quark model, chpt 9 in Thomson | She will post exercises that might be covered in tutorials

God given units (Natural units)

$\hbar = c = 1$, $\epsilon_0, \mu_0 = 1$

Electron mass: $m_e = 0.511\ MeV$

Proton: $m_p = 938\ MeV$

- Fine structure constant

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

Relativistic Kinematics

$$\eta_{\mu\nu} = diag(1,-1,-1,-1)$$

$$p^\mu = (E, \vec{p})^\mu = m\gamma(1,\vec{\beta})^\mu$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E_p = (p + m^2)^{\frac{1}{2}} \qquad p^2 = m^2$$

If $E = E_p \Rightarrow$ particle is on-shell

Decay of Massive particle in its rest frame

$\hat{3} \quad \xleftarrow{1} (M) \xrightarrow{2} \quad p_M^\mu = (M, 0,0,0)$

Case 1:

$m_1 = m_2 = 0$
$\Rightarrow p_1^2 = p_2^2 = 0$

$\Rightarrow$ $p_1^\mu = (\frac{M}{2}, 0, 0, \frac{M}{2})$

$p_2^\mu = (\frac{M}{2}, 0, 0, \frac{M}{2})$

$+ \quad p_M^\mu = p_1^\mu + p_2^\mu$

Case 2:

$m_1 \neq 0, m_2 = 0 \quad p_1^\mu = (E_p, 0, 0, p)$

$p_2^\mu = (p, 0, 0, -p)$

where $E_p = \frac{M^2 + m_1^2}{2M}$, $p = \frac{M^2 - m^2}{2M}$

Case 3:

$m_1 \neq 0, m_2 \neq 0$

$p_1^\mu = (E_1, 0, 0, p)$

$p_2^\mu = (E_2, 0, 0, -p)$

where $E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$, $E_2 = \frac{M_1^2 - m_1^2 + m_2^2}{2M}$

$$p = \sqrt{\frac{\lambda(M, m_1, m_2)}{2M}}$$

$$\lambda(a,b,c) = a^4 + b^4 + c^4 - 2cb - 2ac - 2bc$$

Discrete groups

only really need $\mathbb{Z}_2$

- Unitary representation:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

for particles this is related to parity and charge conjugation

QM system with 2 states

$|\pi^+\rangle \quad |\pi^-\rangle$

$C|\pi^+\rangle = |\pi^-\rangle$, $C|\pi^-\rangle = |\pi^+\rangle$

$$C\begin{pmatrix} |\pi^+\rangle \\ |\pi^-\rangle \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} |\pi^+\rangle \\ |\pi^-\rangle \end{pmatrix}$$

If $[C,H] = 0 \Rightarrow \pi^+$ and π^- have equal masses and decay rates

"H has $\mathbb{Z}_2$ symmetry"

Since $C^2 = 1 \Rightarrow \lambda = \pm 1$

$\Rightarrow \lambda = 1 \qquad \frac{1}{2}(|\pi^+\rangle + |\pi^-\rangle)$

$\lambda = -1 \qquad \frac{1}{2}(|\pi^+\rangle - |\pi^-\rangle)$

Discrete symmetries

det = -1

Parity: $(x^0, \vec{x}) \longrightarrow (x^0, -\vec{x})$

Time reversal: $(x^0, \vec{x}) \to (-x^0, \vec{x})$

QM intrinsic Parity $\quad P|A(\vec{k})\rangle = \pm|A(-\vec{k})\rangle$ (Parity ±1)

& intrinsic time reversal $\quad \Rightarrow$ anti particles (Conjugate fields)

C charge conjugation QFT $\quad$ Particle $\xleftrightarrow{C}$ anti particle, Opposite charge, same mass

C, P, T are not required to be respected by the Hamiltonian. QFT has a theorem for conservation of CPT

Continuous groups

Space time: translations - $\mathbb{R}^4$ abelian

& Lorentz transformations $SO(3,1)$ non-abelian

- For Standard model (SM) we also want intrinsic $U(1)$, $SU(2)$, $SU(3)$

$U(n)$ = unitary $n \times n$ matrices

$U(1) = \{e^{i\alpha}, \alpha \in \mathbb{R}\}$ Rotations of a circle S^1

$SU(n)$ = Special unitary $n\times n$ matrices

$$SU(n) = \{U_{n\times n} \mid U^\dagger U = 1 \ \&\ \det U = 1\}$$

$SO(3)$ rotation group of $\mathbb{R}^3$

$$U(\vec{\alpha}) = \exp[-i\vec{\alpha}\cdot\vec{J}]$$

$\vec{J}$ angular momentum

J_i are the generators of rotations

$$[J^i, J^j] = i\epsilon^{ijk} J^k$$

$SO(3)$ has a d-dimensional rep for every integer d

$d = 2j + 1 \qquad$ "spin j"

More generally generators of Lie groups satisfy a Lie algebra with commutation relation

$$[t^a, t^b] = i \underbrace{f^{abc}}_{\text{structure constants}} t^c$$

$$U = \exp[-i\alpha^a t^a]$$

Klein-Gordan Eq

Scalar field spin 0

EOM $\quad (\partial^2 + m^2)\phi(x) = 0$

solution $\quad \phi(x) \sim e^{-ip\cdot x}$

where $p^2 = m^2$

Both $E = \pm E_p$ are allowed

QFT! $\phi(x)$ is a field that creates and destroys particles

$|0\rangle$ Vacuum: $|\varphi(p)\rangle$ one particle

$$\langle 0|\phi(x)|\varphi(p)\rangle = e^{-ip\cdot x} \quad \text{with } p^0 = E_p > 0$$

$\Rightarrow$ positive energy sol

$\phi(x)$ is complex

$\Rightarrow \langle \varphi(p)|\phi^\dagger(x)|0\rangle = e^{ipx}$

creation of a particle

If $\phi^\dagger(x) \neq \phi(x)$, then we also have

$$\langle 0|\phi^\dagger(x)|\varphi'(p)\rangle = e^{-ipx}$$

$\Rightarrow \varphi'(p)$ is the anti-particle

If $\phi^\dagger(x) = \phi(x) \Rightarrow \varphi$ can be its own anti-particle.

Maxwells eqs Spin 1

vector field $V^i(x)$ (Vector representation of angular mom)

Particle destruction:

$$\langle 0|V^i(x)|v(p,\epsilon)\rangle = \epsilon^i e^{-ipx}$$

- Need $\epsilon^i \to R_{ij}\epsilon^j$

(where $V^i \to R^i_{\ j} V^j$

Relelivistic

$\langle 0|V^\mu(x)|v(p,\epsilon)\rangle = \epsilon^\mu e^{ipx}$

Notice: Norm of this state is $\epsilon^\mu\epsilon_\mu$ can be <0

- This is solved by photons being massless

Variational pricible

4-Vector potential $A^\mu(x)$

Define a tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \underbrace{J^\mu}_{\text{external current}} A_\mu$$

with $\partial_\mu J^\mu = 0$

$$\partial_\mu F^{\mu\nu} = -J^\nu$$

Dirac Eq spin $\frac{1}{2}$

Dirac Matricies

must have $d \geq 4$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

- We will use Dirac basis

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \to \text{for non-rel applications}$$

or Weyl basis

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \to \text{rel applications}$$

Dirac Eq

$$(i\gamma^\mu\partial_\mu - M)\psi = 0$$

- Solution of Dirac eq is solution of KG eq.

