PHY 291- Computational Physics
Homework 5
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N^{th} Body orbits

Abstract: Orbit, the path of a body revolving around an attracting center of mass, as a planet around the Sun or a satellite around a planet. In the 17th century, Johannes Kepler and Isaac Newton discovered the basic physical laws governing orbits; in the 20th century, Albert Einstein's general theory of relativity supplied a more exact description. In this experiment we investigate these laws using matlab and various mathematical concepts to model the orbits of planets. The results of this experiment are shown below.

Introduction: For this experiment, we were first tasked to find the orbits for a one planet-one sun system, then we were asked to find the orbits for a one sun and 2 planet system, and finally the sun, Jupiter and 3 asteroids. Random initial parameters were assigned to each of the 3 exercises. To complete these tasks, we revisit the Euler-Cromer method from the past assignment. From the past assignment, we learned that the Euler-Cromer is used to propagate position-velocity and acceleration equations. This method does not increase the energy of a given system and hence gives accurate solutions. In partnership with the euler cromer method, we choose Newton's Law of Gravitation to bridge the gap. Newton's Law of Gravitation states that the force of gravitational attraction between any two massive bodies is proportional to their masses and inversely proportional to the square of the distance between their centers. This relationship can be observed in **Formula 1.** Next we combined the law of gravitation with the Euler-Cromer equations shown in **Formula 2.** This was achieved by deriving a planet's experienced acceleration from the force of gravitation. With this acceleration we were able to complete the Euler-Cromer equations given in **Formula 2.** These equations were then implemented within matlab, using For loops to propagate these equations through time.

Formula 1: Newton's Law of Gravitation

$$\mathrm{G} = 6.672 \cdot 10^{-11} \ \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$$

$$\vec{F}_i = \frac{-G \cdot M_i \cdot M_j}{R^2{}_{ij}} \hat{R}_{ij}$$

All formulas created using iMathEQ

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Formula 2: Euler-Cromer method

$$V_{i+1} = V_i - (a_i \cdot X_i \cdot \Delta t)$$

$$X_{i+1} = X_i + \left(V_{i+1} \cdot \Delta t\right)$$

All formulas created using iMathEQ

Methods: To begin, the problems must first be initialized.

1. For the first exercise, we were tasked to find the orbits for a one sun and one planet system. Mass and distance matrices were created with values synonymous to our own solar systems values. The components of their initial positions were calculated using random values of theta. This matlab code is demonstrated within **Figure 1.** Next, the initial velocities were determined using the centripetal acceleration formula equated to the force of gravitation. This derived formula can be observed within **Formula 3.** Using the Euler-Cromer equation we propagated the orbits within matlab using For loops and indice If statements.

Figure 1: Matlab Code for Random Initial Conditions

$$\mathrm{theta} = \mathrm{rand}(2,1) * 2 * \mathrm{pi};$$

$$\mathrm{X}(:,1) = \mathrm{abs}(\mathrm{r}. * \cos(\mathrm{theta}));$$

$$\mathrm{Y}(:,1) = \mathrm{abs}(\mathrm{r}. * \sin(\mathrm{theta}));$$

All formulas created using iMathEQ

Formula 3: Initial Velocity Derivation

$$\mathrm{V}_1 = \sqrt{\frac{\mathrm{G} \cdot \mathrm{M}_{\mathrm{sun}}}{\mathrm{R}_{\mathrm{planet}}}}$$

All formulas created using iMathEQ

2. For the second exercise, we were tasked to find the orbits for a one sun and two planet system with a smallish inner planet and a large outer planet. Additionally, we were asked to explore how the orbit of the smaller planet varies as the mass of the large planet

