is an EWMA forecast with smoothing parameter θ and if $\lambda = 1 - \theta$, then using (15.2.3)

$$x_t = (1 - B)X_{t+} = -\frac{\lambda(1 - \delta B)}{g(1 - \delta)}\varepsilon_t = -\frac{\lambda(1 - \delta) + \lambda\delta\nabla}{g(1 - \delta)}\varepsilon_t \tag{15.2.14}$$

Finally, if N_t can be represented by an IMA(0, 1, 1) process with parameter θ, then $\varepsilon_t = a_t$, and this adjustment will yield MMSE control. After summing (15.2.14), we obtain

$$X_t = k_0 + k_P\varepsilon_t + k_I \sum_{i=1}^{t} \varepsilon_i \tag{15.2.15}$$

in which

$$k_P = -\frac{\lambda}{g}\xi \quad \text{and} \quad k_I = -\frac{\lambda}{g}$$

The control equation (15.2.15) yields the discrete analog of continuous PI control mentioned earlier and will hereafter be referred to as (discrete) PI control.

Notice that despite their interesting ramifications, the adjustment equations corresponding to discrete integral control and PI control are extremely simple and intuitive. For discrete integral control

$$x_t = c_1\varepsilon_t \qquad (\text{with } c_1 = k_I)$$

and for PI control

$$x_t = c_1\varepsilon_t + c_2\varepsilon_{t-1} \quad (\text{with } c_1 = k_I + k_P \text{ and } c_2 = -k_P)$$

They, thus, make the adjustment x_t depend linearly on the last error and the last two errors, respectively.

15.2.4 General Minimum Mean Square Error Feedback Control Schemes

Arguing as earlier, it is not difficult to derive theoretical minimum mean square error feedback control schemes for the more general stochastic and linear dynamic models discussed in Chapters 4 and 11. Suppose the response to the series of adjustments in the manipulable input variable X_t is represented by the dynamic transfer function relation (11.2.3), written as

$$\mathcal{Y}_t = L_1^{-1}(B)L_2(B)B^{f+1}X_{t+}$$

where $L_1(B)$ and $L_2(B)$ are polynomials in B. This relation allows for f periods of pure dead time in the response. In addition, assume the noise or process disturbances $\{N_t\}$ may be represented by the linear stochastic ARIMA process defined by

$$N_t = \varphi^{-1}(B)\theta(B)a_t = \left(1 + \sum_{i=1}^{\infty} \psi_i B^i\right) a_t$$

where a_t is a white noise process. Then the error at the output, $\varepsilon_{t+f+1} = Y_{t+f+1} - T$, at time $t + f + 1$ can be written

$$\varepsilon_{t+f+1} = \mathcal{Y}_{t+f+1} + N_{t+f+1} = L_1^{-1}(B)L_2(B)X_{t+} + N_{t+f+1}$$

Clearly, the effect of the disturbance at time $t + f + 1$ would be canceled if it were possible to set $X_{t+} = -L_1(B)L_2^{-1}(B)N_{t+t+1}$. Since $f + 1$ is positive, this is not possible, but intuitively we can obtain minimum mean square error control by replacing N_{t+f+1} by its optimal forecast $\hat{N}_t(f + 1)$ at origin t. Now we can write $N_{t+f+1} = \hat{N}_t(f + 1) + e_t(f + 1)$, where $\hat{N}_t(f + 1)$ is the *forecast* at time t of N_{t+f+1} and $e_t(f + 1)$ is the error of the forecast for $f + 1$ steps ahead. The noise N_{t+f+1} is not known at time t, but its minimum mean square error forecast $\hat{N}_t(f + 1)$ can be deduced from the error sequence $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2} \ldots$, which is observed. Thus, it follows that the control equation $X_{t+} = -L_1(B)L_2^{-1}(B)\hat{N}_t(f + 1)$ will produce at time $t + f + 1$ a level at the output that will cancel out the forecast of the noise $f + 1$ periods ahead, and the error at the output will then be $\varepsilon_{t+f+1} = e_t(f + 1)$, the error of the forecast. To express the control equation in terms of the error sequence ε_t's, we can write

$$\varepsilon_t = e_{t-f-1}(f + 1) = a_t + \psi_1 a_{t-1} + \cdots + \psi_f a_{t-f} = L_4(B)a_t$$

and

$$\hat{N}_t(f + 1) = \psi_{f+1}a_t + \psi_{f+2}a_{t-1} + \cdots = L_3(B)a_t$$

where the operators $L_3(B)$ and $L_4(B)$ are determined from knowledge of the model $N_t = \varphi^{-1}(B)\theta(B)a = \psi(B)a_t$ for the noise process. Hence, we have

$$\hat{N}_t(f + 1) = L_3(B)L_4^{-1}(B)\varepsilon_t$$

Therefore, the MMSE feedback control equation is then

$$X_{t+} = -\frac{L_1(B)L_3(B)}{L_2(B)L_4(B)}\varepsilon_t \tag{15.2.16}$$

Alternatively, as is usually convenient, we can define the control action in terms of the *adjustment* $x_t = X_{t+} - X_{t-1+}$ to be made at time t as

$$x_t = -\frac{L_1(B)L_3(B)(1 - B)}{L_2(B)L_4(B)}\varepsilon_t$$

Example: Model with Dead Time. In particular, one more general dynamic model used above allows for ''dead time''—that is, pure delay in response to adjustment. To illustrate the application of equation (15.2.16), consider a first-order system affected by between f and $f + 1$ unit intervals of pure delay so that

$$(1 - \delta B)\mathcal{Y}_t = g(1 - \delta)[(1 - v) + vB]B^f X_{t-1} \tag{15.2.17}$$

Combining this with the IMA(0, 1, 1) disturbance model of equation (15.2.10), we can use the general derivation above to obtain the MMSE control scheme. In terms of the general model, we have $L_2(B)/L_1(B) = g(1 - \delta)(1 - v\nabla)/(1 - \delta B)$, and the IMA noise model yields $\hat{N}(f + 1) - \hat{N}_{t-1}(f + 1) = \lambda a_t$, so that $L_3(B) = \lambda/(1 - B)$, and also

$$e_{t-f-1}(f + 1) = [1 + \lambda(B + B^2 + \cdots + B^f)]a_t \equiv L_4(B)a_t$$

Hence, for the adjustment x_t, we have the relation

$$L_2(B)L_4(B)x_t = -L_1(B)L_3(B)(1 - B)\varepsilon_t$$

and we obtain the MMSE control equation as

$$(1 - \nu\nabla)[1 + \lambda(B + B^2 + \cdots + B^f)]x_t = -\frac{\lambda}{g(1-\delta)}(1 - \delta B)\varepsilon_t$$

Thus, this optimal control scheme is not PI but is of the form

$$x_t = c_1 x_{t-1} + c_3 x_{t-2} + \cdots + c_f x_{t-f-1} + c(\varepsilon_t - \delta\varepsilon_{t-1}) \tag{15.2.18}$$

where $c = -\lambda/[g(1 - \delta)] = k_I + k_P$.

An interesting example by Fearn and Maris (1991) describes an MMSE scheme of this kind applied to the control of gluten addition to bread-making flour in a flour mill where the object was to maintain the protein content of the flour as close as possible to the target value. A careful process study showed that to an adequate approximation for this process $\delta = 0$, $\nu = 0$, $f = 1$, and $\lambda = 0.25$ ($\theta = 0.75$). The adjustment equation was thus

$$x_t = -0.25x_{t-1} - \frac{0.25}{g}\varepsilon_t \tag{15.2.19}$$

The scheme was tested extensively and the authors remarked that it worked well over a wide range of manufacturing conditions and was robust to moderate changes in the parameters.

The flour milling example does not yield a PI scheme. Notice, however, that the adjustment equation can be written $x_t = -(1 + \lambda B)^{-1}(\lambda/g)\varepsilon_t = -(1 - \lambda B + \lambda^2 B^2 - \cdots)(\lambda/g)\varepsilon_t$. For the rather small value $\lambda = 0.25$, if we truncate the expansion after the first-order term, we obtain the PI scheme $x_t = c_1\varepsilon_t + c_2\varepsilon_{t-1}$ with $c_1 = -\lambda/g$ and $c_2 = \lambda^2/g$. In practice, the behavior of this PI scheme will be almost identical to that of (15.2.19). More generally, we will find that PI schemes have an importance in addition to that conferred on them by their producing MMSE schemes for certain simple models. We therefore next consider how PI schemes can be put in effect using simple *feedback control charts*.

15.2.5 Manual Adjustment for Discrete Proportional–Integral Schemes

The equation for the adjustment $x_t = X_t - X_{t-1}$ for the discrete PI scheme (15.2.15) may also be written

$$x_t = -G(1 + P\nabla)\varepsilon_t \tag{15.2.20}$$

where

$$-G = k_I \quad \text{and} \quad P = \frac{k_P}{k_I} \tag{15.2.21}$$

or equivalently, $k_I = -G$ and $k_P = -PG$, and P is zero for pure integral control. In the special case where the stochastic and dynamic models are defined by (15.2.10) and (15.2.12), respectively, the PI control equation (15.2.15) yields MMSE when $G = \lambda/g$ and $P = \xi$.

Equation (15.2.20) shows how we can make a manual adjustment chart to put PI control into effect. We have already illustrated the use of such a chart for the metallic thickness example in Figure 15.3. For further illustration, we adapt an example discussed by Box et al. (1978). In a dyeing process, the quality characteristic of interest was the color index. Deviations ε_t from the desired target value of $T = 9$ were compensated by changing the dye addition rate X. For this example, the disturbance in the color index was approximated by an IMA(0, 1, 1) model with $\lambda = 0.3$, and a change of 1 unit in the dye addition rate X eventually produced a change of 0.06 unit in the color index so that $g = 0.06$.

Suppose at first that ξ were zero so that the dynamic model was simply $\mathcal{Y}_t = gBX_{t+}$, implying that a change in the input X_t was fully effective at the output in one time interval. Then,

$$-G = k_I = -\frac{\lambda}{g} = -\frac{0.30}{0.06} = -5 \quad \text{and} \quad k_P = 0 \tag{15.2.22}$$

The MMSE integral feedback equation would be

$$X_t = k_0 - G\sum_{i=1}^{t} \varepsilon_t = k_0 - 5\sum_{i=1}^{t} \varepsilon_i \tag{15.2.23}$$

and at time t the corresponding *adjustment* would be

$$x_t = -G\varepsilon_t = -5\varepsilon_t \tag{15.2.24}$$

Appropriate action is read off the manual adjustment chart in Figure 15.7 with scales such that one unit deviation in the color index corresponds to $-G = -5$ units of adjustment of the dye addition rate. Action is taken after each observation by recording the value of the color index (indicated by a filled dot) and reading off on the left-hand scale the required adjustment to the dye addition rate. Thus, in the diagram at time 1:30 p.m., the color index was 9.14 calling for a reduction of -0.7 in the dye addition rate.

Now consider the case where, due perhaps to incomplete mixing of the dye, the process was subject to inertia, which was approximated by a first-order dynamic system as in (15.2.13) with $\delta = 0.2$ and consequently $\xi = \delta/(1-\delta) = 0.25$. Thus, as before, $G = 0.3/0.06 = 5$ and now $P = \xi = 0.25$. Thus, the appropriate MMSE control equation (15.2.15) would call for proportional–integral action such that

$$X_t = k_0 - 1.25\varepsilon_t - 5\sum_{i=1}^{t} \varepsilon_i \tag{15.2.25}$$

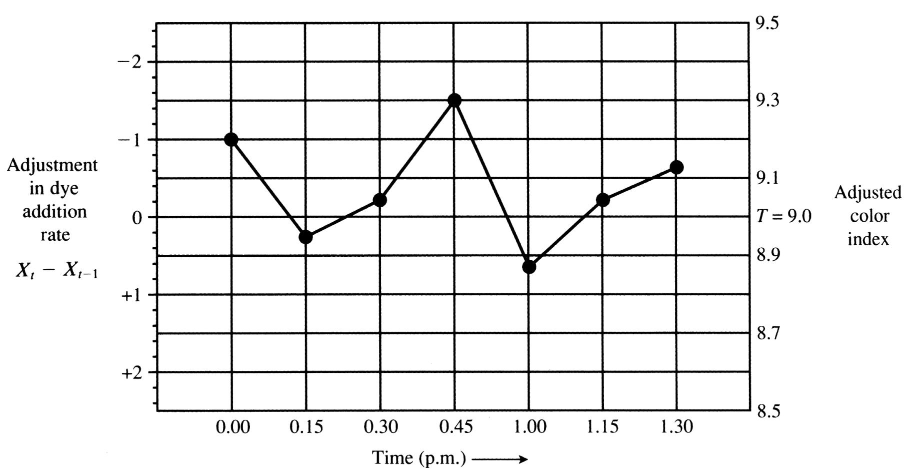

FIGURE 15.7 Manual adjustment chart for discrete integral control.

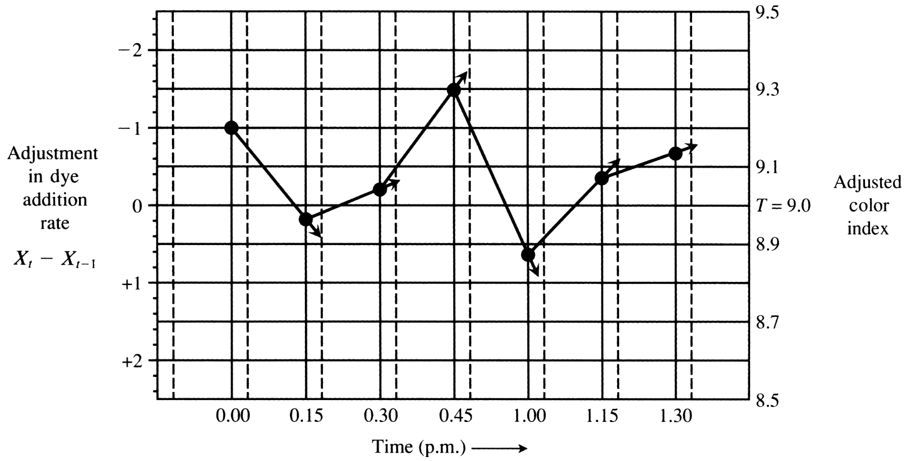

FIGURE 15.8 Manual adjustment chart putting into effect discrete integral plus proportional control.

The corresponding adjustment equation is

$$x_t = -5(1 + 0.25\nabla)\varepsilon_t \qquad (15.2.26)$$

To put this into effect manually, the chart in Figure 15.8 may be employed with the vertical dashed lines placed at a fraction $P = k_P/k_I = 0.25$ within each sampling interval. At each step the operator extrapolates the line through the last two points to the next dashed line and reads off the appropriate adjustment. Thus, in this figure, the last two readings, at 1:15 and 1:30 p.m., were 9.06 and 9.14. The projected value of 9.16 requires reduction of the dye addition rate by −0.8 unit. No exactness is required. A line extrapolated by eye is good enough. As we later explore other uses of PI charts, we will sometimes use schemes in which P is negative. This calls for *interpolation* between the last two points rather than extrapolation.

Rounded Adjustment. The feedback schemes as so far discussed require that we take *some* action at every opportunity—in this example, every 15 minutes. In practice, usually little is lost if the ''rounded'' adjustment chart indicated in Figure 15.9 is used. Such a chart is easily constructed from the original chart by dividing the action scale into bands. The adjustment made when an observation falls within the band is that appropriate to the middle point of the band on an ordinary chart. Figure 15.9 shows a rounded chart in which possible action is limited to −2-, −1-, 0-, 1-, or 2-unit catalyst formulation changes. The increase in mean square error (usually small), which results from using the rounded scheme, is often outweighed by the convenience of working with a small number of standard adjustments. A convenient width for the rounded bands is about one standard deviation σ_ε or a little less. Justification for the use of such charts was provided by Box and Jenkins (1976, Section 13.1), where consideration is given to the effects of errors in the adjustment x_t. Note that the use of all these manual adjustment charts requires no calculation—they are simple and entirely graphical.

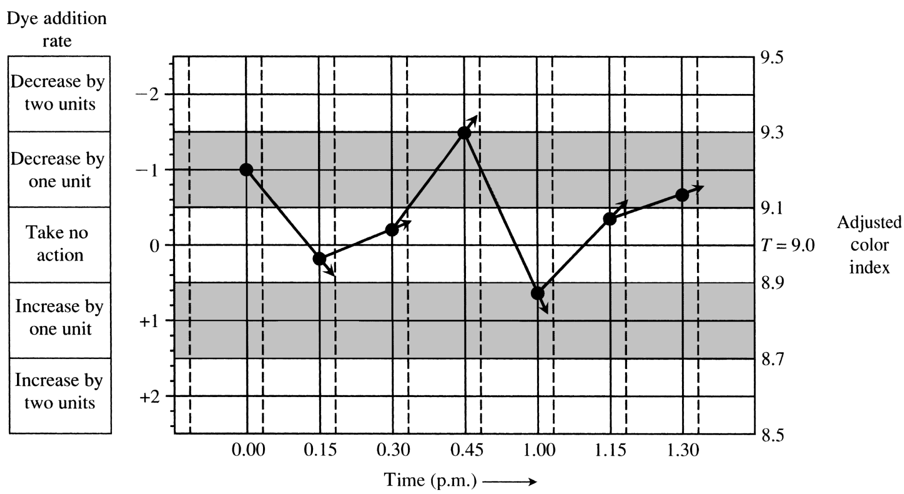

FIGURE 15.9 Rounded adjustment chart for proportional–integral control.

15.2.6 Complementary Roles of Monitoring and Adjustment

It is sometimes complained that feedback control can conceal the nature of a compensated disturbance that otherwise might be eliminated. However, when combined with appropriate monitoring, this need not happen. Adjustment schemes and monitoring schemes are complementary and should be used in consort. Figure 15.10 illustrates the point. This shows the behavior of a simulated feedback scheme in which the disturbance is an IMA(0, 1, 1) process with $\lambda = 0.2$ and the process dynamics are represented by a first-order system (15.2.13) with $\delta = 0.5$ and $g = 1.0$. The calculations were made assuming that the system is controlled by the PI controller,

$$-X_t = \text{constant} + 0.20\varepsilon_t + 0.20\sum_{i=1}^{t}\varepsilon_i \tag{15.2.27}$$

which, for these stated parameter values, produces MMSE. Although this is not usually done, the control action X_t in Figure 15.10(b), as well as the deviation from target $\{\varepsilon_t\}$ in Figure 15.10(d), can be charted (or better still, displayed on the screen of a process computer). Assuming the dynamics known, the exact compensation $\mathcal{Y}_t$ shown in Figure 15.10(c) can also be computed and hence the original disturbance N_t of Figure 15.10(a) can be reconstructed.

Examination of these monitoring displays motivates a generalized concept of common and special causes. The disturbance and the dynamic system together define the *common cause* system, which is taken account of in the design of the controller. But management action could change the system and hence the appropriate form of control. For example, suppose it was discovered that in the operation of the system, the pattern of the feedback control action X_t shown in Figure 15.10(b) mirrored that of a particular impurity in the feedstock. If this correlation checked out as a causative relation, management might decide to change the control system either by removing the impurity from the feedstock before it reached the process, or if that were impossible or too expensive, by measuring it and compensating for it by appropriate feedforward control.

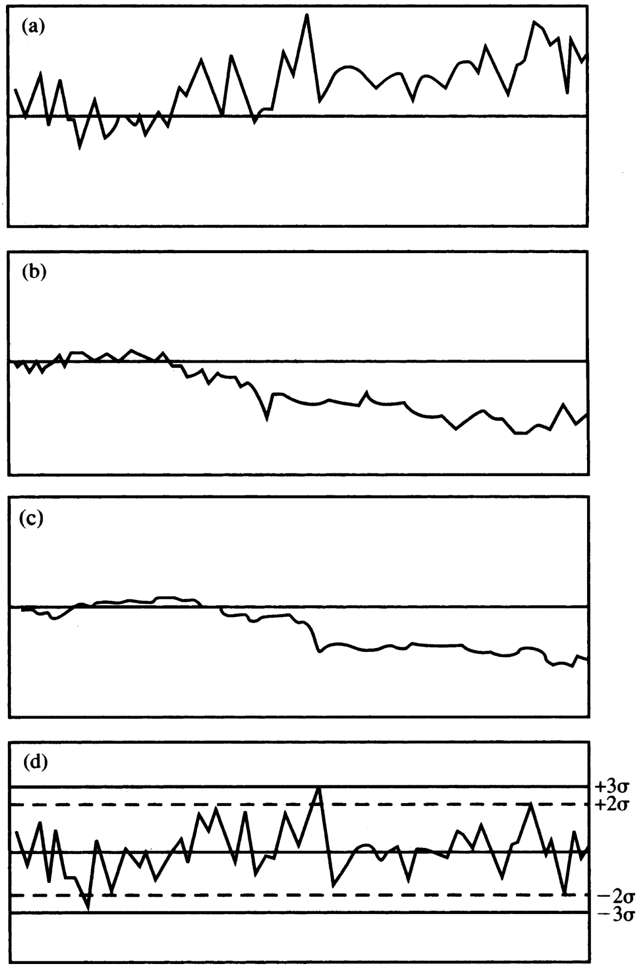

FIGURE 15.10 (a) Disturbance N_t, (b) feedback control action X_t, (c) compensation of the disturbance $\mathcal{Y}_t$, and (d) resulting deviation ε_t from the target value.

In addition, a *special cause* producing a temporary deviation from the underlying system model, induced perhaps by misoperation of the controller or a mistake by the operator, can be evidenced in the residual sequence $\{\varepsilon_t\}$ leading to remedial action. To illustrate this, we have added a deviation of size $3\sigma_a$ to the 30th value of the disturbance N_t in Figure 15.10(a). After the disturbance has been subjected to feedback control, this outlier is clearly visible in the record of the deviations ε_t from target plotted as a Shewhart chart in Figure 15.10(d). The control limits can be calculated directly from the models used to design the controller or from the record of the ε_t's during stable operation. Also, as noted later in Section 15.6, more specific checks may be applied to detect possible changes in the system parameters.

