

The Trajectory of a Baseball Pitch

Abstract: Various mathematical and physical ideas were used to approximate the trajectory of a baseball pitch. These methods were implemented within matlab to acquire these results. These results are shown below.

Introduction: Physics and painting are similar. The level of depiction desired, is proportional to the level of techniques used. In this assignment, we were asked to use Euler's method to represent a baseball pitch. Euler's method would be described as an intermediate method of depiction. While other methods can be applied for detailed results, this method will give a moderate depiction of the components. In this experiment the trajectory of a baseball pitch will be calculated using Euler's method and various other components. Substituting position, velocity and acceleration into **Formula 1**. The trajectory will be calculated without air resistance, with air resistance, and finally with the Magnus force present. For a 3 dimensional depiction, the formula will be applied to the x(forward,backward), y(left,right), and z(up,down) axes.

Formula 1: Euler's Method

$$Y_n = Y_{n-1} + hF(X_{n-1}, Y_{n-1})$$

Methods: To begin, the problems must first be initialized.

1. Assuming experiment one was performed on earth with no air resistance, the gravitational acceleration of earth was used for the z component. Omitting the y components of the initial position and velocity metrics given in **Figure 1**, a 2 dimensional representation can be created. These values were based on an average MLB pitcher. In matlab, position and velocity functions were created by substituting these metrics into **Formula 1** componentially. Within matlab, the for loop function was used to calculate the position of the ball using timesteps. The smaller the timestep, the more accurate the results. The time step chosen was $dt = 0.001$ seconds.

Figure 1: Initial Conditions

$$\begin{aligned} X_{position}(0) &\leftrightarrow = 18.4404 \text{ meters (60.5 feet)} \\ Y_{position}(0) &= -0.61 \text{ meters (2 feet from center of body)} \\ Z_{position}(0) &\updownarrow = 1.8796 \text{ meters (6 feet 2 inches)} \\ X_{velocity}(0) &= 43.3629 \frac{\text{meters}}{\text{second}} \left(97 \frac{\text{miles}}{\text{hour}} \right) \\ Y_{velocity}(0) &= 0 \frac{\text{meters}}{\text{second}} \left(0 \frac{\text{miles}}{\text{hour}} \right) \\ Z_{velocity}(0) &= 0 \frac{\text{meters}}{\text{second}} \left(0 \frac{\text{miles}}{\text{hour}} \right) \end{aligned}$$

- Assuming experiment two was performed on earth with air resistance. The force acting on the ball was calculated with gravity in addition to air resistance. The derivation of acceleration with air resistance is given by **Formula 2**. The air resistance coefficient was dialed in until realistic results were obtained. The initial position and velocity metrics are given in **Figure 1**.

Formula 2: Acceleration with air resistance

$C_d = \text{Air resistance Coefficient}$ $m = \text{Mass of the baseball}$ $V = \text{Velocity}$ $Acceleration_{(x,y,z)} = \frac{C_d \times V^2 \times \frac{\vec{v}}{ v }}{m}$

- Assuming experiment 3 was performed on earth with air resistance and with the addition of the magnus force. The force acting on the ball was calculated as the product of the force of gravity, air resistance, and the magnus force. The magnus force is a force applied to a body with rotational acceleration. For this experiment we assume the pitcher gave the ball an initial angular velocity of 2800 rpms in the X-direction. The angular force applied to a rotating baseball is given by the cross product of the angular velocity vectors and the velocity vectors. The cross product coefficient was dialed in as well until realistic results were obtained. The initial position and velocity metrics are given in **Figure 1**. All these values were based on an average MLB pitcher. In matlab, position and velocity functions were created by substituting these metrics into **Formula 1** componentially. Within matlab, the for loop function was used to calculate the position of the ball using timesteps. The smaller the timestep, the more accurate the results. The time step chosen was $dt = 0.001$ seconds.

Results:

- Comparing experiment 1 and 2, it was hypothesized that the ball with no air friction will travel further, have a higher final velocity and end higher in the air. These results were then depicted in **Graph 1** (3rd Base View of Pitches) and **Figure 2**.

