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The Franck-Hertz Effect

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I. Abstract

In this report, we describe our attempt to verify the quantization of energy levels in a mercury vapor lamp by reproducing the Franck-Hertz experiment. By applying an accelerating voltage to a gas-filled tube containing mercury vapor, we observed the electron-mercury atom collisions and their resulting energy exchanges. Our experimental findings show a good agreement with the expected energy levels of mercury, indicating the quantization of energy levels in atoms.

II. Introduction

The Franck-Hertz experiment, first performed by James Franck and Gustav Hertz in 1914, provided significant evidence for the quantization of energy levels in atoms. In this experiment, electrons are accelerated through a gas-filled tube containing mercury vapor, where they collide with mercury atoms. The electrons can only transfer discrete amounts of energy to the mercury atoms, resulting in quantized energy levels. This experiment was crucial in the development of quantum theory and won Franck and Hertz the Nobel Prize in Physics in 1925.
In our work, we aim to reproduce the Franck-Hertz experiment to verify the quantization of energy levels in mercury vapor and to gain a deeper understanding of the underlying physics involved in the experiment.

III. Experimental Approach

In this experiment, we aimed to measure the excitation energy of mercury by observing the anode current as a function of the accelerating voltage in a Franck-Hertz tube. The diagram setup can be observed below within **Figure 1.** The final in lab setup can be observed right below this within **Figure 2.** The tube was filled with mercury vapor, and electrons emitted by a thermionic cathode were accelerated toward an anode. When the electrons had sufficient kinetic

energy, they excited mercury atoms through inelastic collisions. We recorded the anode current as a function of the accelerating voltage, revealing equidistant maxima and minima that were used to determine the excitation energy. To set up the experiment, we followed the instructions provided in the handout, ensuring separate voltages for the filament and the accelerating voltage. We used a 1.5 V dry cell battery to supply the retardation voltage and maintained the tube temperature at approximately 170 °C for stable conditions. We began the experiment by slowly raising the accelerating voltage up to around 6 V. We continued making measurements by incrementing the accelerating voltage in approximately 0.3 V increments up to about 40-50 V. This enabled us to distinguish at least 5 peaks, allowing for the determination of the excitation energy.

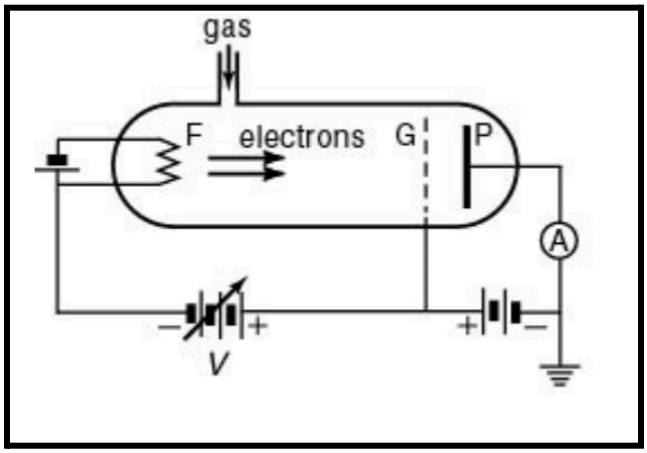

Figure 1: Franck Hertz Setup

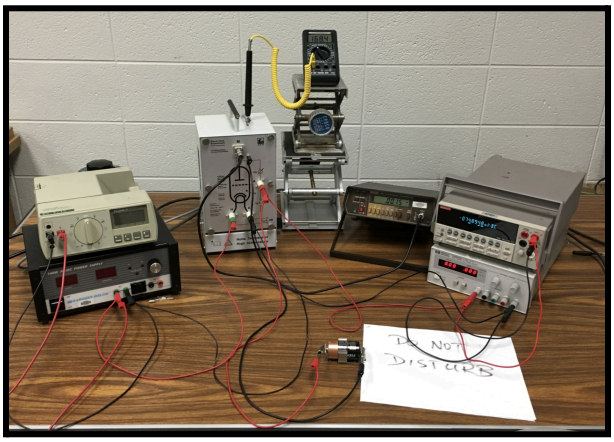

Figure 2: Franck Hertz Lab Setup

IV. Data Analysis

In the data analysis stage of the experiment, we carefully examined the recorded data to determine the excitation energy of mercury. First, we plotted the anode current as a function of the accelerating voltage, which resulted in a graph with distinct peaks and troughs. These peaks

