PHY 291- Computational Physics
Homework 7
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05/09/2022

Ising Model

Abstract: Ferromagnetism arises when a collection of atomic spins align such that their associated magnetic moments all point in the same direction, yielding a net magnetic moment which is macroscopic in size. The simplest theoretical description of ferromagnetism is called the Ising model.

Introduction: For this experiment, we will explore the essence of the ising model. The Ising model plays a central role in the theory of phase transitions. It is a mathematical model of ferromagnetism. For example, iron can be magnetized in a magnetic field, but if heated, it loses magnetization beyond Curie temperature. Within Formula 1 we observe the Hamiltonian representation of spin interaction within a magnetic field for a 1 dimensional model. It must be known that J>0 is the strength of exchange interaction and that all spins "S" given within the formula can be given a value of +1 or -1. This formula tells us that the spin "S_i" interacts with its neighboring spin "S_{i+1}", the subtraction of the second hamiltonian tells us that the spin is also affected by the applied magnetic field. This Hamiltonian can further be used to describe how the heat of a system within 2 dimensions, changes with respect to the spins. Using **Formula 1,** we arrive at **Formula 2,** which will be used within our experiment to decide the changes within our system.

Formula 1: 1-D Hamiltonian for spin interaction within a field

$$\boxed{\Delta E = H = -J\sum_{i=1}^{N} S_i S_{i+1} - H\sum_{i=1}^{N} S_i}$$

All formulas created using iMathEQ

Formula 2: Probability used to except a spin

$$\boxed{e^{\frac{\Delta E}{kT}}}$$

All formulas created using iMathEQ

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Methods: To begin, the problems must first be initialized.

1. For this experiment, we were tasked to develop a model for the 2D Ising model on a square lattice. We were also asked to calculate the magnetization as a function of time for a given temperature. Finally we were asked to plot both magnetization and energy as a function of temperature, over a range of temperatures which spans the critical T. To accomplish these tasks within matlab we created the code given within **Figure 1.** First we created a for loop which ranged from 1-100 for temperature, contained within this for loop was a 10x10 matrix "M", as well as the denominator of the probability function as a function of the temperature divided by 10. This allowed us to increase the temperature of the system with respect to time. We then created three more for loops contained within the previous for loop. Their ranges were 1-1000, 1-10, and 1-10, respectively. Within these three for loops we created a second spin particle "F1". We also created the hamiltonian summation of the change in energy. We set "J" equal to 2 and multiplied the first matrix times the spin particle "F1". We then used if statements to decide whether or not the energy of the system would flip negative based on whether or not the change in energy was negative, or if **Formula 2** was greater than a random number ranging from 0-1. We then created magnetization as a function of temperature and time by summing the first matrix "M". We also created a second spin particle "F2" which used the updated "M" matrix. We then created a final energy function, setting "J" equal to ½ and multiplying the matrix "M" times the second spin particle "F2". Finally we plotted the magnetization and energy functions.