Assuming the models correct, in this particular example the residual ε_t's will be a white noise sequence. For control schemes that are not MMSE or that allow for dead time, however, the sequence $\{\varepsilon_t\}$ will, in general, be autocorrelated. One way to allow for this is to filter $\{\varepsilon_t\}$ suitably to produce a sequence that, given the assumed model, will be white noise. Appropriate checks may then be applied to that series.

15.3 EXCESSIVE ADJUSTMENT SOMETIMES REQUIRED BY MMSE CONTROL

One rationalization for the use of integral control and proportional–integral control is that for perhaps the simplest models for disturbance [equation (15.2.10)] and dynamics [equations (15.2.12) and (15.2.13)], which approximate reality, these forms of feedback adjustment can produce minimum mean square error.[2] Unfortunately, MMSE control sometimes requires unacceptably large manipulations of the compensating variable X_t. For illustration, consider again the situation where to an adequate approximation the disturbance model is the IMA(0, 1, 1) model of equation (15.2.10) with parameter θ and the dynamic model is the first-order difference equation (15.2.13) with parameters δ and g. Then, the MMSE feedback control adjustment scheme can be written (see (15.2.14)) as

$$x_t = -\frac{\lambda}{g}\frac{1-\delta B}{1-\delta}\varepsilon_t = -\frac{\lambda}{g(1-\delta)}(\varepsilon_t - \delta\varepsilon_{t-1}) \tag{15.3.1}$$

where $\lambda = 1-\theta$ and $\varepsilon_t = a_t$. If δ is negligibly small, MMSE control will be obtained with $x_t = -(\lambda/g)\varepsilon_t$ and let us then write

$$\sigma_x^2 = \text{var}[x_t] = \frac{\lambda^2}{g^2}\sigma_a^2 = k \tag{15.3.2}$$

But then, when δ is *not* negligible,

$$\sigma_x^2 = k\left[\frac{1+\delta^2}{(1-\delta)^2}\right]$$

Thus, if δ were near its upper limit of unity, σ_x^2 could become very large. For example, with $\delta = 0.9$ (so that only 1/10 of the eventual change produced by a step input is experienced in the first interval), $\sigma_x^2 = 181k$. In fact, as δ approaches unity, the MMSE control action in equation (15.3.1) takes on more and more of an ''alternating'' character,[3] the adjustment made at time t reversing a substantial portion of the adjustment made at time $t-1$. The reason for such alternating and variable adjustment can also be understood from the consideration that with $\delta = 0.9$, the constant $P = \xi = 9$ of the manual adjustment chart for MMSE control would call for *extrapolation* of the line joining ε_{t-1} and ε_t by *nine sampling intervals!* In practice, constrained schemes can be used that at the expense of rather small increases in MSE at the output require much less compensatory manipulation.

[2]This theoretical formulation, which results in a discrete PI controller yielding MMSE, is, however, not unique. For example, a PI controller giving MMSE can be obtained from the models $\mathcal{Y}_t = gBX_t$ and $N_t = (1-\theta_1 B - \theta_2 B^2)a_t$, as well as the dynamics model (15.2.13) with IMA(0, 1, 1) noise model (15.2.10).

[3]A value of $\delta = 0.9$ corresponds to a time constant for the system of over nine sampling intervals. The occurrence of such a value would immediately raise the question as to whether the sampling interval being taken was too short; whether in fact the inertia of the process was so large that little would be lost by less frequent surveillance. Now (see Appendix A15.2) the question of the choice of sampling interval must depend on the nature of the noise that infects the system. Because the properties of the noise usually reflect system inertia as well, in many cases it would be concluded that the sampling interval should be increased.

15.3.1 Constrained Control

When the adjustments x_t form a stationary time series, such constrained control schemes can be obtained by finding an unconstrained minimum of the expression

$$\sigma_\varepsilon^2 + \alpha\sigma_x^2 \tag{15.3.3}$$

where α can be regarded as an undetermined multiplier that allocates the relative *quadratic costs* of variations of ε_t and x_t. Such a scheme will be called a constrained MMSE scheme or CMMSE scheme. In particular, we have seen that for an IMA(0, 1, 1) disturbance and first-order dynamics, the *unconstrained* MMSE scheme calls for an adjustment of

$$x_t = -\frac{\lambda}{g}(1 + \xi\nabla)\varepsilon_t = -\frac{\lambda(1 - \delta B)}{g(1 - \delta)}\varepsilon_t \tag{15.3.4}$$

It is shown in Appendix A15.1 (see equation (A15.1.27)) that the corresponding CMMSE is of the form

$$x_t = [k_1 + (1 - \lambda)k_0]x_{t-1} - (1 - \lambda)k_1 x_{t-2} - \frac{\lambda(1 - k_0)(1 - \delta B)}{g(1 - \delta)}\varepsilon_t \tag{15.3.5}$$

where k_0 and k_1 are fairly complicated functions of the parameters g, λ, δ, and α. A table for applying such control is also given in Appendix A15.1.

For illustration suppose that $\lambda = 0.6$, $\delta = 0.5$, and $g = 1$; then the optimal *unconstrained* MMSE scheme is

$$x_t = -1.2(1 - 0.5B)\varepsilon_t \tag{15.3.6}$$

with

$$\sigma_x^2 = (0.6)^2\left[\frac{1 + (0.5)^2}{(1 - 0.5)^2}\right]\sigma_a^2 = 1.80\sigma_a^2$$

from (15.3.2)–(15.3.2a), and $\sigma_\varepsilon^2 = \sigma_a^2$. Suppose that this amount of variation in the adjustment x_t produced difficulties in process operation and it was desired to reduce it so that σ_x^2 was about $0.50\sigma_a^2$. Use of Table A15.2 shows that this can be achieved with the scheme

$$x_t = 0.32x_{t-1} - 0.06x_{t-2} - (0.57 \times 1.2)(1 - 0.5B)\varepsilon_t \tag{15.3.7}$$

which reduces σ_x^2 to $0.47\sigma_a^2$ with $\sigma_\varepsilon^2 = 1.07\sigma_a^2$. Thus, an almost fourfold reduction in σ_x^2 is produced for an increase of only 7% in the output variance. Such optimal constrained schemes are extremely attractive since they often produce a very large reduction in σ_x^2 for only a small increase in σ_ε^2. See, for example, Whittle (1963), Tunnicliffe Wilson (1970a, 1970b), MacGregor (1972), Box and Jenkins (1976), Harris et al. (1982), Aström and Wittenmark (1984), Rivera et al. (1986), and Bergh and MacGregor (1987). Unfortunately, such schemes can become complicated.

In practice, however, exact ''optimality'' is to some extent an illusion because assumptions are never true. It turns out that a form of constrained control, which is almost as good as CMMSE control, can often be obtained using an *appropriately tuned* PI controller. Such a controller has the advantage that it is simple and, in particular, is easily adapted to manual control. The following example shows how suitably tuned PI controllers can do almost as

TABLE 15.1 Illustrative Results Comparing Different Control Schemes for Models (15.2.13) and (15.2.10), with g= 0.4, $\delta = 0.5$, $\lambda = 0.4$, and $\sigma_a^2 = 1$

		σ_ε^2	σ_x^2
(a) MMSE control	$-x_t = (1+\nabla)\varepsilon_t$	1	5
(b) Optimal constrained control	$-x_t = -0.82x_{t-1} - 0.21x_{t-2}$ $-0.39\varepsilon_t + 0.19\varepsilon_{t-1}$	1.20	0.25
(c) Optimal constrained PI control	$-x_t = 0.52(1-0.25\nabla)\varepsilon_t$	1.20	0.25

well as optimal constrained schemes in producing great reductions in the variance σ_x^2 of the adjustment for only modest increases in the output variance σ_ε^2.

As an illustration, consider once again the situation where the process disturbance is represented by an IMA(0, 1, 1) process of (15.2.10) and the process dynamics by the first-order system (15.2.13), that is,

$$(1-\delta B)\mathcal{Y}_t = (1-\delta)gBX_{t+}$$

and suppose that $\lambda = 0.4$, $\sigma_a^2 = 1$, $g = 0.4$, and $\delta = 0.5$, so that $\xi = \delta/(1-\delta) = 1$. Then minimum mean square error control is achieved by the PI scheme (a) shown in Table 15.1, yielding an output variance σ_ε^2 of 1.00 with $\sigma_x^2 = 5$. Using the optimal constrained control equation (b) in Table 15.1, it is possible to achieve a 20-fold reduction in σ_x^2 (to 0.25) at the expense of a 20% increase in σ_ε^2 to 1.20. But almost nothing is lost by, instead, using the much simpler optimal constrained PI controller (c) in Table 15.1 for which, to two-decimal accuracy, the same result is obtained. Notice that if we use a manual adjustment chart for the MMSE PI scheme (a), it would be necessary to extrapolate one whole time period ahead from the current time t. However, for the constrained PI control (c), we must *interpolate* a quarter of a period back from the current time t. This accounts for the much greater stability of the latter scheme. A fuller discussion of this topic can be found in Box and Luceño (1993).

15.4 MINIMUM COST CONTROL WITH FIXED COSTS OF ADJUSTMENT AND MONITORING

From the point of view of cost, we can summarize the discussion so far as follows. If we assume that the *only* control cost we need to consider is that of being off target and that this cost is proportional to the square of the deviation from target, unconstrained minimum mean square error control implies minimization of the total cost of the scheme. Suppose, however, that there is an additional quadratic loss associated with the size of the adjustment x_t, and that α is some measure of the *relative* cost of being off target and of making adjustments. Then, $\sigma_\varepsilon^2 + \alpha\sigma_x^2$ can be a measure of the overall cost of the scheme, and minimization of this quantity can produce a control scheme yielding minimum cost, and, as we have seen, suitably chosen PI schemes can often do almost as well. In either case, in practice, it is rarely easy to gauge α, in terms of relative costs. Instead, choice of a suitable scheme can be made by empirical judgment of what constitutes a satisfactory reduction of σ_x^2 in exchange for an acceptable increase in σ_ε^2. The same kinds of considerations apply to systems for which there are fixed adjustment and monitoring costs.

15.4.1 Bounded Adjustment Scheme for Fixed Adjustment Cost

Especially in the ''parts'' industries, situations occur where an adjustment often has immediate effect but entails a *fixed* cost incurred, for example, by stopping a machine or changing a tool.

Bounded Adjustment Charts. It was shown by Box and Jenkins (1963) that in the latter case, on the assumption of a quadratic off-target loss and an IMA disturbance, the minimum cost feedback control is *not* achieved by repeated adjustment after each observation. Instead, it requires that an adjustment be made only when an exponentially weighted average $\hat{\varepsilon}_t(1)$ of the deviations from target falls outside some fixed limits, $\pm L$, say. We call this *bounded adjustment*. The adjustment that should then be made is the one that will produce a change $-\hat{\varepsilon}_t(1)$ at the output. Such an adjustment can be put into effect manually using a ''bounded adjustment chart'' such as that discussed below, or automatically.

A bounded adjustment chart such as that shown in Figure 15.11 is superficially similar to that proposed for process monitoring by Roberts (1959). However, its purpose and design are different. The purpose is to decide when, and by how much, to *adjust* the process. The boundary lines are designed to minimize the overall cost, taking into account both the cost of making adjustments and the cost of being off target. Their purpose is *not* to discover statistically significant deviations from target. As the cost of adjustment approaches zero, the lines come closer together, converging on the target value when the cost of adjustment is zero and so yielding the ''repeated adjustment'' MMSE scheme.

Figure 15.11 shows an example of such a chart for the metallic thickness control problem that would be appropriate if there had been a fixed cost for changing the deposition rate X. As before, $\lambda = 0.2$, $g = 1.2$, and $\sigma_a = 11$. At time t, an open circle represents the deviation from target ε_t obtained after periodically changing the deposition rate X_t as required by the chart. A filled circle represents an appropriate exponentially weighted moving average

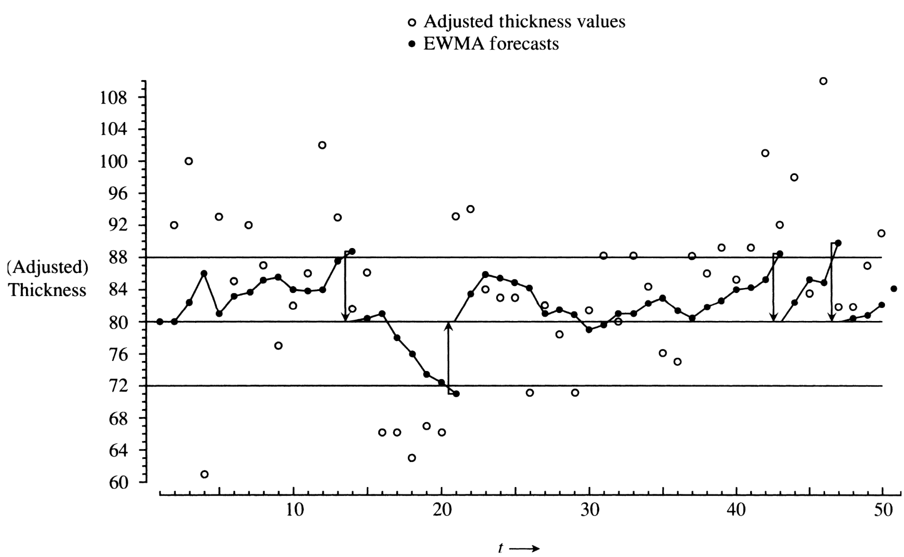

FIGURE 15.11 Bounded adjustment chart: the open circles are the thickness deviations ε_t (after adjustment), the filled circles are their EWMA forecasts $\hat{\varepsilon}_{t-1}(1)$ of these deviations.

forecast. This is conveniently updated using the formula

$$\hat{\varepsilon}_t(1) = \lambda\varepsilon_t + \theta\hat{\varepsilon}_{t-1}(1)$$

The particular chart shown has boundary lines at 80 ± 8, that is, at $T \pm 0.720\sigma_a$. We discuss the rationale for this choice below. To understand how the chart operates, suppose initially that the deposition rate is some value X_0. This will remain unchanged until time $t = 13$, when the forecasted value 88.7 (i.e., $\hat{\varepsilon}_t(1) = 8.7$) falls outside the upper limit and the chart signals that a change is needed in the deposition rate that will reduce the thickness by -8.7. An adjustment of

$$X_{13} - X_0 = -8.7/1.2$$

is now made in the deposition rate. Notice that such an adjustment does not upset the calculation of the next EWMA. For example, the forecasted thickness at time $t = 14$ is

$$(0.2 \times 81.3) + (0.8 \times 80.0) = 80.3$$

where 80 is the appropriate previous forecasted value *after the adjustment has been made to bring the process on target.*

15.4.2 Indirect Approach for Obtaining a Bounded Adjustment Scheme

Tables for calculating the positions of the appropriate limit lines for minimum cost schemes in terms of the *cost of being off target* and the *cost of adjustment* were provided by Box and Jenkins (1963), Box et al. (1974), and Box and Kramer (1992). However, as we said earlier, these costs are not always easy to assess, and it seems more practical to use these results to provide an envelope of minimum cost schemes and then to choose among them empirically by considering the increased standard deviation at the output obtained in exchange for a longer interval between making adjustments. This approach was illustrated by Box (1991b). Table 15.2 shows theoretical average adjustment intervals (AAIs) and percent increase in standard deviation (ISD) of the adjusted process for various values of λ and L/σ_a, where limit lines of the bounded adjustment scheme are at $T \pm L$.

For illustration, consider again the thickness adjustment example. Entering Table 15.2 with $\lambda = 0.2$ shows how much inflation in the error standard deviation would occur for a bounded scheme for various choice of L/σ_a. Thus, if L/σ_a were set equal to 0.5, a 2.6% increase in the standard deviation would occur, but on the average, adjustments would be needed only every 10 intervals. If L/σ_a were set equal to 1.0, a 9% increase in standard deviation would result, but the AAI would be 32. The scheme depicted in Figure 15.11 is a compromise in which L/σ_a was set equal to 0.72, which rough interpolation shows would give a 5% increase in the standard deviation with an AAI of about 20. To achieve this, L was set equal to $8 \approx 0.72 \times 11$. A Monte Carlo study using the 100 observations of metallic thickness graphed in Figure 15.2 shows an actual inflation of the standard deviation of 8.5% for this example with an AAI of 14. In view of the rather limited sample size, the agreement must be considered quite good.

Interpolation Chart. Any degree of technological sophistication can be used in applying these ideas: anything from transducers taking actions calculated by computers to operators taking actions based on a simple interpolation chart such as that shown in Figure 15.12, which used a pushpin and a piece of thread to indicate the appropriate *manual* adjustment.

TABLE 15.2 Average Adjustment Interval (AAI) and Percent Increase in Standard Deviation of Output (ISD) for Various Choice of L/σ_a Where the Limit Lines Are at $T \pm L$

λ	L/σ_a	AAI	Percent Increase in Standard Deviation ISD
0.1	0.5	32	2.4
	1.0	112	9
	1.5	243	18
	2.0	423	30
0.2	0.5	10	2.6
	1.0	32	9
	1.5	66	20
	2.0	112	32
0.3	0.5	5	2.6
	1.0	16	10
	1.5	32	20
	2.0	52	33
0.4	0.5	4	2.6
	1.0	10	10
	1.5	19	21
	2.0	32	34
0.5	0.5	3	2.5
	1.0	7	10
	1.5	13	21
	2.0	21	35

Source: Box (1991b).

In the situation depicted, a previous forecast made at time $t - 1$ was 86 and the observation, which has just been made at time t, is 66. Just before the current time t, therefore, the location of the pushpin on the current forecast scale would be at 86 with the thread hanging down from the pin. As soon as the actual value 66 became available, the thread would be pulled tightly to join the point 66 on the right-hand scale. The updated forecast of 82 would then be read off on the intermediate scale. This value lies within the boundaries, so that pushpin would be moved down to this new current forecast value with the thread hanging loose again until the next observation became available to produce a new updated forecast. As soon as an updated forecast fell outside either boundary, the appropriate adjustment in deposition rate to cancel out the forecasted deviation would be made, and the pushpin would then be *placed on the target value* ready for the next interpolation.

15.4.3 Inclusion of the Cost of Monitoring

It was shown by Box and Kramer (1992) how these results could be extended to the case where the cost of monitoring the process had also to be taken into account. They considered the possibility of further reducing cost by less frequent monitoring at an interval m instead of at a unit interval. They provided charts for obtaining minimum cost schemes given that in addition to σ_a and λ (estimated from plant data), three cost constants were known:

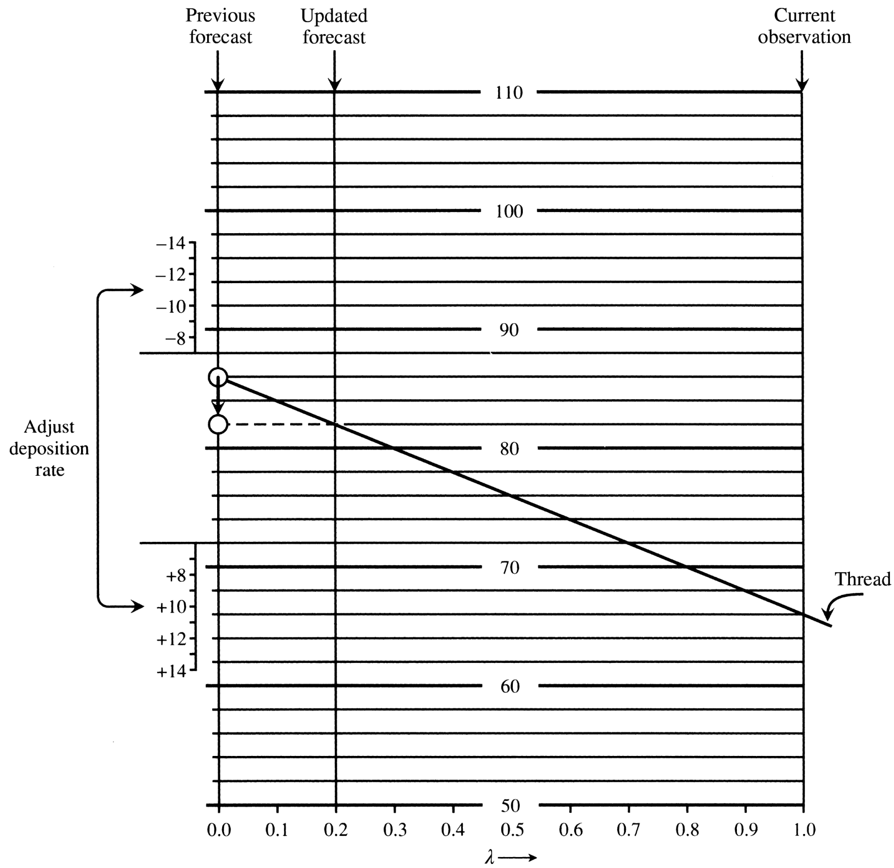

FIGURE 15.12 Interpolation chart to update the forecasted value of thickness and to indicate when and by how much the deposition rate should be adjusted.

(1) the (assumed quadratic) cost of being off target, (2) the fixed cost of making a change, and (3) the fixed monitoring cost of taking an observation. Given this information, the corresponding values of L/σ_a and of m yielding minimum cost could be read off their charts.

Again, these three individual costs may not be easy to determine, and Box and Luceño (1993) used their results to allow the choice of scheme to be based on empirical judgment. The charts shown in Figure 15.13 give the values of the AAI and the percent ISD with respect to σ_a corresponding to value of the nonstationarity measure $\lambda = 0.1(0.1)0.6$, 0.8, and 1.0, the standardized action limit $L/\sigma_a = 0.0(0.25)\ 2.5$, and the monitoring interval $m = 1, 2, 3, \ldots$. The charts cover small to moderate increases in the output standard deviation such as might be needed in practice. Thus, the larger values of m appear only with smaller values of λ.