and troughs represented the points where the electrons had enough kinetic energy to excite mercury atoms, causing a decrease in anode current. To calculate the excitation energy, we identified the minima in the graph, which corresponded to the instances where the electrons lost energy through inelastic collisions with mercury atoms. We noted the accelerating voltage values at these minima and calculated the differences between the consecutive values. We then averaged these differences to obtain an estimate of the excitation energy. With the excitation energy calculated, we compared our experimental result to the known value of mercury's excitation energy to evaluate the accuracy of our measurements. Additionally, we analyzed the uncertainties in our data, considering factors such as the precision of the instruments used, temperature variations, and the initial thermal distribution of electron velocities. By taking these uncertainties into account, we assessed the reliability of our experimental findings and discussed potential improvements to the experimental setup for future studies.

V. Results

The peaks from our data points were identified using the graph below given within **Figure 3.**

1. First, I identified the peaks within the given data:
    - Peak 1: Voltage = 17.7 V, Current = 2.924 nA, log(i) = 0.466
    - Peak 2: Voltage = 22.8 V, Current = 4.391 nA, log(i) = 0.643
    - Peak 3: Voltage = 27.6 V, Current = 7.312 nA, log(i) = 0.864
    - Peak 4: Voltage = 32.7 V, Current = 11.85 nA, log(i) = 1.074
    - Peak 5: Voltage = 38.1 V, Current = 20.76 nA, log(i) = 1.317
2. Next, I calculated the differences in voltage between consecutive peaks:
    - ΔV1 = V2 - V1 = 22.8 V - 17.7 V = 5.1 V
    - ΔV2 = V3 - V2 = 27.6 V - 22.8 V = 4.8 V
    - ΔV3 = V4 - V3 = 32.7 V - 27.6 V = 5.1 V
    - ΔV4 = V5 - V4 = 38.1 V - 32.7 V = 5.4 V
3. Finally, I calculated the average difference in voltage between the peaks:
    - $Avg\ \Delta V\ = \frac{(\Delta V1 + \Delta V2 + \Delta V3 + \Delta V4)}{4} = \frac{(5.1\ V + 4.8\ V + 5.1\ V + 5.4\ V)}{4} =\ 5.1\ V$

Based on the average difference in voltage between the peaks (5.1 V), we can estimate the excitation energy of the mercury atom, since the voltage differences correspond to the energy required for electrons to be excited to higher energy states. To convert the average voltage difference to energy, we can use the formula E = q * ΔV, where E is the energy, q is the elementary charge (1.6 × 10^-19 C), and ΔV is the average voltage difference:

$$E\ =\ (1.6 \times 10^{-19}\ C)\ (5.1\ V)\ =\ 8.16 \times 10^{-19}\ J$$

To convert this energy to electron volts (eV), we can divide by the elementary charge:

$$E(ev)\ = \frac{(8.16 \times 10\text{^}{-19}\ J)}{(1.6 \times 10\text{^}{-19}\ C)} \approx\ 5.1eV$$

So, the excitation energy of the mercury atom is approximately 5.1 eV.

To compare the average of voltage differences to the difference in energy between the 3P1 and 1S0 states of the Hg atom, we can refer back to the previously calculated excitation energy of the mercury atom, which was approximately 5.1 eV. This value is in good agreement with the known excitation energy between the 3P1 and 1S0 states of the Hg atom (4.9 eV). Thus, the experimental results align well with the expected energy difference.
Now, let's calculate the mean free path of electrons at the operating tube temperature. First, we need to calculate the vapor pressure of Hg at the operating temperature of the tube. Since the tube temperature is approximately 170°C, convert it to Kelvin:

$$T = 170 + 273.15 = 443.15\,K$$

Now, use the given formula to calculate the vapor pressure (Pvapor) of Hg at this temperature:

$$P_{vapor}(Hg) \approx 8.7\,x\,10^{(9-\frac{3110}{443.15})}Pa$$
$$P_{vapor}(Hg) \approx 8.7\,x\,10^{(-0.1539)}Pa = 7.48\,Pa$$