$$\psi \sim e^{-ipx}$$

- 2 positive energy an 2 negative energy sols

Given in terms of 2-compont Spinors: $s = 1, 2$

$$\psi = u^s(p)e^{-ipx}, \quad \psi = v^s(p)e^{ipx}$$

Matrix elements for destroying/creating one electron

$$\langle 0|\psi(x)|e^-(p,s)\rangle = u^s(p)\, e^{-ipx}$$

$$\langle e^-(p,s)|\psi^\dagger(x)|0\rangle = u^{s\dagger}(p)\, e^{ipx}$$

or for positrons

$$\langle 0|\psi^\dagger(x)|e^+(p,s)\rangle = v^{\dagger s} e^{ipx}$$

$$\langle e^+(p,s)|\psi(x)|0\rangle = v^s e^{-ipx}$$

- couple to Maxwells eqs

Electrons & photons $\to$ QED

- We can couple Dirac eq to EM field

by $\partial_\mu \to D_\mu \equiv \partial_\mu - ieA_\mu$

$$\mathcal{L} = \bar\psi(i\gamma^\mu D_\mu - M)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad \bar\psi = \gamma^0\psi^\dagger$$

Relelivistic Normalization

$$\langle p_1|p_2\rangle = 2E_{p_1}(2\pi)^3\delta^{(3)}(\vec p_1 - \vec p_2)$$

$$\int\frac{d^4p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2) = \int\frac{d^3p}{(2\pi)^3}\int dp^0\,\delta(|p^0|^2 - |p|^2 - m^2) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_0}$$

Spin-Statistics Thm of QFT

Fields w/ integer spin $\to$ particles with BE stats

Fields with half-integer spin $\to$ particles with FD stats

SM: fields of spin $0, \frac{1}{2}, 1$

Also composit particles

Hydrogen Atom (Not to scale)

p e

- Non-rel:

$V(r) = \frac{\alpha}{r}$

Bound state energies $E = -\frac{1}{2}\alpha^2 m\frac{1}{n^2}$ $n = 1, 2, \ldots$

. Orbital wave functions $Y_{lm}(\theta,\phi)$, $l = 0, 1, \ldots n-1$, $m = -l, \ldots, l$

Parity:

$$P|nlm\rangle = (-1)^l|nlm\rangle$$

- Fine structure: electron spin $\frac{1}{2}$

- Spin orbit interactions + relelivistic effects

$\vec L$ = orbital angular mom

$\vec S$ = spin angular mom

$$\Delta H \sim \vec L\cdot\vec S$$

$$\Delta H = \frac{g-1}{2}\frac{\alpha}{M_e^2 r^3}\vec L\cdot\vec S$$

- We will diagonalize ΔH

Let $\vec J = \vec L + \vec S$ $\leftarrow$ Tot angular mom

orbital ang mom; electron spin

$$\vec L\cdot\vec S = \frac{1}{2}(\vec J^2 - \vec L^2 - \vec S^2)$$

Each $(n,l) \longrightarrow \underset{\text{spin}}{2}\,\underset{m}{(2l+1)}$ qm states

- Fine structure splits degeneracy

1s orbital

$n=1, l=0, s=\frac{1}{2}, j=\frac{1}{2}$

$$\vec L\cdot\vec S = \frac{1}{2}\left[\frac{1}{2}\cdot\frac{3}{2} - 0 - \frac{1}{2}\frac{3}{2}\right]$$

2s $n=2$ $l=0$ $s=\frac{1}{2}$ $j=\frac{1}{2}$ $\vec L\cdot\vec S = \frac{1}{2}$

2p $n=2$ $l=1$ $s=\pm\frac{1}{2}$ $j = \begin{cases} \frac{3}{2} & \text{4 states} \\ \frac{1}{2} & \text{2 states} \end{cases}$ $\vec L\cdot\vec S = -1$

Hyperfine structure

Proton spin $\frac{1}{2}$

$$\Delta H \sim \vec S_p\cdot\vec S_e = \frac{1}{2}\left[(\vec S_p + \vec S_e) - \vec S_p^2 - \vec S_e^2\right]$$

1s state is split

$\vec S_p + \vec S_e = 1$ $|\uparrow\uparrow\rangle, \frac{1}{\sqrt2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ $|\downarrow\downarrow\rangle$

$\vec S_p + \vec S_e = 0$ $\frac{1}{\sqrt2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Positronium: Bound state of e^+, e^-

Label eigenstates by J^{PC} (J: ang mom, P: parity, C: charge)

Parity

Y_{lm} gets $(-1)^l$ under spatial inversion

electron and positron have opposit intrinsic parities $P = (-1)^{l+1}$

charge ① C exchanges $e^+ \leftrightarrow e^-$

-1 is wavefunction from femionic exchange

② Co-ordinate reversal in Y_{lm} $(-1)^l$

③ exchange of spins $\vec S_{e^+}$ and $\vec S_{e^-}$

$$\begin{cases} +1 & S=1 \\ -1 & S=0 \end{cases}$$

$$\text{Total } C = (-1)^{l+1}\begin{cases} 1 & S=1 \\ -1 & S=0 \end{cases}$$

Spectrum

"s" is $l=0$ $\Rightarrow P=-1$

2s: 1^{--} $S=1$; 0^{-+} $S=0$

1s: 1^{--} $s=1$; 0^{-+} $s=0$

2P: 1^{+-} $S=0$; 2^{++}, 1^{++}, 0^{++} $S=1$

P is $l=1$ $\Rightarrow P=1$

QED interactions mediated by the photon

Couplings to e^+ and e^- are opposit $\Rightarrow$

$$C|\gamma(\epsilon,p)\rangle = -|\gamma(\epsilon,p)\rangle$$

Possible photon transitions

$2s(1^{--}) \to 2p(J^{++})$

$2p(J^{++}) \to 1s(1^{--})$

$2p(1^{++}) \to 1s(0^{-+})$

$1s(1^{--}) \to 1s(0^{-+})$

Ground states can decay to photons

- But decay to a single photon is not possible (momentum cons)

Decay to $\gamma\gamma$ possible only for $C=+$ (i.e. 0^{-+})

1s 1^{--} can decay to $\gamma\gamma\gamma$

The Quark model

- Hadrons are the particles observed in nuclei, they are bound states of quarks

Quarks appear in baryons or mesons, never alone

Pions: The lightest Hadrons are π-mesons

$\pi^\pm$ 139.57 MeV

π^0 134.98 MeV

Charmonium

Any particle + antiparticle

$\rightarrow$ excited EM fields

$\rightarrow$ any other particle + anti-particle

1974 Two groups found a sharp resonance at the same energy level (3.1 GeV).

- One collided e^+, e^- $\xrightarrow{\text{found}}$ "ψ" — Theory need a 4th quark, charm
- other $p+p \rightarrow e^+e^- + X$ "J"

Reasonance called J/ψ "gypsy"

Highest rate comes from annihilation by the EM current $\vec{J} = \bar{\psi}\vec{\gamma}\psi$

$\langle 0|\vec{J}(x)|e^+e^-\rangle$

spin 1 $\quad P=-1,\ C=-1 \quad J^{PC}=1^{--}$

1s, 2S etc higher l-states also observed like positronium

$\Rightarrow$ conclude bound state $c\bar{c}$

- Potential is not $\frac{1}{r}$

but like $A\log r$ or $-\frac{\alpha}{r}+Br$

$\rightarrow$ quarks are confined

Light mesons

Pions found to have $J^P = 0^-$

$\pi^0 \rightarrow 2\gamma \Rightarrow C(\pi^0) = +1$

Lightest hadrons all have $J^P = 0^-$ or 1^- 9 of each $l=0$

- Light psudo scalar Mesons.