For example, we saw earlier that by using a bounded adjustment chart with $L/\sigma_a = 0.72$ instead of a continuous scheme, the average adjustment interval could be increased to about 20 at the cost of an increase of 5% in the standard deviation. This is confirmed by the chart of Figure 15.13 for $\lambda = 0.2$, which also shows, for example, that if we monitor the process

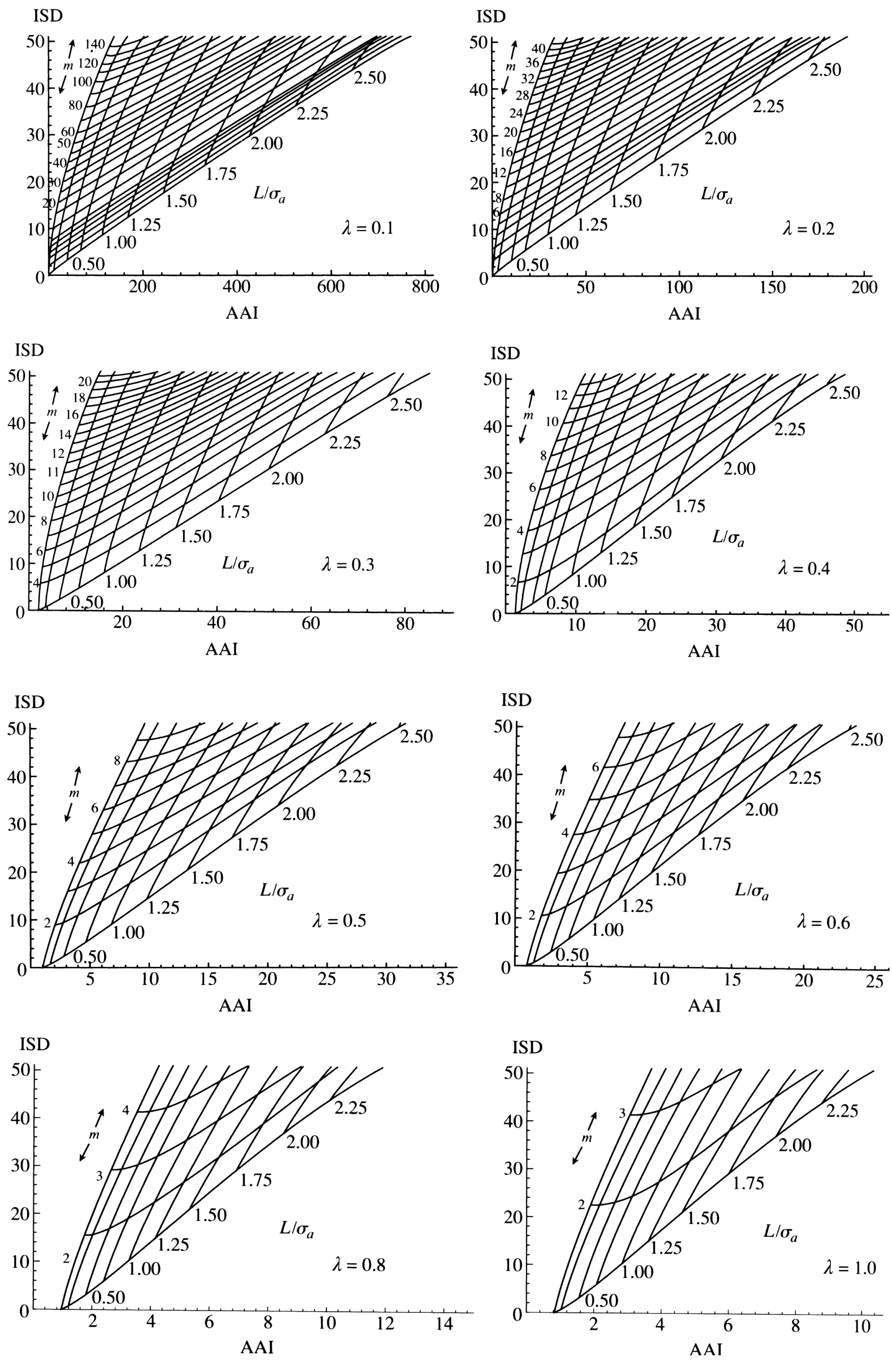

FIGURE 15.13 Charts for $\lambda = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, and 1.0 showing AAIs and ISDs obtained from various choices of L/σ_a and m.

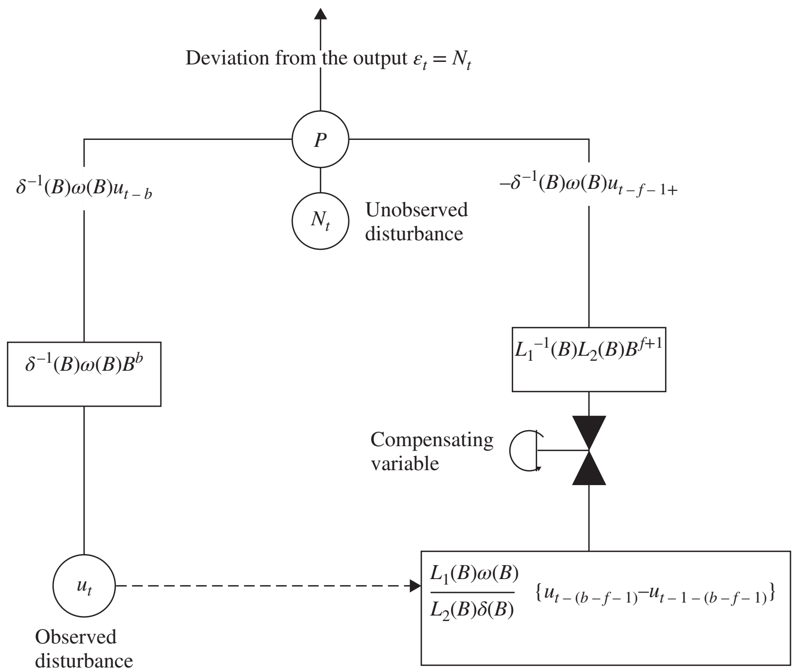

FIGURE 15.14 System at time t subject to an observed input disturbance u_t and unobserved disturbance N_t, with potential compensating variable X_t.

half as frequently ($m = 2$) and we again set $L/\sigma = 0.72$, we could obtain about the same average adjustment interval (20) but with an 8% increase in the standard deviation.

15.5 FEEDFORWARD CONTROL

We now consider the design of discrete *feedforward* control schemes that give minimum mean square error at the output. A situation arising in the manufacture of a polymer is illustrated in Figure 15.14. The viscosity Y_t of the product is known to vary in part due to fluctuations in the feed concentration u_t, which can be observed but not changed. The steam pressure X_t is a control variable that is measured, can be manipulated, and is potentially available to alter the viscosity by any desired amount and hence compensate potential deviations from target. The total effect in the output viscosity of all *other* sources of disturbance at time t is denoted by N_t.

15.5.1 Feedforward Control to Minimize Mean Square Error at the Output

We can suppose that Y_t, u_t, X_t, N_t are deviations from reference values, which are such that if the conditions $u = 0$, $X = 0$, $N = 0$ were continuously maintained, then the process would remain in an equilibrium state such that the output was exactly on the target value $Y = 0$.

The transfer function model, which connects the observed but uncontrollable input disturbance u_t (feed concentration) and the output Y_t (viscosity), is assumed to be

$$\mathcal{Y}_{1t} = \delta^{-1}(B)\omega(B)B^b u_t$$

Now, changes will be made in X at times $t, t-1, t-2, \ldots$ immediately after the observations $u_t, u_{t-1}, u_{t-2}, \ldots$ are taken. Hence, we obtain a ''pulsed'' input, and we denote the level of X in the interval t to $t+1$ by X_{t+}. For this pulsed input, it is assumed that the transfer function model, which connects the compensating variable X_t (steam pressure) and the output Y_t (viscosity), has the effect

$$\mathcal{Y}_{2t} = L_1^{-1}(B)L_2(B)B^{f+1}X_{t+}$$

where $L_1(B)$ and $L_2(B)$ are polynomials in B. Then, if no control is exerted (the potential compensating variable X_t is held fixed at $X_t = 0$), the total error or deviation from target value $T = 0, \varepsilon_t = Y_t - T$, in the output viscosity will be

$$\varepsilon_t = \delta^{-1}(B)\omega(B)u_{t-b} + N_t$$

Clearly, it ought to be possible to compensate the effect of the measured parts of the overall disturbance by manipulating X_t. Now at time t, and at the point P in Figure 15.14,

1. The total effect of the input disturbance (u) is

$$\delta^{-1}(B)\omega(B)u_{t-b}$$

2. The total effect of the compensation (X) is

$$L_1^{-1}(B)L_2(B)X_{t-f-1+}$$

and we assume that the effects of the input influences u and X on the output Y are additive. Then, the effect of the observed input disturbance u will be canceled if we set

$$L^{-1}(B)L_2(B)X_{t-f-1+} = -\delta^{-1}(B)\omega(B)u_{t-b}$$

Thus, the control action at time t should be such that

$$L_1^{-1}(B)L_2(B)X_{t+} = -\delta^{-1}(B)\omega(B)u_{t-(b-f-1)} \tag{15.5.1}$$

Case 1: b ≥ f + 1. Now at time t, the values $u_{t+1}, u_{t+2} \ldots$ are unknown. The control action (15.5.1) is directly realizable, therefore, only if $(b - f - 1) \geq 0$, in which case the desired control action at time t is to set the manipulated variable X to the level

$$X_{t+} = -\frac{L_1(B)\omega(B)}{L_2(B)\delta(B)}u_{t-(b-f-1)} \tag{15.5.2}$$

Alternatively, it is often more convenient to define the control action in terms of the *change* $x_t = X_{t+} - X_{t-1+}$, which is to be made in the level of X immediately after the observation u_t has come to hand. This is

$$x_t = -\frac{L_1(B)\omega(B)}{L_2(B)\delta(B)}(u_{t-(b-f-1)} - u_{t-1-(b-f-1)}) \tag{15.5.3}$$

The situation is illustrated in Figure 15.14. The effect at P from the control action is $-\delta^{-1}(B)\omega(B)u_{t-b}$, and this exactly cancels the effect at P of the input disturbance. The component of the deviation from target due to u_t is (theoretically at least) exactly eliminated at the observation times, and only the component N_t due to the unobserved disturbance remains.

Case 2:* $(b - f - 1)$ *Negative. It can happen that $f + 1 > b$. This means that an observed input disturbance reaches the output before it is possible for compensating action to become effective. In this case the action in (15.5.2) is not realizable because at time t, when the action is to be taken, the relevant value $u_{t+(f+1-b)}$ of the input disturbance is not yet available. One would usually avoid this situation if one could (if some quicker acting compensating variable could be used instead of X), but sometimes such an alternative is not available.

Now with $u'_t = \delta^{-1}(B)\omega(B)u_t$ represented by the linear model (see, for example, Box et al. (1974))

$$u'_t = \left(1 + \sum_{i=1}^{\infty} \psi'_i B^i\right)\alpha_t$$

where α_t is a white noise process with mean zero and variance σ_α^2, then

$$u'_{t+f+1-b} = \hat{u}'_t(f+1-b) + e'_t(f+1-b)$$

In this expression

$$e'_t(f+1-b) = \alpha_{t+f+1-b} + \psi'_1\alpha_{t+f-b} + \cdots + \psi'_{f-b}\alpha_{t+1}$$

is the forecast error. Then, we can write the right-hand side of (15.5.2) in the form

$$-L_1(B)L_2^{-1}(B)\hat{u}'_t(f+1-b) - L_1(B)L_2^{-1}(B)e'_t(f+1-b)$$

Now, $e'_f(f+1-b)$ is a function of the uncorrelated random variates $\alpha_{t+h}(h \geq 1)$, which have not yet occurred at time t and which are uncorrelated with any variable known at time t (and the α_{t+h} are therefore not forecastable). It follows that the optimal (minimum mean square error) action is achieved by setting

$$X_{t+} = -\frac{L_1(B)}{L_2(B)}\hat{u}'_t(f+1-b) \tag{15.5.4}$$

that is, by making the *change* in the compensating variable at time t equal to

$$x_t = -\frac{L_1(B)}{L_2(B)}\{\hat{u}'_t(f+1-b) - \hat{u}'_{t-1}(f+1-b)\} \tag{15.5.5}$$

This results in an additional component in the deviation ε_t from the target, which now becomes

$$\varepsilon_t = N_t + e'_{t-f-1}(f+1-b)$$

If the model for the input disturbance is $\varphi_u(B)u_t = \theta_u(B)\alpha_t$, then the model for $u'_t = \delta^{-1}(B)\omega(B)u_t$ can be written

$$\varphi'_u(B)u'_t = \theta'_u(B)\alpha_t$$

with

$$\varphi'_u(B) = \varphi_u(B)\delta(B) \text{ and } \theta'_u(B) = \theta_u(B)\omega(B)$$

The needed forecasts $\hat{u}'_t(f + 1 - b)$, obtained as in Chapter 5, can then be written conveniently in terms of previous u's and α's obtainable from the u series itself.

15.5.2 An Example: Control of the Specific Gravity of an Intermediate Product

In the manufacture of an intermediate product, used for the production of a synthetic resin, the specific gravity Y_t of the product had to be maintained as close as possible to the value 1.260. This was actually achieved by a mixed scheme of feedforward and feedback control. We consider the complete scheme later and discuss here only the feedforward part. The process has rather slow dynamics, and also the disturbance is known to change slowly, so that observations and adjustments are made at 2-hour intervals. The uncontrolled input disturbance that is fed forward is the feed concentration u_t, which is measured as deviations from an origin of 30 g/L. The relation between specific gravity and feed concentration over the range of normal operation has the effect

$$\mathcal{Y}_{1t} = 0.0016u_t$$

where the effect $\mathcal{Y}_{1t}$ is measured from the target value 1.260.

This relation contains ''no dynamics'' because the feed concentration can only be measured at the inlet to the reactor, so that in our general notation $\delta(B) = 1, \omega(B) = 0.0016, b = 0$. Control is achieved by varying pressure, which is referred to a convenient origin of 25 psi. The transfer function model relating specific gravity and pressure X_t was estimated as having the effect

$$(1 - 0.7B)\mathcal{Y}_{2t} = 0.0024X_{t-1+}$$

so that $L_1(B) = (1 - 0.7B), L_2(B) = 0.0024, f = 0$. So far as could be ascertained, the effects of pressure and feed concentration were approximately additive in the region of normal operation. Therefore, the control equation (15.5.4) is used, since $b - f - 1$ is negative, and yields

$$X_{t+} = -\frac{(1 - 0.7B)0.0016}{0.0024}\hat{u}_t(1) \tag{15.5.6}$$

for, in this particular example, $u'_t = 0.0016u_t$ and hence $\hat{u}'_t(1) = 0.0016\hat{u}_t(1)$. Study of the feed concentration showed that it could be represented by the linear stochastic model of order (0, 1, 1),

$$\nabla u_t = (1 - \theta_u B)\alpha_t$$

with $\theta_u = 0.5$. For such a process,

$$\hat{u}_t(1) = (1 - \theta_u)u_t + \theta_u\hat{u}_{t-1}(1)$$

that is, $(1 - \theta_u B)\hat{u}_1(1) = (1 - \theta_u)u_t$ or

$$\hat{u}_t(1) = \frac{1 - \theta_u}{1 - \theta_u B}u_t$$

TABLE 15.3 Calculation of Adjustments for Feedforward Control Scheme (15.5.7)

	Concentration			Pressure	
t	$u_t + 30$	u_t	X_{t+}	$X_{t+} + 25$	x_t
0	31.6	1.6	−0.63	24.4	
1	31.1	1.1	−0.31	24.7	0.3
2	34.4	4.4	−1.36	23.6	−1.1
3	32.0	2.0	−0.32	24.7	1.1
4	28.2	−1.8	0.90	25.9	1.2

Thus, the control equation (15.5.6) can be written finally as

$$X_{t+} = -\frac{(1 - 0.7B)0.0016(0.5)}{0.0024(1 - 0.5B)}u_t$$

or

$$X_{t+} = 0.5X_{t-1+} - 0.333(u_t - 0.7u_{t-1}) \qquad (15.5.7)$$

Table 15.3 shows the calculation of the first few of a series of settings of the pressure required to compensate the variations in feed concentration, given the starting conditions for time $t = 0$ of $u_0 = 1.6$, $X_{0+} = -0.63$. Once the calculation has been started off, it is sometimes more convenient to work directly with the changes x_t to be made at time t using

$$x_t = 0.5x_{t-1} - 0.333(\nabla u_t - 0.7\nabla u_{t-1}) \qquad (15.5.8)$$

Figure 15.15a shows a section of the feed concentration. Figure 15.15b shows the output after applying feedforward control. Figure 15.15c shows the specific gravity if no control had been applied. These values Y_t are, of course, not directly available but may be obtained in general from the values Y_t', which actually occurred using

$$Y_t = Y_t' + \hat{u}_{t-f-1}'(f + 1 - b)$$

For this example then

$$Y_t = Y_t' + \frac{0.0008}{1 - 0.5B}u_{t-1}$$

that is,

$$Y_t = 0.5Y_{t-1} + Y_t' - 0.5Y_{t-1}' + 0.0008u_{t-1}$$

As a result of feedforward control, the root mean square error deviation of the output from the target value over the sample record shown is 0.003. Over the same period, the root mean square error of the uncorrected series would have been 0.008. The improvement is marked and extremely worthwhile. However, it appears that other unidentified sources of disturbance exist in the process, as evidenced by the drift away from target. This kind of tendency is frequently met in pure feedforward control schemes, but may be compensated by the addition of feedback control, as discussed in Section 15.2. We will briefly indicate the details of the combined scheme later in Section 15.5.4.

Control action is effected in whatever manner is most suited to the situation. If changes are made infrequently, and if the control equation is fairly simple as in the above example,

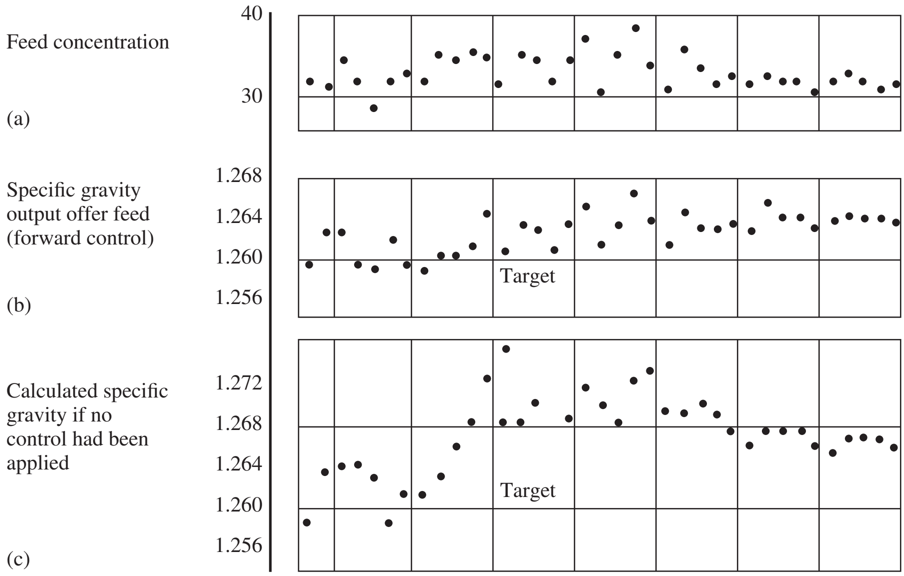

FIGURE 15.15 (a) Feed concentration, (b) Specific gravity after feedforward control, (c) Specific gravity if no control had been applied.

the theory we have outlined may be used to obtain optimal control *manually*. It is then convenient to use some form of control chart or nomogram that can be easily understood by the process operator, similar to charts illustrated in Section 15.2 regarding feedback control.

15.5.3 Feedforward Control with Multiple Inputs

No difficulty arises in principle when the effects of several additive input disturbances $u_1, u_2, \ldots, u_m$ are to be compensated by changes in X using feedforward control. Suppose the combined effect at the output of all the input disturbances is given by

$$\mathcal{Y}_t = \sum_{j=1}^{m} \delta_j^{-1}(B)\omega_j(B)B^{b_j}u_{j,t} = \sum_{j=1}^{m} B^{b_j}u'_{j,t}$$

where $u'_{j,t} = \delta_j^{-1}(B)\omega_j(B)u_{j,t}$, and, as before, the transfer function model for the compensating variable contributes the effect

$$\mathcal{Y}_{2t} = L_1^{-1}(B)L_2(B)B^{f+1}X_{t+}$$

Then, proceeding precisely as before, the required control action is to change X at time t by an amount

$$x_t = -L_1(B)L_2^{-1}(B)\sum_{j=1}^{m}[u'_{j,t+f+1-b_j} - u'_{j,t+f-b_j}] \tag{15.5.9}$$

where

$$[u'_{j,t+f+1-b_j} - u'_{j,t+f-b_j}] \\ = \begin{cases} u'_{j,t+f+1-b_j} - u'_{j,t+f-b_j} & f+1-b_j \leq 0 \\ \hat{u}'_{j,t}(f+1-b_j) - \hat{u}'_{j,t-1}(f+1-b_j) & f+1-b_j > 0 \end{cases} \tag{15.5.10}$$

If, as before, N_t is an unmeasurable disturbance, then the error or deviation from target at the output from this control action in the compensating variable X_t+ will be

$$\varepsilon_t = N_t + \sum_{j=1}^{m} e'_{j,t-f-1}(f+1-b_j) \tag{15.5.11}$$

where $e'_{j,t-f-1}(f+1-b_j) = 0$ if $f+1-b_j \leq 0$, and is the forecast error corresponding to the jth input variable $u_{j,t}$ if $f+1-b_j > 0$.

On the one hand, feedforward control allows us to take prompt action to cancel the effect of input disturbance variables, and if $f+1-b_j \leq 0$, to anticipate completely such disturbances, at least in theory. On the other hand, to use this type of control we must be able to measure the disturbing variables and possess complete knowledge—or at least a good estimate—of the relationship between each input disturbance variable and the output. In practice, we could never measure *all* of the disturbances that affected the system. The remaining disturbances, which we have denoted by N_t and which are not affected by feedforward control, could of course increase the variance at the output or cause the process to wander off target, as in fact occurred in the example discussed in Section 15.5.2. Clearly, we can prevent this from happening by using the deviations ε_t themselves to indicate an appropriate adjustment, that is, by using feedback control as discussed in earlier sections of this chapter. In fact, a combined feedforward–feedback control scheme can be used, which provides for the elimination of identifiable input disturbances by feedforward control and for the reduction of the remaining disturbance by feedback control.

