k)\cdot\epsilon_p \;\epsilon_q^*\cdot\epsilon_k^*\right]$$

(a) clearly symmetric

(b) Colinear approximation:

$q = (zE, q_\perp, 0, zE)$ gotta modify these to have $p^2 = k^2 = 0$

$k = ((1-z)E, -q_\perp, 0, (1-z)E)$

$$p = \left(E, 0, 0, E - \frac{q_\perp^2}{2z(1-z)E}\right)$$

$$q = \left(zE, q_\perp, 0, zE - \frac{q_\perp^2}{2zE}\right)$$

$$k = \left((1-z)E, -q_\perp, 0, (1-z)E - \frac{q_\perp^2}{2(1-z)E}\right)$$

Polarization vectors

$$\epsilon_R(p) = \frac{1}{\sqrt{2}}(0, i, 1, 0)$$

$$\epsilon_L(p) = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$

$$\epsilon_R(q) = \frac{1}{\sqrt{2}}\left(0, 1, i, \frac{-q_\perp}{zE}\right) \quad \mathcal{O}(q_\perp)$$

$$\epsilon_R(k) = \frac{1}{\sqrt{2}}\left(0, 1, i, \frac{q_\perp}{(1-z)E}\right)$$

	$\epsilon_R(p)$	$\epsilon_R(q)$	$\epsilon_R(k)$	$\epsilon_L(p)$	$\epsilon_L(q)$	$\epsilon_L(k)$
p	0	$\frac{q_\perp}{\sqrt{2}z}$	$\frac{-q_\perp}{(1-z)\sqrt{2}}$	0		
q	$\frac{-q_\perp}{\sqrt{2}}$	0	$-\frac{q_\perp}{\sqrt{2}(1-z)}$		0	
k	$\frac{q_\perp}{\sqrt{2}}$	$\frac{q_\perp}{\sqrt{2}z}$	0			0
$\epsilon_R(p)$, $\epsilon_R(q)$, $\epsilon_R(k)$		0			-1	
$\epsilon_L(p)$, $\epsilon_L(q)$, $\epsilon_L(k)$		-1			0	

$g_R \to g_R g_L$

$$g_s f^{abc}\left[(k+p)\cdot\epsilon_R^*(q)\,\epsilon_L^*(k)\cdot\epsilon_R(p) - (p+q)\cdot\epsilon_L^*(k)\,\epsilon_R(p)\cdot\epsilon_R^*(q) + (q-k)\cdot\epsilon_R(p)\,\epsilon_R^*(q)\cdot\epsilon_L^*(k)\right]$$

$$= g_s f^{abc}\left[-\left(2\frac{q_\perp}{\sqrt{2}}\frac{1}{1-z}\right)(-1) + \left(-2\frac{q_\perp}{\sqrt{2}}\right)(-1)\right]$$

$$= g_s f^{abc}\sqrt{2}\,q_\perp \frac{z}{z-1}$$

Similarly:

$$g_R \to g_L g_R \to g_s f^{abc}\sqrt{2}\,q_\perp\frac{z-1}{z}$$

$$g_R \to g_R g_R \to g_s f^{abc}\sqrt{2}\,q_\perp\frac{z}{z(z-1)}$$

Square and sum spins:

$$2q_\perp^2\left[0 + \left(\frac{z}{z-1}\right)^2 + \left(\frac{z-1}{z}\right)^2 + \left(\frac{z}{z(z-1)}\right)^2\right]$$

$$\frac{1}{8}\sum_{abc} f^{abc}f^{abc} = 3 \qquad = 6q_\perp^2 g_s^2 \frac{z^4 + (z-1)^4 + 1}{z^2(z-1)^2}$$

Back to lectures:

If $m_u = m_d \Rightarrow$ Isospin symmetry

If $m_u = m_d = 0 \Rightarrow SU(2)$ chiral symmetry

- For massless fermions:

$$\mathcal{L} = -\frac{1}{4}F^2 + \sum_{f=u,d}\left[\psi_{fL}^{+} i\bar{\sigma} D \psi_{fL} + \psi_{fR}^{+} i\sigma D \psi_{fR}\right]$$

$\Rightarrow$ Seperate $SU(2)_L$ and $SU(2)_R$ isospin symmetries

$$\psi_f = \begin{pmatrix}\psi_{fL}\\ \psi_{fR}\end{pmatrix} \quad \underbrace{\psi \to \exp[i\vec{\alpha}\cdot\tfrac{\vec{\sigma}}{2}]\psi}_{SU(2)\text{ isospin}}, \quad \underbrace{\psi \to \exp[i\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2}]\gamma^5\psi}_{\text{Chiral } SU(2)}$$

where

$$\gamma^5 = \begin{pmatrix}-1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1\end{pmatrix} = i\gamma^0\gamma^1\gamma^2\gamma^3, \text{ so } \{\gamma^5, \gamma^\mu\} = 0$$

ψ_L and ψ_R are eigenstates of γ^5 with e values -1 and $+1$

Two conserved currents:

$$j^{\mu a} = \bar{\psi}\gamma^\mu \frac{\sigma^a}{2}\psi, \quad j^{\mu 5 a} = \bar{\psi}\gamma^\mu\gamma^5\frac{\sigma^a}{2}\psi$$

check conservation:

Dirac Eq: $(i\gamma^\mu D_\mu - m)\psi = 0 \qquad 0 = -iD_\mu\bar{\psi}\gamma^\mu - \bar{\psi}m$

$$\partial_\mu j^{\mu a} = (D_\mu\bar{\psi})\gamma^\mu\frac{\sigma^a}{2}\psi + \bar{\psi}\gamma^\mu\frac{\sigma^a}{2}(D_\mu\psi)$$

$$= i\bar{\psi}m\frac{\sigma^a}{2}\psi - i\bar{\psi}m\frac{\sigma^a}{2}\psi = i\bar{\psi}\left[m, \frac{\sigma^a}{2}\right]\bar{\psi}$$

$\Rightarrow$ isospin conserved if $m_u = m_d$ $\left(\text{recall } m = \begin{pmatrix}m_u & \\ & m_d\end{pmatrix}\right)$

$j^{\mu a} = 0 \Rightarrow [m, \sigma^a] = 0 \Leftrightarrow m_u = m_d$

For the chiral isospin current:

$$\partial_\mu j^{\mu 5 a} = (D_\mu\bar{\psi})\gamma^\mu\gamma^5\frac{\sigma^a}{2}\psi + \bar{\psi}\gamma^\mu\gamma^5\frac{\sigma^a}{2}(D_\mu\psi)$$

$$= i\bar{\psi}m\gamma^5\frac{\sigma^a}{2}\psi + i\bar{\psi}\frac{\sigma^a}{2}\gamma^5 m\psi$$

$$= i\bar{\psi}\left\{m, \frac{\sigma^a}{2}\right\}\gamma^5\psi$$

$\partial_\mu j^{\mu 5 a} \Leftrightarrow \{m, \sigma^a\} = 0 \Rightarrow m_u = m_d = 0$

Spontaneous Symmetry breaking:

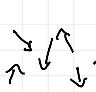
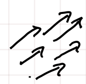
This QCD with 2 massless flavours is invariant under $U(1) \times SU(2)_L \times SU(2)_R$

$U(1)$: Bayron number? why?

Cannot be an exact symmetry. If it were we would have triples of quantum numbers.

- Prevents mass terms for nucleons or else results in doubling L-R of states
- Resolution $SU(2)_L \times SU(2)_R$ is a symmetry of the Hamiltonian, but not respected by states
- Ground states can be transformed by symmetries rather then preserved

CM example

① Magnet  above T_c  below T_c

② Superconductivity Electrons bound into pairs
- Ground state has no definite charge. Has a resevoir of such pairs.

QCD: Massless $q\bar{q}$ can be produced at no energy cost.

$q_R\bar{q}_R$ bound state w/ zero momentum, zero angular momentum

$\Rightarrow$  Same Helicity $\Rightarrow$ Not anti particles

Spontanously break $SU(2)$

u_L can become (mix into) u_R by annihilating $\bar{u}_R$ of the bound pair.

- Order parameter Δ for symmetry breaking

Consider Vacumm state w/ Condensates $|0\rangle$

state $u_L\bar{u}_L$ annihilated by opperators $\psi_{uR}^{+}\psi_{uL}$

$$\underbrace{\langle 0|\psi_{jL}^{+}\psi_{iL}|0\rangle}_{= -\Delta\delta_{ij}} \neq 0 \Leftrightarrow \text{condensate present}$$

Act on $|0\rangle$ with $SU(2)_L$ rotation to get $|\vec{\alpha}\rangle$

Then $|\vec{\alpha}\rangle$ is another vacuum state and

$$\langle\vec{\alpha}|\psi_{jR}^{+}\psi_{iL}|\vec{\alpha}\rangle = -\Delta\left(\exp\left[\frac{i\vec{\alpha}\cdot\vec{\sigma}}{2}\right]\right)_{ij}$$

$\vec{\alpha}$ can take any value, break isospin and parity symmetry.

- State of ground states isomorphic to $SU(2)$ itself.

