Visualizing and analyzing superposition in multi-qubit quantum states

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This project explores the visualization of quantum states, focusing on superposition and entanglement, and demonstrates a user-friendly interface built with Streamlit and Qiskit. The primary objective is to create an interactive tool that allows users to select the number of qubits and instantly view their quantum state through Bloch spheres, Q-sphere diagrams, and probability distributions. By applying a sequence of standard quantum gates (e.g., Hadamard, CNOT) to generate superposition and entangled states, the tool provides visual representations and metrics, such as reduced density matrix purity, to confirm the presence of entanglement. Methods include designing and simulating quantum circuits using Qiskit's Python-based framework, followed by rendering results within a Streamlit web application. The key outcomes are a clear demonstration of how qubits evolve from pure basis states into complex quantum states, and an illustration of entanglement through quantifiable measures. This project is relevant to educational and research settings, serving as an accessible platform for students and practitioners to better understand fundamental quantum principles and the behavior of multi-qubit systems.

Keywords: Quantum computing; Superposition; Qubits; Qiskit; Bell States

1 Introduction

Quantum computing is an emerging computational paradigm that harnesses the principles of quantum mechanics to perform calculations more efficiently than classical counterparts in certain domains. Unlike classical bits, which can only exist in the states 0 or 1, quantum computing uses quantum bits or qubits, which leverage the phenomenon of superposition. A single qubit can be represented as a linear combination of the basis states $|0\rangle$ and $|1\rangle$ such as $|0\rangle + |1\rangle = \alpha|0\rangle + \beta|1\rangle$ where α and β are complex amplitudes, and the probability of measuring the qubit in each state depends on the magnitudes of these amplitudes.

Another key resource in quantum computing is entanglement, a uniquely quantum correlation between qubits. When two or more qubits are entangled, the state of each qubit cannot be fully described without reference to the others. Such states enable powerful computational tasks like quantum teleportation and can greatly influence the efficiency of algorithms like Shor's factoring algorithm and Grover's search.

Despite these fascinating concepts, a persistent challenge remains: understanding and visualizing complex quantum states can be difficult for beginners. Many educational materials rely heavily on mathematical formalism, making it challenging for beginners to develop an intuitive grasp of phenomena like superposition and entanglement [1]. The lack of accessible, user-friendly tools for visualizing quantum states and entanglement poses a barrier to learning and teaching quantum computing fundamentals.

This project focuses on developing an interactive visualization platform that allows users to create quantum circuits, apply gates, and immediately view the resulting qubit states. Through graphical representations such as Bloch spheres, probability distributions, and Q-spheres, learners can gain a clearer understanding of superposition and identify when multiple qubits are entangled. The primary aims of this project are to:

1. Provide a dynamic interface for constructing and viewing quantum states.
2. Demonstrate superposition and quantify entanglement in multi-qubit systems.
3. Offer educational insights and intuitive visualizations that enhance comprehension of the fundamental principles of quantum computing.

By bridging the gap between abstract theory and tangible visualization, this project aspires to make quantum computing concepts more approachable and engaging for students, educators, and enthusiasts.

2 Literature Review

Antje Kohnle, Charles Baily, and Scott Ruby explore the impact of visual representations in interactive simulations on students' understanding of quantum superposition, with a focus on single-photon experiments. Leveraging the QuVis Quantum Mechanics Visualization Project, the study redesigns visualizations used in a Mach-Zehnder interferometer setup to tackle persistent misconceptions, such as the belief that a beam splitter divides a photon into two distinct, half-energy components. Through iterative trials involving interviews and classroom evaluations, the authors developed a revised visualization labeled "Visualization III," which effectively emphasized the connections between components of a superposition state using dashed lines and avoided elements suggesting classical particle behavior. This visualization significantly improved students' conceptual grasp of superposition, as evidenced by comparative trials conducted in 2013 and 2014. Statistical analysis confirmed that the updated visualizations reduced incorrect interpretations and enhanced students' ability to form accurate mental models of quantum phenomena. The study underscores the critical role of thoughtfully designed visual tools in teaching abstract quantum concepts, demonstrating how interactive simulations can bridge the gap between mathematical descriptions and intuitive understanding, thereby fostering deeper learning in quantum mechanics[1].

