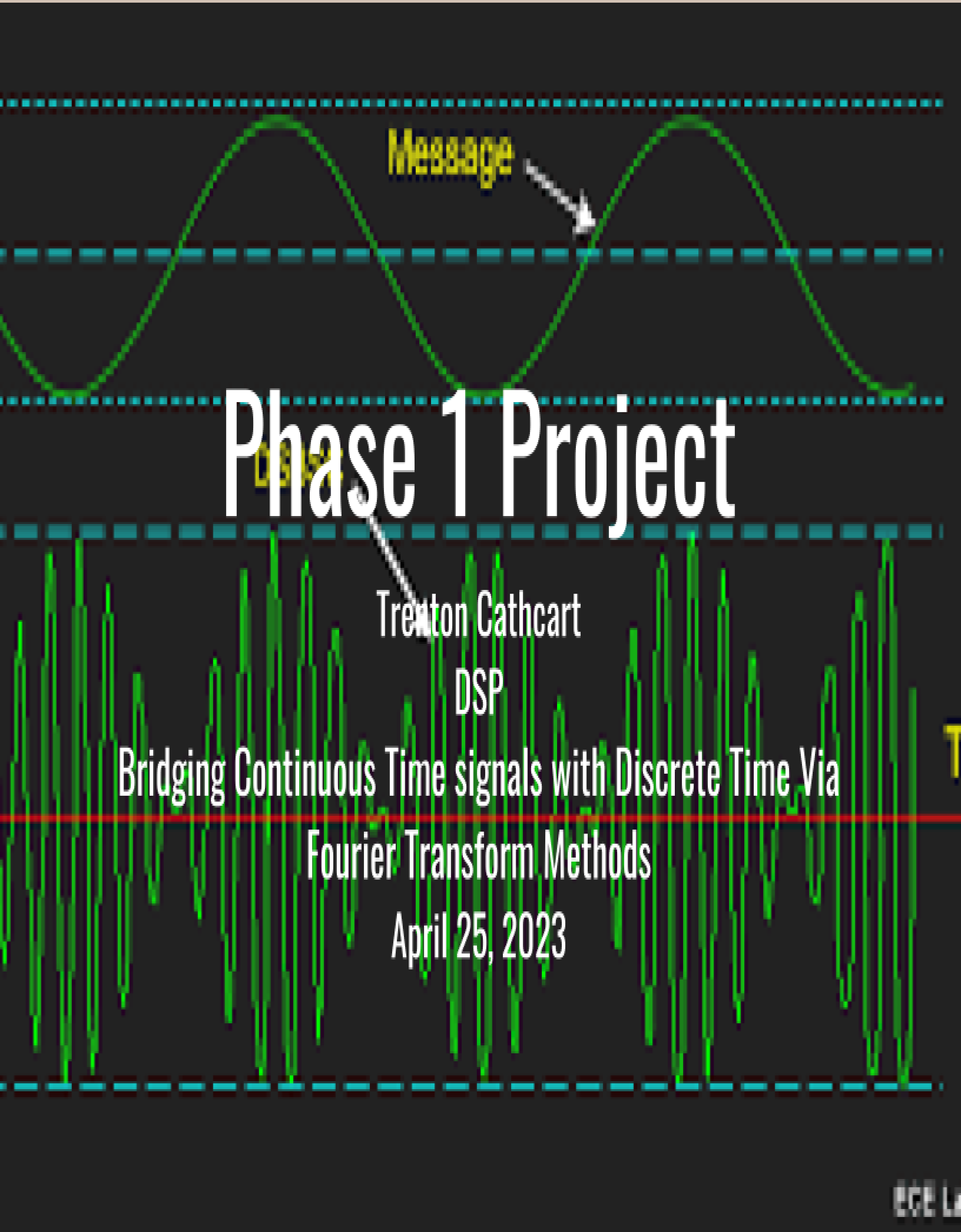

Phase 1 Project

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DSP

Bridging Continuous Time signals with Discrete Time Via Fourier Transform Methods

April 25, 2023

EE-3410
Trenton Cathcart
Date: 04/25/2023

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III. Introduction:

This report focuses on the first phase of a semester project aimed at bridging the gap between continuous-time (CT) and discrete-time (DT) signals using Fourier Transform methods. The main goal of Phase 1 is to generate various DT signals by varying the sampling frequency (fs) and the DT time range while exploring the use of window functions, such as Rectangle, Triangle, and Hamming/Hanning/Blackman, to select a set of samples for signal processing. This process is essential due to the limited computational resources available for processing signals. The findings of this phase will lay the foundation for the subsequent analysis of these DT signals using various Fourier Transform techniques in Phase 2.

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IV. Background Theory:

In the context of signal processing, continuous-time (CT) signals represent a continuous stream of data in the time domain. However, modern digital signal processing systems require discrete-time (DT) signals, which are sequences of samples taken from the CT signals. The process of converting a CT signal to a DT signal is called sampling, and the choice of the sampling frequency (fs) plays a crucial role in the accurate representation of the CT signal. Nyquist's criterion states that the sampling frequency should be at least twice the highest frequency present in the signal. By using window functions, such as Rectangle, Triangle, and Hamming/Hanning/Blackman, we can select a limited number of samples (N) from the DT signal for further processing. This process is essential because real-world computational resources are limited, and the choice of window function can affect the representation of the DT signal. Understanding these concepts is fundamental to bridge the gap between CT and DT signals and perform spectral analysis using various Fourier Transform methods.

V. Methodology:

In this project, we generate discrete-time (DT) signals from continuous-time (CT) signals using various sampling frequencies and time ranges. We focus on three window functions (Rectangle, Triangle, and Hamming/Hanning/Blackman) to process the Amplitude Modulated wave (DSBSC) signals. For each DT signal, we provide a high-quality figure with two subplots, analyze the periodicity, verify the Nyquist's criteria, and derive mathematical equations. We create 10 different DT signals for each window function, resulting in 30 DT signals. Our approach aims to bridge the gap between observing the spectral content of CT signals and performing spectral analysis using FFT, DTFT, and DFT algorithms.