Goldstone Bosons

Goldstone Theorem For every spontanously broken symmetry there is a massless particle created by the symmetry current

In QCD

$\underbrace{j^{05a}(x)|0\rangle}$

$Q^{5a} \leftarrow$ charge generating global chiral $SU(2)$

- State with momentum $\vec{p}$: $\int d^3x\, e^{-i\vec{p}\cdot\vec{x}} j^{05a}(x)|0\rangle$

As $\vec{p} \to 0$, goes to a ground state.

hence particle π has 0 mass

Annihilation:

$$\langle 0| j^{\mu 5} |\pi(\vec{p})\rangle = i \underset{\uparrow}{f} p^\mu e^{-ipx}$$

some constant

- current conservation:

$$0 = \langle 0| \partial_\mu j^{\mu 5}(x)|\pi(\vec{p})\rangle$$
$$= f p^2 e^{-ipx}$$

$\Rightarrow p^2 = 0$ hence massless

QCD

$$\langle 0| j^{\mu 5}(x) | \pi^b(\vec{p})\rangle = i \underset{\uparrow}{f_\pi} p^\mu \delta^{ab} e^{-ipx}$$

some constant

pion decay constant

measured to be 93MeV

3 chiral $SU(2)$ currents $j^{\mu 5 a}$

- Isospin triplet $\Rightarrow I = 1$
- Parity of j^{05a} is $-1 \Rightarrow P = -1$
- Spin 0

$\longrightarrow$ Corresponds to pions π^-, π^0, π^+

Pions as Goldstone Bosons:

Example Matrix element of chiral isospin current in Nucleon state

For Vector Isospin; $\langle N(p')| j^{\mu a} |N(p)\rangle = 1 \cdot \bar{u}(p')\gamma^\mu u(p)\cdot \frac{\sigma^a}{2}$

in aproximation of low mom transfer

For chiral Isospin $\langle N(p')| j^{\mu 5a} |N(p)\rangle = \underset{\uparrow}{g_A}\, \bar{u}(p')\gamma^\mu\gamma^5 u(p) \frac{\sigma^a}{2} + \cdots$

dim-less constant

If $\partial_\mu j^{\mu 5 a} = 0$, then $g_A = \frac{f_\pi}{M_N} g_{\pi NN}$ "Goldberger-Treiman relation"

Approx true

Add small quark masses

$$\Delta H = m_u \bar{\Psi}_u \Psi_u + m_d \bar{\Psi}_d \Psi_d$$
$$= m_u \Psi_{uR}^\dagger \Psi_{uL} + m_d \Psi_{dR}^\dagger \Psi_{dL} + (R \leftrightarrow L)$$

$$\langle 0|\Delta H|0\rangle = -2\Delta(m_u + m_d)$$

Recall: $\partial_\mu j^{\mu 5 a} = i \bar{\Psi} \{ m, \frac{\sigma^a}{2} \} \gamma^5 \Psi$

take $m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \frac{m_u + m_d}{2}\mathbb{1} + \frac{m_u - m_d}{2}\sigma^3$

$$\Rightarrow \{ m, \frac{\sigma^3}{2} \} = \frac{m_u + m_d}{2}\sigma^a + \frac{m_u - m_d}{2}\delta^{3a}\mathbb{1}$$

$$\partial_\mu j^{\mu 5 a} = i\left(\frac{m_u + m_d}{2}\right)\bar{\Psi}\sigma^a\gamma^5\Psi + i\left(\frac{m_u - m_d}{2}\right)\delta^{3a}\bar{\Psi}\gamma^5\Psi$$

Annihilation of pion:

$$\langle 0| \partial_\mu j^{\mu 5 a} |\pi^b\rangle = i(m_u + m_d)\langle 0|\bar{\Psi}\frac{\sigma^a}{2}\Psi|\pi^b\rangle + 0$$

$$f_\pi p^2 \delta^{ab} = (m_u + m_d)\Delta' \delta^{ab}$$

some constant

$$p^2 = m_\pi^2 = \frac{m_u + m_d}{f_\pi}\Delta'$$

- Estimate $\Delta' \sim$ QCD scale $\sim (500\,\text{MeV})^2$

$f_\pi = 93\,\text{MeV}$

$m_u + m_d \approx 7\,\text{MeV}$

Spontaneous Symmetry breaking

Mass of Hadrons mass comes from SSB, Interaction with condensate.

- From chiral $SU(3)$ with strange quark

$$M_{K^+}^2 = \frac{(m_u + m_s)}{f_\pi}\Delta' \qquad m_\eta^2 = \frac{(4m_s + m_u + m_d)}{3 f_\pi}\Delta'$$

$$M_{K^0}^2 = \frac{(m_d + m_s)}{f_\pi}\Delta' \qquad \frac{m_s}{(m_u + m_d)/2} = \frac{2 m_K^2}{m_\pi^2} \sim 27$$

$$\frac{m_u}{m_d} \approx 0.6$$

Bare masses, $m_u = 2.2\,\text{MeV}$, $m_d = 4.7\,\text{MeV}$, $m_s = 96\,\text{MeV}$

Observed masses increased slightly by EM affects.

Sujested exercises

Thomson: 11.1, 11.2, 11.7

Read 11.1 $\to$ 11.7

Weak interaction: The current-current model

Beta decay $n \to p e^- \bar{\nu}_e$

$$\tau = 880 \text{ s}$$

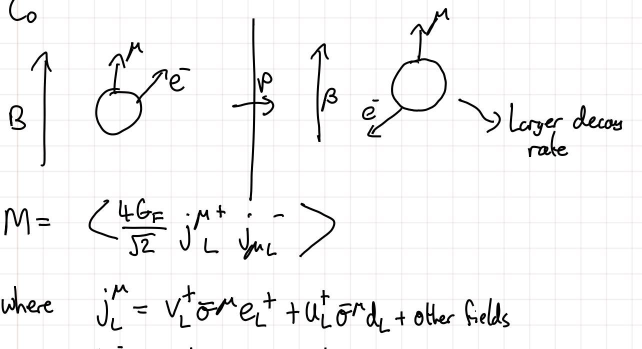
New interaction to change flavour

1950's: $K^0 \to \pi^+ \pi^-$ and $K^0 \to \pi^+ \pi^- \pi^0$

$P = +1$ $\qquad$ $P = -1$

P = Parity $\Rightarrow$ This is evidence that parity is not fundamental and that strangeness is not conserved

Weak interaction proposed as a new fundamental interaction and found to violate parity in β decay of polarized ^{60}Co



$$M = \left\langle \frac{4G_F}{\sqrt{2}} j_L^{\mu +} j_{\mu L}^{-} \right\rangle$$

where $j_L^{\mu +} = \nu_L^{\dagger} \bar{\sigma}^{\mu} e_L + u_L^{\dagger} \bar{\sigma}^{\mu} d_L$ + other fields

$$j_L^{\mu -} = e_L^{\dagger} \bar{\sigma}^{\mu} \nu_L + d_L^{\dagger} \bar{\sigma}^{\mu} u_L + \cdots$$

V-A vector axial theory

$$u_L^\dagger \bar{\sigma}^\mu d_L = \bar{u}\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)d$$

$$= \frac{1}{2}\left[\underbrace{\bar{u}\gamma^\mu d}_{V} - \underbrace{\bar{u}\gamma^\mu\gamma^5 d}_{A}\right]$$

Fermi's constant:

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}

V-A current violates P and C, preserves CP

- Full weak interactions (ch 19) vilates CP

Could consider other bilinears

S	$\bar{\Psi}\phi$
P	$\bar{\Psi}\gamma^5\phi$
V	$\bar{\Psi}\gamma^\mu\phi$
A	$\bar{\Psi}\gamma^\mu\gamma^5\phi$
T	$\bar{\Psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$

Only V and A mediated by spin 1 bosons

V & A seperatly preserve P

Any non-trivial linear combination violates P

V-A violates parity maximally

① Polarization of e^- in β decay

Preferentially L polarized $m_e \neq 0$

For e^- with $p^\mu = (E, 0, 0, p)^\mu$

Dirac eq $\begin{pmatrix} -m & E - p\sigma^3 \\ E + p\sigma^3 & -m \end{pmatrix} u(p) = 0$

Solutions:

$$u_R = \begin{pmatrix} \sqrt{E-p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E+p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \qquad u_L = \begin{pmatrix} \sqrt{E+p} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E-p} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$

e_L^+ reads top components of Dirac spinors

$$Pol = \frac{Prob(e_L) - Prob(e_R)}{Prob(e_L) + Prob(e_R)}$$

V-A → $Prob(e_L) \sim E+p$

$Prob(e_R) \sim E-p$

→

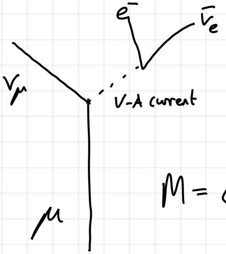
$$Pol = \frac{E+p - (E-p)}{E+p + E-p} = \frac{p}{E} = \frac{v}{c} < 1$$

observe $Pol \to 1$ for most relativistic interactions

② Muon decay

μ interacts with its own neutrino ν_μ

Decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



$$M = \langle \nu_\mu \bar{e} \bar{\nu}_e | \frac{4G_F}{\sqrt{2}} \nu_{\mu L}^\dagger \bar{\sigma}^\mu \mu_L e_L^\dagger \bar{\sigma}_\mu \nu_{e_L} | \mu \rangle$$

book has a typo?

$$= \frac{4G_F}{\sqrt{2}} u_L^\dagger(p_\nu) \bar{\sigma}^\mu u_L(p_\mu) u_L^\dagger(p_e) \bar{\sigma}_\mu u_L(p_{\bar{\nu}})$$

$$|u^\dagger \bar{\sigma} u u^\dagger \bar{\sigma} u|^2 = \frac{1}{2} 4 (2 p_e p_\nu)(2 p_\mu \cdot p_{\bar{\nu}})$$

Derivation – Fierz identity

– Two factors obtained in two different frames

Peskin 15.1

Fierz identity

3 body phase space

CM: frame for $e, \bar{\nu}_e, \nu_\mu$

$E_{cm}^2 = m_\mu^2 = s$

$x_e = \frac{2E_e}{m_\mu}$ $x_{\bar{\nu}} = \frac{2E}{m_\mu}$ $x_\nu = \frac{2E_\nu}{m_\mu}$

