PHY 291- Computational Physics
Homework 4
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# Euler Method vs. Euler Cromer Method

**Abstract:** Various mathematical and physical ideas were used to show the calculation differences between the euler method and euler cromer method. These methods were compared within matlab by using numerical observations and graphical comparisons to depict the differences. The results of the experiment are shown below.

**Introduction:** The Euler schemes are compared within this experiment. These schemes have both been observed to have an error proportional to your time-step, deltaT. However, the Euler method is an energy increasing method. This means that as you iterate over time, the sequence of solutions you produce will have increasing numerical oscillations. Since each updated solution will have more and more oscillations, the energy of the system will increase artificially and generate a solution that is less and less accurate. In contrast, the Euler-Cromer doesn't increase the energy of the solution and hence solutions are more accurate. While still having errors, this method is more accurate over a longer time-iteration range. It should be noted that the scheme does technically contribute a little artificial oscillations. But it can be shown that the change in energy of the system for this scheme is bounded and when averaged over time, it's essentially 0. The boundedness of the energy is what makes the scheme much more stable in comparison to standard Euler. The differences in the formulas are shown within **Formula 1.**

**Formula 1:** Euler vs. Euler-Cromer Method

$$\text{Euler Method}$$

$$V_{i+1} = V_i - (A \cdot X_i \cdot \Delta t)$$

$$X_{i+1} = X_i + (V_i \cdot \Delta t\ )$$

$$\text{Euler Cromer Method}$$

$$V_{i+1} = V_i - (A \cdot X_i \cdot \Delta t)$$

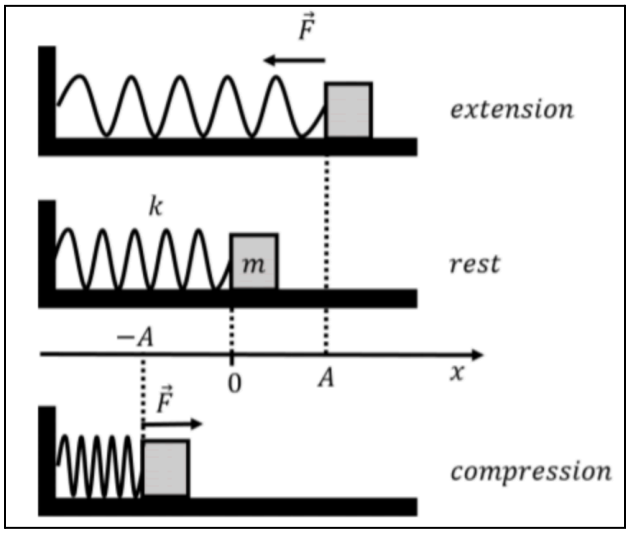
$$X_{i+1} = X_i + (V_{i+1} \cdot \Delta t)$$

**Methods:** To begin, the problems must first be initialized.

1. For this experiment, a block with a mass 1 kg was connected to a spring attached to a fixed surface. A picture was provided within **Figure 1** for better understanding. The accelerations within **Formula 1** were derived from the spring constant divided by the mass of the block. The spring constant “K” was chosen to be 10. Assuming the

scenarios were performed in a vacuum with no air resistance, the systems were extended 0.5 meters forward with no initial velocity. The initial position would be at “A” represented within **Figure 1.** These values were all chosen to demonstrate these error propagations. Within matlab, the for loop function was used to calculate the position, velocity, time, and energy of the object using timesteps**.** The smaller the timestep, the more accurate the results. The time step chosen can be derived from the period divided by the number of iterations. Finally, energy equations were added to these loops to compare the systems final total energies. These energy formulas are given within **Formula 2.**

**Figure 1:** Initial Conditions picture




**Formula 2:** Energy of Systems

$$Potential\ Energy_{i+1} = \frac{1}{2} K X_i^2$$

$$Kinetic\ Energy_{i+1} = \frac{1}{2} M V_i^2$$

**Results:**

1. Comparing these experiments, it was hypothesized that the euler method would return a system of increasing energy and the cromer method would not. This comparison is depicted within **Graph 1** and **Figure 2. Graph 1** demonstrated how the velocity changes with time for both methods. This graph shows that the euler method's velocity seems to increase without bound. Also, it can be observed within **Figure 2,** that the final position,

velocity, and energy of the system for the euler method increased to unrealistic amounts. While the euler cromer method stayed within a realistic domain for these results.

**Graph 1:** Comparison of Euler vs Euler Cromer

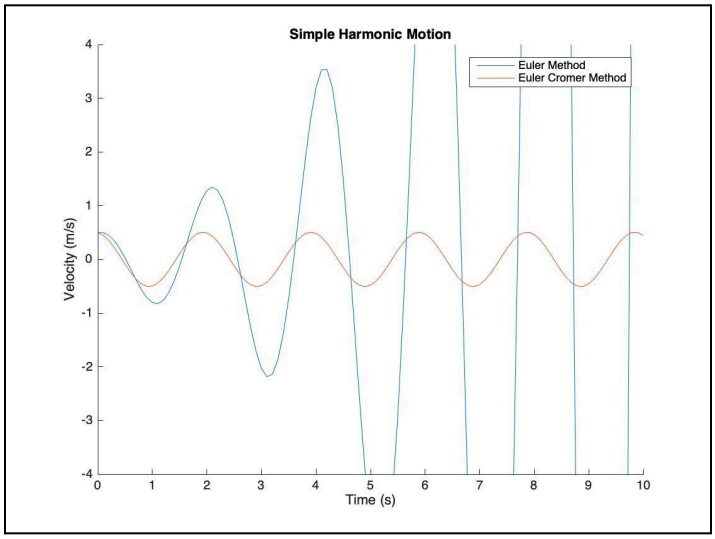



**Figure 2:** Final numerical results

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The initial x position with the euler method is 0.500000 meters
The Final x position with the euler method is 25.660576 meters
The initial x position with the euler cromer method is 0.500000 meters
The Final x position with the euler cromer method is 0.498067 meters
The initial Velocity with the euler method is 0.000000 m/s
The Final Velocity with the euler method is 157.273398 m/s
The initial Velocity with the euler cromer method is 0.000000 m/s
The Final Velocity with the euler cromer method is -0.036115 m/s
The Final energy with the euler method is 14236.169773 Joules
The Final energy with the euler cromer method is 1.366782 Joules
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**Conclusion:** This method presents the basic underlying physics of simple harmonic motion using euler's method and euler cromer method. These results reveal the differences between these methods and their possible applications. While these methods do return results, both are still fundamentally flawed, and there exists better tools to model physical systems.