VI. Procedure/Result/Analysis:

I. Ideal Sampling Frequency and Number of Samples

In **Figure 1**, we observe the results for the ideal sampling frequency and number of sample cases. The chosen sampling frequency (Fs) is significantly greater than twice the highest frequency of the signal, ensuring that the Nyquist's criterion is satisfied, and the continuous-time (CT) signal is accurately represented in the discrete-time (DT) domain. The number of samples (N) is sufficiently large to display multiple periods of the signal, providing a comprehensive view of the signal's behavior. The figure presents four subplots, where the first three show the DT signals generated using the Rectangular, Triangular, and Hamming window functions, respectively. In these subplots, we can observe that the windowed DT signals closely resemble the original CT signal, indicating that the chosen window functions effectively preserve the signal's essential characteristics. Moreover, the DT signals appear to be periodic, which is expected given the ideal choice of sampling frequency and number of samples. The fourth subplot displays the original CT DSBSC signal, serving as a reference for comparison with the windowed DT signals. Overall, the analysis of **Figure 1** demonstrates that the selection of an ideal sampling frequency and an appropriate number of samples plays a crucial role in accurately representing CT signals in the DT domain, while the choice of window functions influences the preservation of the signal's characteristics during the sampling process.

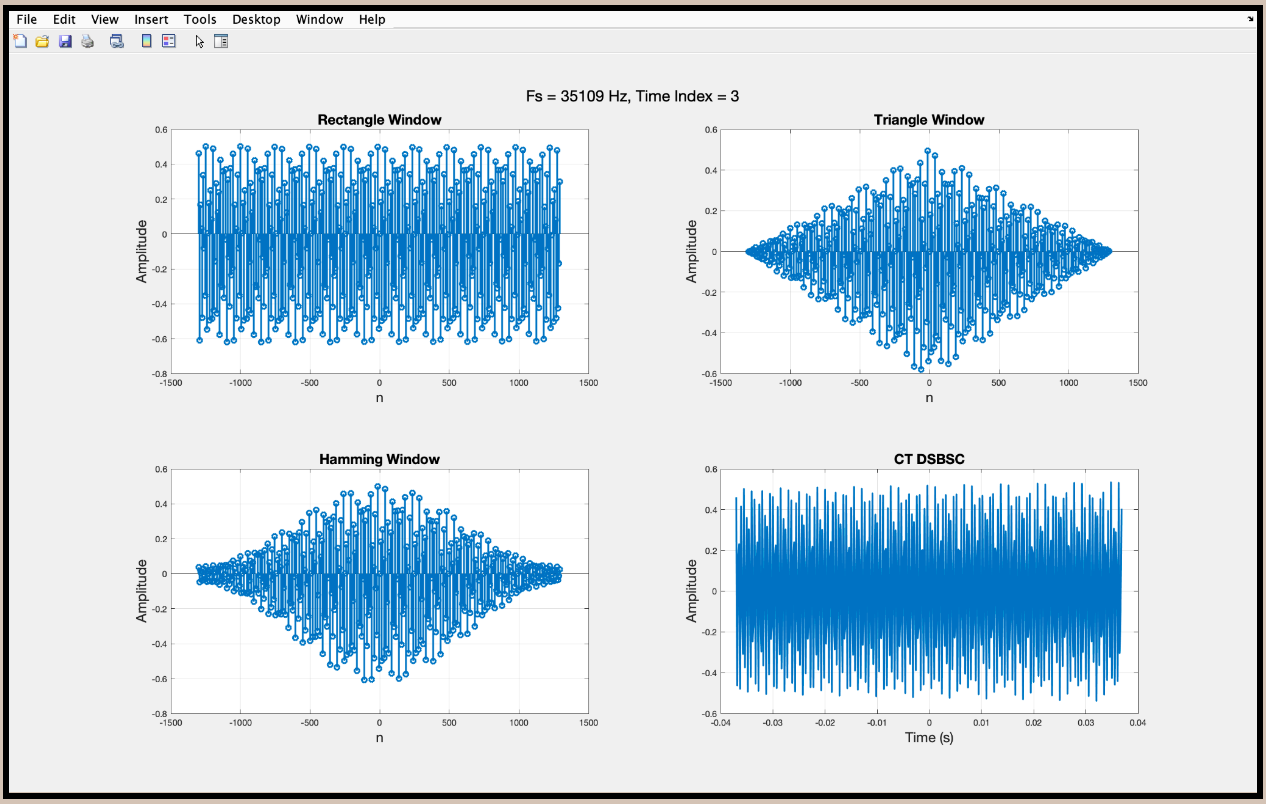

Figure 1: Ideal sampling and period

II. Satisfying Nyquist's Criteria with Periodic Signals

In Figure 2, we explore the scenario where Nyquist's criteria is satisfied with periodic signals. To achieve this, we have chosen a sampling frequency (Fs) that is at least twice the highest frequency present in the signal, as required by Nyquist's criterion. This ensures that the continuous-time (CT) signal is accurately represented in the discrete-time (DT) domain without any loss of information or aliasing.

The figure consists of four subplots, where the first three represent the DT signals generated using the Rectangular, Triangular, and Hamming window functions, respectively. In these subplots, we can observe that the windowed DT signals exhibit clear periodicity, indicating that the chosen sampling frequency and window functions effectively capture the periodic nature of the original CT signal. The periodicity in the DT signals is a direct consequence of satisfying Nyquist's criterion, as it guarantees the preservation of the signal's frequency content.

The fourth subplot displays the original CT DSBSC signal, which serves as a reference for comparison with the windowed DT signals. The analysis of Figure 2 highlights the importance of satisfying Nyquist's criterion in maintaining the periodicity of the original CT signal when converting it to the DT domain. The choice of window functions further influences the accuracy of the representation and the preservation of the signal's essential characteristics during the sampling process.