$x_e + x_{\bar{\nu}} + x_\nu = 2$

$0 < x_i < 1$

$$2\, p_\mu p_{\bar\nu} = 2 E_\mu E_\nu = m_\mu^2 X_{\bar\nu} \qquad \Big|\quad m_\nu = 0$$

$$2 p_e \cdot p_\nu = (p_e + p_\nu)^2 = (p_\nu - p_{\bar\nu})^2$$

$$= m_\mu^2 - m_\mu^2 X_{\bar\nu}$$

$$= (1 - X_{\bar\nu})\, m_\mu^2$$

$$\Rightarrow |u^\dagger \bar\sigma u\, \bar u \bar\sigma u|^2 = 2 m_\mu^4 (1 - X_{\bar\nu}) X_{\bar\nu}$$

Then we want to integrate this over phase space. Recall

$$\int d\Pi_3 = \frac{E_{cm}^2}{128\pi^3} \int dX_1\, dX_2$$

and
$$\Gamma = \frac{1}{2m_\mu} \frac{m_\mu^2}{128\pi^2} \int_0^1 dX_e \int_0^1 dX_{\bar\nu}\, 8 G_F^2\, 2 m_\mu^2 X_{\bar\nu}(1 - X_{\bar\nu})$$

$$= \frac{G_F^2 m_\mu^5}{16\pi^3} \int_0^1 dX_{\bar\nu} \left[\frac{X_e^2}{2} - \frac{X_e^3}{3} \right]$$

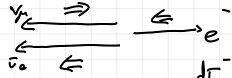
$$= \frac{G_F^2 m_\mu^2}{192\pi^3}$$

G_F = Universal strength of V-A interaction

Electron energy distribution

Can use the experiment $\tau \sim 2\mu s$ to measure $G_F \simeq 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2}$

- when $X_e = 1$



e^- emitted in a direction opposit spin of initial μ

- Predict and observe angular distribution

$$\frac{d\Gamma}{d\cos\theta} \sim (1 - \cos\theta)$$

0 at $\theta = 0$
max at $\theta = \pi$

$|D_\mu \varphi|^2$

$$\frac{1}{2}v^2\left(\frac{1}{2}\right)^2\left[g^2\left((A^1_\mu)^2+(A^2_a)^2\right)+\left(-gA^3_\mu+g'B_\mu\right)\right]$$

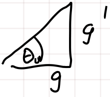
Mass terms $\left(\frac{gv}{2}\right)^2 W^+_\mu W^{-\mu} \equiv m_\omega^2 W^+_\mu W^{-\mu}$

$$\frac{1}{2}v^2\left(-gA^3+g'B\right)^2 \to \frac{1}{2}M_Z^2\left(Z_\mu\right)^2$$

Define the <u>weak mixing angle</u> θ_ω

$\tan\theta_\omega = \frac{g'}{g}$ so that

$$c_\omega = \cos\theta_\omega = \frac{g}{\sqrt{g^2+(g')^2}}, \quad s_\omega = \sin\theta_\omega = \frac{g'}{\sqrt{g^2+(g')^2}}$$



Define $Z_\mu = c_\omega A^3_\mu - s_\omega B_\mu$

$A_\mu = s_\omega A^3_\mu + c_\omega B_\mu$ photon

Then: $m_\omega = \frac{gv}{2}$ $\quad m_Z = \sqrt{g^2+(g')^2}\,\frac{v}{2}$ $\quad m_A = 0$

Unified electroweak model

θ_W satifies $\quad \frac{M_W}{M_Z} = c_W$

Couplings of W and Z bosons (and γ)

$$A_\mu^3 = c_W Z_\mu + s_W A_\mu$$
$$B_\mu = -s_W Z_\mu + c_W A_\mu$$

quark or lepton

$$D_\mu \Psi = \left(\partial_\mu - ig A_\mu^a I^a - ig' B_\mu Y\right)\Psi$$

$$= \left[\partial_\mu - \frac{ig}{\sqrt{2}}\left(W_\mu^+ \sigma^+ + W_\mu^- \sigma^-\right) - ig\left(c_W Z_\mu + s_W A_\mu\right) I^3 \right.$$
$$\left. - ig'\left(-s_W Z_\mu + c_W A_\mu\right) Y\right]\Psi$$

$$= \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ \sigma^+ + W_\mu^- \sigma^-\right) - ieA_\mu Q - i\frac{g}{c_W} Z_\mu Q_Z\right]\Psi$$

where $e = g s_W = g' c_W = \frac{gg'}{\sqrt{g^2 + (g')^2}}$

$Q = I^3 + Y$ $\qquad \Rightarrow Q_Z = c_W^2 I^3 - s_W^2 Y$

$\frac{Q_Z}{c_W} = c_W I^3 - \frac{g'}{g} s_W Y$ $\qquad = \boxed{I^3 - s_W^2 Q}$

Identify I^3 and Y quantum numbers of quarks and leptons.

- Ignore all masses so that L, R are indep
- Crucial L couples to W bosons, R dont (V-A)

L are in SU(2) doublet, $I = \frac{1}{2}$

R are in SU(2) singlets $I = 0$

Match electric charges to values of Y

	Q	I^3	$Y = Q - I^3$
u_L	$2/3$	$1/2$	$1/6$
d_L	$-1/3$	$-1/2$	$1/6$
ν_{e_L}	0	$1/2$	$-1/2$
e^-_L	-1	$-1/2$	$-1/2$
u_R	$2/3$	0	$2/3$
d_R	$-1/3$	0	$-1/3$
ν_{eR}	0	0	0
e^-_R	1	0	-1

The representations are generations

$$\begin{pmatrix}\nu_{eL}\\ e^-_L\end{pmatrix} \quad e^-_R \quad \begin{pmatrix}u_L\\ d_L\end{pmatrix} \quad u_R \quad d_R$$

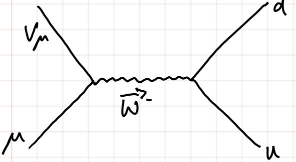
$$\begin{pmatrix}\nu_\mu\\ \mu^-_L\end{pmatrix} \quad \mu^-_R \quad \begin{pmatrix}c_L\\ s_L\end{pmatrix} \quad c_R \quad s_R$$

$$\begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix} \quad \tau^-_R \quad \begin{pmatrix}b_L\\ T_L\end{pmatrix} \quad b_R \quad T_R$$

All rows have some quantum numbers

The Neutral Current weak interaction

V-A current is charged

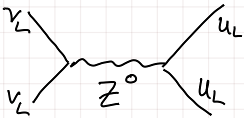


$$\frac{g^2}{2} j^{\mu -}\frac{1}{q^2-M_W^2} j^+_\mu$$

For $M_W^2 \gg q^2$ $\quad \frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2}$

g of SU(2)

New neutral current interaction



$$\sim \frac{1}{2}\frac{g^2}{c_W^2}\left(j^\mu_L - s_W^2\, j^\mu_Q\right)\frac{1}{q^2-M_Z^2}\left(j^{\mu 3}_L - s_W^2\, j_{\mu Q}\right)$$

electric charge current

For $q^2 << M_\omega^2, M_Z^2$

$$M = \left\langle \frac{4G_F}{\sqrt{2}} \left(j_L^{\mu +} j_L^{\mu -} + \left(j_L^{\mu 3} - s_\omega^2 j_Q^\mu \right)^2 \right) \right\rangle$$

Recall DIνS:

$$\frac{d^2\sigma}{dx\,dy}(\nu_L p \to \mu^- X) = \frac{G_F^2}{\pi} s \left[x f_d(x) + x f_{\bar{u}}(x)(1-y^2)^2 \right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu}_R p \to \mu^+ X) = \frac{G_F^2}{\pi} s \left[x f_u(x)(1-y)^2 + x f_{\bar{d}}(x) \right]$$

Now include Z charges and R quarks

$$Q_Z = I^3 - s_\omega^2 Q$$

$$Q_{u_L} = \frac{1}{2} - \frac{2}{3} s_\omega^2 \qquad Q_{u_R} = -\frac{2}{3} s_\omega^2$$

$$Q_{d_L} = -\frac{1}{2} + \frac{1}{3} s_\omega^2 \qquad Q_{d_R} = \frac{1}{3} s_\omega^2$$

"Adapting this formula"

$$\frac{d^2\sigma}{dxdy}(\nu_L p \to \nu X)\Big|_q = \frac{G_F^2 s}{\pi}\Big[x f_u(x)\Big(\underbrace{\big(\tfrac{1}{2}-\tfrac{2}{3}s_\omega^2\big)^2}_{u_L} + \underbrace{\big(-\tfrac{2}{3}s_\omega^2\big)^2(1-y)^2}_{u_R}\Big)$$

$$+ x f_d(x)\Big(\underbrace{\big(-\tfrac{1}{2}+\tfrac{1}{2}s_\omega^2\big)^2}_{d_L} + \underbrace{\big(\tfrac{1}{3}s_\omega^2\big)(1-y^2)}_{d_R}\Big)\Big]$$

$$\frac{d^2\sigma}{dxdy}(\nu_L p \to \nu X)\Big|_{\bar q} = \frac{G_F^2 s}{\pi}\Big[x f_{\bar u}(x)\Big(\underbrace{\big(\tfrac{1}{2}-\tfrac{2}{3}s_\omega^2\big)^2(1-y)^2}_{\bar u_L} + \underbrace{\big(-\tfrac{2}{3}s_\omega^2\big)^2}_{\bar u_R}\Big)$$

$$+ x f_d(x)\Big(\underbrace{\big(-\tfrac{1}{2}+\tfrac{1}{2}s_\omega^2\big)^2(1-y)^2}_{\bar d_L} + \underbrace{\big(\tfrac{1}{3}s_\omega^2\big)}_{\bar d_R}\Big)\Big]$$