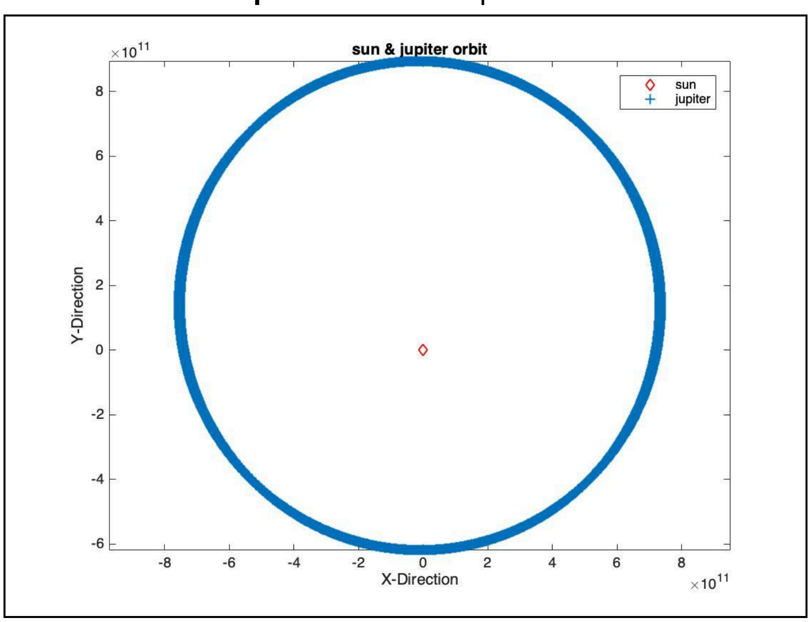
changes. Given the additional planet, the planets experience more than 1 force. Therefore, the acceleration experienced by both planets was subtracted from the velocity Euler-Cromer equation given in **Formula 1.** Mass and distance matrices were created with values synonymous to our own solar systems values. The components of their initial positions and velocity were calculated the same way demonstrated in **Figure 1** & **Formula 3.** Using the Euler-Cromer equation we propagated the orbits within matlab using For loops and indice If statements. Given the additional planet, The for loop indices were made larger.

3. Finally the last exercise, we were tasked to find orbits for a sun, Jupiter and three small asteroids. Additionally we were asked to place the initial orbits of the asteroids into various places and look for resonances. orbital resonance occurs when orbiting bodies exert regular, periodic gravitational influence on each other, usually because their orbital periods are related by a ratio of small integers. Within matlab, the asteroids were created using a for loop to assign random initial parameters. These initial parameters were run through a second for loop, with the exact same approach as step 1 & 2.

Results:

1. For exercise 1, the program was executed and the results can be observed within **Graph 1.** With random initial parameters, some program executions return graphs that seem to run away. This could be due to an unreasonable initial velocity or position of Jupiter. With too large of an initial velocity or position the planet will overcome the force of gravity and fly off. There are also cases in which certain values of theta return complex velocity values. This was fixed by implementing the absolute value function within matlab.

Graph 1: Sun and Jupiter Orbit



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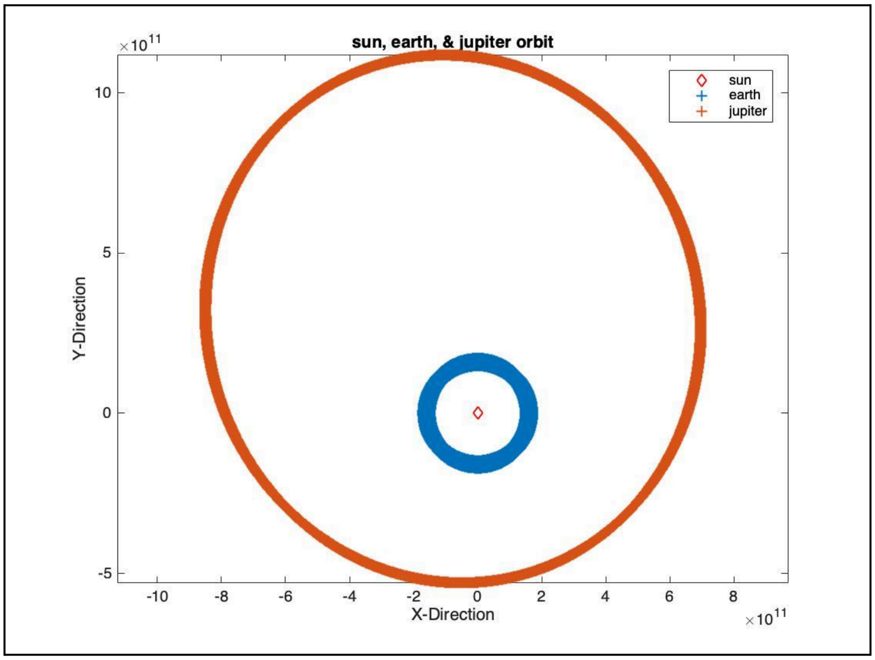
2. For exercise 2, the program was executed and the results can be observed within **Graph 2.** These results are synonymous with exercise 1. The difference occurs with the addition of another planet. With the increase of the outer planets mass, the outer planet tends to slingshot the inner planet more frequently. This is due to the larger gravitational force experienced due to higher mass. It was observed that to maintain an orbit, the inner and outer planet must not have mass differences higher than plus or minus 10^2 .