$J^P = 0^-$ $\quad \eta'$ 958 MeV

η 548 MeV

$K^-\ \bar{K}^0\ K^0\ K^+$ 498 MeV

$\pi^-\ \pi^0\ \pi^+$ 140 MeV

- Light vector Mesons $\quad \phi$ 1020 MeV $\rightarrow \gamma\pi^0$

$J^P = 1^-$ $\quad K^{*-}\ \bar{K}^{*0}\ K^{*+}$ 892 MeV

$\rho^-\ \omega\ \rho^0\ \rho^+$ 781, 720 MeV $\rightarrow \gamma\pi^0$

K, K^*, Λ^0 dont appear single only with another "Strange Particles"

(Λ^0: excited state proton)

Vaguely correct: the 9 1^- states are bound states of $(\bar{u},\bar{d},\bar{s})\cdot(u,d,s)$

$m_u \sim m_d$ 3-4 MeV $\quad m_s \sim 95$ MeV

- Posit discrete symmetry $u \leftrightarrow d$

and approx symmetry $u \leftrightarrow s$, $d \leftrightarrow s$

- exstend to continuous symmetry SU(2) rotation "isospin"
- SU(3) rotation

Quark flavour Isospin (Thomson convention chp 9?)

$u = \begin{pmatrix}1\\0\end{pmatrix},\ d = \begin{pmatrix}0\\1\end{pmatrix}$

- Useful if EM << strong interactions

Isospin:

$\vec{T} = \frac{1}{2}\vec{\sigma}$ $\quad$ Doublet $\begin{pmatrix}u\\d\end{pmatrix}$ $\quad$ Antiquark doublet $\begin{pmatrix}\bar{d}\\ \bar{u}\end{pmatrix}$

$2 \otimes \bar{2} = 3 \oplus 1$ (spin $\frac{1}{2}$, spin 3, spin 0)

$q\bar{q}$ Isospin states

$\phi(1,1) = -u\bar{d} \sim \pi^+, \rho^+$

$\phi(1,0) = \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) \sim \pi^0, \rho^0$

$\phi(1,-1) = d\bar{u} \sim \pi^-, \rho^-$

$\phi(0,0) = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) \sim \eta, \omega$

SU(3) flavour Symmetry

Triplet $u = \begin{pmatrix}1\\0\\0\end{pmatrix},\ d = \begin{pmatrix}0\\1\\0\end{pmatrix}\ s = \begin{pmatrix}0\\0\\1\end{pmatrix}$

8 generators of SU(3)

Gell mann matricies, prefer $u \leftrightarrow d$ isospin

$\lambda_1 = \begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\ \lambda_2 = \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix},\ \lambda_3 = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$ SU(2) of u,d

$\lambda_4 = \begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\ \lambda_5 = \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}$ $\quad u \leftrightarrow s$

$\lambda_6 = \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}\quad \lambda_7 = \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}$ $\quad d \leftrightarrow s$,

$\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$ equal treatment of u,d

Normalized so that $\mathrm{tr}(\lambda_i\lambda_j) = 2\delta_{ij}$

Isospin:

$\hat{T}_i = \frac{1}{2}\lambda_i$ eg $\hat{T}_3 u = \frac{1}{2}u$, $\hat{T}_3 d = -\frac{1}{2}d$, $\hat{T}_3 s = 0$

Total Isospin

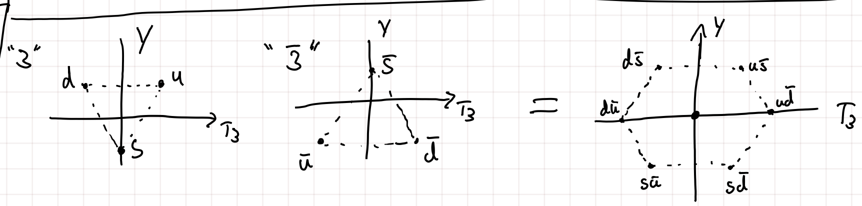
$\hat{T}^2 = \sum_i T_i^2 = \frac{1}{4}\sum_i \lambda_i^2 = \frac{4}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

2 - commuting generators $\hat{T}_3\ \hat{T}_8$

Quantum numbers $\hat{T}_3 = \frac{1}{2}\lambda_3 \leftarrow$ 3rd comp isospin

$\hat{Y} = \frac{1}{\sqrt{3}}\lambda_8 \leftarrow$ Hypercharge

$\hat{Y}u = \frac{1}{3}u \quad \hat{Y}d = \frac{1}{3}d \quad \hat{Y}s = -\frac{2}{3}s$



Combine remaining generators into ladder operators

$\hat{T}_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$ $\hat{V}_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$ $\hat{U}_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$

$u \uparrow -\bar{d}$, $d \downarrow \bar{u}$ $\quad$ $d \uparrow -\bar{s}$, $s \downarrow \bar{d}$ $\quad$ $u \uparrow -\bar{s}$, $s \downarrow \bar{u}$

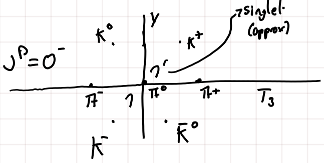
3 states with $T_3 = Y = 0$

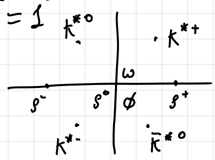
$T_-|u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle$
$V_-|u\bar{s}\rangle = |s\bar{s}\rangle - |u\bar{u}\rangle$
$U_-|d\bar{s}\rangle = |s\bar{s}\rangle - |d\bar{d}\rangle$

2 lin indep

$3 \otimes \bar{3} = 8 \oplus 1$ — octet, singlet

Singlet $\frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$





$|\eta'\rangle \simeq \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$|\eta\rangle \simeq \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$

$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$|\omega\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$
$|\phi\rangle \simeq s\bar{s}$

G-Parity

Pions are not eigenstates of charge conjugation

$C|\pi^+\rangle = |\pi^-\rangle$

Define: $G = C e^{i\pi I_2}$ — rotation in isospin about $\hat{2}$ axis

$T_1 \to T_1$
$T_2 \to T_2$
$T_3 \to -T_3$

$G|\pi^+\rangle = -|\pi^+\rangle$
$G|\pi^-\rangle = -|\pi^-\rangle$
$G|\pi^0\rangle = -|\pi^0\rangle$

$G = -1$ for pions $\to$ selection rules

Ex J/ψ has $C = -1$ and $I = 0$
$\Rightarrow G = -1$
$\Rightarrow$ can decay into an odd No of pions

Heavy mesons with b, c

$c\bar{s}$
$c\bar{u}$ $c\bar{d}$ + anti particles

$J^P = 0^-$ D_s^+ 1968 MeV
D^0 D^+ 1869 MeV

$b\bar{u}$ $b\bar{s}$ $b\bar{d}$ + anti

$J^P = 0^-$ B_s^0 5367 MeV
B^- B^0 5227 MeV

$J^P = 1^-$ D_s^{*+} 2112 MeV
D^{*0} D^{*-} 2010 MeV

$J^P = 1^-$ B_s^{*0} 5415 MeV
B^{*-} B^{*0} 5325 MeV

. No mesons with top quark $M_t \sim 173$ GeV decays too rapidly

Baryons: Baryons are fermions

Baryon number B is conserved

$\to$ The lightest baryon is stable. It's the proton!