15.5.4 Feedforward–Feedback Control

A combined feedforward–feedback control scheme provides for the elimination of identifiable input disturbances by feedforward control and for the reduction of the remaining disturbance by feedback control. We briefly discuss a combined feedforward–feedback scheme in which m identifiable input disturbances $u_1, u_2, \ldots, u_m$ are fed forward. The combined effects on the output of all the input disturbances and of the compensating input variable X_t are assumed to be additive of the same form as given previously in Section 15.5.3. It is assumed also that N'_t is a further unidentified disturbance and that the *augmented noise* N_t is made up of N'_t plus that part of the feedforward disturbance that cannot be predicted at time t. Thus, using (15.5.11),

$$N_t = N'_t + \sum_{j=1}^{m} e'_{j,t-f-1}(f+1-b_j)$$

where $e'_{j,t-f-1}(f+1-b_j) = 0$ if $f+1-b_j \leq 0$, and includes any further contributions from errors in forecasting the identifiable inputs. It is assumed that N_t can be represented by a linear stochastic process so that, in the notation of Section 15.2.4, it follows that the relationship between the forecasts of this noise process and the forecast errors may be written as

$$\frac{L_3(B)(1-B)}{L_4(B)}\varepsilon_t = \hat{N}_t(f+1) - \hat{N}_{t-1}(f+1)$$

where $\varepsilon_t = e_{t-f-1}(f+1) = N_t - \hat{N}_{t-f-1}(f+1)$.

Arguing as in (15.2.16) and (15.5.9), the optimal control action for the compensating input variable X_t to minimize the mean square error at the output is

$$x_t = -\frac{L_1(B)}{L_2(B)}\left\{\sum_{i=1}^{m}[u'_{j,t+f+1-b_j} - u'_{j,t+f-b_j}] + \frac{L_3(B)(1-B)}{L_4(B)}\varepsilon_t\right\} \tag{15.5.12}$$

where the $[u'_{j,t+f+1-b_j} - u'_{j,t+f-b_j}]$ are as given in equation (15.5.10). The first term in the control equation (15.5.12) is the same as in (15.5.9) and compensates for changes in the feedforward input variables. The second term in (15.5.12) corresponds exactly to (15.2.16) and compensates for that part N'_t of the augmented noise, which can be predicted at time t.

An Example of Feedforward–Feedback Control. We illustrate by discussing further the example used in Section 15.5.2, where it was desired to control specific gravity as close as possible to a target value 1.260. Study of the deviations from target occurring *after feedforward control* showed that they could be represented by the IMA(0, 1, 1) process

$$\nabla N_t = (1-0.5B)a_t$$

where a_t is a white noise process. Thus,

$$\frac{L_3(B)(1-B)}{L_4(B)}a_t = \hat{N}_t(1) - \hat{N}_{t-1}(1) = 0.5a_t$$

and $\varepsilon_t = e_{t-1}(1) = a_t$. As in Section 15.5.2, the remaining parameters are

$$\delta^{-1}(B)\omega(B) = 0.0016 \quad b = 0$$

$$L_2^{-1}(B)L_1(B) = \frac{1-0.7B}{0.0024} \quad f = 0$$

and

$$\hat{u}_t(1) - \hat{u}_{t-1}(1) = \frac{0.5}{1-0.5B}(u_t - u_{t-1})$$

Using (15.5.12), the minimum mean square error adjustment incorporating feedforward and feedback control is

$$x_t = -\frac{1-0.7B}{0.0024}\left[\frac{(0.0016)(0.5)}{1-0.5B}(u_t - u_{t-1}) + 0.5\varepsilon_t\right] \tag{15.5.13}$$

that is,

$$x_t = 0.5x_{t-1} - 0.333(1-0.7B)(u_t - u_{t-1}) - 208(1-0.7B)(1-0.5B)\varepsilon_t$$

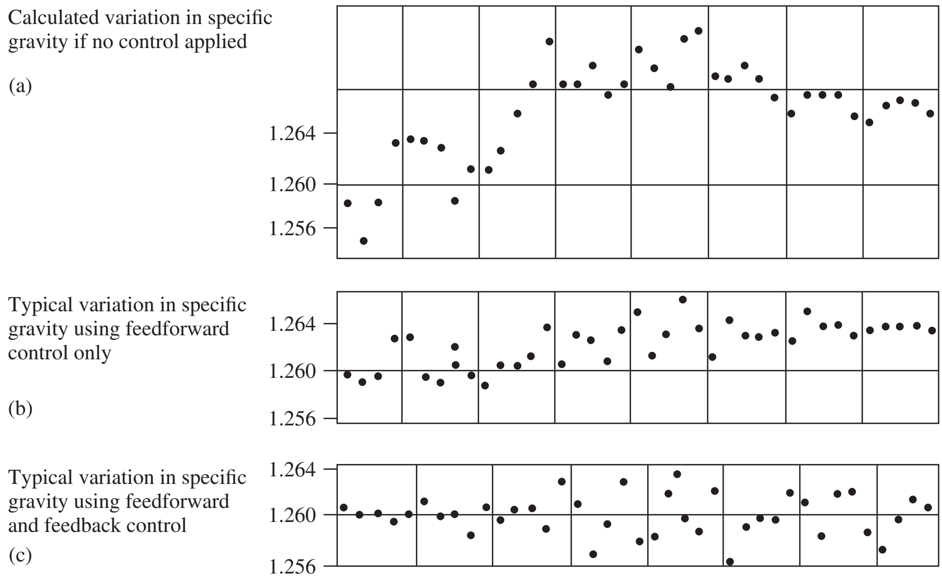

FIGURE 15.16 Typical variation in specific gravity with (a) no control, (b) feedforward control only, and (c) feedforward with feedback control.

or

$$x_t = 0.5x_{t-1} - 0.333u_t + 0.566u_{t-1} - 0.233u_{t-2} - 208\varepsilon_t + 250\varepsilon_{t-1} - 73\varepsilon_{t-2} \quad (15.5.14)$$

Figure 15.16 shows the section of record previously given in Figure 15.15, when only feedforward control was employed, and the corresponding calculated variation that would have occurred if no control had been applied. This is now compared with a record from a scheme using both feedforward and feedback control. The introduction of feedback control resulted in a further substantial reduction in mean square error and corrected the tendency to drift from the target, which was experienced with the feedforward scheme.

Note that with a feedback scheme, the correction employs a forecast having lead time $f + 1$, whereas with a feedforward scheme the forecast has lead time $f + 1 - b$ and no forecasting is involved if $f + 1 - b$ is zero or negative. Thus, feedforward control gains in the immediacy of possible adjustment whenever b is greater than zero. The example we have quoted is an exception in that $b = 0$, and consequently no advantage of immediacy is gained, in this case, by feedforward control. It might be true in this case that equally good control could have been obtained by a feedback scheme alone. In practice, possibly because of error transmission problems, the mixed scheme did rather better than the pure feedback system.

15.5.5 Advantages and Disadvantages of Feedforward and Feedback Control

With feedback control, it is the total disturbance, as evidenced by the error at the output, that actuates compensation. Therefore, it is not necessary to be able to identify and measure the sources of disturbance. All that is needed is that we *characterize* the disturbance N_t at the output by an appropriate stochastic model (and as we have seen in earlier sections, an IMA(0, 1, 1) model would often provide adequate approximation to the noise model). Because we are not relying on ‘‘dead reckoning,’’ unexpected disturbances and moderate errors in identifying and estimating the system’s characteristics will normally result only in greater variation about the target value and not (as may occur with feedforward control) in a consistent drift away from the target value. On the other hand, especially if the delay $f + 1$ is large, the errors about the target (since they are then the errors of a remote forecast) may be large, although they have zero mean. Clearly, if identifiable sources of input disturbance can be partially or wholly eliminated by feedforward control, then this should be done. Then, only the unidentifiable error has to be dealt with by feedback control.

In summary, although we can design a feedback control scheme that is optimal, in the sense that it is the best possible feedback scheme, it will not usually be as good as a combined feedforward–feedback scheme in which sources of error that can be are eliminated before the feedback loop.

15.5.6 Remarks on Fitting Transfer Function–Noise Models Using Operating Data

It is desirable that the parameters of a control system be estimated from data collected under as nearly as possible the conditions that will apply when the control scheme is in actual operation. The calculated control action, using estimates so obtained, properly takes account of noise in the system, which will be characterized as if it entered at the point provided for in the model. This being so, it is desirable to proceed iteratively in the development of a control scheme. Using technical knowledge of the process, together with whatever can be learned from past operating data, preliminary transfer function and noise models are postulated and used to design a pilot control scheme. The operation of this pilot scheme can then be used to supply further data, which may be analyzed to give improved estimates of the transfer function and noise models, and then used to plan an improved control scheme.

For example, consider a feedforward–feedback scheme with a single feedforward input, as in Section 15.5.1, and the case with $b - f - 1$ nonnegative. Then for any inputs u_t and X_{t+}, the output deviation from target is given by

$$\varepsilon_t = \delta^{-1}(B)\omega(B)u_{t-b} + L_1^{-1}(B)L_2(B)X_{t-f-1+} + N_t \tag{15.5.15}$$

and it is assumed that the noise N_t may be described by an ARIMA(p, d, q) model. It is supposed that time series data are available for ε_t, u_t, and X_{t+} during a sufficiently long period of actual plant operation. Often, although not necessarily, this would be a period during which some preliminary pilot control scheme was being operated. Then for specified orders of transfer function operators and noise model, the methods of Sections 12.3 and 12.4 may be used directly to construct the sums of squares and likelihood function and to obtain estimates of the model parameters in the standard way through nonlinear estimation using numerical iterative calculation.

Consider now a pure feedback system that may be represented in the transfer function–noise model form

$$\varepsilon_t = v(B)X_{t+} + N_t \tag{15.5.16}$$

$$X_{t+} = c(B)\varepsilon_t\{+d_t\} \tag{15.5.17}$$

with

$$v(B) = L_1^{-1}(B)L_2(B)B^{f+1}$$

where $c(B)$ is the known operator of the controller, not necessarily optimal, and d_t is either an additional unintended error or an added ''dither'' signal that has been deliberately introduced. The curly brackets in (15.5.17) emphasize that the added term may or may not be present. In either case, estimation of the unknown transfer function and noise model parameters can be performed, as described in Chapter 12.

However, difficulties in estimation of the model under feedback conditions can arise when the added term d_t is not present. To better understand the nature of issues involved in fitting of the model, we can substitute (15.5.17) in (15.5.16) to obtain

$$[1 - v(B)c(B)]\varepsilon_t = \psi(B)a_t\{+v(B)d_t\} \tag{15.5.18}$$

First consider the case where d_t is *zero*. Because, from (15.5.17), X_{t+} is then a deterministic function of the ε_t's, the model (which appears in (15.5.16) to be of the transfer function form) is seen in (15.5.18) to be equivalent to an ARIMA model whose coefficients are functions of the known parameters of $c(B)$ and of the unknown dynamic and stochastic noise parameters of the model. It is then apparent that, with d_t absent, estimation difficulties can arise, as all dynamic and stochastic noise model forms $v_0(B)$ and $\psi_0(B)$, which are such that

$$\psi_0^{-1}(B)[1 - v_0(B)c(B)] = \psi^{-1}(B)[1 - v(B)c(B)] \tag{15.5.19}$$

will fit equally well in theory. In particular, it can be shown (Box and MacGregor, 1976) that as the pilot feedback controller used during the generation of the data approaches near optimality, near singularities occur in the sum-of-squares surface used for estimation of model parameters. The individual parameters may then be estimated only very imprecisely or will be nonestimable in the limit. In these circumstances, however, accurate estimates of those functions of the parameters that are the constants of the feedback control equation may be obtainable. Thus, while data collected under feedback conditions may be inadequate for estimating the *individual* dynamic and stochastic noise parameters of the system, it may nevertheless be used for updating the estimates of the constants of a control equation whose mathematical form is assumed known.

The situation can be much improved by the deliberate introduction during data generation of a random signal d_t as in (15.5.17). To achieve this, the action $c(B)\varepsilon_t$ is first computed according to the control equation and then d_t is added on. The added signal can, for example, be a random normal variate or a random binary variable and should have mean zero and variance small enough so as not to unduly upset the process. We see from (15.5.18) that with d_t present, the estimation procedure based on fitting model (15.5.16) now involves a genuine transfer function model form in which ε_t depends on the random input d_t as well as on the random shocks a_t. Thus, with d_t present, the fitting procedure tacitly employs not only information arising from the autocorrelations of the ε_t's but also additional information associated with the cross-correlations of the ε_t's and the d_t's.

In many examples, data from a pilot scheme are used to re-estimate parameters with the model form *already identified* from open-loop (no feedback control loop) data and from previous knowledge of the system. Considerable caution and care is needed in using closed-loop data in the model identification/specification process itself. In the first place, if d_t is absent, it is apparent from (15.5.16) that cross-correlation of the ''output'' ε_t and the ''input'' X_{t+} with or without prewhitening will tell us (what we already know) about $c(B)$ and not, as might appear if (15.5.16) were treated as defining an open-loop system, about $v(B)$. Furthermore, since the autocorrelations of the ε_t will be the same for all model forms satisfying (15.5.19), unique identification is not possible if nothing is known about the form of either $\psi(B)$ or $v(B)$. On the other hand, if either $\psi(B)$ or $v(B)$ is known, the autocorrelation function can be used for the identification of the other. With d_t present, the form of (15.5.18) is that of a genuine transfer function–noise model considered in Chapter 12 and corresponding methods may be used for identification.

15.6 MONITORING VALUES OF PARAMETERS OF FORECASTING AND FEEDBACK ADJUSTMENT SCHEMES

Earlier we mentioned the complementary roles of process adjustment and process monitoring. This symbiosis is further illustrated if we again consider the need to monitor the adjustment scheme itself. It has often been proposed that the series of residual deviations from the target from such schemes (and similarly the errors from forecasting schemes) should be studied and that a Shewhart chart or more generally a cumulative sum or other monitoring chart should be run on the residual errors to warn of changes. The cumulative sum is, of course, appropriate to look for small changes in mean level, but often other kinds of discrepancies may be feared. A general theory of sequential directional monitoring based on a cumulative Fisher score statistic (Cuscore) was proposed by Box and Ramírez (1992) (see also Bagshaw and Johnson, 1977).

Suppose that a model can be written in the form of deviations e_t that depend on an unknown parameter θ as

$$e_t = e_t(\theta) \tag{15.6.1}$$

and *that if the correct value of the parameter* $\theta = \theta_0$ is employed in the model, $\{e_t\} = \{a_t\}$ is a sequence of Normal iid random variables. Then, the cumulative score statistic appropriate to detect a departure from the value θ_0 may be written

$$Q_t = \sum_{i=1}^{t} e_i r_i \tag{15.6.2}$$

where $r_t = -(de_t/d\theta)|_{\theta=\theta_0}$ may be called the detector signal.

For example, suppose that we wished to detect a shift in a mean from a value θ_0 for the simple model $y_t = \theta + e_t$. We can write

$$e_t = e_t(\theta) = y_t - \theta \quad a_t = y_t - \theta_0 \tag{15.6.3}$$

Then, in this example, the detector signal is $r_t = 1$ and $Q_t = \sum_{i=1}^{t} e_i$, the well-known *cumulative sum* statistic.

In general, for some value of θ close to θ_0, since e_t may be approximated by $e_t = a_t - (\theta - \theta_0)r_t$, the cumulative product in (15.6.2) will contain a part

$$-(\theta - \theta_0) \sum_{i=1}^{t} r_i^2 \tag{15.6.4}$$

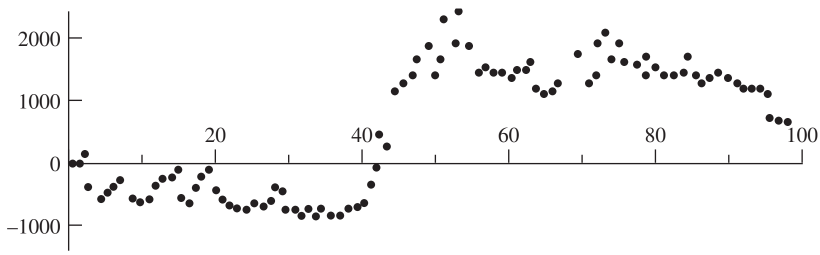

FIGURE 15.17 Cuscore monitoring for detecting a change in the parameter θ used in conjunction with the adjustment chart of Figure 15.3.

which systematically increases in magnitude with sample size t when θ differs from θ_0. For illustration, consider the possibility that in the feedback control scheme for metallic thickness of Section 15.2.1, the value of λ (estimated as 0.2) may have changed during the period $t = 1$ to $t = 100$. For this example,

$$e_t = e_t(\theta) = \frac{1 - B}{1 - \theta B} N_t \tag{15.6.5}$$

Thus,

$$r_t = -\frac{1 - B}{(1 - \theta B)^2} N_{t-1} = -\frac{e_{t-1}}{1 - \theta B} = -\frac{\hat{e}_{t-1}(1)}{\lambda} \tag{15.6.6}$$

where $\hat{e}_{t-1}(1) = \lambda(1 - \theta B)^{-1} e_{t-1}$ is an EWMA of past e_t's. The cumulative score (Cuscore) statistic for detecting this departure is, therefore,

$$Q_t = -\frac{1}{\lambda} \sum_{i=1}^{t} e_i \hat{e}_{i-1}(1) \tag{15.6.7}$$

where the detector signal $\hat{e}_{t-1}(1)$ is, in this case, the EWMA of past values of the *residuals*. These residuals are the deviations from the target plotted on the feedback adjustment chart of Figure 15.3. The criterion agrees with the commonsense idea that if the model is true, then $e_t = a_t$ and e_t is not predictable from previous values. The Cuscore chart shown in Figure 15.17 suggests that a change in parameter may have occurred at about $t = 40$. However, we see from the original data of Figure 15.2 that this is very close to the point at which the level of the original series appears to have changed, and further data and analysis would be needed to confirm this finding.

The important point is that this example shows the *partnership* of two types of control (adjustment and monitoring) and the corresponding two types of statistical inference (estimation and criticism). A further development is to feed back the filtered Cuscore statistic to ‘‘self-tune’’ the control equation, but we do not pursue this further here.

APPENDIX A15.1 FEEDBACK CONTROL SCHEMES WHERE THE ADJUSTMENT VARIANCE IS RESTRICTED

Consider now the feedback control situation where the models for the noise and system dynamics are again given by (15.2.10) and (15.2.13), so that $\varepsilon_t = \mathcal{Y}_t + N_t$ with

$$(1 - B)N_t = (1 - \theta B)a_t \text{ and } (1 - \delta B)\mathcal{Y}_t = (1 - \delta)gX_{t-1+}$$

but some restriction of the input variance var$[x_t]$ is necessary, where $x_t = (1 - B)X_t$. The unrestricted optimal scheme has the property that the errors in the output $\varepsilon_1, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ are the uncorrelated random variables $a_t, a_{t-1}, a_{t-2}, \ldots$ and the variance of the output σ_ε^2 has the minimum possible value σ_a^2. With the restricted schemes, the variance σ_ε^2 will necessarily be greater than σ_a^2, and the errors $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ at the output will be correlated.

We will pose our problem as follows: Given that σ_t^2 be allowed to increase to some value $\sigma_\varepsilon^2 = (1 + c)\sigma_a^2$, where c is a positive constant, we want to find the control scheme that produces the minimum value for $\sigma_x^2 = \text{var}[x_t]$. Equivalently, the problem is to find an (unconstrained) minimum of the expression $\sigma_\varepsilon^2 + \alpha\sigma_x^2$, where α is some specified multiplier that allocates the relative costs of variations in ε_t and x_t.