- Nutrino experiments, use very massive targets

  Can assume $\#p \approx \#n$

  $\hookrightarrow \#u \approx \#d \implies f_q(x) = A\big(f_u(x) + f_d(x)\big)$

$$\frac{d^2\sigma}{dxdy}(\nu \overset{\text{heavy target?}}{A} \to \nu X) = \frac{G_F^2 s}{\pi}\Big[x f_q(x)\Big(\tfrac{1}{2}s_\omega^2\Big) + \tfrac{5}{9}s_\omega^4\big[1+(1-y)^2\big]N$$

$$+ x f_{\bar q}(x)\Big(\big(\tfrac{1}{2}-s_\omega^2\big)(1-y)^2 + \tfrac{5}{9}s_\omega^4\big[1+(1-y)^2\big]$$

For $\bar\nu$ $\quad 1 \longleftrightarrow (1-y)^2$

Measure Ratio charged current

$$r = \frac{\sigma(\bar{\nu}, CC)}{\sigma(\nu, CC)} = \left\langle \frac{x f_q (1-y)^2 + x f_{\bar{q}}(x)}{x f_q(x) + x f_{\bar{q}}(x)(1-y)^2} \right\rangle$$

- Measure ratios:

$$R^{\nu} = \frac{\sigma(\nu, NCC)}{\sigma(\nu, CC)} \quad R^{\bar{\nu}} = \frac{\sigma(\bar{\nu}, NC)}{\sigma(\bar{\nu}, CC)}$$

Values of R^{ν}, $R^{\bar{\nu}}$ lie on a curve

paramatrized by s_w^2, $s_w^2 \approx 0.23$

The W & Z bosons

W, Z observed directly in $p\bar{p}$ collisions Cern 1980's

Sujested exercises

Derive eq 17.4

$\Gamma(W \to \nu e)$ Exercise 18.1 (use references?)

W Bozon

$$W^+ \to \nu_e e^+$$

$$M(W^+ \to \nu_L e^+_R) = \frac{g}{\sqrt{2}} u_L^\dagger(p_\nu)\bar{\sigma}^\mu v_L(p_e)\epsilon_{W\mu}$$

In CM frame: $M = \frac{g}{\sqrt{2}} 2\sqrt{2} E \epsilon_-^* \epsilon_W$

$$\Gamma(W^+ \to \nu_L e^+_R) = \frac{1}{2M_W}\frac{1}{8\pi}\frac{g^2}{2}M_W^2 \, 2 \cdot \frac{1}{3} = \frac{\alpha_W}{12} M_W$$

where $\alpha_W = \frac{g^2}{4\pi} = \frac{e^2}{4\pi S_W^2}$

Also: $\Gamma(W^+ \to \nu_\mu \mu^+) = \Gamma(W^+ \to \nu_\tau \tau^+) = \frac{\alpha_W}{12} M_W$

Quark decays similarly with colour factor 3.

correction $3 \to 3.1$

$$\Gamma(W^+ \to u\bar{d}) = \Gamma(W^+ \to \bar{c}s) = \frac{\alpha_W}{12} M_W \, 3 \cdot \left(1 + \frac{\alpha_s M_W}{\pi}\right)$$

Parameters in EW Theory

We can get M_W & M_Z from couplings and $s_W^2 \approx 0.23$

$$\alpha_W = \frac{g^2}{4\pi} = \frac{\alpha}{s_W^2} \qquad \alpha' = \frac{g'^2}{4\pi} = \frac{\alpha}{c_W^2}$$

- Take α at EW-scale $\alpha(M_Z) = \frac{1}{129}$ (bigger than $\frac{1}{137}$!)

$$\alpha_W = \frac{1}{29.8}, \quad \alpha' = \frac{1}{99.1}$$

$\Rightarrow$ Weak interaction is weak!

Weakness comes from G_F

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \qquad m_W = \frac{gv}{2} \qquad G_F = \frac{1}{\sqrt{2} v^2}$$

$v = 246$ GeV $\gg$ 1 GeV proton mass

$M_W = 80.2$ GeV $\qquad M_Z = 91.5$ GeV

$$\Gamma_W = \frac{\alpha_W}{12} M_W \left[3 + 2\,(3 \cdot 1) \right] = 2.1\ \text{GeV}$$

3 channels leptons

2 quarks (Top too heavy)

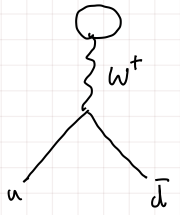
colour + correction

$$BR(e\nu_e) = BR(\mu\nu_\mu) = BR(\tau\nu_\tau) = 11\%$$

$$BR(u\bar{d}) = BR(c\bar{s}) = 34\%$$

W production in pp collisions:

Drell-Yan Process



$$\sigma(u\bar{d} \to W^+) = \frac{1}{2\hat{s}} \int d\Pi\, |M|^2$$

$$\int d\Pi_1 = \int \frac{d^3 p_W}{(2\pi)^3 2E_W} (2\pi)^4 \delta^{(4)}(p_u + p_d - p_W)$$

$$= \int \frac{d^4 p_W}{(2\pi)^4} (2\pi)\, \delta(p_W^2 - m_W^2)\, (2\pi)^4 \delta^{(4)}(p_u + p_d - p_W)$$

$$= 2\pi \, \delta(\hat{s} - M_\omega^2)$$

Average over initial spins & colours, sum over final polarigation states

$$\frac{1}{2\cdot 2} \frac{1}{3\cdot 3} \sum_{\substack{\text{colour} \\ \text{spin}}} |M|^2 = \frac{g^2}{12} M_\omega^2$$

→ Same element as decay

$$\sigma(u\bar{d} \to W^+) = \frac{\pi^2 \alpha_\omega}{3} \delta(\hat{s} - M_\omega^2)$$

$$\sigma(p\bar{p} \to W^+) = \int dx \, f_u(x_1) \int dx_2 \, f_{\bar{d}}(x_2)$$

$$\times \Big[\sigma(u(x_1, P_1)\, \bar{d}(x_2, P_2) \to W^+) + (1 \leftrightarrow 2) \Big] \overbrace{dx_1 dx_2 \, \delta(\hat{s} - M_\omega^2)}^{\frac{dY}{s}}$$

+ all other $q, \bar{q}$

$x_1 = x_1(M_\omega, Y)$
$x_2 = x_2(M_\omega, Y)$

$$\frac{d\sigma}{dY}(pp \to W^+ X) = \frac{\pi^2 \alpha_\omega}{3s} \Big[f_u(x_1) f_{\bar{d}}(x_2) + f_{\bar{d}}(x_1) f_u(x_2) \Big]$$

→ anti quark pdf

Z Boson

$$\Gamma(Z \to f\bar{f}) = \dots = \frac{1}{2M_Z}\frac{1}{8\pi}\frac{g^2}{c_\omega^2} M_Z^2 \frac{2}{3} Q_Z^2$$

$$= \frac{\alpha_\omega}{6c_\omega^2} M_Z Q_Z^2$$

L Prefered over R

Species	Q_{ZL}	Q_{ZR}	S_f	A_f
ν	$1/2$	—	0.256	1.00
e	$-\frac{1}{2} + s_\omega^2$	s_ω^2	0.126	0.15
u	$\frac{1}{2} - \frac{2}{3}s_\omega^2$	$-\frac{2}{3}s_\omega^2$	0.144	0.67
d	$-\frac{1}{2} + \frac{1}{3}s_\omega^2$	$\frac{1}{3}s_\omega^2$	0.185	0.74

- If we define $S_f = Q_{ZL}^2 + Q_{ZR}^2$

$$A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{Q_{ZR}^2 + Q_{ZL}^2}$$

then total decay rate $\sim S_f$
while A_f measurs L-R anti-symmetry
i.e polarization of emmited quarks / leptons

Total width $\Gamma_Z = 2.49$ GeV

$BR(u\bar{u}) = 11.9\%$ $BR(d\bar{d}) = 15.3\%$

$BR(\nu_e \bar{\nu}_e) = 6.7\%$ $BR(e^+e^-) = 3.3\%$

+ other generations

- Precission tests of EW model:

Measure S_f and A_f in $e^+e^- \to Z \to \begin{cases} \mu^+\mu^- \\ q\bar{q} \end{cases}$

S_f from total width

- Line shape of Z is sensitive to number of light neutrinos

$$n_\nu = 2.9840 \pm 0.0082$$

A_f from measurements sensitive to polarization
Find very different values for leptons, u quarks, d quarks

Quark Mixing Angles

How can s change to u?

Don't want a new term for $s \to d$

in $K^0 \to \pi^- e^+ \nu$ $\quad \Lambda^0 \to p e^- \bar{\nu}$

$d\bar{s}$ $\quad d\bar{u}$ $\quad uds$ $\quad uud$

Want term like $u_L^\dagger \bar{\sigma}^\mu s_L$ changing term

The Cabibbo mixing angle

relitive strength of $s \leftrightarrow u$ compared to leptons

$$j^{\mu +} = \nu^\dagger \bar{\sigma} \mu_L + \dots + V_{us} u_L^\dagger \bar{\sigma}^\mu s_L$$
$$+ V_{ud} u_L^\dagger \bar{\sigma}^\mu d_L$$

$K \to \pi l \nu \quad \mathcal{M} \sim \langle \pi | \bar{u} \gamma^\mu (1-\gamma^5) s | K \rangle V_{us}$

Measure $V_{us} = 0.2249 \pm 0.001$

Precise measurments from p decay $\to V_{ud} = 0.97425 \pm 0.0002$

Notice!: $|V_{ud}|^2 + |V_{us}|^2 = 1$

i.e. $V_{ud} = \cos\theta_c \quad V_{us} = \sin\theta_c$ for a Cabibbo angle θ_c

SU(2) couples quarks and Leptons at same strength but mixes the quarks.