Light families

- Spin $\frac{1}{2}$ octet

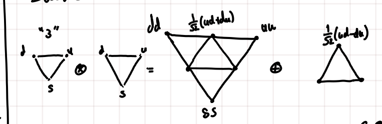
Ξ^- Ξ^0 1313 MeV
Σ^- $\Lambda^0 \Sigma^0$ Σ^+ 1192 1176
n p 938 MeV

Spin $\frac{3}{2}$

Ω^- 1672
Ξ^{*-} Ξ^{*0} 1532
Σ^{*-} Σ^{*0} Σ^{*+} 1385
Δ^- Δ^0 Δ^+ Δ^{++} 1232

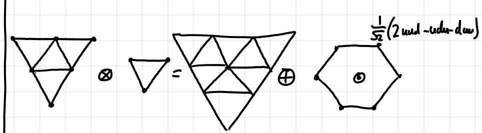
- All states of $P = +1$ (by convention)

- Can be understood as qqq bound states

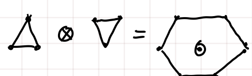


$3 \otimes 3 = 6 \oplus \bar{3}$

$6 \otimes 3 = 10 \oplus 8$



$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$



$\oplus \frac{1}{\sqrt{6}}(uds - usd + dsu - sud - sdu \ldots)$

Suggested exercise:
- work through equations (9.8) — (9.15)
- Thomson exercise 9.3

- work through spin algebra

Total wave function for bound qqq state:

$\psi = \phi_{flavor}\, \chi_{spin}\, \xi_{color}\, \eta_{space}$

$L = 0$ symmetric — what we look at

- ψ must be anti-symmetric
- χ turns out needs to be anti-symm (SU(3) invariants are $\delta_a^b, \epsilon_{abc}\, \epsilon^{abc}$)

$\Rightarrow \phi \cdot \chi$ must be symm

Spin states

qqq spin $\frac{3}{2}$ quadruplet

$\chi(\frac{3}{2}, \frac{3}{2}) = |\uparrow\uparrow\uparrow\rangle$
$\chi(\frac{3}{2}, \frac{1}{2}) = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$
$\chi(\frac{3}{2}, -\frac{1}{2}) = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)$
$\chi(\frac{3}{2}, -\frac{3}{2}) = \downarrow\downarrow\downarrow$

Electric charges of quarks

It follows that
$Q_d = -\frac{1}{3}$, $Q_u = \frac{1}{3}$
$Q_s = -\frac{1}{3}$, $Q_c = \frac{2}{3}$
$Q_b = -\frac{1}{3}$ $(Q_t = +\frac{2}{3})$

Detectors in a nutshell

2-types: - trackers $\to$ trajectory & momentum
- calorimeters $\to$ total energy (particle destroyed)

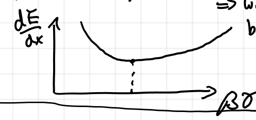
- Energy loss by ionisation
Em interactions that kick electrons

- Bethe-Bloch eq:

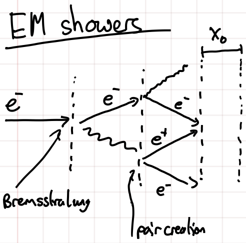
$$\frac{dE}{dx} \approx -4\pi\alpha^2 \frac{nZ}{MeV^2}\left[\log \frac{2\gamma^2\beta^2 m_e}{\omega} - \beta^2\right]$$

- n = number density
- Z = atomic number
- ω = atomic frequency
- γ, β relativistic quantities

- path depends on momentum
- $\frac{dE}{dx}$ depends on velocity
$\Rightarrow$ with measurement of both we can derive particle mass



EM showers



X_0 = radiation length

EM calorimeters are crystals containing these showers in compact regions

- Energy is measured from scintillation light

Hadronic showers

- far more irregular
Not all energy is detectable
Can go into the nucleus
Hadronic calorimeter are cruder by an order of mag

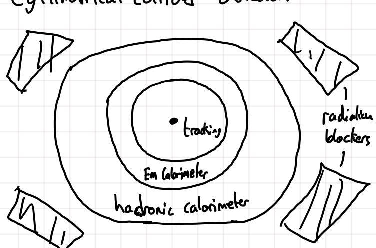
- Transition radiation
effect of crossing vacuum - medium boundary, Can distinguish electrons from pions

Cherenkov radiation

Superluminal velocity in medium $\to$ shockwave

Discriminates particle velocity.

Cylindrical collider detector:



Sujested exercises

Peskin 2.1, 2.5, 7.2, 8.1

Tools for calculation

two phenomena:
- scattering
- decay

- Unstable particle A
Constant rate of decay
Probability of existance

$dP = -P\Gamma_A dt$

$\Rightarrow P(t) = e^{-t\Gamma_A}$

" total width Γ_A "

$\Gamma_A = \sum_F \Gamma(A-f)$ for multible decay processes

Chapter 7 Peskin

Scattering

n_A $\xrightarrow{v_B}$ target $\bullet B$

- Beam of particles with density n_A, velocity v_A

Rate of scattering:

$$\frac{\text{events}}{\text{sec}} = n_A v_A \sigma$$ cross section, units cm^2

differential cross section

$$\frac{d\sigma}{d^3p_1 \cdots d^3p_n}$$ final state momenta

Total cross section:

$$\sigma = \int d^3p_1 \cdots d^3p_n \frac{d\sigma}{d^3p_1 \cdots d^3p_n}$$

Master formula Fermis Golden rule

QM transition matrix element

$$\langle 1\,2 \ldots n | T | A(p_A)\rangle = M(A \to |1 \ldots n\rangle)(2\pi)^4\delta^{(4)}(p_A - \sum_j p_j)$$

Decay:

$$\Gamma(A \to F) = \frac{1}{2m_A}\int d\Pi_n |M(A\to f)|^2$$ - phase space integral

mass of A

where $d\Pi_n = \frac{d^3p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3p_n}{(2\pi)^3 2E_n}(2\pi)^4\delta^{(4)}(p - \sum_i p_i)$

- Spin: sum $\sum$ over final spin states
specify initial state if known, average if not.
- Cross section:

$$\sigma(A+B \to f) = \frac{1}{2E_A 2E_B |v_A - v_B|}\int d\Pi_n |M(A+B\to f)|^2$$

$$\int d\Pi_2 \int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta(P - p_1 - p_2)$$ (2 particle collision)

CoM frame: $\vec{p}_1 = -\vec{p}_2 = \vec{p}$

carry out 3 integrals?

$$\int d\Pi_2 = \int \frac{d^3p\,(2\pi)}{(2\pi)^3 2E_1 2E_2}\delta(E_{cm} - E_1 - E_2)$$

$d^3p = p^2 dp \sin\theta\, d\theta\, d\varphi = p^2 dp\, d\Omega$

$$\int dp\, \delta(E_{cm} - E_1(p) - E_2(p)) = \frac{1}{\left|\frac{dE_1}{dp} + \frac{dE_2}{dp}\right|}$$

$E_{12}^2 = p^2 + m^2$

$\Rightarrow E_1 dE_1 = p\,dp$

$$= \frac{1}{\left|\frac{p}{E_1} + \frac{p}{E_2}\right|} = \frac{E_1E_2}{\underbrace{(E_1+E_2)}_{E_{cm}}p}$$

$$\int d\Pi_2 = \int \frac{p^2 d\Omega}{16\pi^2 E_1E_2}\frac{E_1E_2}{E_{cm}p}$$

$$= \frac{1}{8\pi}\frac{2p}{E_{cm}}\int\frac{d\Omega}{4\pi}$$

Relativistic Limit

$E \gg m_1, m_2$

$E_{cm} \to 2p$ $\quad \int d\Pi_2 \to \frac{1}{8\pi}\int\frac{d\Omega}{4\pi}$

If two particles are identical
$\Rightarrow$ Cant distiguish final states
$\Rightarrow$ need extra factor of $\frac{1}{2}$ in sum over states and hence in $\int d\Pi_2$

- For 3 particles we find:

$$\int d\Pi_3 = \frac{E_{cm}^2}{128\pi^2}\int dx_1 dx_2$$

$\Rightarrow x_1 + x_2 + x_3 = 2$, where $x_i = \frac{2E_i}{E_{cm}}$

- Boundaries given implicitly

$-1 \leq \cos\theta_{ij} \leq 1$

θ_{ij} and between p_i, p_j

Resonance

Breit-Wigner formula for resonance

$$M \sim \frac{1}{E - E_R + \frac{i\Gamma}{2}}$$

E_R = Energy of resonant state

Γ = decay rate.