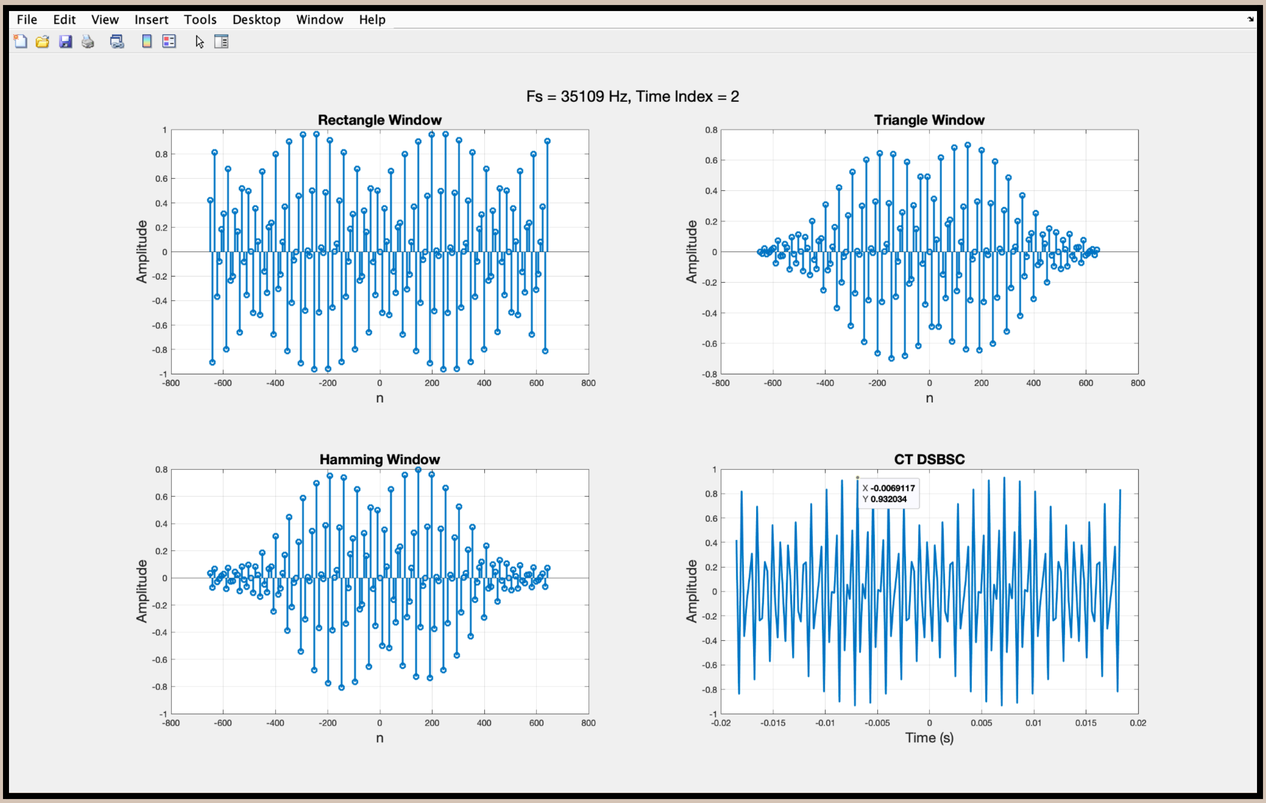

Figure 2: Nyquist with Periodic

III. Satisfying Nyquist's Criteria with Partial and Complete Periods

In this section, we analyze two different scenarios where the Nyquist's criterion is satisfied, but the chosen number of samples results in either complete or partial periods of the DT signals. **Figures 3 and 4** demonstrate these cases, respectively. **Figure 3** represents the case where the chosen number of samples allows for complete periods of the DT signals. The subplots show the DT signals generated using the Rectangular, Triangular, and Hamming window functions, all of which exhibit complete periods. This implies that the chosen sampling frequency and number of samples effectively capture the periodic nature of the original CT signal without any distortion or loss of information. The fourth subplot displays the original CT DSBSC signal, serving as a reference for comparison with the windowed DT signals. On the other hand, **Figure 4** illustrates the case where the chosen number of samples results in partial periods of the DT signals. The first three subplots show the DT signals generated using the Rectangular, Triangular, and Hamming window functions. In these subplots, we observe that the DT signals exhibit partial periods, indicating that the chosen sampling frequency and number of samples only partially capture the periodic nature of the original CT signal. Although Nyquist's criterion is satisfied, the limited number of samples leads to an incomplete representation of the signal's periodicity. The fourth subplot in **Figure 4** shows the original CT DSBSC signal for reference. The analysis of **Figures 3 and 4** emphasizes the significance of not only satisfying Nyquist's criterion but also choosing an appropriate number of samples to accurately represent the periodicity of the original CT signal when converting it to the DT domain. The choice of window functions further impacts the preservation of the signal's essential characteristics during the sampling process.