Jorge Miguel-Ramiro et al. introduce Superposed Quantum Error Mitigation (SQEM), a novel set of protocols designed to enhance quantum computation fidelity by leveraging the principle of superposition. Addressing the challenge of noise and decoherence in quantum systems, SQEM enables computations to occur coherently across multiple branches in superposition, allowing errors to interfere destructively and improve output fidelity. The protocols, applicable across gate-based, measurement-based, and interferometric models, offer both probabilistic and deterministic implementations, the latter ensuring high-fidelity corrections. The authors propose a nested SQEM approach, utilizing auxiliary qubits in superposition to achieve near-perfect fidelity, and an interferometric model that minimizes auxiliary resource requirements. Analytical derivations and simulations demonstrate SQEM's robustness against noise types such as dephasing and depolarization, providing a resource-efficient alternative to traditional error correction methods. This study significantly contributes to the mitigation of quantum errors, offering practical solutions to improve near-term performance of quantum devices without extensive hardware overhead[2].

Richard Jozsa delves into the pivotal role of quantum entanglement in enabling computational advantages that are unattainable with classical systems. The paper underscores that entanglement, more than superposition alone, is the key to the exponential speed-ups observed in quantum algorithms like Shor's integer factorization and the quantum Fourier transform. By allowing quantum systems to exhibit non-classical correlations, entanglement facilitates quantum parallelism, enabling the simultaneous processing of multiple inputs in superposition. This parallelism, combined with the unique properties of entangled states, leads to computational efficiencies that classical methods cannot replicate without exponential resources. The paper also explores the practical implications of entanglement in quantum error correction and quantum teleportation, demonstrating its ability to delocalize information across subsystems, thereby protecting it from localized disturbances. Jozsa highlights the physical and theoretical implications of these phenomena, emphasizing how entanglement reshapes our understanding of computational complexity and the physical limits of computation. This study not only reinforces the foundational importance of entanglement in quantum computing but also broadens its significance in advancing information science and the future of computational paradigms[3].

Ali Javadi-Abhari et al. presents a comprehensive overview of Qiskit, a robust and versatile software development kit designed for quantum computing. The authors detail the software's modular and extensible architecture, which supports workflows for quantum circuit construction, optimization, and execution on various hardware platforms. Qiskit emphasizes scalability, enabling the efficient management of large quantum circuits and parameterized computations, while maintaining a balance between high performance and ease of use through integration with Python and selective utilization of performance-critical languages like Rust. Its transpiler is highlighted as a key component, offering retargetable and hardware-aware circuit transformations that optimize performance and reduce execution complexity. The paper showcases Qiskit's adaptability across abstraction levels, facilitating high-level algorithm design as well as low-level hardware optimization. Advanced features like dynamic circuits, classical-quantum integration, and built-in visualization tools make it suitable for a wide range of quantum computing applications, from algorithm development to experimental research. Additionally, the paper demonstrates Qiskit's capabilities through an end-to-end example of Hamiltonian simulation, illustrating the software's effectiveness in solving real-world problems. By fostering an extensive ecosystem of tools and plugins, Qiskit has become a cornerstone in quantum computing, supporting both academic research and practical applications. The authors underscore its significance as a bridge between theoretical quantum mechanics and experimental implementation, paving the way for advancements in quantum computation[4].

3 Methodology

The project uses Qiskit[5], a Python-based quantum computing framework developed by IBM, and Streamlit, an interactive web application framework, to create a dynamic platform for quantum visualization and exploration. The methodology is broken into the following steps:

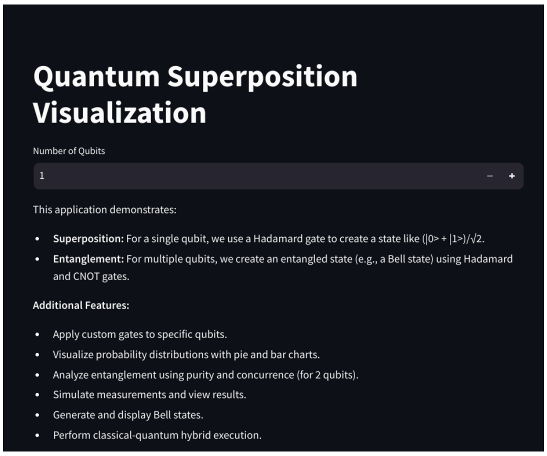

Figure 1: Users can enter the number of qubits.