Fourier transform:

$$\psi(t) = \int\frac{dE}{2\pi}\frac{e^{-iEt}}{E - E_R + i\frac{\Gamma}{2}} = -i\,e^{-iE_Rt}e^{-\Gamma t/2}$$

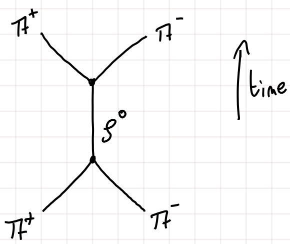
Probability of maintaining Resonance decays exponentially $|\psi(t)|^2 = e^{-\Gamma t}$

lifetime $\tau_R = \frac{1}{\Gamma}$

We need a relativistic version

Ex $\pi^-\pi^+ \to \rho^0 \to \pi^+\pi^-$ | $M_{\rho_0} = 770$ MeV

Feynman diagrams
-convention is time goes upwards



step ①: $\pi^+\pi^- \to \rho^0$
step ②: $\rho^0 \to \pi^+\pi^-$] matrix elements
③ ρ^0 resonance
Breit wigner + sum over polarizations

① $M(\pi^+\pi^- \to \rho^0)$

$\pi^+(p_A)\pi^-(p_B) \to \rho^0(p_c)$

spin 1
$\Rightarrow$ polarization vectors ϵ^μ to describe the states, with $\epsilon^\mu \cdot p_c = 0$

- Lorentz invariance

$\to M(\pi^+\pi^- \to \rho^0)$

$= g\epsilon^* \cdot (p_A - p_B)$

const; spin-1 field eqs

- Can argue this is the only quantity that can be formed?

CoM frame

$p_A = (E, \vec{p})$ $\quad E = \frac{1}{2}m_\rho$ $\quad p^2 = \frac{m_\rho^2}{4} - m_\pi^2$

$p_B = (E, -\vec{p})$

$\epsilon^* = -\epsilon \Rightarrow \epsilon^*(p_A - p_B) = -2\vec{e}\cdot\vec{p}$

$$\sigma(\pi^+\pi^- \to \rho_0) = \frac{1}{2E_AE_B|v_A - v_B|}\int\frac{d^3p_c}{(2\pi)^3}\frac{1}{2E_c}|M|^2\delta(p_c - p_A - p_B)$$

$$\left|\frac{p_A}{E_A} - \frac{p_B}{E_B}\right| = 2\frac{p}{m_\rho/2}$$

$$\int\frac{d^4p}{(2\pi)^4}2\pi\,\delta(p_c^2 - m_\rho^2)$$

$$\Rightarrow \frac{1}{m_\rho^2\frac{4p}{m_\rho}}2\pi\,\delta\left((p_A+p_B)^2 - m_\rho^2\right)g_\rho^2 4|\vec{e}\cdot\vec{p}|^2$$

$\sum_\epsilon |\vec{e}\cdot\vec{p}|^2 = p_x^2 + p_y^2 + p_z^2 = p^2$ $\quad \Rightarrow \sigma = g_\rho^2\frac{p}{m_\rho}(2\pi)\,\delta\left((p_A+p_B)^2 - m_\rho^2\right)$

Step 2

$M(\rho_0 \to \pi^+\pi^-)$

$$\Gamma_\rho = \frac{1}{2M_\rho}\int d\Pi_2 |M|^2$$

average instead of sum

$$= \frac{1}{2M_\rho}\frac{1}{8\pi}\frac{2p}{M_\rho} g_\rho^2 \underbrace{\langle 4|\vec{\epsilon}\cdot\vec{p}|^2\rangle}_{\frac{4}{3}p^2} = \frac{g_\rho^2 p^3}{6\pi M_\rho^2}$$

- Measure ρ^0 decay Find

$$\frac{g_\rho^2}{4\pi} \simeq 2.9$$

(reletivly large) process likes to occur.

Step 3

Resonance $M \sim \dfrac{1}{p^2 - M_R^2 + iM_R\Gamma_R}$

CM frame: $P = (M_R + \Delta E, \vec{0})$

$$\sigma(\pi^+(p_A)\pi^-(p_B) \to \rho_0 \to \pi^+(p_A')\pi^-(p_B'))$$

$$= \frac{1}{2E_A 2E_B|v_A - v_B|}\int d\Pi_2 \left| \sum_{pol} \frac{M(\pi^+\pi^- \to \rho^0(\epsilon))M(\rho^0(\epsilon)\to\pi^+\pi^-)}{(p_A+p_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho}\right|^2$$

$$= \frac{1}{4M_\rho p\cdot 8\pi}\frac{2p}{M_\rho}\int\frac{d\Omega}{4\pi}\frac{1}{(E_{cm}^2 - M_\rho^2) + M_\rho^2\Gamma_\rho^2}\left|\sum_\epsilon (2g_\rho \vec{\epsilon}^*\cdot\vec{p})(2g_\rho \vec{\epsilon}\cdot\vec{p}')\right|^2$$

$$\boxed{\sigma = \frac{1}{4M_\rho^2}\frac{g_\rho^2}{(E_{cm}^2 - M_\rho^2) + M_\rho^2\Gamma_\rho^2}\int\frac{d\Omega}{4\pi}|\vec{p}\cdot\vec{p}'|^2}$$

check limit of long lived narrow resonance

$\sigma(\pi^+\pi^- \to \rho \to f)$

$$= \frac{1}{4M_\rho p}\int d\Pi \left|\sum_\epsilon \frac{(2g_\rho\vec{\epsilon}^*\cdot\vec{p})M(\rho_\epsilon \to f)}{(p_A+p_B)^2 - M_\rho^2 + iM_\rho\Gamma_\rho}\right|^2$$

$$\sum_f \sigma(\pi^+\pi^- \to \rho \to f) = \frac{1}{4M_\rho p} 2\pi g_\rho^2 p^2 \left[\frac{\Gamma_\rho/\pi}{(p_c^2 - M_\rho^2) + M_\rho\Gamma_\rho^2}\right] \to \delta(p_c^2 - M_\rho^2) \text{ as } \Gamma_\rho \to 0$$

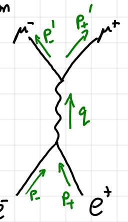
$$\lim_{\Gamma_\rho\to 0}\sigma = \frac{g_\rho^2 p}{M_\rho} 2\pi\, \delta((p_A+p_B)^2 - m_\rho)^2$$

$$\delta(x) = \frac{1}{\pi}\lim_{\epsilon\to 0}\frac{\epsilon}{x^2+\epsilon^2}$$

Strong interaction:

e^+e^- anhilation

$e^+e^- \to \mu\mu$



- Resonance = EM current
  Breit-Wigner at zero mass
  $M \sim \frac{1}{q^2}$ "virtual photon
  Virtual particle ⇒ off shell

$$M(e^+e^- \to \mu^+\mu^-) = (-e)\langle\mu^+\mu^-|j^\mu|0\rangle\frac{1}{q^2}(-e)\langle 0|j_\mu|e^+e^-\rangle$$

Massless spin ½ particles

e, μ treated as massless (Also light quarks)