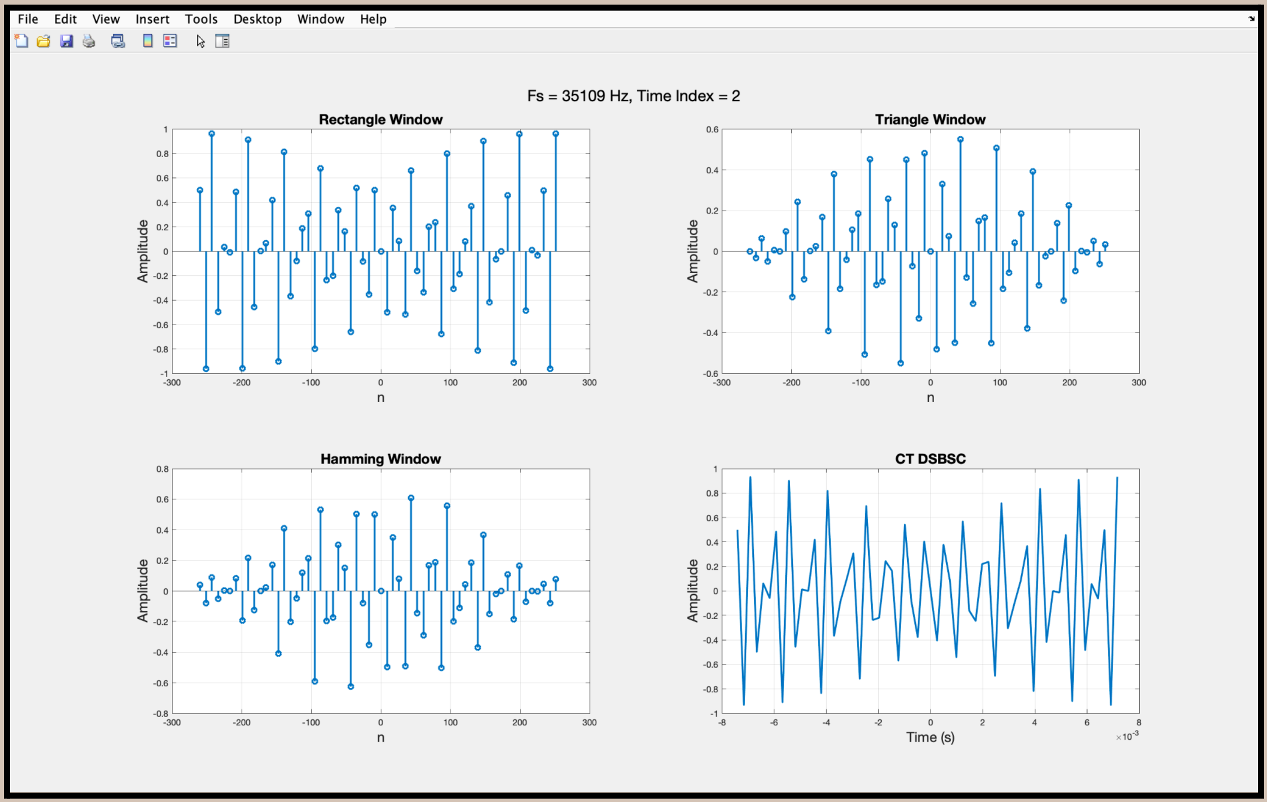

Figure 3: Nyquist with Complete Period

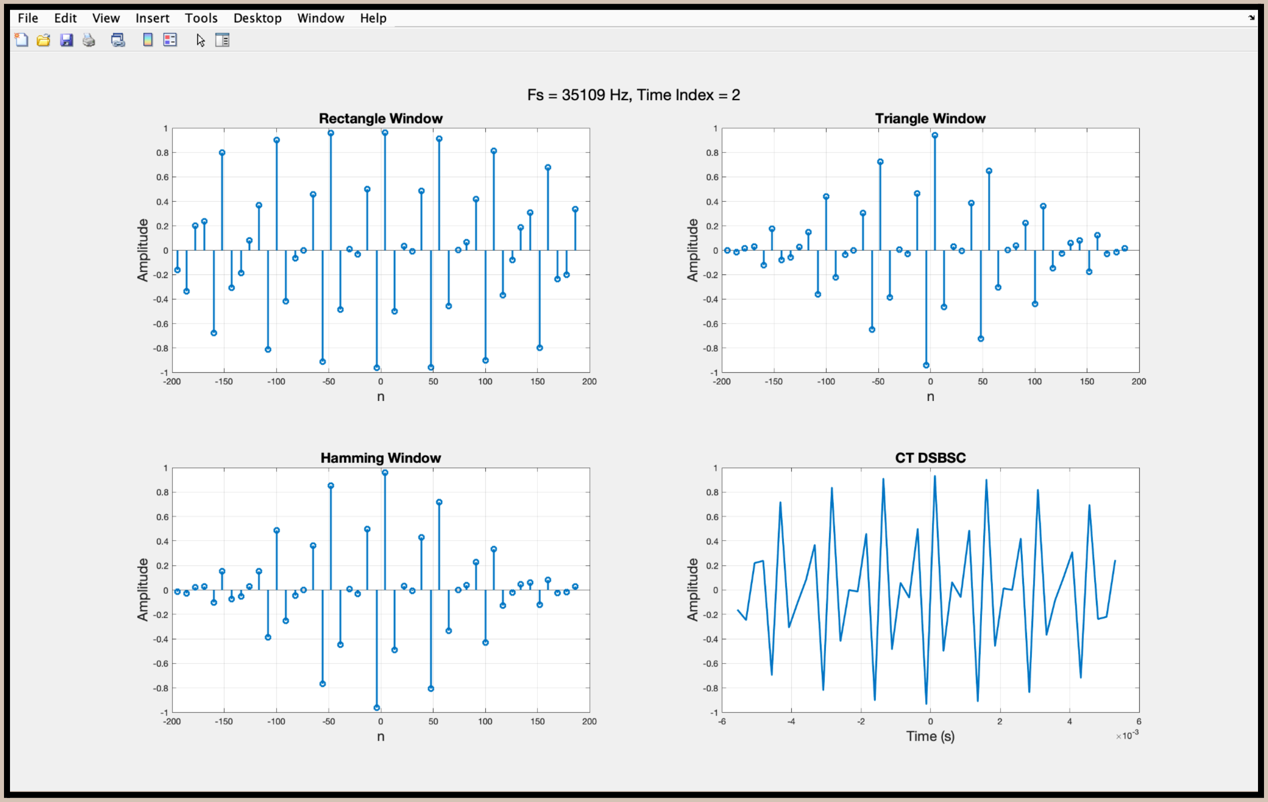

Figure 4: Nyquist with Partial Period

IV. Sampling Frequency at Nyquist's Boundary

In this section, we explore the case where the sampling frequency is set exactly at the Nyquist's boundary, which is 2*(f0+fc). **Figure 5** demonstrates this scenario, with the subplots showing the DT signals generated using the Rectangular, Triangular, and Hamming window functions, as well as the original CT DSBSC signal. When the sampling frequency is precisely at the Nyquist's boundary, the generated DT signals are on the verge of violating Nyquist's criterion. As a result, the DT signals are more susceptible to aliasing effects and may inadequately represent the original CT signal. This can lead to distortions in the reconstructed CT signal, compromising the accuracy of the signal representation. Upon analyzing **Figure 5**, we observe that the DT signals generated using the different window functions appear to capture the periodic nature of the original CT signal. However, the quality of the representation may not be as good as when the sampling frequency is higher than the Nyquist's boundary. The fourth subplot in Figure 5 shows the original CT DSBSC signal for reference.