A15.1.1 Derivation of Optimal Adjustment

Let the optimal adjustment, *expressed in terms of the a_t's*, be

$$x_t = -\frac{1}{g}L(B)a_t \tag{A15.1.1}$$

where

$$L(B) = l_0 + l_1 B + l_2 B^2 + \cdots$$

Then, we see that the error ε_t at the output is given by

$$\begin{aligned}
\varepsilon_t &= \frac{(1-\delta)g}{1-\delta B}X_{t-1+} + N_t \\
&= -\frac{1-\delta}{1-\delta B}(1-B)^{-1}L(B)a_{t-1} + (1-B)^{-1}(1-\theta B)a_t \\
&= a_t + \left[\lambda - \frac{L(B)(1-\delta)}{1-\delta B}\right] S a_{t-1}
\end{aligned} \tag{A15.1.2}$$

where $S = (1 - B)^{-1}$. The coefficient of a_t in this expression is unity, so we can write

$$\varepsilon_t = [1 + B\mu(B)]a_t \tag{A15.1.3}$$

where

$$\mu(B) = \mu_1 + \mu_2 B + \mu_3 B^2 + \cdots$$

Furthermore, in practice, control would need to be exerted in terms of the observed output errors ε_t rather than in terms of the a_t's, so that the control equation actually used would be of the form

$$x_t = -\frac{1}{g}\frac{L(B)}{1 + B\mu(B)}\varepsilon_t \tag{A15.1.4}$$

Equating (A15.1.2) and (A15.1.3), we obtain

$$(1-\delta)L(B) = [\lambda - (1-B)\mu(B)](1-\delta B) \tag{A15.1.5}$$

Since δ, g, and σ_a^2 are constants, we can proceed conveniently by finding an unrestricted minimum of

$$C = \frac{(1-\delta)^2 g^2 V[x_t] + \upsilon V[\varepsilon_t]}{\sigma_a^2} \tag{A15.1.6}$$

where, for example,

$$V[x_t] = \text{var}[x_t]$$

and $\upsilon = (1-\delta)^2 g^2/\alpha$. Now, from (A15.1.3), $V[\varepsilon_t]/\sigma_a^2 = 1 + \sum_{j=1}^{\infty} \mu_j^2$, while from (A15.1.1), $(1-\delta)gx_t = -(1-\delta)L(B)a_t = -\tau(B)a_t$, so that

$$\frac{(1-\delta)^2 g^2 V[x_t]}{\sigma_a^2} = \sum_{j=0}^{\infty} \tau_j^2$$

where

$$\tau(B) = \sum_{j=0}^{\infty} \tau_j B^j = (1-\delta)L(B) = [\lambda - (1-B)\mu(B)](1-\delta B)$$

from (A15.1.5). The coefficients $\{\tau_i\}$ are thus seen to be functionally related to the μ_i by the difference equation

$$\mu_i - (1+\delta)\mu_{i-1} + \delta\mu_{i-2} = -\tau_{i-1} \qquad \text{for } i > 2 \tag{A15.1.7}$$

with $\tau_0 = -(\mu_1 - \lambda)$, $\tau_1 = -[\mu_2 - (1+\delta)\mu_1 + \lambda\delta]$. Hence, we require an unrestricted minimum, with respect to the μ_i, of the expression

$$C = \sum_{j=0}^{\infty} \tau_j^2 + \upsilon\left(1 + \sum_{j=1}^{\infty} \mu_j^2\right) \tag{A15.1.8}$$

This can be obtained by differentiating C with respect to each μ_i ($i = 1, 2, \ldots$), equating these derivatives to zero and solving the resulting equations. Now, a given μ_i only influences the values τ_{i+1}, τ_i, and τ_{i-1} through (A15.1.7), and we see that

$$\frac{\partial \tau_j}{\partial \mu_i} = \begin{cases} -1 & j = i-1 \\ 1+\delta & j = i \\ -\delta & j = i+1 \\ 0 & \text{otherwise} \end{cases} \tag{A15.1.9}$$

Therefore, from (A15.1.8) and (A15.1.9), we obtain

$$\begin{aligned} \frac{\partial}{\partial \mu_i} C &= 2\left(\tau_{i+1}\frac{\partial \tau_{i+1}}{\partial \mu_i} + \tau_i \frac{\partial \tau_i}{\partial \mu_i} + \tau_{i-1}\frac{\partial \tau_{i-1}}{\partial \mu_i} + \upsilon\mu_i\right) \\ &= 2[-\delta\tau_{i+1} + (1+\delta)\tau_i - \tau_{i-1} + \upsilon\mu_i] \qquad \text{for } i = 1, 2, \ldots \end{aligned} \tag{A15.1.10}$$

Then, after substituting the expressions for the τ_j in terms of the μ_i from equation (A15.1.7) in (A15.1.10) and setting each of these equal to zero, we obtain the following equations:

$$(i=1): \qquad -\lambda(1+\delta+\delta^2)+2(1+\delta+\delta^2)\mu_1-(1+\delta)^2\mu_2+\delta\mu_3+\nu\mu_1=0 \tag{A15.1.11}$$

$$(i=2): \qquad \lambda\delta-(1+\delta)^2\mu_1+2(1+\delta+\delta^2)\mu_2-(1+\delta)^2\mu_3+\delta\mu_4+\nu\mu_2=0 \tag{A15.1.12}$$

$$(i>2): \qquad [\delta B^2-(1+\delta)^2B+2(1+\delta+\delta^2)-(1+\delta)^2F+\delta F^2+\nu]\mu_i=0 \tag{A15.1.13}$$

A15.1.2 Case Where δ Is Negligible

Consider first the simpler case where δ is negligibly small and can be set equal to zero. Then the equations above can be written as

$$(i=1): \qquad -(\lambda-\mu_1)+(\mu_1-\mu_2)+\nu\mu_1=0 \tag{A15.1.14}$$

$$(i>1): \qquad [B-(2+\nu)+F]\mu_i=0 \tag{A15.1.15}$$

These difference equations have a solution of the form

$$\mu_i = A_1\kappa_1^i + A_2\kappa_2^i$$

where κ_1 and κ_2 are the roots of the characteristic equation

$$B^2-(2+\upsilon)B+1=0 \tag{A15.1.16}$$

that is, of

$$B+B^{-1}=2+\nu$$

Evidently, if κ is a root, so is κ^{-1}. Thus, the solution is of the form $\mu_i = A_1\kappa^i + A_2\kappa^{-i}$. Now if κ has modulus less than or equal to 1, κ^{-1} has modulus greater than or equal to 1, and since $\varepsilon_t = [1 + B\mu(B)]a_t$ must have finite variance, A_2 must be zero with $|\kappa| < 1$. By substituting the solution $\mu_i = A_1\kappa^i$ in (A15.1.14), we find that $A_1 = \lambda$.

Finally, then, $\mu_i = \lambda\kappa^i$, and since μ_i and λ must be real, so must the root κ. Hence,

$$\mu(B)=\frac{\lambda\kappa}{1-\kappa B} \qquad 0<\kappa<1 \tag{A15.1.17}$$

$$1+B\mu(B)=1+\frac{\lambda\kappa B}{1-\kappa B}=\frac{1-\theta\kappa B}{1-\kappa B} \tag{A15.1.18}$$

where $\theta = 1-\lambda$. Thus,

$$\varepsilon_t=\frac{1-\theta\kappa B}{1-\kappa B}a_t$$

so that

$$\frac{V[\varepsilon_t]}{\sigma_a^2}=1+\frac{\lambda^2\kappa^2}{1-\kappa^2} \tag{A15.1.19}$$

Also, using (A15.1.5) with $\delta = 0$,

$$L(B) = \lambda - \frac{(1-B)\lambda\kappa}{1-\kappa B} = \frac{\lambda(1-\kappa)}{1-\kappa B} \tag{A15.1.20}$$

Thus,

$$x_t = -\frac{\lambda}{g}\frac{1-\kappa}{1-\kappa B}a_t$$

and

$$\frac{V[x_t]}{\sigma_a^2} = \frac{\lambda^2}{g^2}\frac{(1-\kappa)^2}{1-\kappa^2} = \frac{\lambda^2}{g^2}\frac{1-\kappa}{1+\kappa} \tag{A15.1.21}$$

Using (A15.1.4) with (A15.1.18) and (A15.1.20), we now find that the optimal control action, in terms of the observed output error ε_t, is

$$x_t = -\frac{1}{g}\frac{\lambda(1-\kappa)}{1-\theta\kappa B}\varepsilon_t$$

that is,

$$x_t = (1-\lambda)\kappa x_{t-1} - \frac{1}{g}\lambda(1-\kappa)\varepsilon_t \tag{A15.1.22}$$

Note that the constrained control equation differs from the unconstrained one in two respects:

1. A new factor $(1-\lambda)\kappa x_{t-1}$ is introduced, thus making present action depend partly on previous action.
2. The constant determining the amount of integral control is reduced by a factor $1-\kappa$.

We have supposed that the output variance is allowed to increase to some value $\sigma_a^2(1+c)$. It follows from (A15.1.19) that

$$c = \frac{\lambda^2\kappa^2}{1-\kappa^2}$$

that is,

$$\kappa = \sqrt{\frac{c}{\lambda^2 + c}}$$

where the positive square root is to be taken. It is convenient to write $Q = c/\lambda^2$. Then, $Q = \kappa^2/(1-\kappa^2)$ and $\kappa^2 = Q/(1+Q)$ and the output variance becomes $\sigma_a^2(1+\lambda^2 Q)$.

In summary, suppose that we are prepared to tolerate an increase in variance in the output to some value $\sigma_a^2(1+\lambda^2 Q)$; then

1. We compute $\kappa = \sqrt{Q/(1+Q)}$.
2. Optimal control will be achieved by taking action given by (A15.1.22).

Table A15.1 Values of Parameters for a Simple Constrained Control Scheme

$c/\lambda^2 = Q$	κ	W	$c/\lambda^2 = Q$	κ	W
0.10	0.302	53.7	0.60	0.612	24.0
0.20	0.408	42.0	0.70	0.641	21.9
0.30	0.480	35.1	0.80	0.667	20.0
0.40	0.535	30.3	0.90	0.688	18.5
0.50	0.577	26.8	1.00	0.707	17.2

3. The variance of the input will be reduced to

$$V[x_t] = \frac{\lambda^2}{g^2}\frac{1-\kappa}{1+\kappa}\sigma_a^2$$

that is, it will reduce to a value that is $W\%$ of that for the unconstrained scheme, where

$$W = 100\left(\frac{1-\kappa}{1+\kappa}\right)$$

Table A15.1 shows κ and W for values of Q between 0.1 and 1.0. For illustration, suppose that $\lambda = 0.4$. Then the optimal unconstrained scheme will employ the control action

$$x_t = -\frac{0.4}{g}\varepsilon_t$$

with $\varepsilon_t = a_t$. The variance of x_t would be $V[x_t] = (\sigma_a^2/g^2)0.16$. Suppose that it was desired to reduce this by a factor of 4, to the value $(\sigma_a^2/g^2)0.04$. Thus, we require W to be 25%. Table A15.1 shows that a reduction of the input variance to 24% of its unconstrained value is possible with $Q = 0.60$ and $\kappa = 0.612$. If we use this scheme, the output variance will be

$$\sigma_\varepsilon^2 = \sigma_a^2(1 + 0.16 \times 0.60) = 1.10\sigma_a^2$$

Thus, by the use of the control action

$$x_t = 0.37x_{t-1} - \frac{1}{g}0.16\varepsilon_t$$

instead of $x_t = -(0.4/g)\varepsilon_t$, the variance of the input is reduced to about 1/4 of its previous value, while the variance of the output is increased by only 10%.

Case Where δ Is Not Negligible. Consider now the more general situation where δ is not negligible and the system dynamics must be taken account of. The difference equation (A15.1.13) is of the form

$$(\alpha B^{-2} + \beta B^{-1} + \gamma + \beta B + \alpha B^2)\mu_i = 0$$

and if κ is a root of the characteristic equation, so is κ^{-1}. Suppose that the roots are $\kappa_1, \kappa_2, \kappa_1^{-1}, \kappa_2^{-1}$ and that κ_1 and κ_2 are a pair of roots with modulus < 1. Then, in the solution

$$\mu_i = A_1\kappa_1^i + A_2\kappa_2^i + A_3\kappa_1^{-i} + A_4\kappa_2^{-i}$$

A_3 and A_4 must be zero, because ε_t is required to have a finite variance.

Hence, the solution is of the form

$$\mu_i = A_1\kappa_1^i + A_2\kappa_2^i \qquad |\kappa_1| < 1 \quad |\kappa_2| < 1$$

The A's satisfying the initial conditions, defined by (A15.1.11) and (A15.1.12), are obtained by substitution to give

$$A_1 = \frac{\lambda\kappa_1(1-\kappa_2)}{\kappa_1 - \kappa_2} \qquad A_2 = -\frac{\lambda\kappa_2(1-\kappa_1)}{\kappa_1 - \kappa_2}$$

If we write $k_0 = \kappa_1 + \kappa_2 - \kappa_1\kappa_2$, $k_1 = \kappa_1\kappa_2$, then

$$\mu(B) = \lambda\left[\frac{k_0 - k_1 B}{1 - (k_0 + k_1)B + k_1 B^2}\right] \tag{A15.1.23}$$

and

$$1 + B\mu(B) = \frac{1 - k_1 B - (1-\lambda)(k_0 B - k_1 B^2)}{1 - (k_0 + k_1)B + k_1 B^2} \tag{A15.1.24}$$

Now substituting (A15.1.23) in (A15.1.5),

$$L(B) = \frac{\lambda(1-\delta B)(1-k_0)}{(1-\delta)[1 - (k_0 + k_1)B + k_1 B^2]} \tag{A15.1.25}$$

and

$$\frac{L(B)}{1 + B\mu(B)} = \frac{\lambda(1-\delta B)(1-k_0)}{(1-\delta)[1 - k_1 B - (1-\lambda)(k_0 B - k_1 B^2)]}$$

Therefore, using (A15.1.4), we find that the optimal control action in terms of the error ε_t is

$$x_t = -\frac{\lambda}{g}\frac{(1-\delta B)(1-k_0)}{(1-\delta)[1 - k_1 B - (1-\lambda)(k_0 B - k_t B^2)]}\varepsilon_t \tag{A15.1.26}$$

or

$$x_t = [k_1 + (1-\lambda)k_0]x_{t-1} - (1-\lambda)k_1 x_{t-2} - \frac{\lambda(1-k_0)(1-\delta B)}{g(1-\delta)}\varepsilon_t \tag{A15.1.27}$$

Thus, the modified control scheme makes x_t depend on both x_{t-1} and x_{t-2} (only on x_{t-1} if $\lambda = 1$) and reduces the standard integral and proportional action by a factor $1 - k_0$.

Variances of Output and Input. The actual variances for the output and input are readily found since

$$\varepsilon_t = a_t + \lambda\left[\frac{k_0 - k_1 B}{1 - (k_0 + k_1)B + k_1 B^2}\right] a_{t-1}$$

The second term on the right defines a mixed autoregressive–moving average process of order (2, 0, 1), the variance for which is readily obtained to give

$$\frac{V[\varepsilon_t]}{\sigma_a^2} = 1 + \lambda^2 \left\{ \frac{(k_0 + k_1)^2(1 - k_1) - 2k_1(k_0 - k_1^2)}{(1 - k_1)(1 + k_1)^2 - (k_0 + k_1)^2]} \right\} = 1 + \lambda^2 Q \tag{A15.1.28}$$

Also,

$$\frac{V[x_t]}{\sigma_a^2} = \frac{\lambda^2}{g^2(1-\delta)^2} \frac{(1 - k_0)[(1 + \delta^2)(1 + k_1) - 2\delta(k_0 + k_1)]}{(1 + k_0 + 2k_1)(1 - k_1)} \tag{A15.1.29}$$

Computation of k_0 and k_1. Returning to the difference equations (A15.1.13), the characteristic equation may be written

$$B^4 - MB^3 - NB^2 - MB + 1 = 0$$

where $M = (1 + \delta)^2/\delta$ and $N = [(1 + \delta^2) + (1 + \delta^2) + \upsilon]/\delta$. It may also be written in the form

$$(B^2 - TB + P)(B^2 - P^{-1}TB + P^{-1}) = 0$$

where

$$T = \kappa_1 + \kappa_2 \quad \text{and} \quad P = \kappa_1\kappa_2$$

Equating coefficients of B gives

$$T + P^{-1}T = M$$

that is, $T = PM/(1 + P)$, and

$$P + P^{-1} + P^{-1}T^2 = N$$

Thus, $P + P^{-1} + PM^2/(1 + P)^2 = N$, that is,

$$(P + 2 + P^{-1})(P + P^{-1}) + M^2 = N(P + 2 + P^{-1})$$

or

$$(P + P^{-1})^2 + (2 - N)(P + P^{-1}) + M^2 - 2N = 0$$

For suitable vales of υ, this quadratic equation will have two real roots:

$$u_1 = \kappa_1\kappa_2 + \kappa_1^{-1}\kappa_2^{-1} \quad u_2 = \kappa_1\kappa_2^{-1} + \kappa_1^{-1}\kappa_2$$

the root u_1 being the larger. The required quantity P is now the smaller root of the quadratic equation

$$P^2 - u_1P + 1 = 0$$

and T is given by

$$T = [P(u_2 + 2)]^{1/2}$$

Table A15.2 Table to Facilitate the Calculation of Optimal Constrained Control Schemes

		100Q				
δ		20	40	60	80	100
0.9	100 W	21.7	11.3	6.7	4.5	3.1
	k_0	0.44	0.585	0.68	0.74	0.78
	k_1	0.18	0.27	0.34	0.39	0.44
0.8	100 W	22.0	11.7	7.2	4.8	3.4
	k_0	0.44	0.585	0.68	0.74	0.78
	k_1	0.18	0.27	0.33	0.38	0.43
0.7	100 W	22.7	12.4	8.0	5.6	4.1
	k_0	0.44	0.585	0.68	0.74	0.78
	k_1	0.17	0.25	0.32	0.36	0.40
0.6	100 W	24.1	13.6	9.0	6.6	5.0
	k_0	0.44	0.58	0.67	0.73	0.78
	k_1	0.16	0.24	0.29	0.33	0.365
0.5	100 W	26.5	15.5	10.5	7.9	6.2
	k_0	0.43	0.58	0.67	0.72	0.77
	k_1	0.15	0.21	0.26	0.29	0.32
0.4	100 W	28.5	17.7	12.7	9.8	7.9
	k_0	0.43	0.57	0.66	0.72	0.76
	k_1	0.13	0.18	0.22	0.245	0.265
0.3	100 W	31.5	20.5	15.2	12.0	9.9
	k_0	0.43	0.57	0.65	0.71	0.75
	k_1	0.105	0.145	0.17	0.19	0.20
0.2	100 W	34.8	23.6	18.0	14.5	12.2
	k_0	0.42	0.56	0.64	0.69	0.73
	k_1	0.07	0.10	0.12	0.13	0.14
0.1	100 W	38.2	26.7	21.0	17.3	14.6
	k_0	0.42	0.55	0.63	0.68	0.72
	k_1	0.04	0.05	0.06	0.065	0.07

Table of Optimal Values for Constrained Schemes

Construction of the Table. Table A15.2 is provided to facilitate the selection of an optimal control scheme. The tabled values were obtained as follows for each chosen value of the parameter δ in the transfer function model:

1. Compute $M = (1+\delta)^2/\delta$ and $N = ((1+\delta)^2 + (1+\delta^2) + \upsilon)/\delta$ for a series of values of υ chosen to provide a suitable range for Q.
2. Compute $u_1 = 1/2(N-2) + \left[((N-2)/2)^2 + 2N - M^2\right]^{1/2}$ and $u_2 = 1/2(N-2) - \left[((N-2)/2)^2 + 2N - M^2\right]^{1/2}$
3. Compute $k_1 = P = 1/2u_1 - \left[\left(1/2u_1\right)^2 - 1\right]^{1/2}$ and $k_0 = T - P = [k_1(u_2+2)]^{1/2} - k_1$.
4. Compute $Q = \dfrac{(k_0+k_1)^2(1-k_1) - 2k_1(k_0-k_1^2)}{(1-k_1)[(1+k_1)^2 - (k_1+k_1)^2]}$.

5. Compute $W = \dfrac{(1-k_0)[(1+\delta^2)(1+k_1) - 2\delta(k_0+k_1)]}{(1+k_0+2k_1)(1-k_1)(1+\delta^2)}$.

6. Interpolate among the W, k_0, k_1 values at convenient values of Q.

Use of the Table. Table A15.2 may be used as follows. The value of δ is entered in the vertical margin. Using the fact that $V[\varepsilon_t] = (1+\lambda^2 Q)\sigma_a^2$, the percentage increase in output variance is $100Q\lambda^2$. A suitable value of Q is entered in the horizontal margin. The entries in the table are then (1) $100W$, the percentage reduction in the variance of x_t, (2) k_0, and (3) k_1.

For illustration, suppose that $\lambda = 0.6, \delta = 0.5$, and $g = 1$. The optimal unconstrained control equation is then

$$x_t = -1.2(1-0.5B)\varepsilon_t = -1.2(1-0.5B)a_t$$

and $\mathrm{var}[x_t] = 1.80\sigma_a^2$. Suppose that this amount of variation in the input variable produces difficulties in process operation and it is desired to reduce $\mathrm{var}[x_t]$ to about $0.50\sigma_a^2$, that is, to about 28% of the value for the unconstrained scheme. Inspection of Table A15.2 in the row labeled $\delta = 0.5$ shows that a reduction to 26.5% can be achieved by using a control scheme with constants $k_0 = 0.43, k_1 = 0.15$, that is, by employing the control equation (A15.1.27) to give

$$x_t = 0.32x_{t-1} - 0.06x_{t-2} - (0.57 \times 1.2)(1-0.5B)\varepsilon_t$$

This solution corresponds to a value $Q = 0.20$. Therefore, the variance at the output will be increased by a factor of

$$1 + \lambda^2 Q = 1 + 0.6^2(0.2) = 1.072$$

that is, by about 7%.

APPENDIX A15.2 CHOICE OF THE SAMPLING INTERVAL

In comparison to continuous systems, discrete systems of control, such as those discussed here, can be very efficient provided that the sampling interval is suitably chosen. Roughly speaking, we want the interval to be such that not too much change can occur during the sampling interval. Usually, the behavior of the disturbance that has to pass through all or part of the system reflects the inertia or dynamic properties of the system, so that the sampling interval will often be chosen tacitly or explicitly to be proportional to the time constant or constants of the system. In chemical processes involving reaction and mixing of liquids, rather infrequent sampling, say at hourly intervals and possibly with operator surveillance and manual adjustment, will be sufficient. By contrast, where reactions between gases are involved, a suitable sampling interval may be measured in seconds and automatic monitoring and adjustment may be essential.

In some cases, experimentation may be needed to arrive at a satisfactory sampling interval, and in others rather simple calculations will show how the choice of sampling interval will affect the degree of control that is possible.

A15.2.1 Illustration of the Effect of Reducing Sampling Frequency

To illustrate the kind of calculation that is helpful, suppose again that we have a simple system in which, using a particular sampling interval, the noise is represented by a (0, 1, 1) process $\nabla N_t = (1 - \theta B)a_t$ and the transfer function model by the first-order system $(1 - \delta B)\mathcal{Y}_t = g(1 - \delta)X_{t-1}$. In this case, if we employ the MMSE adjustment

$$x_t = -\frac{1-\theta}{g(1-\delta)}(1 - \delta B)\varepsilon_t \tag{A15.2.1}$$

then the deviation from target is $\varepsilon_t = a_t$ and has variance $\sigma_a^2 = \sigma_1^2$, say.

In practice, the question has often arisen: How much worse off would we be if we took samples less frequently? To answer this question, we consider the effect of sampling the stochastic process involved.