Massless Dirac Eq: $i\gamma^\mu\partial_\mu\psi = 0$

$\psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$ Weyl spinors

- Dirac eq factorizes for $M=0$ into:
  $i\bar{\sigma}\cdot\partial\psi_L = 0$, $i\sigma\cdot\partial\psi_R = 0$
- Couple to EM $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$

plane wave solutions, for $\vec{p} = p\hat{3}$

$\psi_R = \begin{pmatrix}1\\0\end{pmatrix}e^{-iEt+iEx^3}$ with $E>0$ distruction of right handed electron. $u_R(p)$

$\psi_L = \begin{pmatrix}0\\1\end{pmatrix}e^{+i|E|t+i|E|x^3}$ with $E<0$ Creation of left handed positron. $v_R(p)$

$\psi_L = \begin{pmatrix}0\\1\end{pmatrix}e^{-iEt+iEx^3}$ $E>0$ destruction of left-handed electron $u_L(p)$

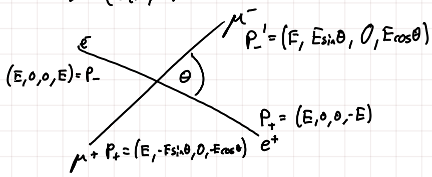
$\psi_L = \begin{pmatrix}1\\0\end{pmatrix}e^{i|E|t+i|E|x^3}$ $E<0$ creation of right handed positron $v_L(p)$

$$j^\mu = \bar{\psi}\gamma^\mu\psi = \psi_L^\dagger\bar{\sigma}\psi_L + \psi_R^\dagger\sigma^\mu\psi_R$$

Helicity conservation e_R^- can scatter into e_R^- or annihilate e_L^+, nothing else.

CM frame

$q = (E_{cm}, \vec{0})$



EM

$$\vec{\epsilon}_\pm = \frac{1}{\sqrt{2}}(1, \pm i, 0) \qquad \vec{\epsilon}_\pm' = \frac{1}{\sqrt{2}}(\cos\theta, \pm i, -\sin\theta)$$

$$|M(e_R^- e_L^+ \to \mu_R^-\mu_L^+)|^2 = |M(e_L^- e_R^+ \to \mu_L^-\mu_R^+)|^2 = e^4(1+\cos\theta)^2$$

$$|M(e_R^- e_L^+ \to \mu_L^-\mu_R^+)|^2 = |M(e_L^- e_R^+ \to \mu_R^-\mu_L^+)|^2 = e^4(1-\cos\theta)^2$$

cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2E_{cm}^2}(1+\cos^2\theta)$$

$$\Rightarrow \sigma = \frac{4\pi\alpha^2}{3E_{cm}^2}$$

peaks at $\cos^2\theta = 1$

we Average over 4 initial spin states including $e_R^+e_R^-$ and $e_L^+e_L^-$
Sum over 2 final spin states

Sujested exercises

PeskIn 8.1

$e^+e^- \to$ hadrons

For $E_{cm} \gg$ Light meson masses quark model predicts the following modification

1. sum over light quark species uds (c, b)
2. $Q_\mu = -1 \to Q_f = \begin{cases} 2/3 & u, c \\ -1/3 & d\,s\,b \end{cases}$

$M \sim Q_f \Rightarrow \sigma \sim Q_f^2$

3. sum over final colour states

Result is:

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_f 3Q_f^2 \frac{4\pi\alpha}{3E_{cm}^2}$$

$$\frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f 3Q_f^2 = \begin{cases} 2 & u,d,s \\ 3\frac{1}{3} & u,d,s,c \\ 3\frac{2}{3} & u,d,s,c,b \end{cases}$$

- Strong interaction can be nearly neglected at this level

Deep inelastic Electron scattering

probing proton structure (quarks, gluons, anti-quarks)

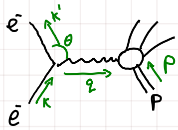
Elastic: $ep \to ep$

Inelastic: break proton, produce hadrons
High momentum transfer

Deep: Probe internal structure
Total mass of final hadrons $\gg m_p$

- Observation shows DIES is well approximated by $ep \to$ hadrons
- Also: p is not just a uud bound state

$\to$ Parton Distribution Functions (PDFs)



Define W = mass of final hadronic system

$$W^2 = (p+q)^2 = m_p^2 + 2P_q + q^2$$

Assume large momentum transfer

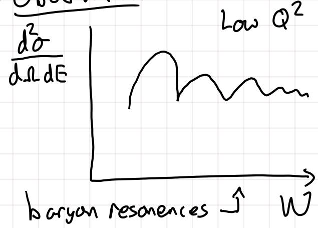
$|q|^2 \gg m_p^2$ i.e. $m_p^2 \to 0$

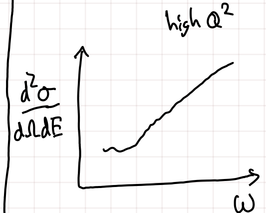
Define $Q^2 = -q^2$

Since q is space like $\Rightarrow Q^2 > 0$

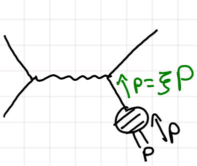
Large $Q^2 \Leftrightarrow$ large momentum transfer to proton

Observation:





clue for interpretation: Parton model

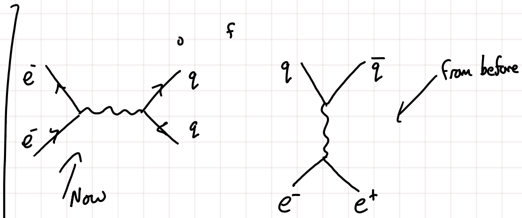


Assume that the parton is a spin $\frac{1}{2}$ quark carrying a flavour label f (or $\bar{f}$ for an anti-quark)

Def: $f_i(\xi)d\xi$ = probability of finding parton of type i carrying momentum fraction $\in (\xi, \xi + d\xi)$

- Sum rule $\int_0^1 d\xi \sum_i f_i(\xi) = 1$ — sum over $f, \bar{f}$

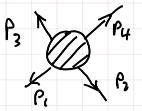
$$\sigma(e^-p \to e^-X) = \int_0^1 d\xi \sum_f \left[f_f(\xi) + f_{\bar{f}}(\xi) \right] \times \sigma(e^-q(\xi P) \to e^-q)$$



- These have the same M (invariant matrix element)

Crossing Symmetry

General $2 \to 2$ Scattering



Orient all momenta outwards
$P_1 + P_2 + P_3 + P_4 = 0$

M only depend on Lorentz invariant

Mandelstam invariants

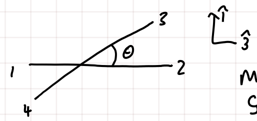
$p_i^2 = m_i^2$

Def: $(P_1+P_2)^2 = s$, $(P_1+P_3)^2 = t$, $(P_1+P_4)^2 = u$

These are not even independent due to momentum conservation

$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

CM frame



massless scattering

$P_1 = (-E, 0, 0, -E)$ $P_3 = (E, E\sin\theta, 0, E\cos\theta)$

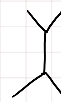
$P_2 = (-E, 0, 0, E)$ $P_4 = (E, -E\sin\theta, 0, -E\cos\theta)$

$s = (2E)^2 = E_{cm}^2$ $\quad s + t + u = 0$

$t = -2E^2(1-\cos\theta)$ $\quad u = -2E^2(1+\cos\theta)$

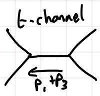
Intermediate state:

S-channel



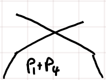
$P_1 + P_2 \sim \frac{1}{s - M_r^2 + iM_r\Gamma_R}$; $|M|^2 \sim \frac{1}{s^2} = \frac{1}{E_{cm}^4}$

t-channel



$|M|^2 \sim \frac{1}{t^2} = \frac{1}{E_{cm}^4 \sin^4(\frac{\theta}{2})}$

$\Rightarrow$ strong peak in forward dir $\theta = 0$



$|M|^2 \sim \frac{1}{u^2} = \frac{4}{E_{cm}^4 (1+\cos\theta)^2}$

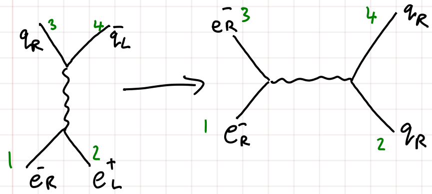
Strong peak in backwards dir $\theta = \pi$

$$|M(e^-_R e^+_L \to q_R \bar{q}_L)|^2 = Q_f^2 e^4 (1+\cos\theta)^2 = 4Q_f^2 e^4 \frac{u^2}{s^2} \quad ①$$

$$|M(e^-_R e^+_L \to q_L \bar{q}_R)|^2 = Q_f^2 e^4 (1-\cos\theta)^2 = 4Q_f^2 e^4 \frac{t^2}{s^2} \quad ②$$

Lorentz invariant

①



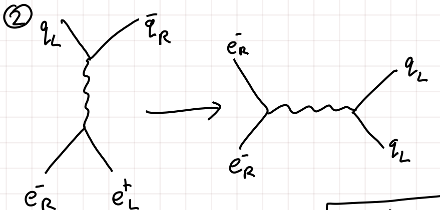
$P_1 \to P_1$, $P_2 \to P_3$, $P_3 \to P_4$, $P_4 \to P_2$

or: $s \to t$, $t \to u$, $u \to s$

$\Rightarrow$ (for ①)

$$|M|^2 = 4Q_f^2 e^4 \frac{s^2}{t^2}$$

②



$$\Rightarrow |M|^2 = 4Q_f^2 e^4 \frac{u^2}{t^2}$$

Consistency check

$|M|^2 \sim u^2 \sim (1+\cos\theta)^2 \Rightarrow |M|^2 = 0$

for $\theta = \pi$ (no backwards Scattering)

- Makes sense with Spin conservation

Total cross section:

$$\sigma(eq\to eq) = \frac{1}{2E}\frac{1}{2E}\frac{1}{8\pi}\int\frac{d\cos\theta}{2}\frac{1}{4}\sum_{spins}|M(\bar{e}q\to\bar{e}q)|^2$$

$$\frac{d\sigma}{d\cos\theta} = \pi\frac{Q_f^2\alpha^2}{s}\frac{s^2+u^2}{t^2}$$

- can use $t=-2E^2(1-\cos\theta)$

$\Rightarrow dt = \frac{1}{2}s\,d\cos\theta$

$$\frac{d\sigma}{dt}(eq\to eq) = \frac{2\pi Q_f^2\alpha^2}{s^2}\frac{s^2+u^2}{t^2}$$

Back to D.I.S Hats ^ refer to partonic kinimatics

$$\sigma(e^-p\to e^-X) = \int d\xi\int d\hat{t}\sum_f\left[f_f(\xi)+f_{\bar{f}}(\xi)\right]\frac{2\pi Q_f\alpha^2}{\hat{s}^2}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Kinematics of DIS

introduce Q^2, x, y

$\hat{t} = q^2 = -Q^2$ momentum transfer

$\hat{s} = (k+p)^2 = 2k\cdot p = 2k\xi P$ (recall we neglect proton mass)

$= \xi S$

define $y = \frac{2P\cdot q}{2P\cdot K}$ - In proton rest frame $y = \frac{q_0}{K_0}$ = fraction of initial e^- energy transfered to proton

$$y = \frac{2\xi P\cdot q}{2\xi P\cdot K} = \frac{2p(k-k')}{2pk} = \frac{\hat{s}+\hat{u}}{\hat{s}} = 1+\frac{\hat{u}}{\hat{s}}$$

$$\Rightarrow \frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2} = 1+(1-y)^2$$

Masslessness of the final state quark means $0=(p+q)^2$

$= 2p\cdot q + q^2 = 2\xi P\cdot q - Q^2$

define:

$x = \frac{Q^2}{2P\cdot q}$ Final quark massless $\Rightarrow \xi = x$

Can measure x to find ξ

Relation

$Q^2 = xys \Rightarrow dQ^2 = xs\,dy$

$$\frac{d^2\sigma}{d\xi d\hat{t}} = \sum_f\left[f_f(\xi)+f_{\bar{f}}(\xi)\right]2\pi Q_f\alpha^2\frac{(1+(1-y)^2)}{Q^4}$$

$d\hat{t} = dQ^2 = xs\,dy$

$$\Rightarrow \frac{d^2\sigma}{dy\,dx} = \sum_f xQ_f^2\left[f_f(\xi)+f_{\bar{f}}(\xi)\right]\frac{2\pi\alpha^2}{Q^4}(1+(1-y)^2)$$ with $0<x<1$, $0<y<1$

Bjorken scaling

On general grounds, we expected the elementary QED cross section

$$\frac{d^2\sigma}{dx\,dy}(e^-p\to e^-X) = \underbrace{f_2(x,Q)}_{\text{unknown form factor.}}\frac{2\pi\alpha^2 s}{Q^4}(1+(1-y)^2)$$

Parton model $\Rightarrow f_2(x,Q) = F_2(x)$

$$= \sum_f Q_f^2 x\left[f_f(x)+f_{\bar{f}}(x)\right]$$

- independence of F_2 on Q^2 is called Bjorken scaling

Q^2 dependence is not fully constant but slow logarithmic

The Gluon

$f_f(x)$, $f_{\bar{f}}(x)$ are parton distribution functions (pdfs) of the proton | (or any other hadron)

- Measurment of pdfs: Quark model

$$F_2(x) = \sum_f Q_f^2 x\left[f_f(x)+f_{\bar{f}}(x)\right]$$

$$= \frac{4}{9}x\,f_u(x) + \frac{1}{9}x\,f_d(x)$$

- preserving charge and Isospin, any extra quark must appear in $q\bar{q}$ pairs and satisfy spin rules

$$\int_0^1 dx\left[f_u(x)-f_{\bar{u}}(x)\right] = 2$$

$$\int_0^1 dx\left[f_d(x)-f_{\bar{d}}(x)\right] = 1$$

$$\int_0^1 dx\left[f_s(x)-f_{\bar{s}}(x)\right] = 0$$

- These cannot be derived from first princibles, only experiment
- DIS of $e^-p\to e^-X$ gives only one compination
- Get an independent combination from scattering on a deuterium target

Isospin Rotation:

Proton and neutron pdfs are related:

$f_u^{(n)}(x) = f_d(x)$ $f_d^{(n)}(x) = f_u(x)$

$f_{\bar{u}}^{(n)} = f_{(\bar{d})}(x)$ $f_{\bar{d}}^{(n)}(x) = f_{\bar{u}}(x)$

$\Rightarrow F_n^{(2)}(x) = \frac{4}{9}x f_d(x) + \frac{1}{9}x f_u(x)$

$\Rightarrow$ can fix $f_u(x)$ and $f_d(x)$

More info from DIS by nutrinos. (Weak interactions)

Sum rules: do not require

$f_f(x) = f_{\bar{f}}(x)$

Ex. quantum fluxuation $p \leftrightarrow \Lambda^0 + K^+$

uud uds $u\bar{s}$

- relativly strong f_s at higher x compared to $f_{\bar{s}}$

$p \leftrightarrow n + \pi^+$

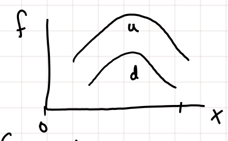
uud udd $u\bar{d}$

- relativly higher $f_{\bar{d}}$ compared to $f_{\bar{u}}$

ud can form a low energy $I=0$ $s=0$ leaves the other u with higher momentum

$\rightarrow$ relativly higher f_u compared to f_d

Valence quark pdfs



Sea quarks:

Found to diverge as $x\to 0$

- divergences cancle between any $f,\bar{f}$ for the sum rules to hold.