Quark and Lepton masses:

$$\Delta \mathcal{L} = -m_f \left(f_R^\dagger f_L + f_L^\dagger f_R \right)$$

Violate SU(2) gauge symmetry

We have SSB by the Higgs field which aquires a Vacuum Expectation Value (VEV)

Can add terms:

$$\Delta \mathcal{L} = -y_e L_a^\dagger \varphi_a e_R - y_d Q_a^\dagger \varphi_a d_R - y_u Q_a^\dagger \epsilon_{ab} \varphi_b^* u_R + h.c$$

where a, b = 1, 2 $\quad L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

(

y_e, y_d, y_u are Yukawa couplings

Each term is Isospin invariant and has $Y=0$

Since $I(\varphi) = \frac{1}{2}$ $Y(\varphi) = \frac{1}{2}$, Replace Higgs field by its VEV

$\varphi \to \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

$\to$

$$\Delta L = -\frac{y_e v}{\sqrt{2}} e_L^+ e_R - \frac{y_d v}{\sqrt{2}} d_L^+ d_R - \frac{y_u v}{\sqrt{2}} u_L^+ u_R + h.c$$

Mass terms for e, d, u $\quad m_f = y_f \frac{v}{\sqrt{2}}$

- Most general couplings compatable with $SU(2) \times U(1)$ symmetry. Gauge couplings must be identical but not Yukawa couplings.

$$\Delta L = -y_e^{ij} L_a^{+i} \varphi_a e_R^j + y_d^{ij} Q_a^{+i} \varphi_a d_R^j$$
$$- y_u^{ij} Q_a^{+i} \epsilon_{ab} \varphi_b^* u_R^j + h.c.$$

$a, b = 1, 2 \qquad i, j = 1, 2, 3$ generations

$y_f^{ij} \to 3 \times 3$ complex valued Yukawa Matricies

Since both $y_f y^+$ and $y_f^+ y_f$ are Hermitian with the same positive eigenvalues

$\exists$ Unitary $U_L^{(f)}, U_R^{(f)}$ st: $yy^+ = U_L Y U_L^+$

and $y^\dagger y = U_R Y U_R^\dagger$ for some:

$$Y = \begin{pmatrix} Y_1 & & \\ & Y_2 & \\ & & Y_3 \end{pmatrix} \qquad y_i > 0$$

Thus $y = U_L \sqrt{Y} U_R^\dagger$. For Leptons, move U onto the leptons

$$e_R^i \to U^{(e)}_{R\,ij} e_R^j \qquad L^i \to U^{(e)}_{L\,ij} L^j$$

New fields are mass eigen states, no effect on kinetic terms.

Lepton number is conserved in each generation

Quarks similar, find mass eigenstates

U matricies cancle from kinetic terms & Z couplings.

But for W coupling

$$u_L^\dagger (i\bar{\sigma}^a) d_L \to u_L^\dagger U_L^{(u)\dagger} (i\bar{\sigma}^a) U_L^{(d)} d_L$$

$$= u_L^\dagger (i\bar{\sigma}^a) V_{CKM} d_L$$

where $V_{CKM} = U_L^{(u)\dagger} U_L^{(d)}$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

generation changing interactions

Cabibbo-Kobayashi Maskawa

Lets count dof

3x3 complex matrix $\to$ 18 real parameters

Unitary $\Rightarrow$ 9 constraints $\to$ 9

If real matrix, 3 Euler angles

9 = 3 Euler angles + 6 phases

Quark phases arbitrary, incl 5 relative phases

$\Rightarrow$ 4 dof = 3 euler angles + 1 phase

$\hookrightarrow$ CP violation

The standard Model

Assume $SU(3) \times SU(2) \times U(1)$ guage symmetry

$\downarrow$ colour $\quad \downarrow$ Isospin $\quad \downarrow$ hypercharge

Spontenous breaking of $SU(2) \times U(1)$

$$\mathcal{L} = \frac{1}{4}\left(\sum_a (F^a_{\mu\nu})^2 + m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right.$$

$$+ \sum_f \bar{\Psi}_f (i\gamma^\mu D_\mu - m_f)\Psi_f + \frac{1}{2}(\partial_\mu h)^2 - V(h)$$

$$V(\varphi) = -\mu^2|\varphi|^2 + \lambda|\varphi^2|^2$$

$$D_{\mu f} = \partial_\mu - ieQ_f A_\mu - \frac{ig}{c_W} Q_{Zf} Z_\mu - \frac{ig}{\sqrt{2}} W^\pm_\mu \sigma^\pm - ig_s A^a_\mu t^a$$

- Interactions of $h(x)$ generated by $V \to V + h(x)$ in mass terms.

Consequences:

- Conservation of quark/Bayron number and lepton number
- P, C, T conserved except through W, Z couplings

to fermions conserved in strong interactions and EM interactions

- Fermion number of each type is conserved except through W couplings.

- W, Z couplings violate C and P maximally through (V-A) current but preserve CP if real valued.

CP is violated by phase of CKM matrix.

Parameters of Standard Model:

19 (assuming massless neutrinos)

4 CKM

6 q masses

3 lepton masses

3 gauge couplings

1 Higgs VEV

1 Higgs mass

1 θ_{QCD} for possible CP violating term $\sim g^2 \theta_{QCD} F_{\mu\nu} \tilde{F}^{\mu\nu}$

3 Euler angles for V_{CKM}

Found V_{us} from K decay

Get 2 more Euler angles from B meson decay

B^+ $\quad u\bar{b}$ $\qquad$ b quark decay ~ μ decay

B^0 $\quad d\bar{b}$

Recall:

$$\Gamma_\mu = \frac{G_F^2 M_\mu^5}{192\pi^3}$$

This is the same for b quark as $b \to c f\bar{f}$ like $\mu \to \nu_\mu e \bar{e}$

$$\Gamma(b \to c f\bar{f}) = |V_{cb}|^2 \frac{G_F^2 M_b^5}{192\pi^3} (3 + 2 \cdot 3)$$

$e\bar{\nu}_e$, $\mu\bar{\nu}_\mu$, $\tau\bar{\nu}_\tau$ $\quad$ $u\bar{d}$, $c\bar{s}$ $\quad$ colour

$$= 4 \times 10^{-10}\,\text{GeV}\, |V_{cb}|^2$$

$$\tau(b) = 1.7 \times 10^{-15}\,\text{s}\, |V_{cb}|^{-2}$$

$$\tau(B) = 1.5 \times 10^{-12}\,\text{s}$$

Also find

$V_{cb} = 4 \times 10^{-2}$

$V_{ub} = 4 \times 10^{-3}$

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda = 0.225$

$A = 0.81$

$|\rho - i\eta| = 0.37$

Sujesled exercises:

Thomson

14.1 – 14.3, 14.10

Flavour changing Nuetral current (FCNC)

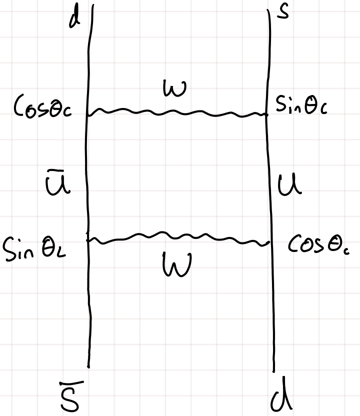
No FCNC terms in Lagrangian

but FC effects for quarks are generated by charged currents

Ex: $K^0 \to \bar{K}^0$

$\bar{s}d \qquad \bar{d}s$

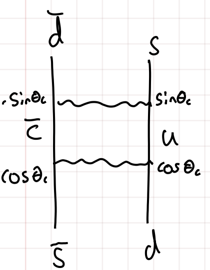
Unitarity of V_{CKM} $\to$ suppression of FCNC

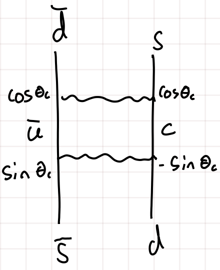


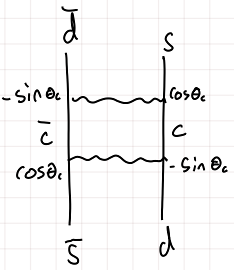
$d \to u : V_{ud}$

$u \to s \quad V_{us}^{*} = V_{us}$

suppresion by including processes with c







- Cancellation when $q^2 \gg m_c^2$ giving $m_W^2 \to m_c^2$ in M agreeing with experiment?
(Constraint in BSM)

CP violation

The $K_0 - \bar{K}_0$ system

$K^0, \bar{K}^0$ are lightest hadrons with strangeness

so stable wrt strong interaction

Nuetral K is a 2-state quantum system.
physical states that are observed are

K_S^0 and K_L^0

- Evolution by $e^{-iM\tau}$, where M is a mass matrix. If CP was conserved → This M would be symmetric.
(particle-anti-particle symmetry)

$$M = \begin{pmatrix} \bar{m} - \frac{i}{2}\bar{\Gamma} & \delta m - \frac{i}{2}\delta\Gamma \\ \delta m - \frac{i}{2}\delta\Gamma & \bar{m} - \frac{i}{2}\bar{\Gamma} \end{pmatrix}$$

$P|K^0\rangle = -|K^0\rangle \qquad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$

$C|K^0\rangle = |\bar{K}^0\rangle \qquad C|\bar{K}^0\rangle = |K^0\rangle$

$\Rightarrow K^0, \bar{K}^0$ are not in CP eigenstates

S for short

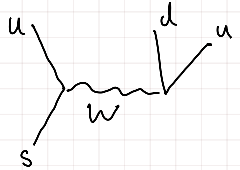
$CP = +1 \quad |K_S^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$

