

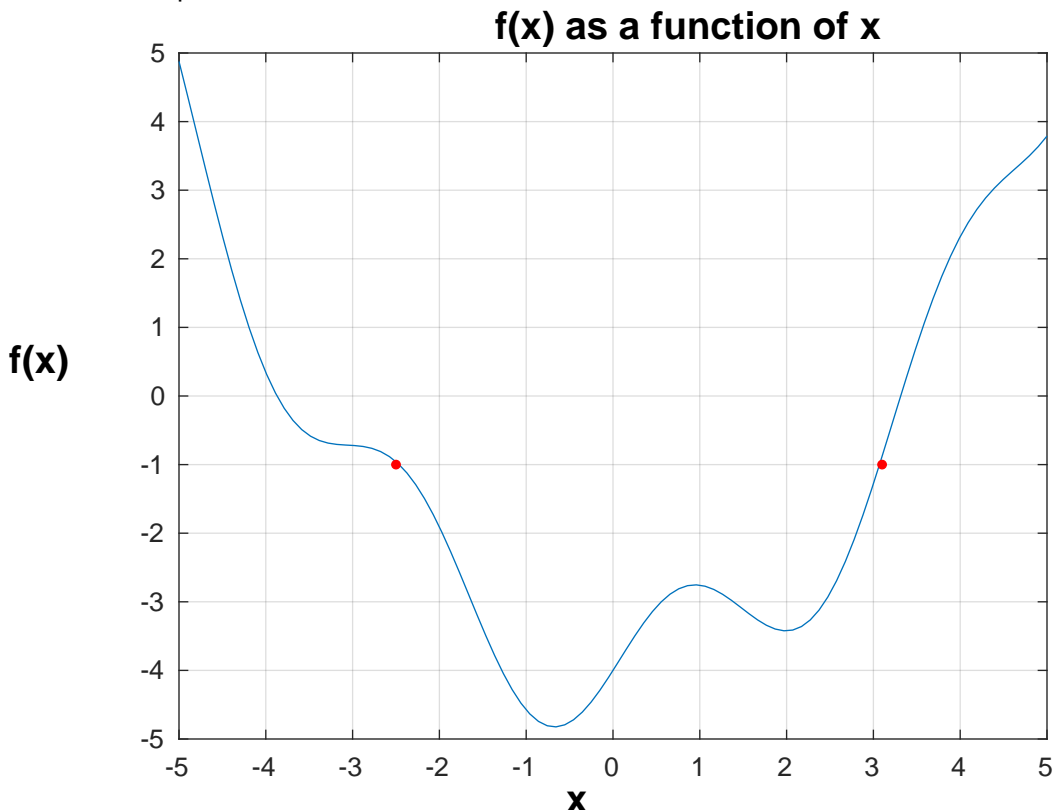
With fixed point iteration, we are taking a general function $y(x) = k$ where k is the "y-value" we are interested in. We can then look at the hyperfunction $f(x) = y(x) - k$ and numerically estimating where $f(x) = 0$ (it will ALWAYS be zero as we are subtracting that constant k to one side)

In this specific case, $k = -1$ and $y(x) = \sin(2x) + (1/3) x^2 - 4$
 $y(x) = -1$ so $f(x) = y(x) + 1$ or

$$f(x) = \sin(2x) + (1/3) x^2 - 3$$

The plot below is of $y(x)$. $f(x)$ would be the same thing shifted up by 1. In both cases, we would see two points of interest (crossing $t(x) = -1$ or equivalently $f(x) = 0$).

With $f(x)$, we can rearrange for some x such that x equals some function of x , namely $x = g(x)$
 (note, there are usually multiple possibilities with which to proceed. It's only important to check the convergence criteria which states that for some point x_0 we're interested in, provided $|g'(x_0)| < 1$, we are guaranteed a convergence, otherwise it MAY converge, it is just not a guarantee)
 with this in mind, we can proceed with the estimation



The logic is pretty straightforward. If we knew exactly what x_0 was, the equation $x = g(x)$ would be true, i.e. the left and right side would equate. If we chose a value close to x_0 and put it into $g(x)$, then the corresponding $g(x_0)$ would be very close to x_0 . Iterating and placing the output of this would approach the actual value

$$\begin{aligned} x_1 &= g(x_0) \\ x_2 &= g(x_1) \\ x_3 &= g(x_2) \\ &\vdots \\ x_{n+1} &= g(x_n) \end{aligned}$$

givenn $f(x) = \sin(2x) + (1/3) x^2 - 3$, $x = \sqrt{3(3+\sin(2x))} = g(x)$ looks like a good choice. The upper bound looks to be about 3.1, so we can choose $x_0 = 3.1$

$$\begin{aligned} x_1 &= \sqrt{3(3-\sin(2 \cdot 3.1))} = 3.0413 \\ x_2 &= \sqrt{3(3-\sin(2 \cdot 3.0413))} = 3.0981 \\ &\vdots \\ x_{40} &= \sqrt{3(3-\sin(2 \cdot 3.0758))} = 3.0758 \end{aligned}$$

out of curiosity, I plotted the behavior of this function;