A15.2.2 Sampling an IMA(0, 1, 1) Process

Suppose that with observations being made at some ''unit'' interval, we have a noise model

$$\nabla N_t = (1 - \theta_1 B)a_t$$

with $\text{var}[a_t] = \sigma_a^2 = \sigma_1^2$, where the subscript 1 is used in this context to denote the choice of sampling interval. Then, for the differences ∇N_t, the autocovariances γ_k are given by

$$\begin{aligned} \gamma_0 &= (1 + \theta_1^2)\sigma_1^2 \\ \gamma_1 &= -\theta_1 \sigma_1^2 \\ \gamma_j &= 0 \qquad j \geq 2 \end{aligned} \tag{A15.2.2}$$

Writing $\zeta = (\gamma_0 + 2\gamma_1)/\gamma_1$, we obtain

$$\zeta = -\frac{(1-\theta_1)^2}{\theta_1}$$

so that, given γ_0 and γ_1, the parameter $\lambda = 1 - \theta_1$ of the IMA process may be obtained by solving the quadratic equation

$$(1 - \theta_1)^2 - \zeta(1 - \theta_1) + \zeta = 0$$

selecting that root for which $-1 < \theta_1 < 1$. Also,

$$\sigma_1^2 = -\frac{\gamma_1}{\theta_1} \tag{A15.2.3}$$

Suppose now that the process N_t is observed at intervals of h units (where h is a positive integer) and the resulting process is denoted by M_t. Then,

$$\begin{aligned} \nabla M_t = N_t - N_{t-h} &= (a_t + a_{t-1} + \cdots + a_{t-h+1}) \\ &\quad - \theta_1(a_{t-1} + a_{t-2} + \cdots + a_{t-h}) \\ \nabla M_{t-h} = N_{t-h} - N_{t-2h} &= (a_{t-h} + a_{t-h-1} + \cdots + a_{t-2h+1}) \\ &\quad - \theta_1(a_{t-h-1} + \cdots + a_{t-2h}) \end{aligned}$$

and so on. Then, for the differences ∇M_t, the autocovariances $\gamma_k(h)$ are

$$\begin{aligned}\gamma_0(h) &= [(1+\theta_1^2) + (h-1)(1-\theta_1)^2]\sigma_1^2 \\ \gamma_1(h) &= -\theta_1\sigma_1^2 \\ \gamma_j(h) &= 0 \qquad j \geq 2\end{aligned} \tag{A15.2.4}$$

It follows that the process M_t is also an IMA process of order (0, 1, 1),

$$\nabla M_t = (1 - \theta_h B)e_t$$

where e_t is a white noise process with variance σ_h^2. Now

$$\frac{\gamma_0(h) + 2\gamma_1(h)}{\gamma_1(h)} = -\frac{h(1-\theta_1)^2}{\theta_1}$$

so that

$$\frac{h(1-\theta_1)^2}{\theta_1} = \frac{(1-\theta_h)^2}{\theta_h} \tag{A15.2.5}$$

Also, since $\gamma_1(h) = -\theta_h\sigma_h^2 = -\theta_1\sigma_1^2$, it follows that

$$\frac{\sigma_h^2}{\sigma_1^2} = \frac{\theta_1}{\theta_h} \tag{A15.2.6}$$

Therefore, we have shown that the sampling of an IMA process of order (0, 1, 1) at interval h produces another IMA process of order (0, 1, 1). From (A15.2.5), we can obtain the value of the parameter θ_h for the sampled process, and from (A15.2.6) we can obtain the variance $\sigma_h^2 = \text{var}[e_t]$ of the corresponding white noise generating process in terms of the parameters θ_1 and $\sigma_t^2 = \text{var}[a_t]$ of the original process.

In Figure A15.1, θ_n is plotted against log h, a scale of h being appended. The graph enables one to find the effect of increasing the sampling interval of a (0, 1, 1) process by any given multiple. For illustration, suppose that we have a process for which $\theta_1 = 0.5$ and $\sigma_1^2 = 1$. Let us use the graph to find the values of the corresponding parameters $\theta_2, \theta_4, \sigma_2^2, \sigma_4^2$ when the sampling interval is (a) doubled and (b) quadrupled. Marking on the edge of a piece of paper the points $h = 1, h = 2, h = 4$ from the scale of the graph, we set the paper

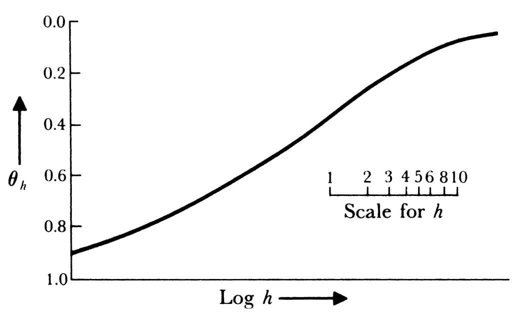

FIGURE A15.1 Sampling of IMA(0, 1, 1) process: parameter θ_h plotted against log h.

horizontally so that $h = 1$ corresponds to the point on the curve for which $\theta_1 = 0.5$. We then read off the ordinates for θ_2 and θ_4 corresponding to $h = 2$ and $h = 4$. We find that

$$\theta_1 = 0.5 \qquad \theta_2 = 0.38 \qquad \theta_4 = 0.27$$

Using (A15.2.6), the variances are in inverse proportion to the values of θ, so that

$$\sigma_1^2 = 1.00 \qquad \sigma_2^2 = 1.32 \qquad \sigma_4^2 = 2.17$$

Suppose now that for the original scheme with unit interval, the dynamic constant was δ_1 (again we will use the subscript to denote the sampling interval). Then, since in real time the same fixed time constant $T = -h/\ln(\delta)$ applies to all the schemes, we have

$$\delta_2 = \delta_1^2 \qquad \delta_4 = \delta_1^4$$

The scheme giving minimum mean square error for a *particular* sampling interval h would be

$$x_t(h) = -\frac{1-\theta_h}{g(1-\delta_1^h)}(1-\delta_1^h B)\varepsilon_t(h)$$

or

$$x_t(h) = -\frac{1-\theta_h}{g}\left(1 + \frac{\delta_1^h}{1-\delta_1^h}\nabla\right)\varepsilon_t(h) \tag{A15.2.7}$$

Suppose, for example, with $\theta_1 = 0.5$ as above, $\delta_1 = 0.8$, so that $\delta_2 = 0.64$, $\delta_4 = 0.41$. Then the optimal schemes would be

$$h = 1: \quad x_t(1) = -\frac{0.5}{g}(1 + 4\nabla)\varepsilon_t(1) \qquad \sigma_\varepsilon^2 = 1.00 \qquad g^2\sigma_x^2 = 10.25$$

$$h = 2: \quad x_t(2) = -\frac{0.62}{g}(1 + 1.78\nabla)\varepsilon_t(2) \qquad \sigma_\varepsilon^2 = 1.32 \qquad g^2\sigma_x^2 = 5.50$$

$$h = 4: \quad x_t(4) = -\frac{0.73}{g}(1 + 0.69\nabla)\varepsilon_t(4) \qquad \sigma_\varepsilon^2 = 2.17 \qquad g^2\sigma_x^2 = 3.84$$

In accordance with expectation, as the sampling interval is increased and the dynamics of the system have relatively less importance, the amount of ''integral'' control is increased and the ratio of proportional to integral control is markedly reduced. We noted earlier that an excessively large adjustment variance σ_x^2 would usually be a disadvantage. The values of $g^2\sigma_x^2$ are indicated to show how the schemes differ in this respect. The smaller value for σ_x^2 would not of itself, of course, justify the choice $h = 4$. Using an optimal constrained scheme, as is described in Appendix A15.1, with $h = 1$, a very large reduction in σ_x^2 would be produced with only a small increase in the output variance. For example, entering Table A15.2 with $\delta = 0.8$, $100Q = 20$, we find that for a 5% increase of output variance to the value $(1 + \lambda^2 Q)\sigma_1^2 = 1.05\sigma_1^2$, the input variance for the scheme with $h = 1$ could be reduced to 22% of its unconstrained value, so that $g^2\sigma_x^2 = 10.25 \times 0.22 = 2.26$.

Using (A15.1.27), we obtain for the constrained scheme with $h = 1$,

$$x_t = 0.40x_{t-1} - 0.09x_{t-2} - 0.56\left[\frac{0.5}{g}(1 + 4\nabla)\right]\varepsilon_t(1)$$
$$\sigma_\varepsilon^2 = 1.05 \qquad g^2\sigma_x^2 = 2.26$$

In practice, various alternative schemes could be set out with their accompanying characteristics and an economic choice made to suit the particular problem. In general, the increase in output variance that comes with the larger interval would have to be balanced off against the economic advantage, if any, of less frequent surveillance.

EXERCISES

15.1. In a chemical process, 30 successive values of viscosity N_t that occurred during a period when the control variable (gas rate) X_t *was held fixed* at its standard reference origin were recorded as follows:

Time	Viscosities									
1–10	92	92	96	96	96	98	98	100	100	94
11–20	98	88	88	88	96	96	92	92	90	90
21–30	90	94	90	90	94	94	96	96	96	96

Reconstruct and plot the error sequence (deviations from target) ε_t and adjustments x_t, which would have occurred if the optimal feedback control scheme

$$x_t = -10\varepsilon_t + 5\varepsilon_{t-1} \tag{1}$$

had been applied during this period. It is given that the dynamic model is

$$y_t = 0.5y_{t-1} + 0.10x_{t-1} \tag{2}$$

and that the error signal may be obtained from

$$\varepsilon_t = \varepsilon_{t-1} + \nabla N_t + y_t \tag{3}$$

Your calculation sequence should proceed in the order (2), (3), and (1) and initially you should assume that $\varepsilon_1 = 0, y_1 = 0, x_1 = 0$. Can you devise a more direct way to compute ε_t from N_t?

15.2. Given the following combinations of disturbance and transfer function models:

$$\begin{aligned}
&(1) & \nabla N_t &= (1 - 0.7B)a_t \\
& & (1 - 0.4B)\mathcal{Y}_t &= 5.0X_{t-1+} \\
&(2) & \nabla N_t &= (1 - 0.5B)a_t \\
& & (1 - 1.2B + 0.4B^2)\mathcal{Y}_t &= (20 - 8.5)X_{t-1+} \\
&(3) & \nabla^2 N_t &= (1 - 0.9B + 0.5B^2)a_t \\
& & (1 - 0.7B)\mathcal{Y}_t &= 3.0X_{t-1+} \\
&(4) & \nabla N_t &= (1 - 0.7B)a_t \\
& & (1 - 0.4B)\mathcal{Y}_t &= 5.0X_{t-2+}
\end{aligned}$$

(a) Design the minimum mean square error feedback control schemes associated with each combination of disturbance and transfer function model.

(b) For case (4), derive an expression for the error ε_t and for its variance in terms of σ_a^2.

(c) For case (4), design a nomogram suitable for carrying out the control action manually by a process operator.

15.3. In a treatment plant for industrial waste, the strength u_t of the influent is measured every 30 minutes and can be represented by the model $\nabla u_t = (1 - 0.5B)\alpha_t$. In the absence of control, the strength of the effluent Y_t is related to that of the influent u_t by an effect $\mathcal{Y}_{1t}$ that can be represented as

$$\mathcal{Y}_{1t} = \frac{0.3B}{1 - 0.2B}\tilde{u}_t$$

An increase in strength in the waste may be compensated by an increase in the flow X_t of a chemical to the plant, whose effect on Y_t is represented by the effect

$$\mathcal{Y}_{2t} = \frac{21.6B^2}{1 - 0.7B}\tilde{X}_t$$

Show that minimum mean square error feedforward control is obtained with the control equation

$$\tilde{X}_t = -\frac{0.3}{21.6}\left[\frac{(0.7 - 0.2B)(1 - 0.7B)}{(1 - 0.2B)(1 - 0.5B)}\right]\tilde{u}_t$$

that is, $\tilde{X}_t = 0.7\tilde{X}_{t-1} - 0.1\tilde{X}_{t-2} - 0.0139(0.7\tilde{u}_t - 0.69\tilde{u}_{t-1} + 0.14\tilde{u}_{t-2})$.

15.4. A pilot feedback control scheme, based on the following disturbance and transfer function models:

$$\begin{aligned}
\nabla N_t &= a_t \\
(1 - \delta B)\mathcal{Y}_t &= \omega_0 X_{t-1+} - \omega_1 X_{t-2+}
\end{aligned}$$

was operated, leading to a series of adjustments x_t and errors ε_t. It was believed that the noise model was reasonably accurate, but that the parameters of the transfer function model were of questionable accuracy.

(a) Given the first 10 values of the x_t, ε_t series shown below:

t	x_t	ε_t	t	x_t	ε_t
1	25	−7	6	−30	1
2	42	−7	7	−25	3
3	3	−6	8	−25	4
4	20	−7	9	20	0
5	5	−4	10	40	−3

set out the calculation of the residuals a_t $(t = 2, 3, \ldots, 10)$ for $\delta = 0.5$, $\omega_0 = 0.3, \omega_1 = 0.2$, and for arbitrary starting values y_1^0 and x_0^0.

(b) Calculate the values $y_1, \hat{x}_0$ of y_1^0 and x_0^0 that minimize the sum of squares $\sum_{t=2}^{10}(a_t|\delta = 0.5, \omega_0 = 0.3, \omega_1 = 0.2, y_1^0, x_0^0)^2$ and the value of this minimum sum of squares.

15.5. Consider (Box and MacGregor, 1976) a system for which the process transfer function is gB and the noise model is $(1 - B)N_t = (1 - \theta B)a_t$ so that the error ε_t at the output satisfies

$$(1 - B)\varepsilon_t = g(1 - B)X_{t-1+} + (1 - \theta B)a_t$$

Suppose that the system is controlled by a known discrete ''integral'' controller

$$(1 - B)X_{t+} = -c\varepsilon_t$$

(a) Show that the errors ε_t at the output will follow the ARMA(1, 1) process

$$(1 - \phi B)\varepsilon_t = (1 - \theta B)a_t \qquad \phi = 1 - gc$$

and hence that the problem of estimating g and θ using data from a pilot control scheme is equivalent to that of estimating the parameters in this ARMA(1, 1) model.

(b) Show also that the optimal control scheme is such that $c = c_0 = (1 - \theta)/g$ and hence that if the pilot scheme used in collecting the data happens to be optimal already, then $1 - \theta$ and g cannot be separately estimated.

PART FIVE

CHARTS AND TABLES

This part of the book is a collection of auxiliary material useful in the analysis of time series. This includes tables and charts for obtaining preliminary estimates of the parameters in autoregressive–moving-average models, together with the usual tail area tables of the normal, χ^2, and t distributions. This is followed by a listing of the time series analyzed in the book, as well as some additional time series that are discussed in the exercises located at the end of the individual chapters.

Time Series Analysis: Forecasting and Control, Fifth Edition. George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, and Greta M. Ljung

COLLECTION OF TABLES AND CHARTS

Charts B, C, and D are adapted and reproduced from Stralkowski (1968) with permission of the author. Tables E, F, and G are condensed and adapted from *Biometrika Tables for Statisticians,* Volume I, with permission from the trustees of Biometrika.

Time Series Analysis: Forecasting and Control, Fifth Edition. George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, and Greta M. Ljung

TABLE A Table Relating ρ_1 to θ for a First-Order Moving Average Process

θ	ρ_1	θ	ρ_1
0.00	0.000	0.00	0.000
0.05	−0.050	−0.05	0.050
0.10	−0.099	−0.10	0.099
0.15	−0.147	−0.15	0.147
0.20	−0.192	−0.20	0.192
0.25	−0.235	−0.25	0.235
0.30	−0.275	−0.30	0.275
0.35	−0.315	−0.35	0.315
0.40	−0.349	−0.40	0.349
0.45	−0.374	−0.45	0.374
0.50	−0.400	−0.50	0.400
0.55	−0.422	−0.55	0.422
0.60	−0.441	−0.60	0.441
0.65	−0.457	−0.65	0.457
0.70	−0.468	−0.70	0.468
0.75	−0.480	−0.75	0.480
0.80	−0.488	−0.80	0.488
0.85	−0.493	−0.85	0.493
0.90	−0.497	−0.90	0.497
0.95	−0.499	−0.95	0.499
1.00	−0.500	−1.00	0.500

Table A may be used to obtain first estimates of the parameters in the $(0, d, 1)$ model $w_t = (1 - \theta B)a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ for ρ_1.

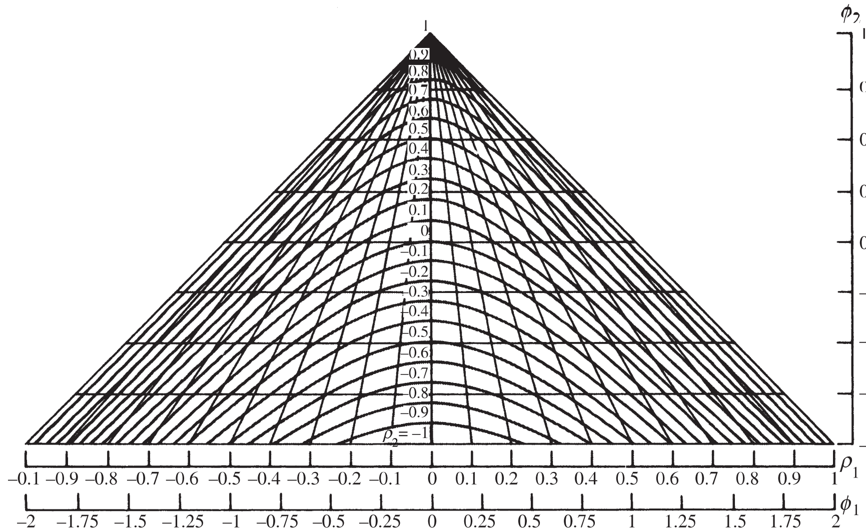

CHART B Chart relating ρ_1 and ρ_1 to ϕ_1 and ϕ_2 for a second-order autoregressive process.

The chart may be used to obtain estimates of the parameters in the $(2, d, 0)$ model $(1 - \phi_1 B - \phi_2 B^2)w_t = a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

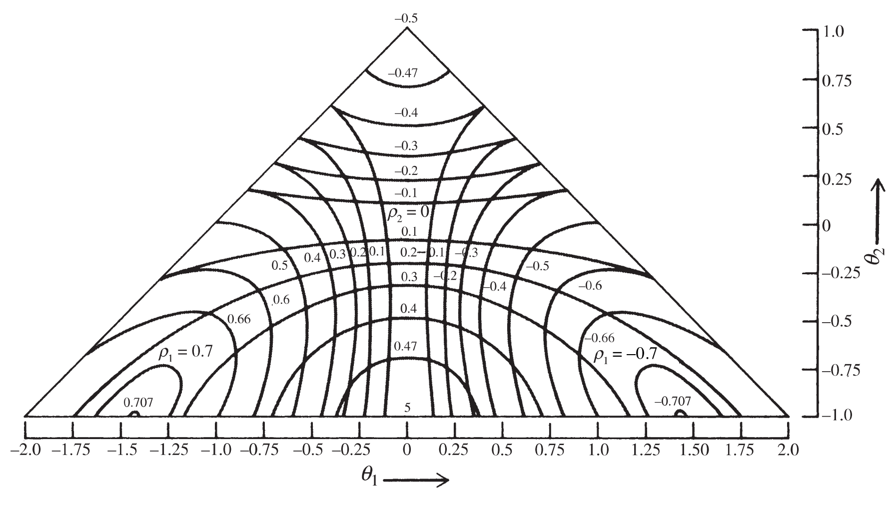

CHART C Chart relating ρ_1 and ρ_1 to θ_1 and θ_2 for a second-order autoregressive process.

The chart may be used to obtain estimates of the parameters in the $(0, d, 2)$ model $w_t = (1 - \theta_1 B - \theta_2 B^2)a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

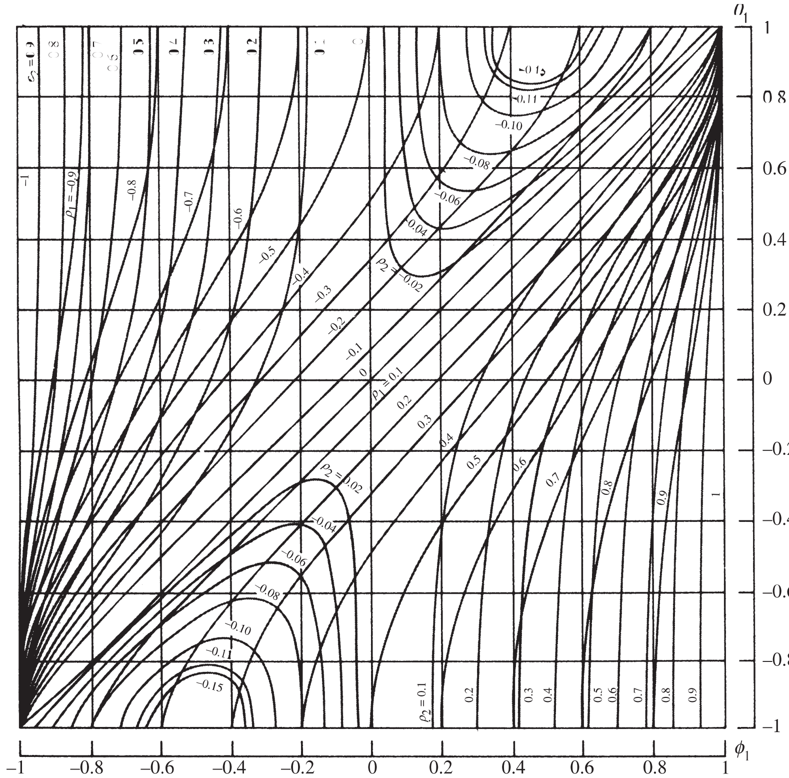

CHART D Chart relating ρ_1 and ρ_2 to ϕ and θ for a mixed first-order autoregressive–moving average process.

The chart may be used to obtain estimates of the parameters in the $(1, d, 1)$ model $(1 - \phi B)w_t = (1 - \theta B)a_t$, where $w_t = \nabla^d z_t$, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1 and ρ_2.

TABLE E Tail Areas and Ordinates of Unit Normal Distribution[a]

u_ε	ε	$p(u_\varepsilon)$	u_ε	ε	$p(u_\varepsilon)$
0.0	0.500	0.3989	1.6	0.055	0.1109
0.1	0.460	0.3969	1.7	0.045	0.0940
0.2	0.421	0.3910	1.8	0.036	0.0790
0.3	0.382	0.3814	1.9	0.029	0.0656
0.4	0.345	0.3683	2.0	0.023	0.0540
0.5	0.309	0.3521	2.1	0.018	0.0440
0.6	0.274	0.3322	2.2	0.014	0.0355
0.7	0.242	0.3123	2.3	0 011	0.0283
0.8	0.212	0.2897	2.4	0.008	0.0224
0.9	0.184	0.2661	2.5	0.006	0.0175
1.0	0.159	0.2420	2.6	0.005	0.0136
1.1	0.136	0.2179	2.7	0.003	0.0104
1.2	0.115	0.1942	2.8	0.003	0.0079
1.3	0.097	0.1714	2.9	0.002	0.0059
1.4	0.081	0.1497	3.0	0.001	0.0044
1.5	0.067	0.1295			

[a] Shown are the values of the unit normal deviate u_ε such that $\Pr\{u > u_\varepsilon\} = \varepsilon$; also shown are the ordinates $p(u = u_\varepsilon)$.