Total sum rule: $\int_0^1 dx\, x\sum_{i=f,\bar{f}} f_i(x) = 1$

- We observe that:

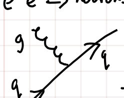
$$\frac{P_{q+\bar{q}}}{P} = \int_0^1 dx\, x\sum_f\left[f_f(x)+f_{\bar{f}}(x)\right] \approx 0.5$$

What is missing? Gluons!

- Posit that their is a field responsible for strong interaction (with particles!)

If gluons exist $\Rightarrow$

$e^+e^-\to$ hadrons should also produce gluons radiated from outgoing $q\bar{q}$



Prediction of observed 3-Jet events

- Found at $E_{cm} = 30$ GeV

4 jet events observed at 91 GeV (at Desy)

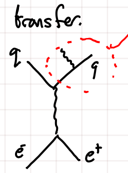
Model Gluons as photons

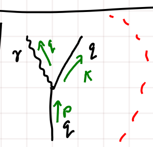
(Spin-1) could try spin-0/2 only this works

- coupling to q as γ couples to e

For Jets, assume colinear splitting i.e. small momentum transfer.

$e^+e^-\to q\bar{q}\gamma$





can be slightly off mass shell?

$P \approx (E,0,0,E)$

let z be momentum fraction transfered to γ

$$q = \left(zE, q_\perp, 0, zE - \frac{q_\perp^2}{2zE}\right)$$

so that $q^2 = 0 + \mathcal{O}\left(\left(\frac{q_\perp}{E}\right)^4\right)$

$$K = \left((1-z)E, -q_\perp, 0, (1-z)E - \frac{q_\perp^2}{2(1-z)E}\right)$$

$\Rightarrow K^2 = 0 + \mathcal{O}(q_\perp^4)$

$$P = \left(E,0,0,E-\frac{q_\perp^2}{2z(1-z)E}\right)$$

$$P^2 = \frac{q_\perp^2}{z(1-z)} + \mathcal{O}(q_\perp^4)$$

$$M(q_R \to \gamma q_R) = Q_f e \langle q_R(k)| j^\mu | q_R(p)\rangle \epsilon^*_\mu(q)$$

For Right handed only:

$$j^\mu = \psi_R^\dagger \sigma^\mu \psi_R$$

$$= Q_R e\, u_R^\dagger(k)\sigma^\mu u_R(p)\,\epsilon^*_\mu(q)$$

- Spinors $u_R(p) = \sqrt{2E}\begin{pmatrix}1\\0\end{pmatrix}$

$$u_R(k) = \sqrt{2(1-z)E}\begin{pmatrix}1\\ -\frac{q_\perp}{2(1-z)E}\end{pmatrix}$$

To $O(q_\perp)$:

photon polarizations $\quad$ So $\epsilon\cdot q = 0$

$$\epsilon_R = \frac{1}{\sqrt2}\left(0,1,i,-\frac{q_\perp}{2E}\right), \quad \epsilon_L = \frac{1}{\sqrt3}\left(0,1,-1,-\frac{q_\perp}{2E}\right)$$

$$u_R^\dagger \sigma\cdot\epsilon_L^* u_R = -\frac{2E}{\sqrt{1-z}}\left(1, -\frac{q_\perp}{2(1-z)E}\right)\frac{1}{\sqrt2}\left[\sigma^1 + i\sigma^2 - \frac{q_\perp}{2E}\sigma^3\right]\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \sqrt2\, q_\perp \frac{\sqrt{1-z}}{z}$$

$$u_R^\dagger\sigma\cdot\epsilon_R^* u_R = \cdots = \sqrt2\, q_\perp \frac{\sqrt{1-z}}{2(1-z)}$$

$$\sum_\epsilon |M|^2 = 2Q_f^2 e^2 q_\perp^2 \frac{1}{z^2(1-z)}\left(1+(1-z)^2\right)$$

$$\sigma(e^+e^- \to q\bar q\gamma) = \frac{1}{2E_A 2E_B 2}\int d\Pi_3\, |M(e^+e^-\to q\bar q\gamma)|^2$$

$$\int \Pi_3 = \int \frac{d^3\overset{\text{from } \bar q}{\bar p}\, d^3k\, d^3q}{(2\pi)^9\, 2\bar p\, 2k\, 2q}(2\pi)^4\delta^{(4)}(Q-\bar p - q - k)$$

$$k = p - q \Rightarrow d^3k = d^3p$$

$$k = (1-z)p + O(q_\perp)$$

$$q = zp + O(p_\perp)$$

$$d^3q = dq^z\, d^2q_\perp = p\, dz\, \pi\, dq_\perp$$

$$\sigma(e^+e^-\to q\bar q\gamma) = \frac{1}{2E_A 2E_B 2}\int d\Pi_2\, |M(e^+e^-\to q\bar q)|^2$$

$$\times \int \frac{dz\,\pi\, dq_\perp}{(2\pi)^2\, 2z(1-z)}\left|\frac{1}{p}\right|^2 |M(q\to\gamma q)|^2$$

$$= \sigma(e^+e^-\to q\bar q)\int \frac{dz\, dq_\perp^2}{16\pi^2 z(1-z)}\left(\frac{z(1-z)}{q_\perp}\right) 2Q_f^2 e^2 q_\perp^2 \frac{1}{z^2(1-z)}\left(1+(1-z)^2\right)$$

- remaining integrals diverge as z or $q_\perp \to 0$ $\quad$ - soft and co-linear divergences.

Weizsacker-Williams distribution

$$\sigma(A\to B+f+\gamma) \approx \sigma(A\to B+f)\int dz \int \frac{dq_\perp}{q_\perp}\frac{Q_f^2\alpha}{\pi}\frac{1+(1-z)^2}{z}$$

$$\sigma(A+f\to B+\gamma)\approx \sigma(A+f\to B)\ \text{''} \qquad \text{''}$$

For f = electrons, cut of these integrals at the scale of $m_e \le q_\perp \le E_{cm}$

$$\frac{m_e}{E_{cm}} \le z \le 1$$

- we get $\frac{2\alpha}{\pi}\log^2\frac{E_{cm}}{m_e}$ for $E_{cm} >> m_e$

$e^+e^- \to 3$ jets

Let g_s = strong interaction coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

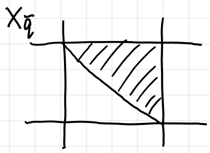
$$\sigma(A\to B+q+g)\approx\sigma(A\to B+f)\int dz\int\frac{dq_\perp}{q_\perp}\overbrace{\frac43}^{\text{added colour}}\frac{\alpha_s}{\pi}\frac{1+(1-z)^2}{z}$$

Relax co-linear assumption

$E_q, E_{\bar q}, E_g$ $\quad E_{cm} = Q = E_q + E_{\bar q} + E_g$

Define fractions

$$x_q = \frac{2E_q}{Q}, \quad x_{\bar q} = \frac{2E_{\bar q}}{Q}, \quad x_g = \frac{2E_g}{Q}$$



Co-linear at boundaries (are favourite)

$$\sigma(e^+e^-\to q\bar q g) = \sigma(e^+e^-\to q\bar q)\int dx_q\, dx_{\bar q}\,\frac{2\alpha_s}{3\pi}\,\frac{x_q^2+x_{\bar q}^2}{(1-x_q)(1-x_{\bar q})}$$