L for long

$CP = -1 \quad |K_L^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$

$M_S = \bar{m} - \delta m - \frac{i}{2}(\bar{\Gamma} - \delta\Gamma)$

$M_L = \bar{m} + \delta m + \frac{i}{2}(\bar{\Gamma} + \delta\Gamma)$



$K^0, \bar{K}^0 \to \pi, \pi$

$K^0 \, \bar{K}^0 \to \pi \, \pi \, \pi$

$CP \, |\pi\pi\rangle = |\pi\pi\rangle$

$CP \, |\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$

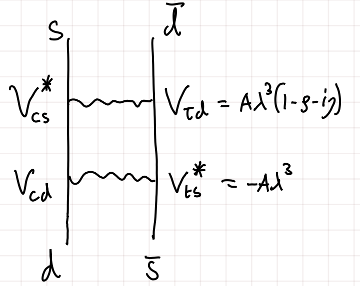
$K_S \to |\pi\pi\rangle \qquad K_L \to |\pi\pi\pi\rangle$

$\tau_S \ll \tau_L$

Nuetral kaons are produced as K^0, $\bar{K}^0$ but propogate (& decay) as K_S, K_L

CP is <u>not</u> conserved as there is some:

$$K_L^0 \to \pi\pi$$



$$M = \begin{pmatrix} \bar{m} - \frac{i}{2}\bar{\Gamma} & \delta m(1+i\xi) - \frac{i}{2}\delta\Gamma \\ \delta m(1-i\xi) + \frac{i}{2}\delta\Gamma & \bar{m} - \frac{i}{2}\bar{\Gamma} \end{pmatrix}$$

M eigenstates $|K_S^0\rangle = \frac{1}{\sqrt{2}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right]$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right]$$

where $\epsilon = \dfrac{\frac{i}{2}\zeta\,\delta m}{\delta m - \frac{i}{2}\delta\Gamma}$

$$\delta m = \tfrac{1}{2}(m_L - m_S)$$
$$\Gamma_S = \bar{\Gamma} - \delta\Gamma$$
$$\Gamma_L = \bar{\Gamma} + \delta\Gamma$$

Since $\Gamma_L << \Gamma_S$

$\Rightarrow \delta\Gamma \approx -\frac{1}{2}\Gamma_S$

so $\epsilon = \dfrac{2i\,\zeta(m_L - m_S)/2}{m_L - m_S + i\frac{\Gamma_S}{2}}$

- It turns out that $m_L - m_S \sim \frac{\Gamma_S}{2}$, phase of $\epsilon \sim 44°$

M eigenstates in terms of CP eigenstates

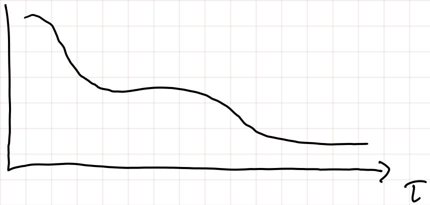
$$|K_S^0\rangle = |K_+^0\rangle + \epsilon|K_-^0\rangle$$
$$|K_L^0\rangle = |K_-^0\rangle + \epsilon|K_+^0\rangle$$

$$\Rightarrow \frac{\Gamma(K_L^0 \to \pi\pi)}{\Gamma(K_S^0 \to \pi\pi)} = |\epsilon|^2$$

Can measure and find

$|\epsilon| = 2.23 \times 10^{-3}$

$\epsilon \neq 0$ results in slow oscillation over exp decay



Legth of scale of oscillation

~ tens of meters (good legth for experiment)

Skipping sec 19.2

- CP violation in $B^0 - \bar{B}^0$. CP violating phase is associated to heavy quarks so a B meson -> larger order affect

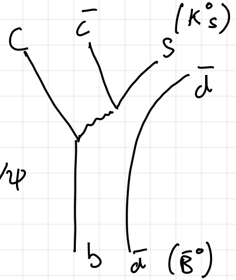
B^0 $\bar{B}^0$ have many decay modes relativly few are common to both

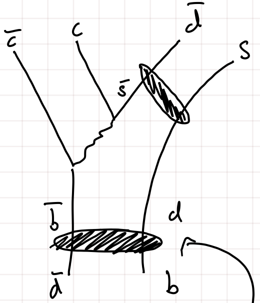
Sujested Ex:

Thomson 13.5

Peskin 21.1 21.3

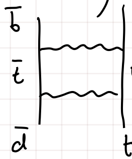
CP Violation in $B^0 - \bar{B}^0$





dominating

$V_{td} = A\lambda^3(1-\rho-i\eta) = Ce^{-i\beta}$



$\sim V_{tb} V_{td}^* V_{td}^* V_{tb}$

Relative phase is $-e^{2i\beta}$

$$M = \begin{pmatrix} m - i\frac{\Gamma}{2} & -\frac{1}{2}e^{2i\beta}\delta m \\ -\frac{1}{2}e^{-2i\beta}\delta m & \bar{m} - i\frac{\Gamma}{2} \end{pmatrix}$$

Life time of $B^0/\bar{B}^0$

$\tau = 1.52 \times 10^{-12}\,s$

Eigenstates of M:

$$|B^0_L\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle + e^{-2i\beta}|\bar{B}^0\rangle\right) \qquad \bar{m} - \frac{\delta m}{2} - \frac{i\Gamma}{2} = M_H$$

$$|B_H^0\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle - e^{-2i\beta}|\bar{B}^0\rangle\right) \qquad \bar{m} + \frac{\delta m}{2} - i\frac{\Gamma}{2} = m_L$$

$$m_H - m_L = \delta m = 3.3 \times 10^{-13}\ \text{GeV}$$

observed accidentally close to Γ

$$|B^0(\tau)\rangle \doteq e^{-i\bar{m}\tau - \frac{\Gamma\tau}{2}}\left(|B^0\rangle \cos\frac{\delta m\,\tau}{2} + i|\bar{B}_0\rangle e^{-2i\beta}\sin\frac{\delta m\,\tau}{2}\right)$$

$$|\bar{B}^0(\tau)\rangle \doteq e^{-i\bar{m}\tau - \frac{\Gamma\tau}{2}}\left(|\bar{B}^0\rangle \cos\frac{\delta m\,\tau}{2} + i|B_0\rangle e^{+2i\beta}\sin\frac{\delta m\,\tau}{2}\right)$$

$$\Gamma(B^0(\tau) \to J/\psi\, K_s^0) \sim e^{-\Gamma\tau}(1 - \sin\delta m\tau\ \sin 2\beta)$$

$$\Gamma(\bar{B}^0(\tau) \to J/\psi\, K_s^0) \sim e^{-\Gamma\tau}(1 + \sin\delta m\tau\ \sin 2\beta)$$

Measure symmetry:

$$\frac{\Gamma(\bar{B}^0 \to J/\psi\, K_s^0) - \Gamma(B^0 \to J/\psi\, K_s^0)}{\Gamma(\bar{B}^0 \to J/\psi\, K_s^0) + \Gamma(B^0 \to J/\psi\, K_s^0)}$$

$$= \sin\delta m\tau\ \sin 2\beta$$

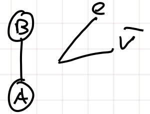
$\sin 2\beta$ controls the amplitude of the oscillation.

LHCB focuses on CP violation and precision B-meson studies.

Sensitive to models of new physics

Neutrino Masses & mixing

Bound on ν masses from β decay



$\Gamma(A \to B\, e\, \bar{\nu})$

$$= \frac{1}{2m_A} \int \frac{d^3p_B\, d^3p_e\, d^3p_\nu}{(2\pi)^9\, 2E_B\, 2E_e\, 2E_\nu} (2\pi)^4 \delta^{(4)}(p_A - p_B - p_e - p_\nu)\, |M|^2$$

B takes up recoil momentum, approx $\hat{p}_e$ & $\hat{p}_\nu$ are uncorrelated

CM frame:

$$E_e + E_\nu = m(A) - m(B) = \Delta m_{AB}$$

solve $\delta^{(4)}$ for $\vec{P}_B \overset{?}{=} 0$

$$\Gamma = \frac{1}{2M_A}\frac{1}{(2\pi)^5 2M_B}\int (4\pi^2)\frac{dP_e\, P_e^2}{2E_e}\int \frac{dP_\nu\, P_\nu^2}{2E_\nu}\delta(\Delta M_{AB} - E_B - E_\nu)|M|^2$$

$$dP_e P_e = dE_e\, E_e \qquad dP_\nu P_\nu = dE_\nu\, E_\nu$$

If $m_\nu = 0 \Rightarrow P_\nu = E_\nu = \Delta m_{AB} - E_e$

Approx: $|M|^2 \sim$ constant

dominated by higher E_e

Then $\Gamma \sim \int_{m_e}^{\Delta m_{AB}} dE_e\, E_e\, (\Delta m_{AB} - E_e)^2$

Kure plot

number of events?

Find bound $m_\nu < 2.05\, eV$

Will argue that ν_μ, ν_τ are also close.