1. **Quantum Circuit Initialization:** The application starts by letting users specify the number of qubits for their quantum circuit. A default circuit is constructed, applying Hadamard gates to the first qubit to create superposition and CNOT gates for entanglement between the first two qubits.

Figure 2: Users can apply custom gates - X, Y, Z, H, CNOT.

2. **Custom Gate Application:** Users can dynamically modify the circuit by selecting gates (e.g., X, Y, Z, H, CNOT) and applying them to specific qubits. This

customization provides flexibility for exploring various quantum states.

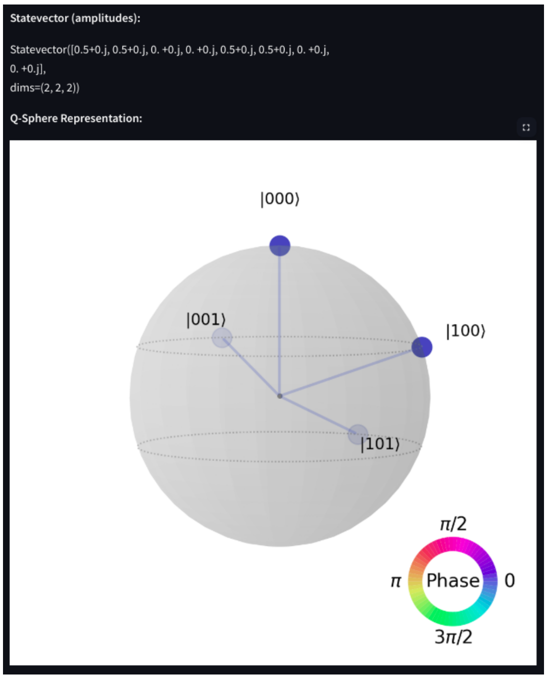

Figure 3: Q-Sphere Representation of Superposition between 3 qubits.

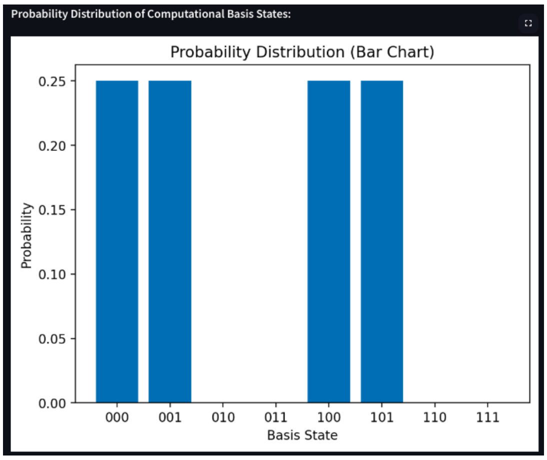

Figure 4: Probability distribution of computational states.

3. **Statevector Simulation and Visualization:** The application computes the quantum statevector for the circuit and provides visual representations:
   - *Bloch Sphere:* Visualizes single-qubit states to demonstrate superposition.
   - *Q-Sphere and Probability Charts:* For multi-qubit states, the Q-sphere shows the superposition and entanglement of all basis states, while bar and pie charts display their probability distributions.
4. **Measurement Simulation:** Users can simulate measurements of the quantum circuit with a specified number of shots (e.g., 100 to 10,000). The results are displayed as a histogram to reflect the distribution of measurement outcomes.

