ENGR20005
Numerical Methods in Engineering

Workshop 6

Part A: Pre-Lecture Problems

Please try to attempt 6.1 before Lecture 11 and 6.2 before Lecture 12.

6.1 Write a MATLAB program the uses the Simpson's rule (Eq. (11.13) in the lecture slides for Lecture 11) to approximate the integral of the function

$$\int_{2}^{10} \frac{1}{x} dx \tag{6.1}$$

Compare with the answer using trapezoidal rule and also with the exact analytical solution.

6.2 Write a MATLAB program that uses the Trapezoidal rule to calculate the double integral

$$\int_{0}^{6} \int_{0}^{2} \left(x^4 + 2y^2x^2 \right) dxdy. \tag{6.2}$$

Part B: MATLAB Livescripts

6.3 The livescript *ENGR20005_Workshop6p3.mlx* runs through Newton interpolation.

(a) Read through the livescript and make sure you understand what each line of code does.

(b) Modify the livescript to approximate the *Gaussian function*

$$f(x) = e^{-x^2} \tag{6.3}$$

within the interval $-1 \leq x \leq 1$. Use as many nodes as you deem fit.

6.4 The livescript *ENGR20005_Workshop6p4.mlx* runs through the use of Lagrange interpolation to approximate functions.

(a) Read through the livescript and make sure you understand what each line of code does.

(b) Modify the livescript to approximate Eq. (6.3)

6.5 The livescript *ENGR20005_Workshop6p5.mlx* covers the use of linear and quadratic splines for interpolation.

(a) Read through the livescript and make sure you understand what each line of code does.

(b) Modify the livescript to approximate Eq. (6.3)

Part C: Problems

6.6 The temperature in lakes usually varies with depth of the water. Near the surface, the temperature of water is usually warmer and more sensitive to the changes in air temperature during the course of the day. As one goes deeper into the lake, the temperature gets progressively cooler. The temperature of the lake measured at various depth is shown in the table below.

z depth of lake (m)	$T(^oC)$
0.0	22.8
2.3	22.8
4.9	22.8
9.1	20.6
13.7	13.9
18.3	11.7
22.9	11.1
27.2	11.1

(a) Use Quadratic spline and plot how the temperature vary with depth. Plot the Lagrange and Quadratic Spline interpolated function at as many points as possible for $0 \le z \le 27.2$.

(b) For quadratic splines, plot both cases for $c_0 = 0$ and $c_n = 0$. Can you explain the differences between these two cases?

(c) Use the MATLAB function `interp1d()` or `spline()` to find the cubic spline interpolation for the data in the table above. Plot the this cubic spline interpolation function.

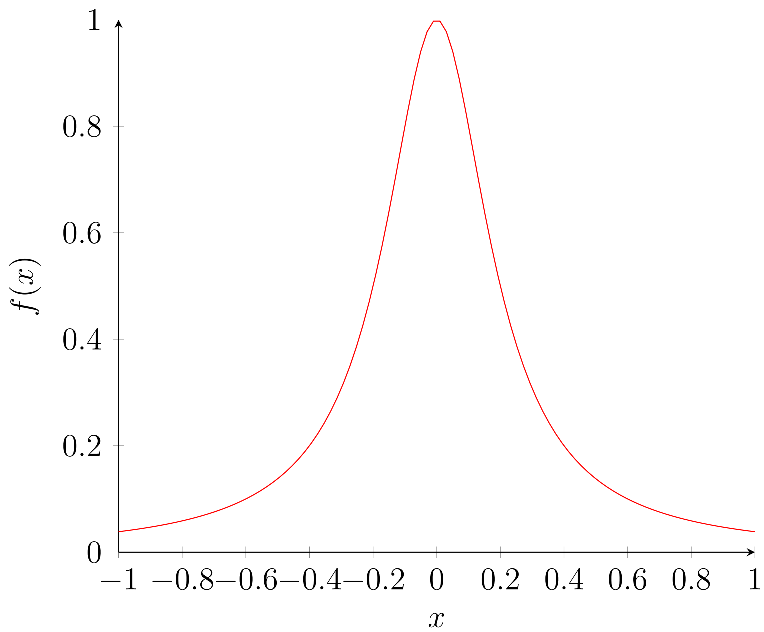
(d) Do the graph look reasonable ? Which interpolation scheme works better in this situation ?

Keep the solution to this question. We will be referring to it in Workshop 8.

6.7 *The *Witch of Agnesi* is a function that appears prominently in probability theory, and after some scaling may be written as

$$f(x) = \frac{1}{1 + 25x^2} \tag{6.4}$$

which has been plotted below.



We'll attempt to use Lagrange interpolation to approximate this function.

(a) Evaluate Eq. (6.4) at the following points

i. $x_i \in \{-1, 0, 1\}$.

ii. $x_i \in \{-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1\}$

(b) Determine the Lagrange polynomial using the points you determined in part (a) and plot it along with Eq. (6.4). What do you notice?

This effect where the ends diverge rapidly as we increase number of Lagrange interpolation nodes is called *Runge's phenomena*.

(c) Repeat parts (a) and (b) with the points

$$x_i \in \{-1.0000, -0.8998, -0.6772, -0.3631, 0.0000, 0.3631, 0.6772, 0.8998, 1.0000\}$$

What happens now?

These are called *Gauss–Lobatto–Legendre (GLL) nodes* and are very nearly the optimal choice of interpolation points.

(d) Repeat the exercise with quadratic Spline interpolation. Which method performs better?

6.8 Consider the function

$$f(x) = \frac{1}{1-x} \tag{6.5}$$

(a) Show that the zeroth and first divided differences are given by

$$f[x_0] = \frac{1}{1-x_0}$$
$$f[x_0, x_1] = \frac{1}{(1-x_0)(1-x_1)}$$

(b) Show that the n^{th} divided difference is given by

$$f[x_0, x_1, \ldots, x_n] = \frac{1}{(1-x_0)(1-x_1)\cdots(1-x_n)}$$

(c) Hence show that the n^{th} order Newton interpolant is given by

$$\frac{1}{1-x} \approx \frac{1}{1-x_0} + \frac{x-x_0}{(1-x_0)(1-x_1)} + \cdots + \frac{(x-x_0)\cdots(x-x_{n-1})}{(1-x_0)\cdots(1-x_n)} \tag{6.6}$$

(d) Take the limit of Eq. (6.6) as $x_i \to 0$, $i = 0, \ldots, n$. Compare your answer with the Taylor series of Eq. (6.5) about the point $x = 0$.