More constraints from structure of early universe

$$\sum_i m_{\nu_i} < 0.23\, eV$$

Neutrino Masses (theory)

① Simplest mechanisims: Yukawa couplings

$$\Delta \mathcal{L} = - y_\nu L^{+i}_a \epsilon_{ab} \varphi^*_b \nu^j_R + h.c$$

Since the masses are tiny, prefer <u>weak</u> eigenstates over mass eigenstates

$$L^i \to U^{(e)}_{L_{ij}} L^j$$

$$y_\nu \to y'_\nu = U^{(e)\dagger}_L y_\nu$$

ν_e = neutrino state produced with e in weak interaction

Diagonlaize y_ν by $y'_\nu = U^{(\nu)}_L Y_\nu U^{(\nu)}_R$

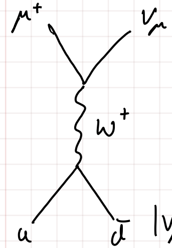
Y_ν real & diagonal

$U_R^{(\nu)}$ can be transformed away, $U_L^{(\nu)}$ btw flavour & mass eigenstates

$\nu_1\ \nu_2\ \nu_3$ label mass eigenstates

$U_L^{(\nu)}$ called PMNS matrix or V_{PMNS}

Consider π^+ decay, $\pi^+ \to \mu^+ \nu_\mu$



3 mass components of ν_μ have momenta $p_i = E - \frac{m_i^2}{2E} + \dots$

$$|\nu_\mu\rangle = \sum_{i=1,2,3} V_{\mu i} e^{i\left(E - \frac{m_i^2}{2E}\right)x} |\nu_i\rangle$$

$$\text{Prob}(\nu_\mu \to \nu_\mu) = \left| \sum_i V_{\mu i} V_{\mu i}^* e^{-i\left(\frac{m_i^2}{2E}\right)x} \right|^2$$

If only two flavours:

$$V = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad \text{Prob}(\nu_\mu \to \nu_\mu) = 1 - \sin^2\theta \sin^2\left[\frac{(m_2^2 - m_1^2)}{4E} x\right]$$

Oscillation b/w flavour eigenstates of length $L = \frac{4\pi E}{m_2^2 - m_1^2} \sim km$

Quarks 1. observe mass eigenstates in massive Hadrons

2 mixing in weak interactions

Neutrinos: 1: observe flavour eigenstates through weak decay

2: time evolution is for mass eigenstates $\rightarrow$ oscillations

---

Alternate mechanisim: Majorana term

$$\Delta L = -\frac{1}{2} M_{ij} (L^i_{a\alpha} \epsilon_{ab} \varphi^*_b) (L^j_{c\beta} \epsilon_{cb} \varphi^*_d) \epsilon_{\alpha\beta}$$

From $m_\nu \neq 0$ get $M_{ij} \frac{v^2}{2}$

ΔL can be generated by very massive $\nu_R \rightarrow$ small values for m_{ν_L} by seesaw mechanisim

- experimentally disfavored by absense of neutrinoless doble beta decay

$Xe^{136} \rightarrow Ba^{136} + e^- \bar{\nu}_e e^- \bar{\nu}_e$ vs:

$Xe^{136} \rightarrow Ba^{136} + e^- e^-$

suppresed by small masses but should be non-zero (Not detected!)

Evidence of Neutrino masses

Oscillations $\rightarrow$ 1. Appearence of wrong flavour charged leptons

Dissapearence of right flavour charged lepton $\nu_\mu \rightarrow$ less μ

Types of experiments:

- ν flux in cosmic rays, through earth or not
- ν flux from sun
- ν flux from Nuclear reactors

Long base line experiments from accellerators.

General agreement $m_2^2 - m_1^2 \simeq 7.6 \times 10^{-5}\,eV^2$

$|m_3^2 - m_2^2| \simeq 2.3 \times 10^{-3}\,eV^2$

$m_\nu \sim 10^{-2}\,eV$ PMNS $\theta_{12} \approx 35^\circ$

$\theta_{23} \simeq 45^\circ$ $\theta_{13} \simeq 10^\circ$ phase unknown

The Higgs Boson

Consider the top quark decay

$$\frac{g}{\sqrt{2}} W_\mu^- b_L^\dagger \bar{\sigma}^\mu t_L$$

$t \to bW$

$$m_t = 173 \text{ GeV}$$
$$m_b = 4 \text{ GeV}$$
$$m_W = 80 \text{ GeV}$$

$$\Gamma_t = \frac{g^2}{64\pi} \frac{m_t^3}{m_W^2} \left(1 + 2\frac{m_W^2}{m_t^2}\right)\left(1 - \frac{m_W^2}{m_t^2}\right)^2$$

$$\sim \alpha_W m_t \frac{m_t^2}{m_W^2}$$

compare to rate with unbroken $SU(2) \times U(1)$ symmetry

$$\Gamma_t = \frac{y_t^2}{32\pi} m_t = \frac{g^2}{64} m_t \left(\frac{m_t^2}{m_W^2}\right)$$

In $\mathcal{L}$ expand $v \to v + h(x)$

$$\Delta L = -\sum_f m_f \bar{f} f \frac{h(x)}{v} + 2 m_w^2 W_\mu^+ W^{-\mu}$$

Exercise derive this

$$+ m_Z^2 Z_\mu Z^\mu \frac{h(x)}{v} + O(h^2)$$

$$- \frac{1}{2} m_h^2 h(x)^2 - \frac{1}{2} m_h^2 \frac{h^3(x)}{v} + O(h^4)$$

- All term conserve P & C so Higgs is Spin 0 with $P = +1$. All couplings suppressed by $v = 246$ GeV

Ehanced by large mass for W, Z, t

Higgs decay:

$h \to W^+ W^-$, $h \to ZZ$ allowed for heavy higgs but not observed

$\Rightarrow M_h \leq 160$ GeV

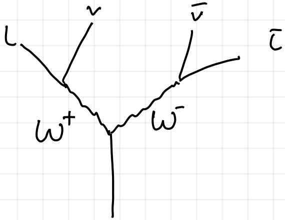
Dominant mode: $h \to b\bar{b}$

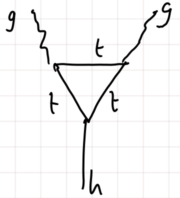
$\Gamma(h \to b\bar{b}) \sim 2\,\mathrm{MeV}$

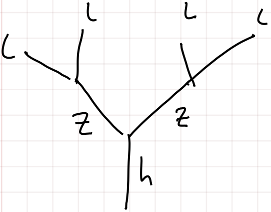
very narrow, hard to measure directly

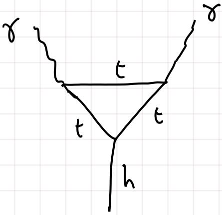
h can decay to offshell W, Z, t

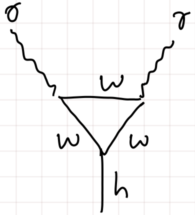
Also $h \to gg$ or $\gamma\gamma$ mediated by t of W loop











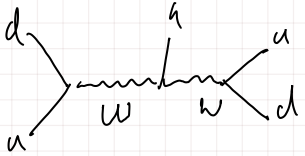
Higgs Production (Hadron colliders)

Reverse decay process

$b\bar{b} \to h$ suppressed by proton pdf's

$gg \to h$ large Gluon pdf. gluon-gluon fusion

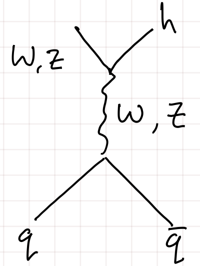
- Vector boson fusion (VBF)



with two forward jets

associated production

associated W/Z production



Decays can be hard to detect in large QCD background

$gg \to h \to b\bar{b}$ $\quad gg \to b\bar{b}$ ← 10^6 + times greater

Similar for hadronic decays from

$h \to WW^*$ $\quad h \to ZZ^*$

Concentrate on modes with visible lepton / photon final states

$h \to \gamma\gamma$ $\quad$ BR $\sim 0.23\%$ ← discovery ← CMS

$h \to ZZ^*$ $\quad$ BR $\sim 0.016\%$ ← Atlas

In 2012 observed at 125 GeV

Test properties:

spin 0 favoured (Spin 1 particle cannot decay to two photons)

check decay to quarks and leptons to probe mass generating mechanism

as of 2017 Decays supports SM to 20 - 30 %

Targeting 1 %.

Exam

- lecture matericel] only this
- sujested exercise]
- may be asked to interpret plots or key formulas

Q1 - short questions, no equations, few lines of text needed

- Ways to prepare:
  - Work through derivations
  - Sujested exercises
  - margin notes (Peskin) + Thomson summaries
  - Interpret formulas and plots
  - not bad to look at past exams.

- can you set up in CoM frame
- can you work with parametrization
- use of mandlestam variables
- crossing sym
- can you work with representations of groups
- where colour/isospin algebra should be applied

work with dirac spinors

- kinematics includes the phase space

- Be ready to <u>use</u> feynman rules
- get comfortable with what spinors to use
  + polarization vector

[Read Peskin Appendix C]

- Things in Appendix D may be derivations
- Hint on 3 body phase space??? *
- Be able to use but not derive things in appendix E

Proper Revision

Field equations

- KG eq for a scalar field (real & complex)
- Particles & fields
- Maxwells eqs for massless spin 1
- Dirac eq for spin $\frac{1}{2}$, massive or massless

in massless case → seperation into L & R fields.

- Hydrogen atom

QM numbers n, l, m $\quad E = -\frac{1}{2}\alpha^2 m \frac{1}{n^2}$

$Y_{lm}(\theta, \varphi)$

$$P|n\,l\,m\rangle = (-1)^l |n\,l\,m\rangle$$

- Fine structure, accounts for spin $\frac{1}{2}$ interactions + reletivistic effects

Diagonalize $\Delta H = \vec{L}\cdot\vec{S}$ $\quad$ Adds qm numbers j, s $\quad \vec{J} = \vec{L} + \vec{S}$

Hyperfine structure

Diagonlize $\Delta H \sim \vec{S}_p \cdot \vec{S}_e$

QM numbers S_p, S_e

Positronium: bound state of e^- and e^+
spectrum of states distinguished by n, l
j^{PC} $\Rightarrow$ Selection rules for decays

Quark Model:

Hadrons, Mesons, baryons, Pions

$m_\pi \sim 140$ MeV

Charmonium $c\bar{c}$, J/ψ ~ positronium spectrum at 3.1 GeV

- Quark flavour: Flavour Isospin SU(2) rotates up quarks into down.

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

- Can be exstended to $SU(3)$ flavour symmetry.

