2.1

$$\begin{bmatrix} \beta_1 & \gamma_1 & 0 & 0 & \cdots & 0 \\ \alpha_2 & \beta_2 & \gamma_2 & 0 & \cdots & 0 \\ 0 & \alpha_3 & \beta_3 & \gamma_3 & \cdots & 0 \\ & & & & & \gamma_{n-1} \\ 0 & \cdots & & & \alpha_n & \beta_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_n \end{bmatrix}$$

Tridiagonal matrix

(use r_1 to remove α_2 in r_2)

(Augmented)

$$\left[\begin{array}{cccccc|c} \beta_1 & \gamma_1 & & 0 & 0 \cdots & 0 & Q_1 \\ 0 & \beta_2 - \frac{\alpha_2}{\beta_1}\gamma_1 & \gamma_2 & 0 & \cdots & 0 & Q_2 - \frac{\alpha_2}{\beta_1} Q_1 \\ 0 & \alpha_3 & \beta_3 & \gamma_3 & \cdots & & \vdots \\ \vdots & & & & & \gamma_{n-1} & \\ 0 & \cdots & & & \alpha_n & \beta_n & Q_n \end{array}\right]$$

To avoid extreme mess, let $\beta_i^* = \beta_i - \frac{\alpha_i}{\beta_{i-1}}\gamma_{i-1}$ and $Q_i^* = Q_i - \frac{\alpha_i}{\beta_{i-1}} Q_{i-1}$ (unless $i=1$ in which case $\beta_1^* = \beta_1$ and $Q_1^* = Q_1$)

So we replace with β_2^* and Q_2^*

But indeed this pattern will repeat until we clear all α_i to the last row leaving us with:

$$\left[\begin{array}{cccccc|c} \beta_1^* & \gamma_1 & 0 & 0 & \cdots & 0 & Q_1^* \\ 0 & \beta_2^* & \gamma_2 & 0 & \cdots & 0 & \vdots \\ 0 & 0 & \beta_3^* & \gamma_3 & & \vdots & \vdots \\ \vdots & & & & & \gamma_{n-1} & \vdots \\ 0 & 0 & \cdots & & 0 & \beta_n^* & Q_n^* \end{array}\right]$$

2.2 As such we can get a simple equation for the ith row:

$$\cdots + (0)\cancel{(x_{i-1})}^{0} + \beta_i^* x_i + \gamma_3 (x_{i+1}) + (0)\cancel{(x_{i+2})}^{0} + \cdots = Q_i^*$$

$$\boxed{x_i = \frac{Q_i^* - \gamma_3 (x_i + 1)}{\beta_i^*}}$$

for $i < n$ else $x_n = \frac{Q_n^*}{\beta_n^*}$ (no γ_{n+1} term)