Graph 1: 3rd Base View of Pitches

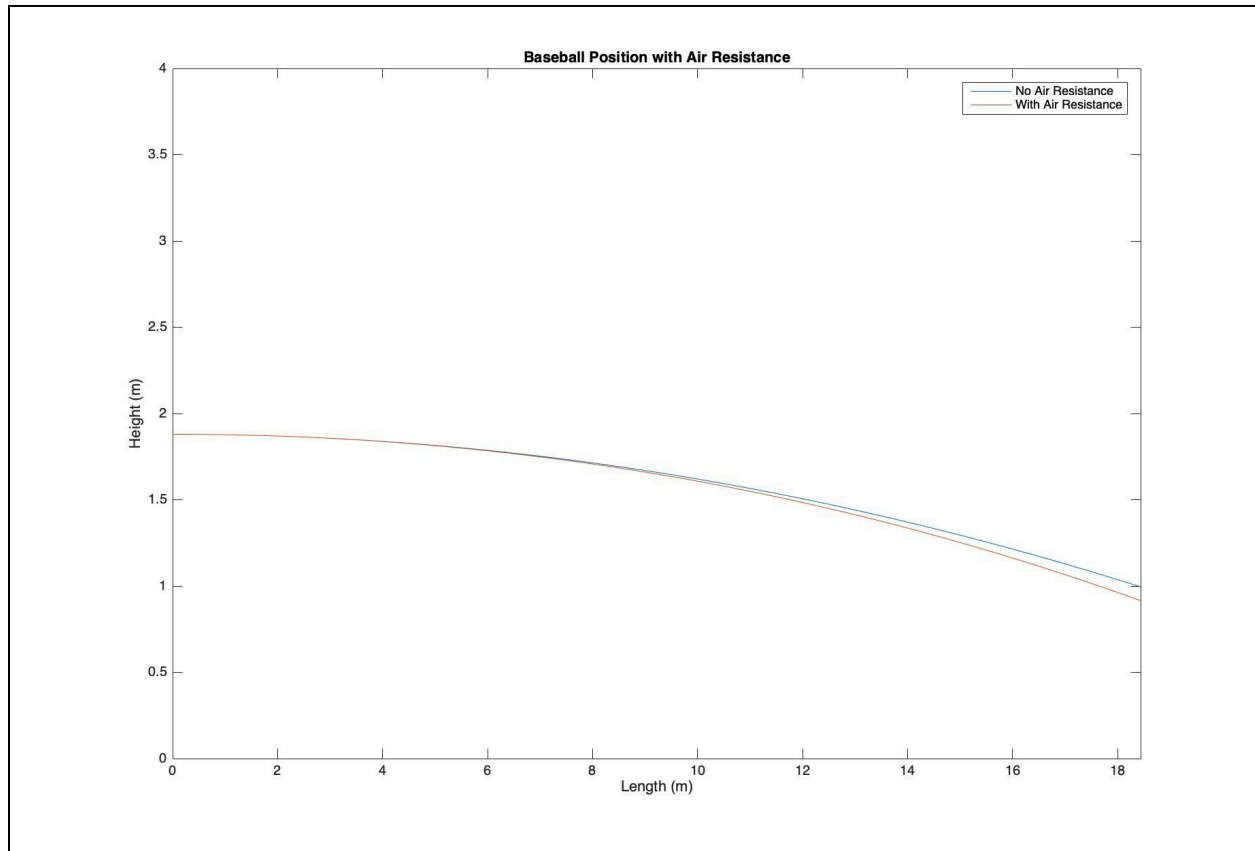


Figure 2: Final Velocities and positions

Part 1 Results

The Final velocity in the x direction, with no air resistance is: 4.336290×10^1 m/s
The Final velocity in the y direction, with no air resistance is: -4.410000×10^0 m/s
The ball with no air resistance displaced 9.900450×10^{-1} m in the z direction
The Final velocity in the x direction, with air resistance is: 3.810862×10^1 m/s
The Final velocity in the y direction, with air resistance is: -4.235202×10^0 m/s
The ball with air resistance displaced 9.497858×10^{-1} m in the z direction

2. Comparing experiment 2 and 3, from the view of the catcher it was hypothesized that the ball with angular velocity would curve to the right by a small amount. This was calculated using the right hand rule of a cross product. The ball with no angular velocity was expected to keep a straight trajectory. These results were then depicted in **Graph 2**(Catchers view of the pitch) and **Figure 3**.

Graph 1: Catchers view of the pitch

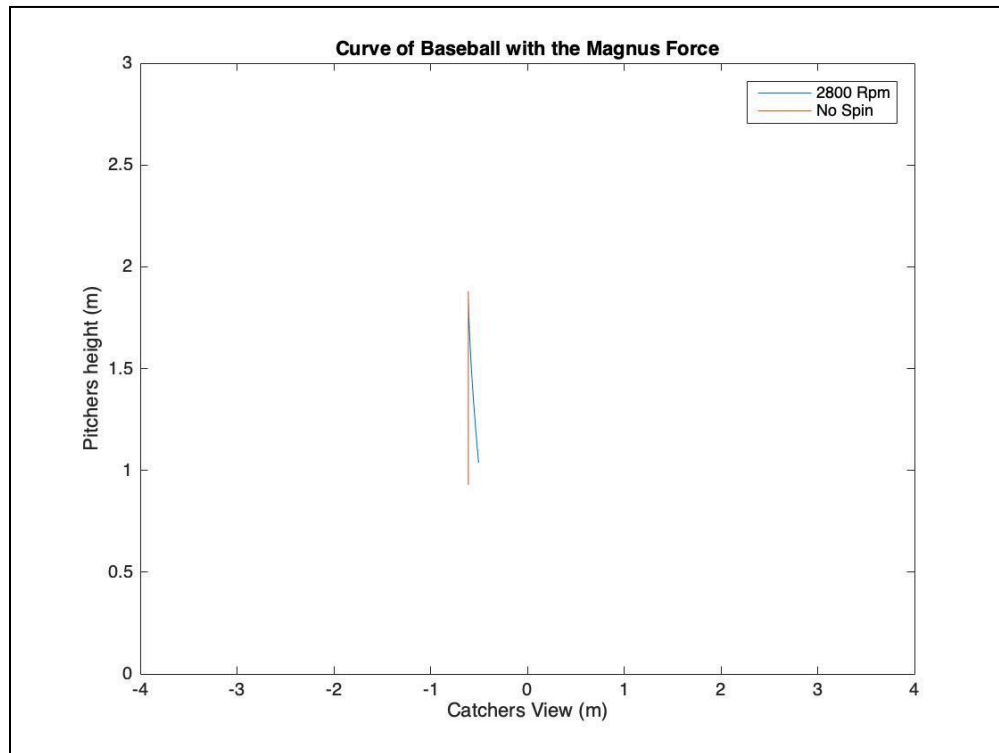


Figure 3: Final Velocities and positions

Part 2 Results

The Ball Traveled 1.841225×10^1 meters to home plate with the magnus force

The Ball Curved 1.067106×10^{-01} meters to the right of the catcher with the magnus force

The Ball Dropped 8.428852×10^{-01} meters during its travel with the magnus force

Conclusion: This method presents the basic underlying physics of a baseball pitch or any projectile using euler's method. These results depict a rough estimate of what a baseball would do under certain parameters. There are still other forces at play, as well as closer approximation methods.