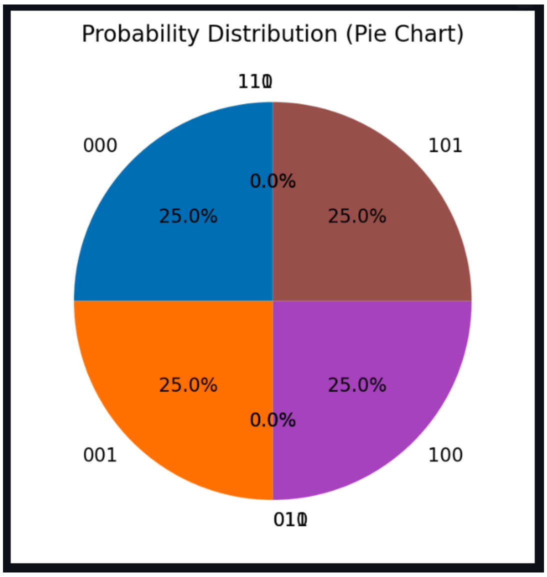

Figure 5: Pie chart depicting probability distribution of computational states.

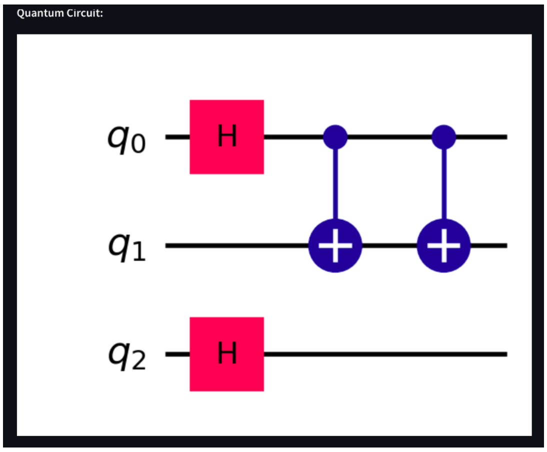

Figure 6: Custom Quantum Circuit created by applying Custom Quantum Gates.

5. **Entanglement Analysis:** The reduced density matrix of one qubit is computed for multi-qubit systems, and metrics like purity and concurrence (for two-qubit systems) are calculated to quantify entanglement.

6. **Bell State Generation:** A dedicated feature allows users to generate and visualize standard Bell states ($|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$) using specific quantum gates.

4 Results and Discussion

7. **Classical-Quantum Hybrid Execution:** Classical inputs are integrated to modify the quantum circuit dynamically, demonstrating hybrid computations. For instance, the RX gate is iteratively applied to the first qubit based on the user-defined classical parameter.

8. **User Interaction and Real-Time Updates:** Streamlit provides a real-time interface

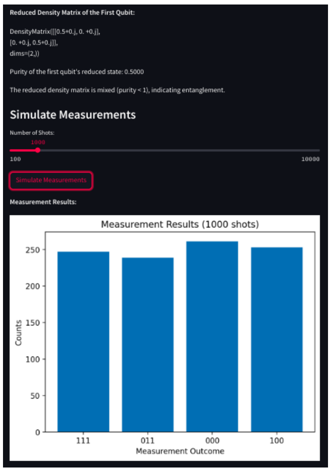

Figure 7: Measurement Simulation.

where circuit modifications, measurements, and visualizations are updated dynamically, enhancing the learning and exploration experience.

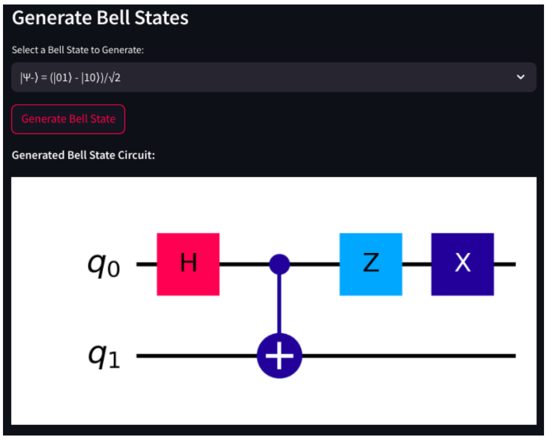

Figure 8: Bell State Visualization.