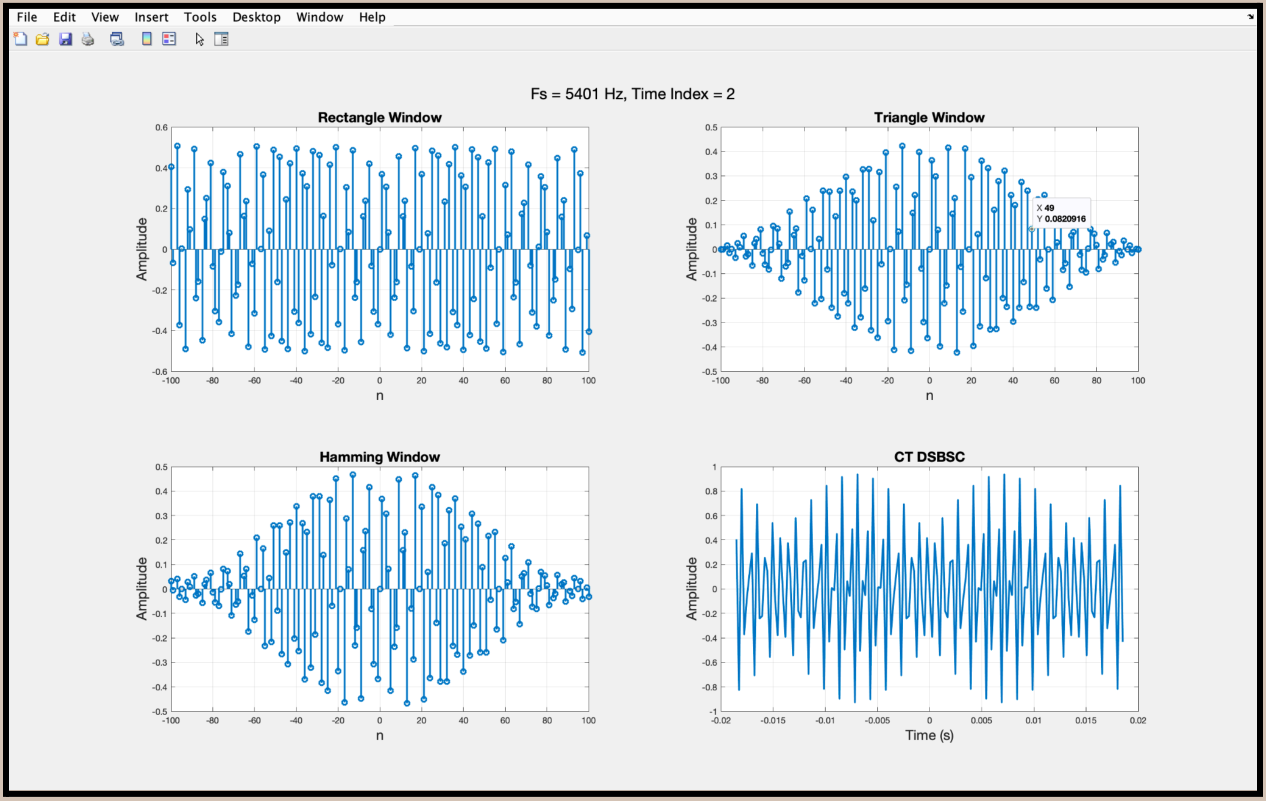

Figure 5: Nyquist Boundary

V. Violating Nyquist's Criteria (Periodic and Aperiodic Cases)

In this section, we analyze the consequences of violating Nyquist's criterion by choosing a sampling frequency that is below Nyquist's boundary. Figure 6 demonstrates a periodic case, while **Figure 7** shows an aperiodic case. Both figures display subplots with DT signals generated using Rectangular, Triangular, and Hamming window functions and the corresponding original CT DSBSC signal. When the sampling frequency is below the Nyquist's boundary, the generated DT signals are highly susceptible to aliasing effects, resulting in a poor representation of the original CT signal. In the periodic case (**Figure 6**), the DT signals seem to maintain some periodic nature, but the representation is distorted due to undersampling. On the other hand, the aperiodic case (**Figure 7**) exhibits even more distortion, and the DT signals lose their periodicity, further compromising the accuracy of the signal representation. This analysis emphasizes the importance of adhering to Nyquist's criterion when choosing a sampling frequency to ensure accurate CT signal representation and avoid aliasing effects that can significantly degrade signal quality.

