

Given we know the boundary condition $\phi(0,t) = 0$, we can remove the first row (which keep in mind is an equation which states what ϕ_0 is) we have a new matrix

$$-\frac{U}{2\Delta x} \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix} + \frac{U}{\Delta x^2} \begin{pmatrix} 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{pmatrix}$$

But this is a downgrade in a sense, its 5x6. Square matrices allow for certain evaluation tricks like inversion, diagonalization. Fortunately, since the first column multiplies ϕ_0 which we know, we can pull out that first column (being careful not to forget the pre-factors)

$$\frac{d}{dt}(\phi) = \underbrace{\left[-\frac{U}{2\Delta x} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -2 & 2 \end{pmatrix} + \frac{U}{\Delta x^2} \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & 1 \end{pmatrix} \right]}_{L'} \underbrace{\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix}}_P + \underbrace{\left[\frac{U}{2\Delta x} + \frac{U}{\Delta x^2} \right]}_{\text{from } (-1)(-\frac{U}{2\Delta x})} \phi_0$$