Figure 1: Matlab code used for Ising model

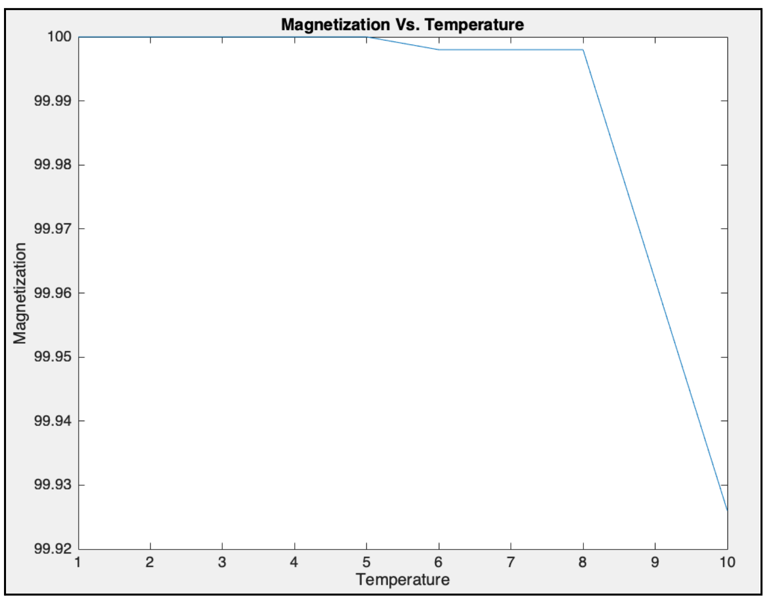
```
clc
close all
clear all

for temp=1:100
    M=ones(10);
    TT=temp/10;
    for t =1:1000
        for j=1:10
            for k=1:10
                F1=circshift(M, [0 1])+circshift(M, [0 -1])+circshift(M, [1 0])+circshift(M, [-1 0]);
                delE=2*M(j,k)*F1(j,k);
                if delE<0
                    M(j,k)=-M(j,k);
                    continue
                end
                if rand < exp(-delE/TT)
                M(j,k)=-M(j,k);
                end
            end
        end
       Mag(temp,t)=sum(sum(M));
       F2=circshift(M, [0 1])+circshift(M, [0 -1])+circshift(M, [1 0])+circshift(M, [-1 0]);
       Energy(temp,t)=sum(sum(-0.5*M.*F2));
    end
      magtrue(temp)=sum(Mag(temp,:))/1000;
      Etrue(temp)=sum(Energy(temp,:));
end
plot(magtrue)
figure(2)
plot(Etrue)
```

Results:

1. For this experiment, we created 3 graphs to calculate the magnetization as a function of time for a given temperature range, small **(Graph 1)**, medium **(Graph 2)**, and large

(Graph 3). It may be noticed that for the small temperature range, 1-10, the magnetization never goes below 99. This is most likely due to the fact that the energy of the system never increased enough to flip the spins of the system. For the medium and large graphs we can observe that as the temperature reached approximately 20, the magnetization of the system became zero. This is likely because the energy of the system increased enough to flip the spins of the system, therefore the magnetization decreased to nothing. The critical temperature point for this system is likely somewhere near 20. It can also be observed that the magnetization dips below 0, this is not accurate because we cannot have negative magnetization. It likely dipped below 0 because at those points the magnetization of the system flipped rapidly and the program counted the flip to be negative. Finally we created a plot that showed the increase of energy of the system with respect to temperature. Given that temperature increase means energy increase, **Graph 4** is an accurate representation of this property. The graph also seems to top out at a certain energy level. This is likely due to it being an isolated system, and conservation of energy takes place given **Formula 2.**

Graph 1: Magnetization with respect to temperature (Small)



