$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} = \nu\frac{\partial^2 \phi}{\partial x^2} \qquad \phi(0,t) = 0, \quad \phi(x,0) = e^{-\left(\frac{x-3}{.3}\right)^2}$$

Using Fwd, central, and reverse differencing, re-express $\frac{\partial \phi}{\partial t}$ and $\frac{\partial^2 \phi}{\partial t^2}$ as deriv. matrices.

$$\frac{\partial \phi}{\partial x} = \begin{pmatrix} f_0' \\ f_1' \\ f_2' \\ \vdots \\ f_{n-3}' \\ f_{n-2}' \\ f_{n-1}' \end{pmatrix} = \frac{1}{2\Delta x}\begin{pmatrix} -2y_0 & 2y_1 & 0 & \cdots & \cdots & 0 & 0 \\ -y_0 & 0 & y_2 & 0 & \cdots & & 0 \\ 0 & -y_1 & 0 & y_3 & 0 & \cdots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & & 0 & -y_{n-4} & 0 & y_{n-2} & 0 \\ 0 & & & 0 & -y_{n-3} & 0 & y_{n-1} \\ 0 & 0 & \cdots & & 0 & -2y_{n-2} & 2y_{n-1} \end{pmatrix} = \frac{1}{2\Delta_x}\begin{pmatrix} -2 & 2 & 0 & \cdots & \cdots & 0 & 0 \\ -1 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 0 & 1 & 0 & \cdots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & & 0 & -1 & 0 & 1 & 0 \\ 0 & & & 0 & -1 & 0 & 1 \\ 0 & 0 & \cdots & & 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \begin{pmatrix} f_0'' \\ f_1'' \\ f_2'' \\ \vdots \\ f_{n-3}'' \\ f_{n-2}'' \\ f_{n-1}'' \end{pmatrix} = \frac{1}{\Delta_x^2}\begin{pmatrix} y_0 & -2y_1 & y_2 & 0 & \cdots & 0 & 0 \\ y_0 & -2y_1 & y_2 & 0 & \cdots & & 0 \\ 0 & y_0 & -2y_1 & y_1 & 0 & \cdots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & & & y_{n-4} & -2y_{n-3} & y_{n-2} & 0 \\ 0 & 0 & \cdots & 0 & y_{n-3} & -2y_{n-2} & y_{n-1} \\ & & & 0 & y_{n-3} & -2y_{n-2} & y_{n-1} \end{pmatrix} = \frac{1}{\Delta_x^2}\begin{pmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & & 0 \\ 0 & 1 & -2 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & -2 & 1 & 0 \\ 0 & & \cdots & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -2 \quad 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}$$

Isolate the $\frac{\partial \phi}{\partial t}$ and add the prefactors to see $\frac{d}{\partial t}(\phi) = [L](\phi)$ where

$$[L] = \frac{-U}{2\Delta_x}\begin{pmatrix} -2 & 2 & 0 & \cdots & \cdots & 0 & 0 \\ -1 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 0 & 1 & 0 & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & 0 & -1 & 0 & 1 & 0 \\ 0 & & & 0 & -1 & 0 & 1 \\ 0 & 0 & \cdots & & 0 & -2 & 2 \end{pmatrix} + \frac{\nu}{\Delta_x^2}\begin{pmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & & & 0 \\ 0 & 1 & -2 & 1 & 0 & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & 0 & 1 & -2 & 1 & 0 \\ 0 & & & 0 & 1 & -2 & 1 \\ 0 & 0 & \cdots & & 0 & 1 & -2 \quad 1 \end{pmatrix}$$

I realized late this isn't an unspecified number of spacing's, but in fact 6 and I don't want to rewrite the above, so here it is as 6:

$$L = -\frac{U}{2\Delta_x}\begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix} + \frac{\nu}{\Delta_x^2}\begin{pmatrix} 1 & -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{pmatrix}$$