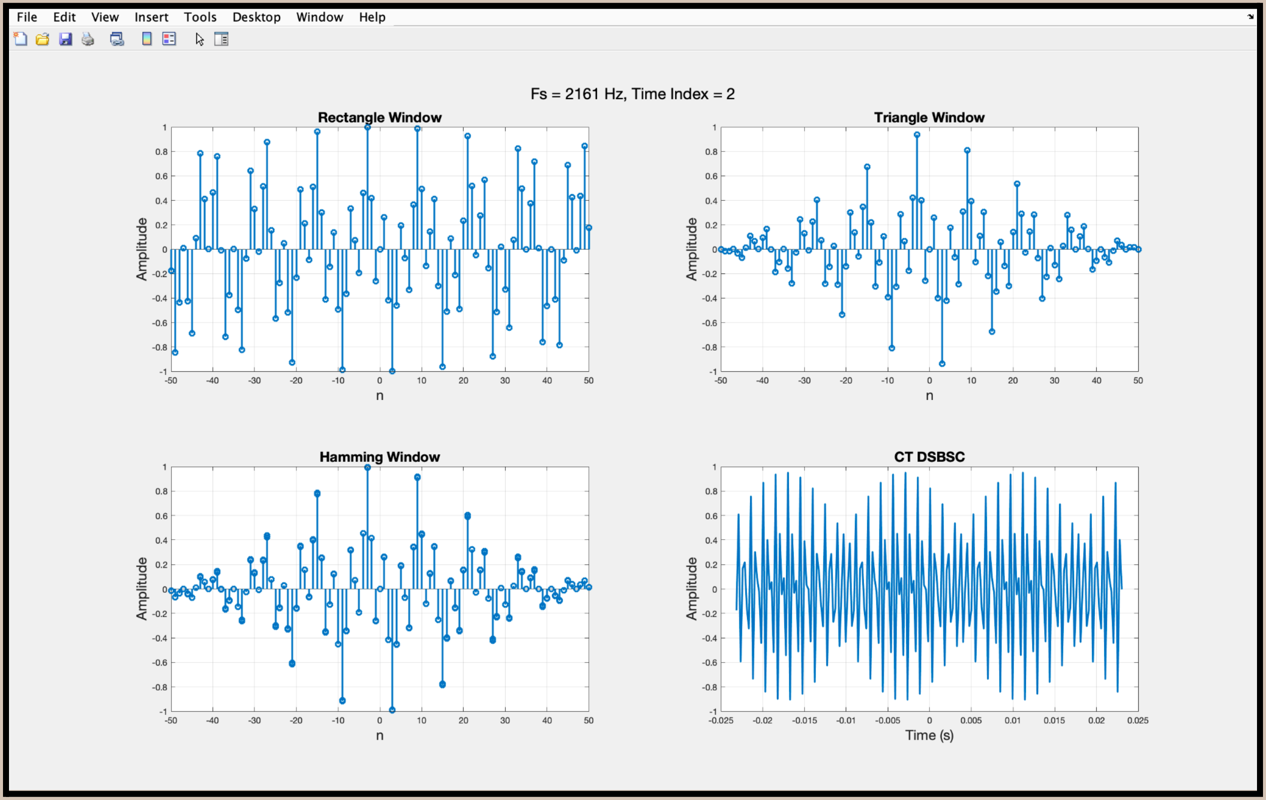

Figure 6: Nyquist violation periodic

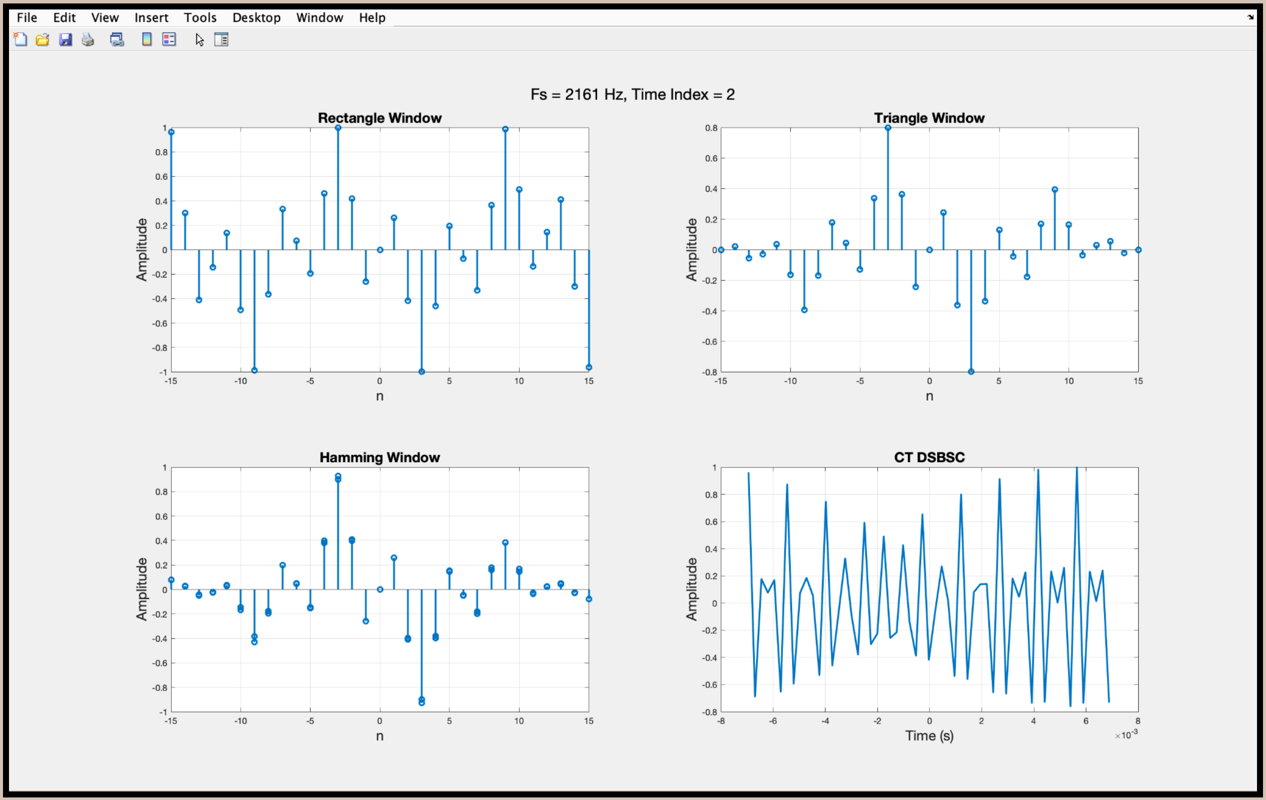

Figure 7: Nyquist violation aperiodic

VI. Sampling Frequency Resulting in Almost Periodic Signals

In **Figure 8**, we observe the results of using a sampling frequency that results in an almost periodic signal. The chosen sampling frequency, approximately 5500 Hz, is slightly above the Nyquist boundary but not as high as the ideal sampling frequency. By selecting this sampling frequency, we obtain a representation of the continuous-time signal that is better than when the Nyquist criterion is violated or at its boundary, but not as accurate as with the ideal sampling frequency. As seen in **Figure 8**, the discrete-time signal appears to be almost periodic, with the signal's peaks and valleys following a similar pattern but not perfectly aligning with the continuous-time signal. The three window functions (Rectangle, Triangle, and Hamming) show a similar trend, indicating that the chosen sampling frequency provides a better representation of the continuous-time signal than when the Nyquist criterion is not satisfied or at its boundary. However, there is still room for improvement in the signal's representation, which could be achieved by increasing the sampling frequency further.