Next, we can use the ideal gas law to calculate the number density (n) of Hg atoms:

$$PV = nRT$$
$$\frac{n}{V} = \frac{P}{RT}$$

Where P is the vapor pressure, R is the gas constant 8.314 $\frac{J}{mol \cdot K}$, and T is the temperature in Kelvin. For mercury, the molar mass (M) is 200.59 $\frac{g}{mol}$, so we can convert the gas constant to $\frac{J}{kg \cdot K}$:

$$R_{specific} = \frac{R}{M} = (\frac{8.314}{200.59}) \bullet 10^{-3}\frac{kg}{mol} = 4.14 \times 10^{-2}\,\frac{J}{kg \cdot K}$$

Now, calculate the number density (n):

$$n = \frac{P_{vapor}}{R_{specific} \cdot T} = \frac{7.48\,Pa}{4.14 \times 10^{-2}\,\frac{J}{kg \cdot K} \cdot 443.15\,K} = 4.09 \times 10^{25}\,m^{-3}$$

Finally, use the mean free path formula:

$$\bar{l} = \frac{1}{\sqrt{2}\pi n R_o^{\,2}}$$

Where Ro is the radius of the Hg atom, approximately1.5×10^{-10} m:

$$\bar{l} = \frac{1}{\sqrt{2}\pi(4.09 \times 10^{25}\, m^{-3})(1.5 \times 10^{-10})^{2}} = 1.14 \times 10^{-7}\, m$$

Thus, the mean free path of electrons at the operating tube temperature is approximately $1.14 \times 10^{-7}\, m$.