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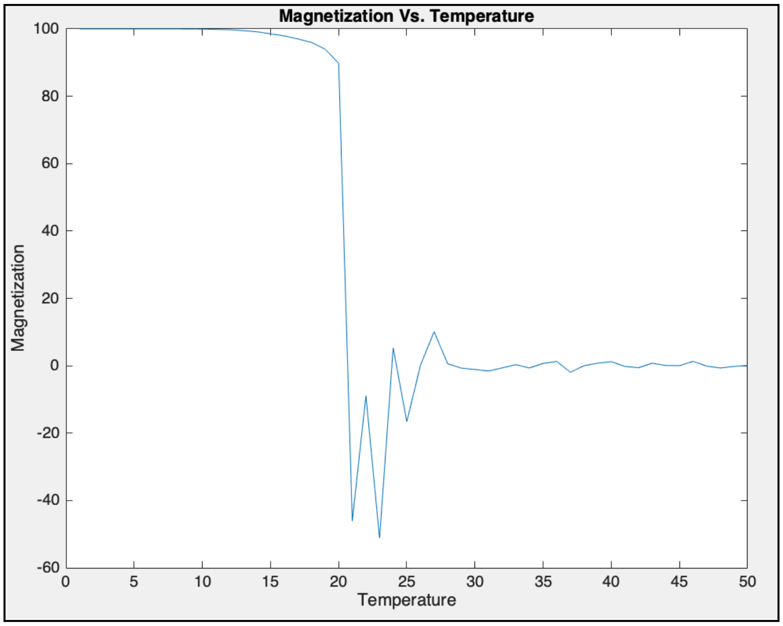
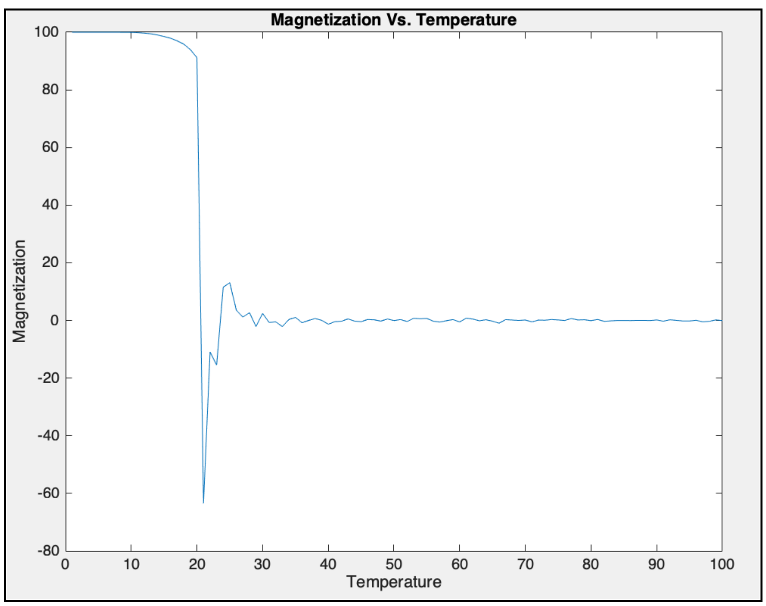
Graph 2: Magnetization with respect to temperature (Medium)



Graph 3: Magnetization with respect to temperature (Large)



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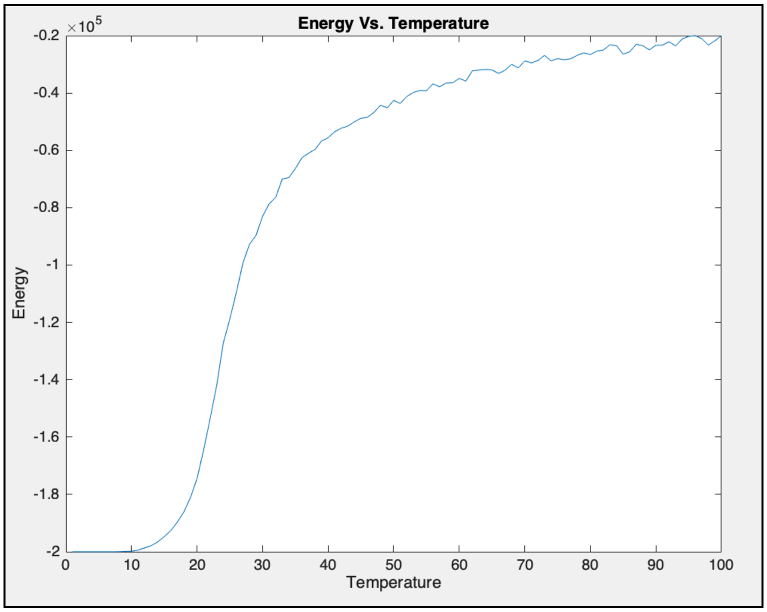
Graph 4: Energy of the system with respect to temperature



Conclusion: This experiment allowed us to explore the phenomena of magnetization and energy with assistance from the Ising model. We learned about the model of spins and how they are the basis of which the phenomena magnetization is modeled off of. This also allowed us to observe the correlation between the energy within a system and the spins of particles. As the energy of a system increased the magnetization or spin alignment tended to decrease. A real world representation of this would be how the magnetization of an object disappears as it is heated past its critical point. This is due to the model of spins flipping or aligning within the object, therefore the object loses magnetization. While there were many errors within the coding and physical concepts, the Ising model aided us to understand energy, magnetization and spins a little better.