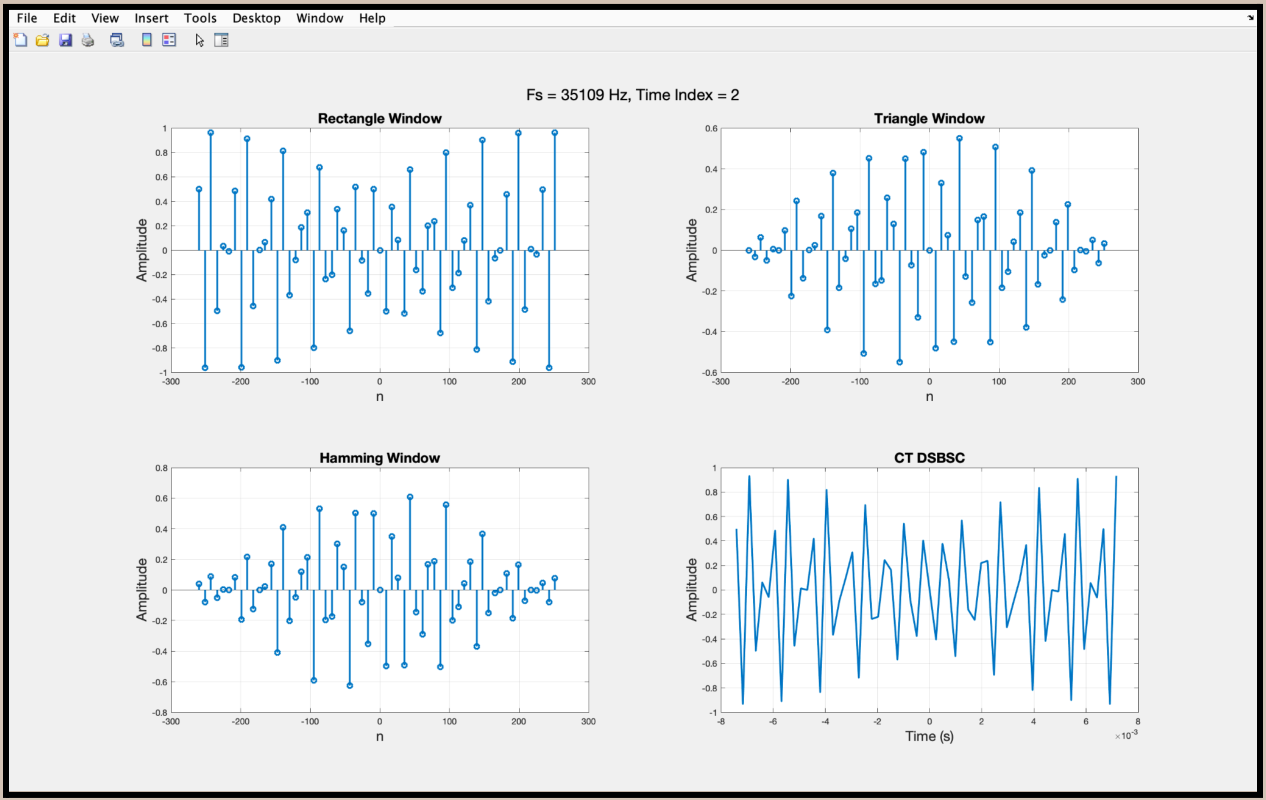

Figure 8: Sampling almost Periodic

VII. Incorporating Symmetrical and Asymmetrical Window Functions

In this section, we explore the effect of using both symmetrical and asymmetrical window functions on the generated DT signals. The window functions play a crucial role in signal processing, as they allow us to capture a limited number of samples from a time-unlimited DT signal. By analyzing these samples, we can gain valuable insights into the characteristics of the underlying CT signal. **Figure 9** demonstrates the result of applying symmetrical window functions, such as Rectangle, Triangle, and Hamming, to the DT signal. It can be observed that these window functions have a smooth, continuous shape. Due to their symmetry, these window functions preserve the spectral characteristics of the original signal, which allows for a more accurate representation of the signal's frequency content. On the other hand, **Figure 10** showcases the effect of using an asymmetrical window function, specifically the Blackman window. Asymmetrical window functions exhibit a non-uniform shape, which can result in a less accurate representation of the signal's frequency content compared to their symmetrical counterparts. However, they may offer certain advantages, such as better sidelobe suppression and a more gradual transition between the passband and stopband regions. This can be beneficial in specific applications where reducing the impact of sidelobes is more critical than preserving the signal's spectral characteristics.