- Mesons: octets and singlets. $3 \otimes \bar{3} = 8 \oplus 1$

chose these because colour invariant?

- Baryons: $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

down to $10 \oplus 8$ once we Include spin wave func with anti-sym fermions

- Observable states are colourless

- Heavy mesons with bottom and charm quarks but not t as it decays to fast

Baryon (quark) number is conserved → stability of the proton.

<u>Detectors:</u> Trackers, Calorimeters

measurments of mass, velocity, particle type. Detector design.

Computation:

Decay width Γ

scattering cross section

Fermi's Golden Rule

Phase space integrals

Breit-Wigner resonance

Average over initial state (unless known)

Sum over final states

The strong interaction

$e^+e^- \to \mu^+\mu^-$ helicity conserved

Computed $\frac{d\sigma}{d\cos\theta}$, σ

- Used this to calculate:

$e^+e^- \to$ hadrons

$e^+e^- \to q\bar{q}$ Changes: - Sum over Quark Species

Therefore

- quark charges
- sum over final states

Don't see quarks ⇒ electron-quark scattering occurs in deep inelastic scattering inside a proton

probes proton structure (PdF's)

$e^+ p \to e^- + \text{hadrons}$

Parton model

Crossing symmetry for $eq \to eq$

Derived σ, $\frac{d\sigma}{d\cos\theta}$

DIS kinematic variables

*

Bjorken scaling : $\underbrace{\text{form factor}}_{\text{ratio to QED process}}$ from parton model

depends on dimensionless scattering kinematics not on energy scale

⇒ Partons are point like constitutions

(There is a slow $\log Q^2$ dependence from splitting ?

Form factor in terms of PdF's

PdF sum rules

Observe mis total energy in pdf sum rules from q and $\bar{q}$

→ Gluon

→ Predict 3-jet & 4-jet events

Computed final state gluon radiation in co-linear approximation.

*

Model Gluon as a photon. (massless spin 1)

Get Weizsacher - Williams distribution

$$\sigma(A \to B + f + \gamma) \approx \sigma(A \to B + f) \int dz \int \frac{dq_\perp}{q_\perp} \frac{Q_f^2 \alpha}{\pi} \frac{1+(1-z)^2}{z}$$

$$\sigma(A + f \to B + \gamma) \approx \sigma(A + f \to B) \int dz \frac{dq_\perp}{q_\perp} \frac{Q_f^2 \alpha}{\pi} \frac{1+(1-z)^2}{z}$$

$e^+e^- \to 3$ Jets

QCD

- Global & Local Gauge invariance of QED

*

- Adjoint & Fundemental representation of SU(N)

· Non-Abelian guage symmetry

- Field strength Tensor $F_{\mu\nu}$, Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu a} F_{\mu\nu}^{a}$$

- With matter coupling $\bar{\Psi}(i\not{\partial} - m)\psi$

SU(3) colour.

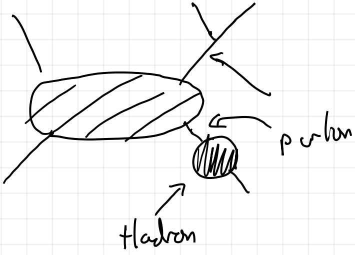
- Colour factors in $M(q \to g q)$ *

Running coupling of constants

Vacuum polarization in QED & <u>asymtotic</u>
<u>Freedom</u> in QCD

consequences for Hadronization
Low E bound states
High E perturbation theory.

<u>Partons</u>
DGLAP differential equations for splitting functions



Deviation from Bjorkin scaling at high energies

Collinear splitting function $P_{h_1 \leftarrow h_2}(z)$

Color factors relate singular behaviour of splitting functions

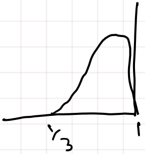
$\longrightarrow$ Gluon jets wider then quark Jets

Jets

Hadronization different from EM showers

irregular and IR-divergent soft & co-linear emmissions

Thrust variable measures 2-jet nature of hadronic final states



Clustering: Pairwise combine nearly collinear particles to an effective particle

QCD at hadron colliders

$(eq \to eq) \to (ud \to ud)$ etc with colour factors

Crossing Symmetry

Partially derived results of Appendix E

Fact: gg prodution dominates at low P_T

qq production dominates at high P_T

$\to$ This is because larger intrinsic σ due to colour Algebra?

- Valence quarks carry largest momentum fractions

- Higher energy $\Rightarrow$ lower $\alpha_s \to$ narrower Jets

Top quark production modes, a big boost in σ from Tevatron to LHC due to larger affects from c quarks and gluons?

Massless QCD Lagrangian & chiral isospin symmetry. (two flavours)

Set up a mass matrix for quarks

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \quad \begin{cases} SU(2) \text{ Isospin if } m_u = m_d \\ SU(2) \text{ chiral isospin if } m_u = m_d = 0 \end{cases}$$

Conserved currents $SU(2)_L \times SU(2)_R$ *

Spontanous Symmetry breaking:

Symmetry of $\mathcal{L}$ or H not respected by ground states.

(SU(2)) Manifold of vacua (ground states)

Goldstone's Theorem: *

SSB (Continuous) $\longrightarrow$ massless particle created by symmetry current

SSB of $SU(2)$ chiral isospin $\rightarrow$
massless pions $\pi^- \pi^0 \pi^+$

$SU(3)$ with - $\rightarrow$ whole octent

Quark masses inputed $\rightarrow$ pion masses as corrections

---

Weak interactions

Parity volations is observed in beta decay but strong & EM interactions conserve P

- need new "Weak" interaction

Also muon decay, pion decay

All consistant with V-A current (vector Axial)

Maximally parity violating

$$u_L^\dagger \bar{\sigma} d_L = \frac{1}{2}\left[\bar{u}\gamma^\mu d - \bar{u}\gamma^\mu\gamma^5 d\right]$$

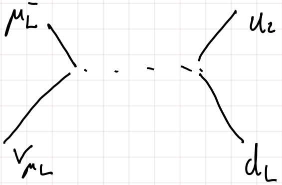
CP still conserved

Computed polarization of e^- in β decay.
preferentially but totally L (due to mass)

Computed $\Gamma(\mu \to e \bar{\nu}_e \mu)$

Computed $\dfrac{BR(\pi^- \to e^- \bar{\nu})}{BR(\pi^- \to \mu^- \bar{\nu})} \sim 10^{-4}$

Deep inelastic Neutrino scattering



- charged current events

*

V-A current-current interaction $\to$ Spin 1

"W" Boson charged & Massive
(no low E resonance)

3 examples of Massive vector Bosons from SSB guage field

(Higgs Mechanism)

① U(1) gauge theory w/ complex scalar field

② SO(3) w/ real scalar field

③ Glashow - Salam - Weinberg electroweak model $SU(2) \times U(1)$ lepton doublet, Higgs doublet

Manifold of degenerate ground states

Vector Boson eats goldstone Boson

→ becomes massive.

From $\underbrace{A^1, A^2, A^3}_{SU(2)}, \underbrace{B}_{U(1)}$ Weak hypercharge

weak Isospin

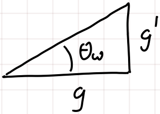
$A^1, A^2 \rightarrow W^+, W^-$

$A^3, B \rightarrow$ mixed into massless photon and massive boson Z.

$$D_\mu \phi = \left(\partial_\mu - i g A_\mu^a I^a - i g' B_\mu Y\right) \psi$$

SU(2) generators

Weak mixing angle

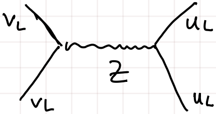


- Massless goldstone Boson is the scalar Higgs field $h(x)$
- Derived W & Z couplings from $D_\mu \Psi$

V−A : L couples to W, R doesnt

L have $I = \frac{1}{2}$, R have $I = 0$

Neutral Current interactions like:



W & Z Bosons: decay and production modes

Mass, lifetime, BR

m_W, m_Z follow from Higgs vev v

Z decay classified by chiral species

Quark Mixing

Terms like $j^{\mu T} = \cdots + V_{us} u_L^{\dagger} \bar{\sigma}^{\mu} s_L + \cdots$

allows decays we observe, loss of strangeness?

Higgs vev $\longrightarrow$ mass terms in new (mixed) basis

$$\Delta = -\frac{y_e v}{\sqrt{2}} e_L^{\dagger} e_R - \frac{y_d v}{\sqrt{2}} d_L^{\dagger} d_R - \frac{y_u v}{\sqrt{2}} u_L^{\dagger} u_R$$

+hc

V_{CKM} mixing matrix $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

With 4 dof 3 euler angles + 1 phase.

$\downarrow$

Connected to CP violation

Descrete symmetries & Conservation Laws of SM Lagrangian

Combuled B/b based on μ decay

$b \to c f \bar{f}$ similar to $\mu \to e \bar{\nu}_e \nu_\mu$

Measurment of $\tau(b)$ gives V_{cb}, V_{ub}

Suppression of FCNC

CP violation in $K^0 - \bar{K}^0$ system

Mass eigenstates vs CP eigenstates

$\uparrow$
evolution

production

Also $B^0 - \bar{B}^0$

<u>Neutrino masses & mixing</u>

Nearly massles: Low upper bound from β decay of Nuclei $M_{\nu_e} < 2.05$ eV, other bounds are on mass difference.

Yukawa couplings are most promising

Flavour vs mass eigenstates.

Oscillation between flavour eigenstates

Experimental evidence for non-zero neutrino mass

Higgs Boson

Saw SM terms in $\mathcal{L}$ explicitly by $v \to v + h(x)$ in higgs doublet

couplings suppresed by v, enhanced by m_W, m_Z, t

Higgs decay & production modes.

QCD background $\longrightarrow$ discovery modes of $\gamma\gamma$ and $4l$