TABLE F Tail Areas of the Chi-Square Distribution[a]

	ε														
m	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005	0.001	m
1	–	–	–	–	0.016	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8	1
2	0.010	0.020	0.051	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6	13.8	2
3	0.072	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8	16.3	3
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13 3	14.9	18.5	4
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7	20.5	5
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5	22.5	6
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3	24.3	7
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0	26.1	8
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6	27.9	9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2	29.6	10
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3	11
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9	12
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5	13
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1	14
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7	15
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3	16
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8	17
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3	18
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8	19
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3	20
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8	21
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3	22
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7	23
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2	24
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6	25
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.1	26
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5	27
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9	28
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3	29
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	59.7	30

[a] Shown are the values of $x_\varepsilon^2(m)$ such that $\Pr\{x^2(m) > x_\varepsilon^2(m)\} = \varepsilon$, where m is the number of degrees of freedom.

TABLE G Tail Areas of the *t* Distribution[a]

	ε					
nu	0.25	0.10	0.05	0.025	0.01	0.005
1	1.00	3.08	6.31	12.71	31.82	63.66
2	0.82	1.89	2.92	4.30	6.96	9.92
3	0.76	1.64	2.35	3.18	4.54	5.84
4	0.74	1.53	2.13	2.78	3.75	4.60
5	0.73	1.48	2.02	2.57	3.36	4.03
6	0.72	1.44	1.94	2.45	3.14	3.71
7	0.71	1.42	1.90	2.36	3.00	3.50
8	0.71	1.40	1.86	2.31	2.90	3.36
9	0.70	1.38	1.83	2.26	2.82	3.25
10	0.70	1.37	1.81	2.23	2.76	3.17
11	0.70	1.36	1.80	2.20	2.72	3.11
12	0.70	1.36	1.78	2.18	2.68	3.06
13	0.69	1.35	1.77	2.16	2.65	3.01
14	0.69	1.34	1.76	2.14	2.62	2.98
15	0.69	1.34	1.75	2.13	2.60	2.95
16	0.69	1.34	1.75	2.12	2.58	2.92
17	0.69	1.33	1.74	2.11	2.57	2.90
18	0.69	1.33	1.73	2.10	2.55	2.88
19	0.69	1.33	1.73	2.09	2.54	2.86
20	0.69	1.33	1.72	2.09	2.53	2.84
30	0.68	1.31	1.70	2.04	2.46	2.75
40	0.68	1.30	1.68	2.02	2.42	2.70
60	0.68	1.30	1.67	2.00	2.39	2.66
120	0.68	1.29	1.66	1.98	2.36	2.62
∞	0.67	1.28	1.64	1.96	2.33	2.58

[a] Shown are the values of $t_\varepsilon(v)$ such that $\Pr\{t(v) > t_\varepsilon(v)\} = \varepsilon$, where v is the number of degrees of freedom.

COLLECTION OF TIME SERIES USED FOR EXAMPLES IN THE TEXT AND IN EXERCISES

SERIES A Chemical process concentration readings: every 2 hours

SERIES B IBM common stock closing prices: daily, May 17, 1961–November 2, 1962

SERIES B′ IBM common stock closing prices: daily, June 29, 1959–June 30, 1960

SERIES C Chemical process temperature readings: every minute

SERIES D Chemical process viscosity readings: every hour

SERIES E Wölfer sunspot numbers: yearly

SERIES F Yields from a batch chemical process: consecutive

SERIES G International airline passengers: monthly totals (thousands of passengers) January 1949–December 1960

SERIES J Gas furnace data

SERIES K Simulated dynamic data with two inputs

SERIES L Pilot scheme data

SERIES M Sales data with leading indicator

SERIES N Mink fur sales of the Hudson's Bay Company: annual for 1850–1911

SERIES P Unemployment and GDP data in UK: quarterly for 1955–1969

SERIES Q Logged and coded U.S. hog price data: annual for 1867–1948

SERIES R Monthly averages of hourly readings of ozone in downtown Los Angeles

Time Series Analysis: Forecasting and Control, Fifth Edition. George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, and Greta M. Ljung

SERIES A Chemical Process Concentration Readings: Every 2 Hours[a]

1	17.0	41	17.6	81	16.8	121	16.9	161	17.1
2	16.6	42	17.5	82	16.7	122	17.1	162	17.1
3	16.3	43	16.5	83	16.4	123	16.8	163	17.1
4	16.1	44	17.8	84	16.5	124	17.0	164	17.4
5	17.1	45	17.3	85	16.4	125	17.2	165	17.2
6	16.9	46	17.3	86	16.6	126	17.3	166	16.9
7	16.8	47	17.1	87	16.5	127	17.2	167	16.9
8	17.4	48	17.4	88	16.7	128	17.3	168	17.0
9	17.1	49	16.9	89	16.4	129	17.2	169	16.7
10	17.0	50	17.3	90	16.4	130	17.2	170	16.9
11	16.7	51	17.6	91	16.2	131	17.5	171	17.3
12	17.4	52	16.9	92	16.4	132	16.9	172	17.8
13	17.2	53	16.7	93	16.3	133	16.9	173	17.8
14	17.4	54	16.8	94	16.4	134	16.9	174	17.6
15	17.4	55	16.8	95	17.0	135	17.0	175	17.5
16	17.0	56	17.2	96	16.9	136	16.5	176	17.0
17	17.3	57	16.8	97	17.1	137	16.7	177	16.9
18	17.2	58	17.6	98	17.1	138	16.8	178	17.1
19	17.4	59	17.2	99	16.7	139	16.7	179	17.2
20	16.8	60	16.6	100	16.9	140	16.7	180	17.4
21	17.1	61	17.1	101	16.5	141	16.6	181	17.5
22	17.4	62	16.9	102	17.2	142	16.5	182	17.9
23	17.4	63	16.6	103	16.4	143	17.0	183	17.0
24	17.5	64	18.0	104	17.0	144	16.7	184	17.0
25	17.4	65	17.2	105	17.0	145	16.7	185	17.0
26	17.6	66	17.3	106	16.7	146	16.9	186	17.2
27	17.4	67	17.0	107	16.2	147	17.4	187	17.3
28	17.3	68	16.9	108	16.6	148	17.1	188	17.4
29	17.0	69	17.3	109	16.9	149	17.0	189	17.4
30	17.8	70	16.8	110	16.5	150	16.8	190	17.0
31	17.5	71	17.3	111	16.6	151	17.2	191	18.0
32	18.1	72	17.4	112	16.6	152	17.2	192	18.2
33	17.5	73	17.7	113	17.0	153	17.4	193	17.6
34	17.4	74	16.8	114	17.1	154	17.2	194	17.8
35	17.4	75	16.9	115	17.1	155	16.9	195	17.7
36	17.1	76	17.0	116	16.7	156	16.8	196	17.2
37	17.6	77	16.9	117	16.8	157	17.0	197	17.4
38	17.7	78	17.0	118	16.3	158	17.4		
39	17.4	79	16.6	119	16.6	159	17.2		
40	17.8	80	16.7	120	16.8	160	17.2		

[a] 197 observations.

SERIES B IBM Common Stock Closing Prices: Daily, May 17, 1961–November 2, 1962[a]

460	471	527	580	551	523	333	394	330
457	467	540	579	551	516	330	393	340
452	473	542	584	552	511	336	409	339
459	481	538	581	553	518	328	411	331
462	488	541	581	557	517	316	409	345
459	490	541	577	557	520	320	408	352
463	489	547	577	548	519	332	393	346
479	489	553	578	547	519	320	391	352
493	485	559	580	545	519	333	388	357
490	491	557	586	545	518	344	396	
492	492	557	583	539	513	339	387	
498	494	560	581	539	499	350	383	
499	499	571	576	535	485	351	388	
497	498	571	571	537	454	350	382	
496	500	569	575	535	462	345	384	
490	497	575	575	536	473	350	382	
489	494	580	573	537	482	359	383	
478	495	584	577	543	486	375	383	
487	500	585	582	548	475	379	388	
491	504	590	584	546	459	376	395	
487	513	599	579	547	451	382	392	
482	511	603	572	548	453	370	386	
479	514	599	577	549	446	365	383	
478	510	596	571	553	455	367	377	
479	509	585	560	553	452	372	364	
477	515	587	549	552	457	373	369	
479	519	585	556	551	449	363	355	
475	523	581	557	550	450	371	350	
479	519	583	563	553	435	369	353	
476	523	592	564	554	415	376	340	
476	531	592	567	551	398	387	350	
478	547	596	561	551	399	387	349	
479	551	596	559	545	361	376	358	
477	547	595	553	547	383	385	360	
476	541	598	553	547	393	385	360	
475	545	598	553	537	385	380	366	
475	549	595	547	539	360	373	359	
473	545	595	550	538	364	382	356	
474	549	592	544	533	365	377	355	
474	547	588	541	525	370	376	367	
474	543	582	532	513	374	379	357	
465	540	576	525	510	359	386	361	
466	539	578	542	521	335	387	355	
467	532	589	555	521	323	386	348	
471	517	585	558	521	306	389	343	

[a] 369 observations (read down).

SERIES B′ IBM Common Stock Closing Prices: Daily, June 29, 1959–June 30, 1960[a]

445	425	406	441	415	461
448	421	407	437	420	463
450	414	410	427	420	463
447	410	408	423	424	461
451	411	408	424	426	465
453	406	409	428	423	473
454	406	410	428	423	473
454	413	409	431	425	475
459	411	405	425	431	499
440	410	406	423	436	485
446	405	405	420	436	491
443	409	407	426	440	496
443	410	409	418	436	504
440	405	407	416	443	504
439	401	409	419	445	509
435	401	425	418	439	511
435	401	425	416	443	524
436	414	428	419	445	525
435	419	436	425	450	541
435	425	442	421	461	531
435	423	442	422	471	529
433	411	433	422	467	530
429	414	435	417	462	531
428	420	433	420	456	527
425	412	435	417	464	525
427	415	429	418	463	519
425	412	439	419	465	514
422	412	437	419	464	509
409	411	439	417	456	505
407	412	438	419	460	513
423	409	435	422	458	525
422	407	433	423	453	519
417	408	437	422	453	519
421	415	437	421	449	522
424	413	444	421	447	522
414	413	441	419	453	
419	410	440	418	450	
429	405	441	421	459	
426	410	439	420	457	
425	412	439	413	453	
424	413	438	413	455	
425	411	437	408	453	
425	411	441	409	450	
424	409	442	415	456	

[a] 255 observations (read down).

SERIES C Chemical Process Temperature Readings: Every Minute[a]

26.6	19.6	24.4	21.1	24.4
27.0	19.6	24.4	20.9	24.2
27.1	19.6	24.4	20.8	24.2
27.1	19.6	24.4	20.8	24.1
27.1	19.6	24.5	20.8	24.1
27.1	19.7	24.5	20.8	24.0
26.9	19.9	24.4	20.9	24.0
26.8	20.0	24.3	20.8	24.0
26.7	20.1	24.2	20.8	23.9
26.4	20.2	24.2	20.7	23.8
26.0	20.3	24.0	20.7	23.8
25.8	20.6	23.9	20.8	23.7
25.6	21.6	23.7	20.9	23.7
25.2	21.9	23.6	21.2	23.6
25.0	21.7	23.5	21.4	23.7
24.6	21.3	23.5	21.7	23.6
24.2	21.2	23.5	21.8	23.6
24.0	21.4	23.5	21.9	23.6
23.7	21.7	23.5	22.2	23.5
23.4	22.2	23.7	22.5	23.5
23.1	23.0	23.8	22.8	23.4
22.9	23.8	23.8	23.1	23.3
22.8	24.6	23.9	23.4	23.3
22.7	25.1	23.9	23.8	23.3
22.6	25.6	23.8	24.1	23.4
22.4	25.8	23.7	24.6	23.4
22.2	26.1	23.6	24.9	23.3
22.0	26.3	23.4	24.9	23.2
21.8	26.3	23.2	25.1	23.3
21.4	26.2	23.0	25.0	23.3
20.9	26.0	22.8	25.0	23.2
20.3	25.8	22.6	25.0	23.1
19.7	25.6	22.4	25.0	22.9
19.4	25.4	22.0	24.9	22.8
19.3	25.2	21.6	24.8	22.6
19.2	24.9	21.3	24.7	22.4
19.1	24.7	21.2	24.6	22.2
19.0	24.5	21.2	24.5	21.8
18.9	24.4	21.1	24.5	21.3
18.9	24.4	21.0	24.5	20.8
19.2	24.4	20.9	24.5	20.2
19.3	24.4	21.0	24.5	19.7
19.3	24.4	21.0	24.5	19.3
19.4	24.3	21.1	24.5	19.1
19.5	24.4	21.2	24.4	19.0
				18.8

[a]226 observations (read down).

SERIES D Chemical Process Viscosity Readings: Every Hour[a]

8.0	8.8	9.3	9.1	9.0	10.0	9.6
8.0	8.6	9.9	9.5	9.0	9.8	8.6
7.4	8.6	9.7	9.4	9.4	9.8	8.0
8.0	8.4	9.1	9.5	9.0	9.7	8.0
8.0	8.3	9.3	9.6	9.0	9.6	8.0
8.0	8.4	9.5	10.2	9.4	9.4	8.0
8.0	8.3	9.4	9.8	9.4	9.2	8.4
8.8	8.3	9.0	9.6	9.6	9.0	8.8
8.4	8.1	9.0	9.6	9.4	9.4	8.4
8.4	8.2	8.8	9.4	9.6	9.6	8.4
8.0	8.3	9.0	9.4	9.6	9.6	9.0
8.2	8.5	8.8	9.4	9.6	9.6	9.0
8.2	8.1	8.6	9.4	10.0	9.6	9.4
8.2	8.1	8.6	9.6	10.0	9.6	10.0
8.4	7.9	8.0	9.6	9.6	9.6	10.0
8.4	8.3	8.0	9.4	9.2	9.0	10.0
8.4	8.1	8.0	9.4	9.2	9.4	10.2
8.6	8.1	8.0	9.0	9.2	9.4	10.0
8.8	8.1	8.6	9.4	9.0	9.4	10.0
8.6	8.4	8.0	9.4	9.0	9.6	9.6
8.6	8.7	8.0	9.6	9.6	9.4	9.0
8.6	9.0	8.0	9.4	9.8	9.6	9.0
8.6	9.3	7.6	9.2	10.2	9.6	8.6
8.6	9.3	8.6	8.8	10.0	9.8	9.0
8.8	9.5	9.6	8.8	10.0	9.8	9.6
8.9	9.3	9.6	9.2	10.0	9.8	9.6
9.1	9.5	10.0	9.2	9.4	9.6	9.0
9.5	9.5	9.4	9.6	9.2	9.2	9.0
8.5	9.5	9.3	9.6	9.6	9.6	8.9
8.4	9.5	9.2	9.8	9.7	9.2	8.8
8.3	9.5	9.5	9.8	9.7	9.2	8.7
8.2	9.5	9.5	10.0	9.8	9.6	8.6
8.1	9.9	9.5	10.0	9.8	9.6	8.3
8.3	9.5	9.9	9.4	9.8	9.6	7.9
8.4	9.7	9.9	9.8	10.0	9.6	8.5
8.7	9.1	9.5	8.8	10.0	9.6	8.7
8.8	9.1	9.3	8.8	8.6	9.6	8.9
8.8	8.9	9.5	8.8	9.0	10.0	9.1
9.2	9.3	9.5	8.8	9.4	10.0	9.1
9.6	9.1	9.1	9.6	9.4	10.4	9.1
9.0	9.1	9.3	9.6	9.4	10.4	
8.8	9.3	9.5	9.6	9.4	9.8	
8.6	9.5	9.3	9.2	9.4	9.0	
8.6	9.3	9.1	9.2	9.6	9.6	
8.8	9.3	9.3	9.0	10.0	9.8	

[a] 310 observations (read down).

SERIES E Wölfer Sunspot Numbers: Yearly[a]

1770	101	1795	21	1820	16	1845	40
1771	82	1796	16	1821	7	1846	62
1772	66	1797	6	1822	4	1847	98
1773	35	1798	4	1823	2	1848	124
1774	31	1799	7	1824	8	1849	96
1775	7	1800	14	1825	17	1850	66
1776	20	1801	34	1826	36	1851	64
1777	92	1802	45	1827	50	1852	54
1778	154	1803	43	1828	62	1853	39
1779	125	1804	48	1829	67	1854	21
1780	85	1805	42	1830	71	1855	7
1781	68	1806	28	1831	48	1856	4
1782	38	1807	10	1832	28	1857	23
1783	23	1808	8	1833	8	1858	55
1784	10	1809	2	1834	13	1859	94
1785	24	1810	0	1835	57	1860	96
1786	83	1811	1	1836	122	1861	77
1787	132	1812	5	1837	138	1862	59
1788	131	1813	12	1838	103	1863	44
1789	118	1814	14	1839	86	1864	47
1790	90	1815	35	1840	63	1865	30
1791	67	1816	46	1841	37	1866	16
1792	60	1817	41	1842	24	1867	7
1793	47	1818	30	1843	11	1868	37
1794	41	1819	24	1844	15	1869	74

[a] 100 observations.

SERIES F Yields from a Batch Chemical Process: Consecutive[a]

47	44	50	62	68
64	80	71	44	38
23	55	56	64	50
71	37	74	43	60
38	74	50	52	39
64	51	58	38	59
55	57	45	59	40
41	50	54	55	57
59	60	36	41	54
48	45	54	53	23
71	57	48	49	
35	50	55	34	
57	45	45	35	
40	25	57	54	
58	59	50	45	

[a] 70 Observations (read down).

SERIES G International Airline Passengers: Monthly Totals (Thousands of Passengers) January 1949–December 1960[a]

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1949	112	118	132	129	121	135	148	148	136	119	104	118
1950	115	126	141	135	125	149	170	170	158	133	114	140
1951	145	150	178	163	172	178	199	199	184	162	146	166
1952	171	180	193	181	183	218	230	242	209	191	172	194
1953	196	196	236	235	229	243	264	272	237	211	180	201
1954	204	188	235	227	234	264	302	293	259	229	203	229
1955	242	233	267	269	270	315	364	347	312	274	237	278
1956	284	277	317	313	318	374	413	405	355	306	271	306
1957	315	301	356	348	355	422	465	467	404	347	305	336
1958	340	318	362	348	363	435	491	505	404	359	310	337
1959	360	342	406	396	420	472	548	559	463	407	362	405
1960	417	391	419	461	472	535	622	606	508	461	390	432

[a] 144 observations.

