$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 15\sin(12x)$$

(boundary conditions: $y(0) = -1$, $y(1) = 2$)

	central differencing	fwd differencing	backward differencing
first deriv	$\frac{dy}{dx}(x_i) \approx \frac{(y_{i+1} - y_{i-1})}{2\Delta}$	$\frac{dy}{dx}x(x_0) \approx \frac{y_1 - y_0}{\Delta}$	$\frac{dy}{dx}(x_{N-1}) \approx \frac{(y_{N-1} - y_{N-2})}{\Delta}$
2nd deriv	$\frac{d^2y}{dx^2}(x_i) \approx \frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta^2}$	$\frac{d^2y}{dx^2}(x_0) \approx \frac{y_0 - 2y_1 + y_2}{\Delta^2}$	$\frac{d^2y}{dx^2}(x_{N-1}) \approx \frac{y_{N-1} - 2y_{N-1} + y_{N-3}}{\Delta^2}$

Using fwd, backward and central differencing schemes, we can express the components as matrices:

$$\frac{d^2y}{dx^2} = \begin{pmatrix} f''_0 \\ f''_1 \\ f''_2 \\ \vdots \\ f''_{n-3} \\ f''_{n-2} \\ f''_{n-1} \end{pmatrix} = \frac{1}{\Delta^2}\begin{pmatrix} y_0 & -2y_1 & y_2 & 0 & \cdots & & 0 & 0 \\ y_0 & -2y_1 & y_2 & 0 & \cdots & & & 0 \\ 0 & y_1 & -2y_2 & y_3 & 0 & \cdots & & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \cdots & 0 & y_{n-4} & -2y_{n-3} & y_{n-2} & 0 \\ 0 & \cdots & & 0 & y_{n-3} & -2y_{n-2} & y_{n-1} \\ 0 & 0 & \cdots & 0 & y_{n-3} & -2y_{n-2} & y_{n-1} \end{pmatrix} = \frac{1}{\Delta^2}\begin{pmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & & 0 \\ 0 & 1 & -2 & 1 & 0 & & \\ \vdots & & & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & -2 & 1 & 0 \\ 0 & \cdots & & 0 & 1 & -2 & 1 \\ 0 & 0 & \cdots & 0 & 1 & -2 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}$$

$$\frac{dy}{dx} = \begin{pmatrix} f'_0 \\ f'_1 \\ f'_2 \\ \vdots \\ f'_{n-3} \\ f'_{n-2} \\ f'_{n-1} \end{pmatrix} = \frac{1}{2\Delta}\begin{pmatrix} -2y_0 & 2y_1 & 0 & \cdots & & 0 & 0 \\ -y_0 & 0 & y_2 & \cdots & & & 0 \\ 0 & -y_1 & 0 & y_3 & 0 & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & -y_{n-3} & y_{n-2} & 0 \\ 0 & & \cdots & 0 & -y_{n-2} & y_{n-1} \\ 0 & 0 & 0 & \cdots & 0 & -2y_{n-2} & 2y_{n-1} \end{pmatrix} = \frac{1}{2\Delta}\begin{pmatrix} -2 & 2 & 0 & \cdots & & 0 & 0 \\ -1 & 0 & 1 & & & & 0 \\ 0 & -1 & 0 & 1 & & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & & -1 & 0 & 1 & 0 \\ 0 & \cdots & & & -1 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-2} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}$$

Combining to create the left side gives:

$$\left[\frac{1}{\Delta^2}\begin{pmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & & 0 \\ 0 & 1 & -2 & 1 & \cdots & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & 1 & -2 & 1 & 0 \\ 0 & \cdots & & 0 & 1 & -2 & 1 \\ 0 & 0 & \cdots & 0 & 1 & -2 & 1 \end{pmatrix} + \frac{1}{2\Delta}\begin{pmatrix} -2 & 2 & 0 & \cdots & & 0 & 0 \\ -1 & 0 & 1 & & & & 0 \\ 0 & -1 & 0 & 1 & \cdots & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & & -1 & 0 & 1 & 0 \\ 0 & \cdots & & & -1 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -2 & 2 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 0 & \cdots & & 0 & 0 \\ 0 & 6 & 0 & & & & 0 \\ 0 & 0 & 6 & & & & \vdots \\ \vdots & & & \ddots & & & \vdots \\ 0 & & & & 6 & 0 & 0 \\ 0 & & & & 0 & 6 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 6 \end{pmatrix}\right]\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}$$

$$
\begin{pmatrix}
\frac{1}{\Delta^2}-\frac{2}{2\Delta}-6 & \frac{-2}{\Delta^2}+\frac{2}{2\Delta} & \frac{1}{\Delta^2} & 0 & \cdots & \cdots & 0 & 0 \\
\frac{1}{\Delta^2}-\frac{1}{2\Delta} & \frac{-2}{\Delta^2}-6 & \frac{1}{\Delta^2}+\frac{1}{2\Delta} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & \frac{1}{\Delta^2}-\frac{1}{2\Delta} & \frac{-2}{\Delta^2}-6 & \frac{1}{\Delta^2}+\frac{1}{2\Delta} & & & & \vdots \\
\vdots & & \vdots & & & & & \vdots \\
\vdots & & & 0 & \frac{1}{\Delta^2}-\frac{1}{2\Delta} & \frac{-2}{\Delta^2}-6 & \frac{1}{\Delta^2}+\frac{1}{2\Delta} & 0 \\
0 & \cdots & \cdots & \cdots & 0 & \frac{1}{\Delta^2}-\frac{1}{2\Delta} & \frac{-2}{\Delta^2}-6 & \frac{1}{\Delta^2}+\frac{1}{2\Delta} \\
0 & 0 & \cdots & \cdots & 0 & \frac{1}{\Delta^2} & \frac{-2}{\Delta^2}-\frac{2}{2\Delta} & \frac{1}{\Delta^2}+\frac{2}{2\Delta}-6
\end{pmatrix}
\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}
$$

Now actually we know the boundary conditions, so the first and last rows can be replaced. Additionally the points in the midlle are all the same, so we may simplify by letting

$$\alpha = \frac{1}{\Delta^2} - \frac{1}{2\Delta} \qquad \beta = -\frac{2}{\Delta^2} - 6 \qquad \gamma = \frac{1}{\Delta^2} + \frac{1}{2\Delta}$$

each solution is of the form $15\sin(12x_i)$, so let $Q_i = 15\sin(12x_i)$ and the B.C.'s $Q_0 = -1 = a$, $Q_{n-1} = 2 = b$.

Altogether, this yields:

$$
\begin{pmatrix}
1 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 \\
\alpha & \beta & \gamma & 0 & \cdots & \cdots & & 0 \\
0 & \alpha & \beta & \gamma & 0 & \cdots & & 0 \\
\vdots & & & 0 & \alpha & \beta & \gamma & 0 \\
0 & & & & 0 & \alpha & \beta & \gamma \\
0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \end{pmatrix}
=
\begin{pmatrix} a \\ Q_1 \\ Q_2 \\ \vdots \\ Q_{n-3} \\ Q_{n-2} \\ b \end{pmatrix}
$$