Graph 2: Sun, Earth, Jupiter Orbit



3. For exercise 3, the program was executed and the results can be observed within **Graph 3.** These results are synonymous with exercise 1. The difference is observed with the asteroid orbits. We were asked to place the initial orbits of the asteroids into various places and look for resonances. Resonance is clearly observed within this simulation. Some of the asteroids experience longer periods of orbit due to the gravitational interaction.

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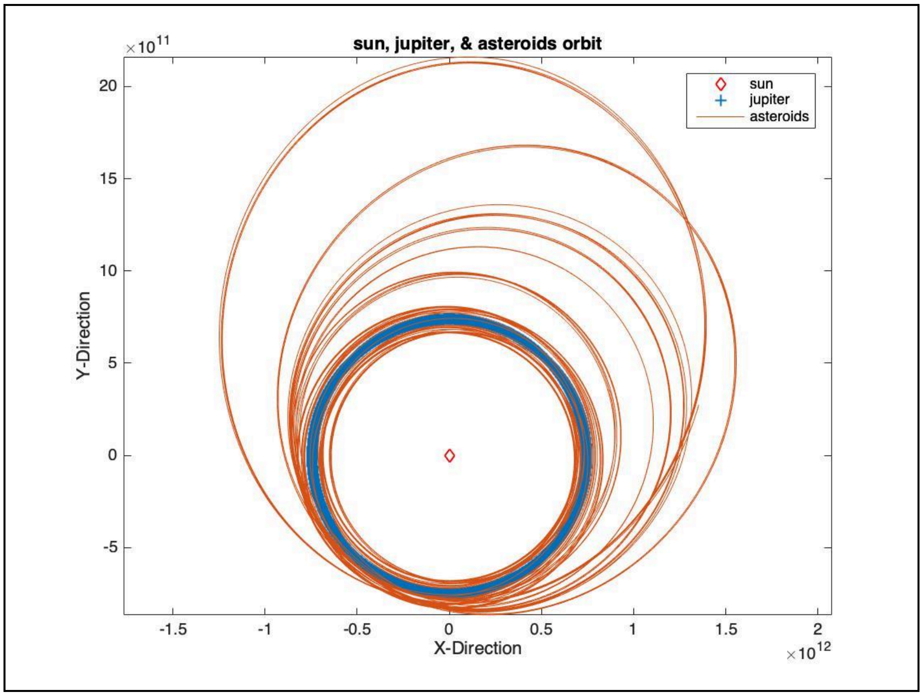
Graph 3: Sun, Jupiter and 3 Asteroids Orbit



Conclusion: This experiment allowed us to observe thousands of planetary orbit simulations. With every simulation something new was learned. It was observed that for each simulation. orbits tend to run away frequently. In our solar system, the planets follow orbits around the Sun that are nearly circular and in the same plane. Most asteroids are found between Mars and Jupiter in the asteroid belt, whereas comets generally follow orbits of high eccentricity. Our solar system in comparison to these simulations, goes to show what a statistical anomaly our solar system went through to create a habitable area. With variations of even the slightest initial parameters these programs seem to go haywire.