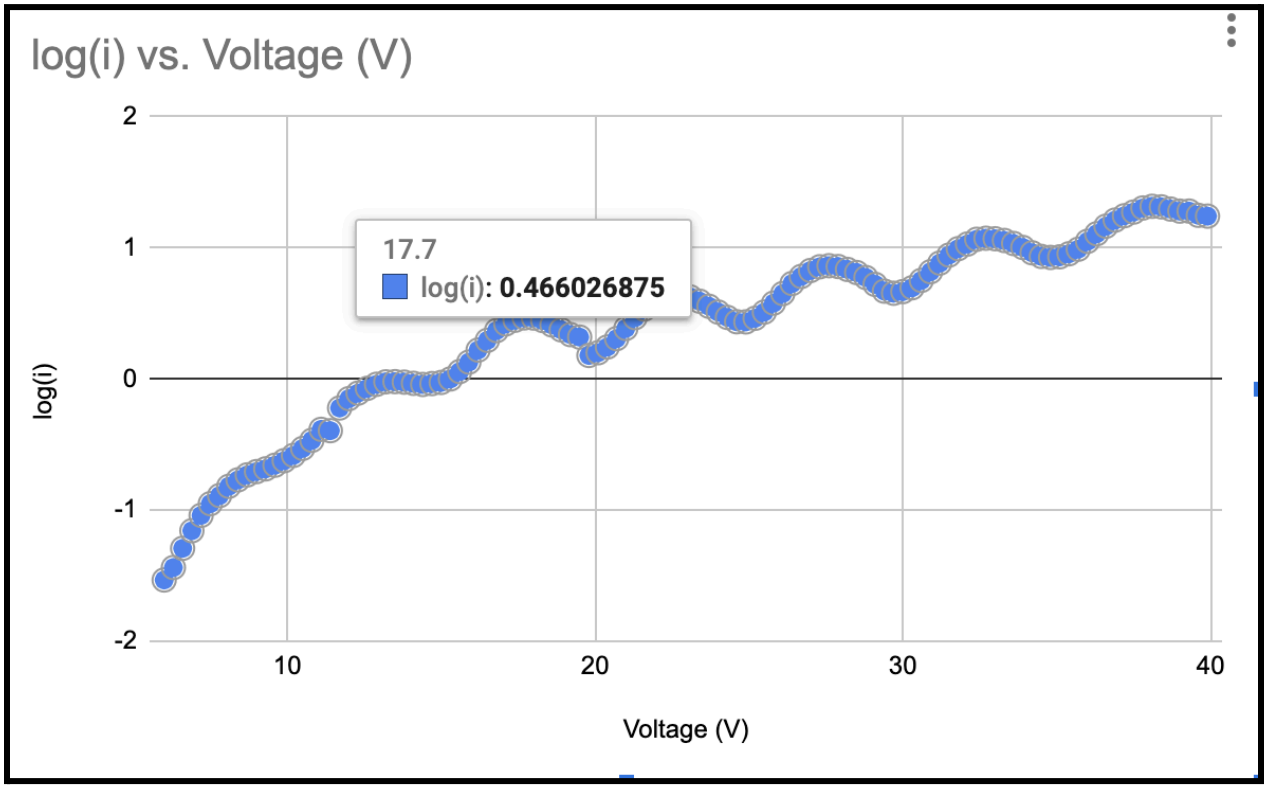

Figure 3: Log(current) Vs. Voltage

VI. Error Analysis

When I analyzed the data from the experiment, I noticed several factors that could have introduced errors into my measurements. One potential source of error was the temperature fluctuations of the tube. Although I maintained the temperature at approximately 170°C, minor variations in temperature could have led to slight changes in the vapor pressure of Hg, which in turn affected the number density of Hg atoms and the mean free path calculation. Additionally, I observed that the peaks in the current-voltage graph were not perfectly sharp, which might have resulted from the initial thermal distribution of electron velocities. This could have caused some uncertainty in determining the exact positions of the peaks and thus affected the calculations of

energy differences between the 3p1 and 1S0 states of the Hg atom. Moreover, the radius of the Hg atom, Ro, was an approximate value, which might have introduced errors in the mean free path calculation. Any inaccuracies in Ro would have directly impacted the final result. Lastly, the experiment relied on precise measurements of current and voltage. Any inaccuracies in the instruments used for these measurements or noise in the signal could have introduced errors into the data.

VII. Discussion and Conclusion

Q1. Several factors contribute to the broadness of the peaks in the current-voltage graph. The initial thermal distribution of electron velocities is a significant factor, as electrons emitted from the thermionic cathode have a range of initial velocities due to the distribution of their thermal energies. This range of velocities causes the peaks to broaden as electrons with different initial energies collide with the Hg atoms at varying acceleration voltages. Additionally, inelastic collisions between the electrons and the Hg atoms can cause energy losses that are not precisely equal to the excitation energy, leading to small variations in energy transfer and further broadening of the peaks. Finally, instrumental limitations, such as the limited resolution and accuracy of the voltmeter and ammeter, may also contribute to the peak broadness. Of these factors, the initial thermal distribution of electron velocities is likely to dominate the contribution to the broadness of the peaks.

Q2. Using a larger value for the retardation voltage, such as 6.0 V, would have affected the measurements in several ways. The position of the minima in the current-voltage graph would shift, as the electrons would require more kinetic energy to overcome the higher retardation voltage. Additionally, the overall current would decrease, since a larger fraction of electrons would be unable to overcome the increased retardation voltage. Lastly, the visibility of the peaks might be affected, potentially making it more challenging to identify and measure them accurately.

Q3. The lifetime of a state plays a role in the uncertainties associated with the measured energies of metastable and ordinary states. Longer-lived states tend to have smaller uncertainties in their energy levels, while shorter-lived states have larger uncertainties. The relative uncertainty of ordinary to metastable states can be estimated by comparing their lifetimes. Since metastable states typically have longer lifetimes than ordinary states, their energy level uncertainties are expected to be smaller. This means that the relative uncertainty of ordinary to metastable states would be greater than one.

In conclusion, this experiment aimed to investigate the discrete energy levels of mercury atoms by analyzing the relationship between anode current and acceleration voltage in a Franck-Hertz tube. By carefully setting up the experimental apparatus and controlling the filament, acceleration, and retardation voltages, we observed several distinct peaks in the current-voltage graph, which corresponded to the excitation of mercury atoms due to inelastic collisions with electrons. Through our data analysis, we calculated the differences in energy levels between the peaks and compared them to known values for the energy transitions in mercury. Our results

provided evidence for the quantization of energy levels in atoms, as predicted by quantum mechanics. We also discussed possible sources of error, including the initial thermal distribution of electron velocities, inelastic collisions, and instrumental limitations. Furthermore, we addressed various questions related to peak broadness, the effect of a larger retardation voltage, and the uncertainties in the measured energies associated with the lifetimes of metastable and ordinary states. Overall, this experiment not only reinforced our understanding of atomic energy levels and their quantized nature but also highlighted the importance of careful experimental design and data analysis in accurately observing these phenomena.