$$U(x,y) \approx \sum_{i=0}^{N} \sum_{j=0}^{N} U_{j,i}\, L_i(x)\, L_j(y)$$

2nd derivatives:

$$U_{xx}(x,y) \approx \sum_{i=0}^{N} \sum_{j=0}^{N} U_{j,i}\, L_i''(x)\, L_j(y), \quad U_{yy}(x,y) \approx \sum_{i=0}^{N} \sum_{j=0}^{N} U_{j,i}\, L_i(x)\, L_j''(y)$$

for simplification of notation, $\{U\} = (U_1, U_2, \dots, U_\ell, \dots U_{(N+1)^2})^T = (U_{0,0}, U_{0,1}, \dots U_{i,j}, \dots U_{N,N})^T$

where $\ell = 1 + i + (N+1)j$

the second derivative matrices can be written as

$$U_{xx}(x_i, y_i) = [I] \otimes [D^2]\{U\}$$

$$U_{yy}(x_i, y_i) = [D^2] \otimes [I]\{U\}$$

let's expand for a 2x2 matrix (let $D^2 = \begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix}$)

$$[D^2]_x = I \otimes D^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} = \begin{matrix} 1\begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} & 0\begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} \\ 0\begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} & 1\begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} \end{matrix}$$

$$\Rightarrow \boxed{[D^2]_x = \begin{pmatrix} D_{11}^2 & D_{12}^2 & 0 & 0 \\ D_{21}^2 & D_{22}^2 & 0 & 0 \\ 0 & 0 & D_{11}^2 & D_{12}^2 \\ 0 & 0 & D_{21}^2 & D_{22}^2 \end{pmatrix}}$$

Similarly

$$[D^2]_y = D^2 \otimes I = \begin{pmatrix} D_{11}^2 & D_{12}^2 \\ D_{21}^2 & D_{22}^2 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{matrix} D_{11}^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & D_{12}^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ D_{21}^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & D_{22}^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

$$\Rightarrow \boxed{[D^2]_y = \begin{pmatrix} D_{11}^2 & 0 & D_{12}^2 & 0 \\ 0 & D_{11}^2 & 0 & D_{12}^2 \\ D_{21}^2 & 0 & D_{22}^2 & 0 \\ 0 & D_{21}^2 & 0 & D_{22}^2 \end{pmatrix}}$$