4.1 Superposition Demonstration

For single-qubit systems, the Bloch sphere visualization effectively shows the superposition state $/(|0\rangle + |1\rangle)/\sqrt{2}$ as a vector not aligned along the z-axis. This aids in understanding how quantum states differ from classical binary states.

4.2 Entanglement Verification

For multi-qubit systems, entanglement is confirmed through the reduced density matrix's purity. For example:

- A two-qubit Bell state ($|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$) shows a mixed reduced density matrix with purity less than 1.
- The concurrence metric quantifies entanglement, returning a non-zero value for entangled states.

4.3 Measurement Insights

Simulated measurements across 1,000 to 10,000 shots demonstrate the probabilistic nature of quantum mechanics. For entangled Bell states, outcomes like $|00\rangle$ and $|11\rangle$ appear with equal probabilities, matching theoretical predictions. Noise simulations highlight how decoherence affects measurement distributions, providing insights into real-world quantum system limitations.

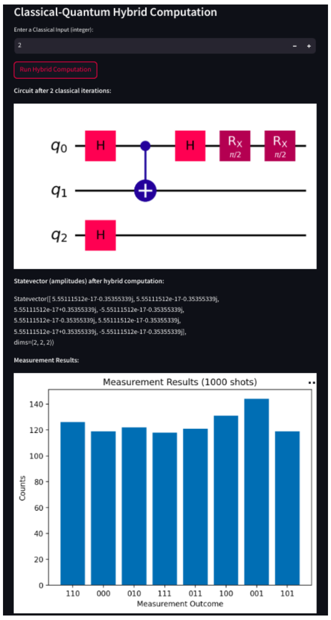

Figure 9: Classical-Quantum Hybrid Computation

4.4 Bell State Generation

The Bell state generation tool validates the application by consistently producing correct circuits and statevectors for all four standard Bell states. Visualizing the Q-sphere for these states reinforces the concept of entanglement, as the amplitudes are distributed across correlated basis states.

4.5 Classical-Quantum Hybrid Computation

Hybrid computations demonstrate the interplay between classical and quantum systems. Iterative applications of the RX gate modify the quantum state in real-time, providing a tangible example of classical parameters influencing quantum dynamics.

4.6 Educational Impact

The application bridges the gap between theoretical quantum mechanics and intuitive understanding by combining dynamic circuit manipulation, visual aids, and metrics like purity and concurrence. It serves as an effective educational tool for students and researchers exploring foundational quantum principles.

4.7 Key Findings

- The Bloch sphere and Q-sphere visualizations make abstract quantum concepts like superposition and entanglement more accessible.
- Entanglement metrics such as purity and concurrence quantitatively validate the presence of entanglement in multi-qubit states.
- Dynamic measurement simulations and probability visualizations offer hands-on insights into the probabilistic behavior of quantum mechanics.
- The integration of classical inputs in hybrid computations demonstrates practical applications in quantum-classical workflows.

4.8 Discussion

The results confirm the application's effectiveness as a learning and experimentation platform for quantum computing. It balances theoretical rigor with visual and interactive elements, fostering a deeper understanding of complex phenomena like superposition, entanglement, and hybrid computation. However, extending the app to include noise modeling and support for advanced algorithms (e.g., Grover's search or Shor's factoring) could further enhance its applicability to real-world quantum computing challenges.

5 Conclusion and Future Work

This project successfully demonstrates the principles of quantum superposition and entanglement using an interactive visualization tool. The application integrates Qiskit and Streamlit to provide a dynamic interface for building and analyzing quantum circuits. By allowing users to customize circuits, simulate measurements, and visualize quantum states through Bloch spheres, Q-spheres, and probability distributions, the project bridges the gap between theoretical quantum mechanics and practical understanding. The results highlight the educational value of the platform, enabling users to explore foundational quantum concepts interactively. Features like entanglement analysis, Bell state generation, and classical-quantum hybrid execution further enhance the tool's applicability to both academic and research contexts. Future extensions could include noise modeling and the implementation of advanced quantum algorithms, broadening the tool's utility for real-world quantum computing challenges. This project represents a step toward making quantum computing concepts more accessible, fostering a deeper understanding of the field for learners and researchers alike.

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