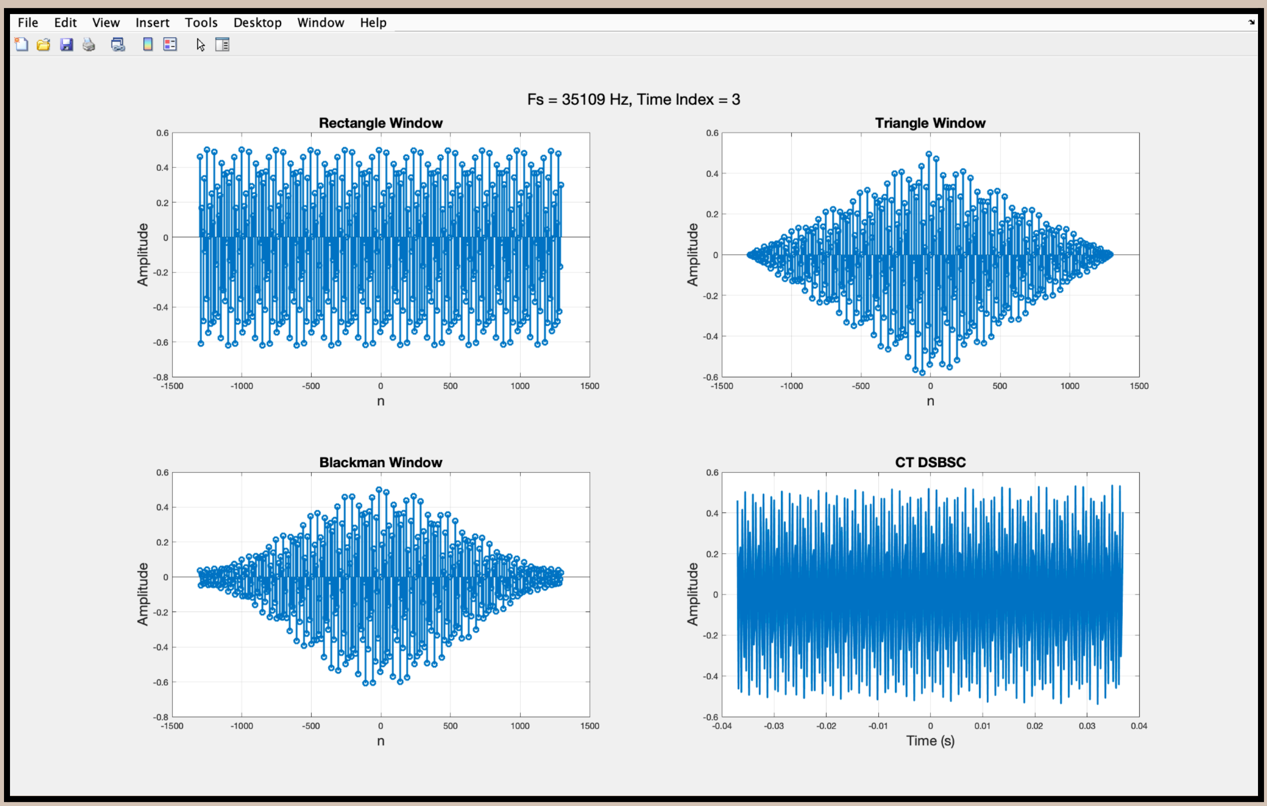

Figure 9: Symmetrical

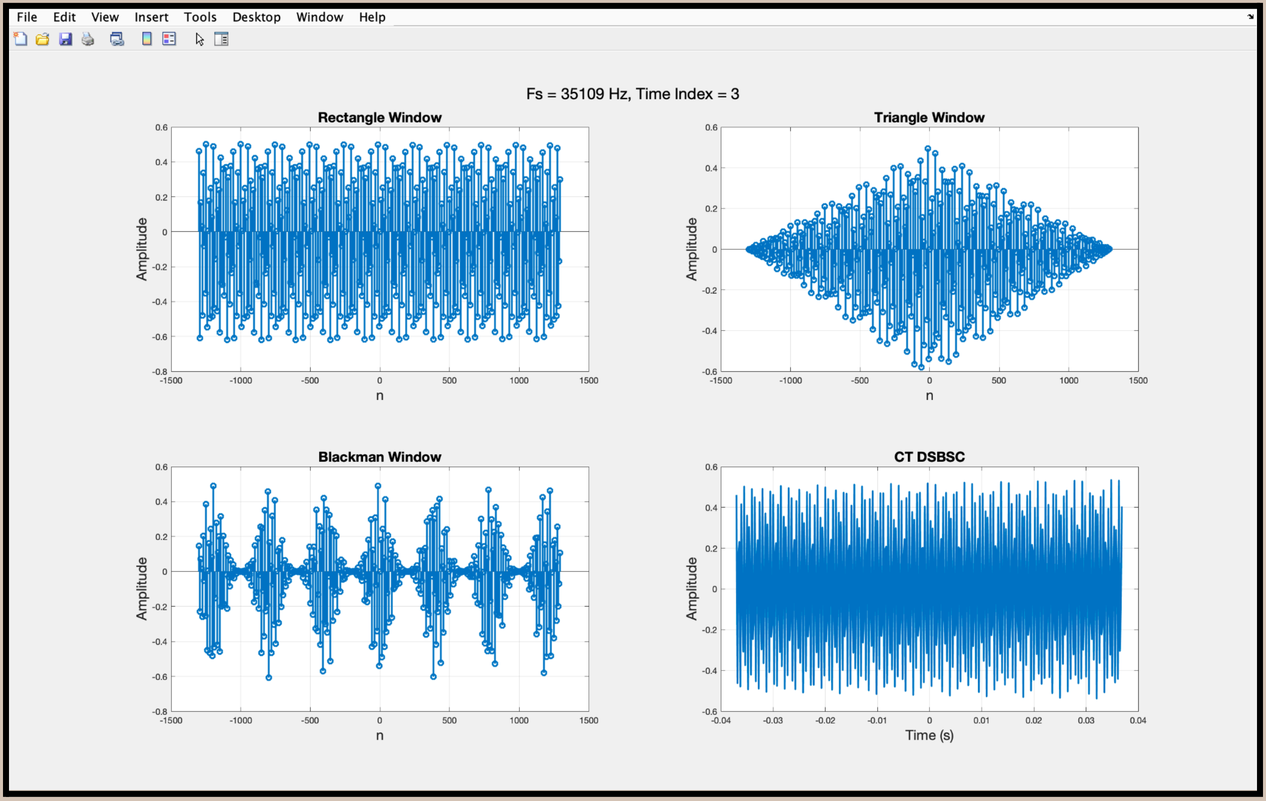

Figure 10: Asymmetrical

VII. Conclusion:

In conclusion, this report has provided an in-depth analysis of the process of generating discrete-time (DT) signals from continuous-time (CT) signals using various sampling frequencies, time ranges, and window functions. We have investigated the effects of satisfying and violating Nyquist's criteria, sampling at Nyquist's boundary, and incorporating symmetrical and asymmetrical window functions on the generated DT signals. Throughout this study, we have gained a deeper understanding of the relationship between CT and DT signals and the crucial role played by the choice of sampling frequency and window function in accurately representing and processing the underlying CT signals.

As we move forward, this foundational knowledge will be invaluable for further analysis in Phase 2 of the project, where we will apply different DT Fourier Transform methods to the generated DT signals. By bridging the gap between observing the spectral content of CT signals and their DT counterparts, we will be able to develop more efficient and accurate signal processing techniques. The lessons learned in this phase will not only enhance our understanding of signal processing but also contribute to the development of innovative solutions for various applications in the field of electrical engineering and beyond.

VIII. References:

None

IX. Appendix:

A. Main MATLAB Code

```
% Message frequency and carrier frequency
f0 = 355.35; % Hz, message frequency
fc = 6.6 * f0; % Hz, carrier frequency

% Sampling frequencies and corresponding time steps
Fs = [round(0.8*(f0+fc)), round(1.5*(f0+fc)), round(2*(f0+fc)), round(13*(f0+fc))];
Ts = 1./Fs;
time_range = [Ts(1), Ts(2), Ts(3)];

% Loop over sampling frequencies
for fs_idx = 1:length(Fs)
    % Loop over time indexes
    for time_idx = 1:length(time_range)
        % Set current sampling frequency and time step
        Fs_curr = Fs(fs_idx);
        Ts_curr = Ts(fs_idx);
        N0 = round(Fs_curr / (f0 + fc));
        fac = [10*N0, 15*N0, 100*N0];
        t = -fac(time_idx) * Ts_curr:time_range(time_idx):fac(time_idx) * Ts_curr;
        n = round(t / Ts_curr);
        N = length(n);

        % Calculate continuous-time and discrete-time DSBSC signals
        dsbsc_ct = dsbsc(t, fc, f0, 1);
        dsbsc_dt = dsbsc(n, fc, f0, Fs_curr);

        % Plot results
        figure;

        % Loop over window functions
        for win_indx = 1:3
            switch win_indx
                case 1
                    win_func = 'Rectangle';
                    window = 1;
                case 2
                    win_func = 'Triangle';
                    window = tri(n, fac(time_idx));

                case 3
                    win_func = 'Blackman';
                    window = ham(n+(N/2), N);
            end

            % Apply window function
            windowed_dt = dsbsc_dt .* window;

            % Subplots 1-3: Windowed discrete-time signal
            subplot(2, 2, win_indx);
```

```
            stem(n, windowed_dt, 'LineWidth', 2);
            title([win_func, ' Window'], 'FontSize', 16);
            xlabel('n', 'FontSize', 16);
            ylabel('Amplitude', 'FontSize', 16);
            grid on;
        end

        % Subplot 4: Continuous-time signal
        subplot(2, 2, 4);
        plot(t, dsbsc_ct, 'LineWidth', 2);
        title('CT DSBSC', 'FontSize', 16);
        xlabel('Time (s)', 'FontSize', 16);
        ylabel('Amplitude', 'FontSize', 16);
        grid on;

        % Add figure title
        sgtitle(['Fs = ', num2str(Fs_curr), ' Hz, Time Index = ', num2str(time_idx)], 'FontSize', 18);
    end
end

function out1 = dsbsc(t,fc,f0,Fs)
    out1 = cos((2*pi*f0*t)/Fs).*sin((2*pi*fc*t)/Fs);
end

function out2 = ham(n,N)
    n=0:N-1;
    out2 = 0.54 - 0.46 * cos(2 * pi * n / (N-1));
end
function out = blackman(n, N)
    a0 = 0.42;
    a1 = 0.5;
    a2 = 0.08;
    out = a0 - a1 * cos(2 * pi * n / (N-1)) + a2 * cos(4 * pi * n / (N-1));
end
```