SERIES J Series J Gas Furnace Data[a]

t	X_t	Y_t	t	X_t	Y_t	t	X_t	Y_t
1	−0.109	53.8	51	1.608	46.9	101	−0.288	51.0
2	0.000	53.6	52	1.905	47.8	102	−0.153	51.8
3	0.178	53.5	53	2.023	48.2	103	−0.109	52.4
4	0.339	53.5	54	1.815	48.3	104	−0.187	53.0
5	0.373	53.4	55	0.535	47.9	105	−0.255	53.4
6	0.441	53.1	56	0.122	47.2	106	−0.229	53.6
7	0.461	52.7	57	0.009	47.2	107	−0.007	53.7
8	0.348	52.4	58	0.164	48.1	108	0.254	53.8
9	0.127	52.2	59	0.671	49.4	109	0.330	53.8
10	−0.180	52.0	60	1.019	50.6	110	0.102	53.8
11	−0.588	52.0	61	1.146	51.5	111	−0.423	53.3
12	−1.055	52.4	62	1.155	51.6	112	−1.139	53.0
13	−1.421	53.0	63	1.112	51.2	113	−2.275	52.9
14	−1.520	54.0	64	1.121	50.5	114	−2.594	53.4
15	−1.302	54.9	65	1.223	50.1	115	−2.716	54.6
16	−0.814	56.0	66	1.257	49.8	116	−2.510	56.4
17	−0.475	56.8	67	1.157	49.6	117	−1.790	58.0
18	−0.193	56.8	68	0.913	49.4	118	−1.346	59.4
19	0.088	56.4	69	0.620	49.3	119	−1.081	60.2
20	0.435	55.7	70	0.255	49.2	120	−0.910	60.0
21	0.771	55.0	71	−0.280	49.3	121	−0.876	59.4
22	0.866	54.3	72	−1.080	49.7	122	−0.885	58.4
23	0.875	53.2	73	−1.551	50.3	123	−0.800	57.6
24	0.891	52.3	74	−1.799	51.3	124	−0.544	56.9
25	0.987	51.6	75	−1.825	52.8	125	−0.416	56.4
26	1.263	51.2	76	−1.456	54.4	126	−0.271	56.0
27	1.775	50.8	77	−0.944	56.0	127	0.000	55.7
28	1.976	50.5	78	−0.570	56.9	128	0.403	55.3
29	1.934	50.0	79	−0.431	57.5	129	0.841	55.0

SERIES J (*continued*)

t	X_t	Y_t	t	X_t	Y_t	t	X_t	Y_t
30	1.866	49.2	80	−0.577	57.3	130	1.285	54.4
31	1.832	48.4	81	−0.960	56.6	131	1.607	53.7
32	1.767	47.9	82	−1.616	56.0	132	1.746	52.8
33	1.608	47.6	83	−1.875	55.4	133	1.683	51.6
34	1.265	47.5	84	−1.891	55.4	134	1.485	50.6
35	0.790	47.5	85	−1.746	56.4	135	0.993	49.4
36	0.360	47.6	86	−1.474	57.2	136	0.648	48.8
37	0.115	48.1	87	−1.201	58.0	137	0.577	48.5
38	0.088	49.0	88	−0.927	58.4	138	0.577	48.7
39	0.331	50.0	89	−0.524	58.4	139	0.632	49.2
40	0.645	51.1	90	0.040	58.1	140	0.747	49.8
41	0.960	51.8	91	0.788	57.7	141	0.900	50.4
42	1.409	51.9	92	0.943	57.0	142	0.993	50.7
43	2.670	51.7	93	0.930	56.0	143	0.968	50.9
44	2.834	51.2	94	1.006	54.7	144	0.790	50.7
45	2.812	50.0	95	1.137	53.2	145	0.399	50.5
46	2.483	48.3	96	1.198	52.1	146	−0.161	50.4
47	1.929	47.0	97	1.054	51.6	147	−0.553	50.2
48	1.485	45.8	98	0.595	51.0	148	−0.603	50.4
49	1.214	45.6	99	−0.080	50.5	149	−0.424	51.2
50	1.239	46.0	100	−0.314	50.4	150	−0.194	52.3
151	−0.049	53.2	201	−2.473	55.6	251	0.185	56.3
152	0.060	53.9	202	−2.330	58.0	252	0.662	56.4
153	0.161	54.1	203	−2.053	59.5	253	0.709	56.4
154	0.301	54.0	204	−1.739	60.0	254	0.605	56.0
155	0.517	53.6	205	−1.261	60.4	255	0.501	55.2
156	0.566	53.2	206	−0.569	60.5	256	0.603	54.0
157	0.560	53.0	207	−0.137	60.2	257	0.943	53.0
158	0.573	52.8	208	−0.024	59.7	258	1.223	52.0
159	0.592	52.3	209	−0.050	59.0	259	1.249	51.6
160	0.671	51.9	210	−0.135	57.6	260	0.824	51.6
161	0.933	51.6	211	−0.276	56.4	261	0.102	51.1
162	1.337	51.6	212	−0.534	55.2	262	0.025	50.4
163	1.460	51.4	213	−0.871	54.5	263	0.382	50.0
164	1.353	51.2	214	−1.243	54.1	264	0.922	50.0
165	0.772	50.7	215	−1.439	54.1	265	1.032	52.0
166	0.218	50.0	216	−1.422	54.4	266	0.866	54.0
167	−0.237	49.4	217	−1.175	55.5	267	0.527	55.1
168	−0.714	49.3	218	−0.813	56.2	268	0.093	54.5
169	−1.099	49.7	219	−0.634	57.0	269	−0.458	52.8
170	−1.269	50.6	220	−0.582	57.3	270	−0.748	51.4
171	−1.175	51.8	221	−0.625	57.4	271	−0.947	50.8
172	−0.676	53.0	222	−0.713	57.0	272	−1.029	51.2
173	0.033	54.0	223	−0.848	56.4	273	−0.928	52.0
174	0.556	55.3	224	−1.039	55.9	274	−0.645	52.8
175	0.643	55.9	225	−1.346	55.5	275	−0.424	53.8
176	0.484	55.9	226	−1.628	55.3	276	−0.276	54.5
177	0.109	54.6	227	−1.619	55.2	277	−0.158	54.9
178	−0.310	53.5	228	−1.149	55.4	278	−0.033	54.9

SERIES J (*continued*)

t	X_t	Y_t	t	X_t	Y_t	t	X_t	Y_t
179	−0.697	52.4	229	−0.488	56.0	279	0.102	54.8
180	−1.047	52.1	230	−0.160	56.5	280	0.251	54.4
181	−1.218	52.3	231	−0.007	57.1	281	0.280	53.7
182	−1.183	53.0	232	−0.092	57.3	282	0.000	53.3
183	−0.873	53.8	233	−0.620	56.8	283	−0.493	52.8
184	−0.336	54.6	234	−1.086	55.6	284	−0.759	52.6
185	0.063	55.4	235	−1.525	55.0	285	−0.824	52.6
186	0.084	55.9	236	−1.858	54.1	286	−0.740	53.0
187	0.000	55.9	237	−2.029	54.3	287	−0.528	54.3
188	0.001	55.2	238	−2.024	55.3	288	−0.204	56.0
189	0.209	54.4	239	−1.961	56.4	289	0.034	57.0
190	0.556	53.7	240	−1.952	57.2	290	0.204	58.0
191	0.782	53.6	241	−1.794	57.8	291	0.253	58.6
192	0.858	53.6	242	−1.302	58.3	292	0.195	58.5
193	0.918	53.2	243	−1.030	58.6	293	0.131	58.3
194	0.862	52.5	244	−0.918	58.8	294	0.017	57.8
195	0.416	52.0	245	−0.798	58.8	295	−0.182	57.3
196	−0.336	51.4	246	−0.867	58.6	296	−0.262	57.0
197	−0.959	51.0	247	−1.047	58.0			
198	−1.813	50.9	248	−1.123	57.4			
199	−2.378	52.4	249	−0.876	57.0			
200	−2.499	53.5	250	−0.395	56.4			

[a]Sampling interval 9 seconds; observations for 296 pairs of data points. X, 0.60 − 0.04 (input gas rate in cubic feet per minute); Y, %CO_2 in outlet gas.

SERIES K Simulated Dynamic Data with Two Inputs[a]

t	X_{1t}	X_{2t}	Y_t	t	X_{1t}	X_{2t}	Y_t
−2	0	0	58.3				
−1			61.8				
0			64.2	30			65.8
1			62.1	31			67.4
2	−1	1	55.1	32	−1	−1	64.7
3			50.6	33			65.7
4			47.8	34			67.5
5			49.7	35			58.2
6			51.6	36			57.0
7	1	−1	58.5	37	−1	1	54.7
8			61.5	38			54.9
9			63.3	39			48.4
10			65.9	40			49.7
11			70.9	41			53.1
12	−1	−1	65.8	42	1	−1	50.2
13			57.6	43			51.7
14			56.1	44			57.4
15			58.2	45			62.6
16			61.7	46			65.8
17	1	1	59.2	47	−1	−1	61.5
18			57.9	48			61.5
19			61.3	49			56.8
20			60.8	50			62.3
21			63.6	51			57.7
22	1	−1	69.5	52	−1	1	54.0
23			69.3	53			45.2
24			70.5	54			51.9
25			68.0	55			45.6
26			68.1	56			46.2
27	1	1	65.0	57	1	1	50.2
28			71.9	58			54.6
29			64.8	59			55.6
				60	0	0	60.4
				61			59.4

[a] 64 observations.

SERIES L Pilot Scheme Data[a]

t	x_t	ε_t	t	x_t	ε_t	t	x_t	ε_t
1	30	−4	53	−60	6	105	55	−4
2	0	−2	54	50	−2	106	0	2
3	−10	0	55	−10	0	107	−90	8
4	0	0	56	40	−4	108	40	0
5	−40	4	57	40	−6	109	0	0
6	0	2	58	−30	0	110	80	−8
7	−10	2	59	20	−2	111	−20	−2
8	10	0	60	−30	2	112	−10	0
9	20	−2	61	10	0	113	−70	6
10	50	−6	62	−20	2	114	−30	6
11	−10	−2	63	30	−2	115	−10	4
12	−55	4	64	−50	4	116	30	−1
13	0	2	65	10	−2	117	−5	0
14	10	0	66	10	−2	118	−60	6
15	0	−2	67	10	−2	119	70	−4
16	10	−2	68	−30	0	120	40	−6
17	−70	6	69	0	0	121	10	−4
18	30	0	70	−10	2	122	20	−4
19	−20	2	71	−10	3	123	10	−3
20	10	0	72	15	0	124	0	−2
21	0	0	73	20	−2	125	−70	6
22	0	0	74	−50	4	126	50	−2
23	20	−2	75	20	0	127	30	−4
24	30	−4	76	0	0	128	0	−2
25	0	−2	77	0	0	129	−10	0
26	−10	0	78	0	0	130	0	0
27	−20	2	79	0	0	131	−40	4
28	−30	4	80	−40	4	132	0	2
29	0	2	81	−100	12	133	−10	2
30	10	0	82	0	8	134	10	0
31	20	−2	83	0	−12	135	0	0
32	−10	0	84	50	−15	136	80	−8
33	0	0	85	85	−15	137	−80	4
34	20	−2	86	5	−12	138	20	4
35	10	−2	87	40	−14	139	20	0
36	−10	0	88	10	−8	140	−10	2
37	0	0	89	−60	2	141	10	0
38	0	0	90	−50	6	142	0	0
39	0	0	91	−50	8	143	−20	2
40	0	0	92	40	0	144	20	−1
41	0	0	93	0	0	145	55	−6
42	0	0	94	0	0	146	0	−3
43	20	−2	95	−20	2	147	25	−4
44	−50	4	96	−30	4	148	20	−4
45	20	0	97	−60	8	149	−60	4
46	0	0	98	−20	6	150	−40	6
47	0	0	99	−30	6	151	10	4
48	40	−4	100	30	0	152	20	0

SERIES L (*continued*)

t	x_t	ε_t	t	x_t	ε_t	t	x_t	ε_t
49	0	−2	101	−40	4	153	60	−6
50	50	−6	102	80	−6	154	−50	2
51	−40	0	103	−40	0	155	−10	2
52	−50	3	104	−20	2	156	−30	4
157	20	0	209	−40	4	261	−25	4
158	0	0	210	40	−2	262	35	−2
159	20	−2	211	−90	8	263	70	8
160	10	−2	212	40	0	264	−10	−5
161	10	−2	213	0	0	265	100	−20
162	10	−22	214	0	0	266	−20	−8
163	50	−6	215	0	0	267	−40	0
164	−30	0	216	20	−2	268	−20	2
165	−30	6	217	90	−10	269	10	0
166	90	12	218	30	−8	270	0	0
167	60	0	219	20	−6	271	0	0
168	−40	4	220	30	−6	272	−20	2
169	20	0	221	30	−6	273	−50	6
170	0	0	222	30	−6	274	50	−2
171	20	−2	223	30	−6	275	30	−4
172	10	−2	224	−90	6	276	60	−8
173	−30	2	225	10	2	277	−40	0
174	−30	4	226	10	2	278	−20	2
175	0	2	227	−30	4	279	−10	2
176	50	−4	228	−20	4	280	10	0
177	−60	4	229	40	−2	281	−110	13
178	20	0	230	10	−2	282	15	4
179	0	0	231	10	−2	283	30	−2
180	40	−8	232	10	−2	284	0	−1
181	80	−12	233	−100	12	285	25	−3
182	20	−8	234	10	6	286	−5	−1
183	−100	6	235	45	−2	287	−15	1
184	−30	6	236	30	−4	288	45	−4
185	30	0	237	30	−5	289	40	−6
186	−20	2	238	−15	−1	290	−50	2
187	−30	4	239	−5	0	291	−10	2
188	20	0	240	10	−1	292	−50	6
189	60	−6	241	−85	8	293	20	1
190	−10	−2	242	0	4	294	5	0
191	30	−4	243	0	0	295	−40	4
192	−40	2	244	60	−4	296	0	6
193	30	−2	245	40	−6	297	−60	8
194	−20	1	246	−30	0	298	40	0
195	5	0	247	−40	4	299	−20	2
196	−20	2	248	−40	6	300	130	−12
197	−30	4	249	50	−2	301	−20	−4
198	20	0	250	10	−2	302	0	−2
199	10	−1	251	30	−4	303	30	−4

SERIES L (*continued*)

t	x_t	ε_t	t	x_t	ε_t	t	x_t	ε_t
200	−15	1	252	−40	2	304	−20	0
201	−75	8	253	10	0	305	60	6
202	−40	8	254	−40	4	306	10	−4
203	−40	6	255	40	−2	307	−10	1
204	90	−6	256	−30	2	308	−25	2
205	90	−12	257	−50	6	309	0	1
206	80	−14	258	0	3	310	15	−1
207	−45	−2	259	−45	6	311	−5	0
208	−10	0	260	−20	5	312	0	0

[a] 312 observations.

SERIES M Sales Data with Leading Indicator[a]

t	Leading Indicator X_t	Sales Y_t	t	Leading Indicator X_t	Sales Y_t	t	Leading Indicator X_t	Sales Y_t
1	10.01	200.1	51	10.77	220.0	101	12.90	249.4
2	10.07	199.5	52	10.88	218.7	102	13.12	249.0
3	10.32	199.4	53	10.49	217.0	103	12.47	249.9
4	9.75	198.9	54	10.50	215.9	104	12.47	250.5
5	10.33	199.0	55	11.00	215.8	105	12.94	251.5
6	10.13	200.2	56	10.98	214.1	106	13.10	249.0
7	10.36	198.6	57	10.61	212.3	107	12.91	247.6
8	10.32	200.0	58	10.48	213.9	108	13.39	248.8
9	10.13	200.3	59	10.53	214.6	109	13.13	250.4
10	10.16	201.2	60	11.07	213.6	110	13.34	250.7
11	10.58	201.6	61	10.61	212.1	111	13.34	253.0
12	10.62	2013	62	10.86	211.4	112	13.14	253.7
13	10.86	201.5	63	10.34	213.1	113	13.49	255.0
14	11.20	203.5	64	10.78	212.9	114	13.87	256.2
15	10.74	204.9	65	10.80	213.3	115	13.39	256.0
16	10.56	207.1	66	10.33	211.5	116	13.59	257.4
17	10.48	210.5	67	10.44	212.3	117	13.27	260.4
18	10.77	210.5	68	10.50	213.0	118	13.70	260.0
19	11.33	209.8	69	10.75	211.0	119	13.20	261.3
20	10.96	208.8	70	10.40	210.7	120	13.32	260.4
21	11.16	209.5	71	10.40	210.1	121	13.15	261.6
22	11.70	213.2	72	10.34	211.4	122	13.30	260.8
23	11.39	213.7	73	10.55	210.0	123	12.94	259.8
24	11.42	215.1	74	10.46	209.7	124	13.29	259.0
25	11.94	218.7	75	10.82	208.8	125	13.26	258.9
26	11.24	219.8	76	10.91	208.8	126	13.08	257.4
27	11.59	220.5	77	10.87	208.8	127	13.24	257.7
28	10.96	223.8	78	10.67	210.6	128	13.31	257.9
29	11.40	222.8	79	11.11	211.9	129	13.52	257.4
30	11.02	223.8	80	10.88	212.8	130	13.02	257.3
31	11.01	221.7	81	11.28	212.5	131	13.25	257.6
32	11.23	222.3	82	11.27	214.8	132	13.12	258.9
33	11.33	220.8	83	11.44	215.3	133	13.26	257.8
34	10.83	219.4	84	11.52	217.5	134	13.11	257.7
35	10.84	220.1	85	12.10	218.8	135	13.30	257.2
36	11.14	220.6	86	11.83	220.7	136	13.06	257.5
37	10.38	218.9	87	12.62	222.2	137	13.32	256.8
38	10.90	217.8	88	12.41	226.7	138	13.10	257.5
39	11.05	217.7	89	12.43	228.4	139	13.27	257.0
40	11.11	215.0	90	12.73	233.2	140	13.64	257.6
41	11.01	215.3	91	13.01	235.7	141	13.58	257.3
42	11.22	215.9	92	12.74	237.1	142	13.87	257.5
43	11.21	216.7	93	12.73	240.6	143	13.53	259.6
44	11 91	216.7	94	12.76	243.8	144	13.41	261.1
45	11.69	217.7	95	12.92	245.3	145	13.25	262.9
46	10.93	218.7	96	12.64	246.0	146	13.50	263.3
47	10.99	222.9	97	12.79	246.3	147	13.58	262.8
48	11.01	224.9	98	13.05	247.7	148	13.51	261.8
49	10.84	222.2	99	12.69	247.6	149	13.77	262.2
50	10.76	220.7	100	13.01	247.8	150	13.40	262.7

[a] 150 observations.

SERIES N Mink Fur Sales of the Hudson's Bay Company: Annual for 1850–1911[a]

1850	29,619	1866	51,404	1882	45,600	1897	76,365
1851	21,151	1867	58,451	1883	47,508	1898	70,407
1852	24,859	1868	73,575	1884	52,290	1899	41,839
1853	25,152	1869	74,343	1885	110,824	1900	45,978
1854	42,375	1870	27,708	1886	76,503	1901	47,813
1855	50,839	1871	31,985	1887	64,303	1902	57,620
1856	61,581	1872	39,266	1888	83,023	1903	66,549
1857	61,951	1873	44,740	1889	40,748	1904	54,673
1858	76,231	1874	60,429	1890	35,596	1905	55,996
1859	63,264	1875	72,273	1891	29,479	1906	60,053
1860	44,730	1876	79,214	1892	42,264	1907	39,169
1861	31,094	1877	79,060	1893	58,171	1908	21,534
1862	49,452	1878	84,244	1894	50,815	1909	17,857
1863	43,961	1879	62,590	1895	51,285	1910	21,788
1864	61,727	1880	35,072	1896	70,229	1911	33,008
1865	60,334	1881	36,160				

[a]62 observations.

SERIES P Unemployment and GDP Data in UK: Quarterly for 1955–1969[a]

		UN	GDP			UN	GDP			UN	GDP
1955	1	225	81.37	1960	1	363	92.30	1965	1	306	108.07
	2	208	82.60		2	342	92.13		2	304	107.64
	3	201	82.30		3	325	93.17		3	321	108.87
	4	199	83.00		4	312	93.50		4	305	109.75
1956	1	207	82.87	1961	1	291	94.77	1966	1	279	110.20
	2	215	83.60		2	293	95.37		2	282	110.20
	3	240	83.33		3	304	95.03		3	318	110.90
	4	245	83.53		4	330	95.23		4	414	110.40
1957	1	295	84.27	1962	1	357	95.07	1967	1	463	111.00
	2	293	85.50		2	401	96.40		2	506	112.10
	3	279	84.33		3	447	96.97		3	538	112.50
	4	287	84.30		4	483	96.50		4	536	113.00
1958	1	331	85.07	1963	1	535	96.16	1968	1	544	114.30
	2	396	83.60		2	520	99.79		2	541	115.10
	3	432	84.37		3	489	101.14		3	547	116.40
	4	462	84.50		4	456	102.95		4	532	117.80
1959	1	454	85.20	1964	1	386	103.96	1969	1	532	116.80
	2	446	87.07		2	368	105.28		2	519	117.80
	3	426	88.40		3	358	105.81		3	547	119.00
	4	402	90.03		4	330	107.14		4	544	119.60

Source: Bray (1971).
[a]60 pairs of data; data are seasonally adjusted; unemployment (UN) in thousands; gross domestic product (GDP) is composite estimate (1963 = 100).

SERIES Q Logged and Coded U.S. Hog Price Data: Annual for 1867–1948[a]

1867	597	1888	709	1909	810	1929	1112
1868	509	1889	763	1910	957	1930	1129
1869	663	1890	681	1911	970	1931	1055
1870	751	1891	627	1912	903	1932	787
1871	739	1892	667	1913	995	1933	624
1872	598	1893	804	1914	1022	1934	612
1873	556	1894	782	1915	998	1935	800
1874	594	1895	707	1916	928	1936	1104
1875	667	1896	653	1917	1073	1937	1075
1876	776	1897	639	1918	1294	1938	1052
1877	754	1898	672	1919	1346	1939	1048
1878	689	1899	669	1920	1301	1940	891
1879	498	1900	729	1921	1134	1941	921
1880	643	1901	784	1922	1024	1942	1193
1881	681	1902	842	1923	1090	1943	1352
1882	778	1903	886	1924	1013	1944	1243
1883	829	1904	784	1925	1119	1945	1314
1884	751	1905	770	1926	1195	1946	1380
1885	704	1906	783	1927	1235	1947	1556
1886	633	1907	877	1928	1120	1948	1632
1887	663	1908	777				

Source: Quenouille (1957).

[a]82 observations; values are $1000 \log_{10}(H_t)$, where H_t is the price, in dollars, per head on January 1 of the year.

SERIES R Monthly Averages of Hourly Readings of Ozone in Downtown Los Angeles[a]

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1955	2.63	1.94	3.38	4.92	6.29	5.58	5.50	4.71	6.04	7.13	7.79	3.83
1956	3.83	4.25	5.29	3.75	4.67	5.42	6.04	5.71	8.13	4.88	5.42	5.50
1957	3.00	3.42	4.50	4.25	4.00	5.33	5.79	6.58	7.29	5.04	5.04	4.48
1958	3.33	2.88	2.50	3.83	4.17	4.42	4.25	4.08	4.88	4.54	4.25	4.21
1959	2.75	2.42	4.50	5.21	4.00	7.54	7.38	5.96	5.08	5.46	4.79	2.67
1960	1.71	1.92	3.38	3.98	4.63	4.88	5.17	4.83	5.29	3.71	2.46	2.17
1961	2.15	2.44	2.54	3.25	2.81	4.21	4.13	4.17	3.75	3.83	2.42	2.17
1962	2.33	2.00	2.13	4.46	3.17	3.25	4.08	5.42	4.50	4.88	2.83	2.75
1963	1.63	3.04	2.58	2.92	3.29	3.71	4.88	4.63	4.83	3.42	2.38	2.33
1964	1.50	2.25	2.63	2.96	3.46	4.33	5.42	4.79	4.38	4.54	2.04	1.33
1965	2.04	2.81	2.67	4.08	3.90	3.96	4.50	5.58	4.52	5.88	3.67	1.79
1966	1.71	1.92	3.58	4.40	3.79	5.52	5.50	5.00	5.48	4.81	2.42	1.46
1967	1.71	2.46	2.42	1.79	3.63	3.54	4.88	4.96	3.63	5.46	3.08	1.75
1968	2.13	2.58	2.75	3.15	3.46	3.33	4.67	4.13	4.73	3.42	3.08	1.79
1969	1.96	1.63	2.75	3.06	4.31	3.31	3.71	5.25	3.67	3.10	2.25	2.29
1970	1.25	2.25	2.67	3.23	3.58	3.04	3.75	4.54	4.46	2.83	1.63	1.17
1971	1.79	1.92	2.25	2.96	2.38	3.38	3.38	3.21	2.58	2.42	1.58	1.21
1972	1.42	1.96	3.04	2.92	3.58	3.33	4.04	3.92	3.08	2.00	1.58	1.21

[a]216 observations; values are in pphm.

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