Combinatorics

Factorials, Permutations and Combinations

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2. Factorials
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What is combinatorics?

Definition

Combinatorics is a branch of mathematics that studies (usu- ally finite) collections of objects that satisfy specified criteria and counting of said objects.

Basic combinatorial concepts and enumerative results appeared throughout the ancient world. In the 6th century BCE, ancient Indian physician Sushruta asserts in Sushruta Samhita that 63 combinations can be made out of 6 different tastes, taken one at a time, two at a time, etc., thus computing all $2^6 - 1$ possibilities. This is the first known story of mathematicians coliding with combinatorics but the study of combinatorics continued through ancient times, through middle ages and is still ongoing today.

Basic objects of study of combinatorics

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Factorials

A factorial of an integer n is the product of all the integers less than or equal to n.

Example:

$5! = 5 * 4 * 3 * 2 * 1 = 120$

The factorial operation is encountered in many areas of mathematics, notably in combinatorics, algebra, and mathematical analysis. Its most basic use counts the possible distinct sequences the permutations of n distinct objects: there are n!.

$0!$

Value of zero factorial is equal to one, or in symbols:

$$0! = 1$$

This is because 0! is a product of no elements and is by convention equal to multiplicative identity which is in case of integers, 1.

Applications of factorials

Factorials can be useful for:

- Calculating the number of permutations
- Calculating the number of combinations
- Algebra
- Calculus
- Much more

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Binomal theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$
$$= \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \cdots + \binom{n}{n-1} ab^{n-1} + \binom{n}{n} b \quad (1)$$

Proof.

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One can establish a bijection between the products of a binomial raised to n and the combinations of n objects. Each product which results in $a^{n-k}b^k$ corresponds to a combination of k objects out of n objects. Thus, each $a^{n-k}b^k$ term in the polynomial expansion is derived from the sum of $\binom{n}{k}$ products. □

Permutations

Permutations are the number of ways we can rearange the order of elements of a set in order to get a new set.

A 1	A 1	A 2	A 2	A 3	A 3
B 2	B 3	B 1	B 3	B 2	B 1
C 3	C 2	C 3	C 1	C 1	C 2

Permutations of set $\{A, B, C\}$

Permutations w/ repetition vs permutations w/o repetition

A permutation of a set of objects is an ordering of those objects. When some of those objects are identical, the situation is transformed into a problem about permutations with repetition.

For instance, it may be desirable to find orderings of boys and girls, students of different grades, or cars of certain colors, without a need to distinguish between students of the same grade. In such a case, the repeated objects are those that do not need to be distinguished.

The permutations without repetition of elements are the different groups of elements that can be done, so that two groups differ from each other only in the order the elements are placed.

Calculating the number of permutations

The following formula is used for calculating the number of permutations of set with n elements.

$$P_n = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1 = n!$$

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