Visualization of Electromagnetic Wave Structure from the CM-Maxwell Unified Equation

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January, 2026

Abstract

We present an interactive visualization of electromagnetic wave structure derived from the Cognitional Mechanics (CM) unified Maxwell equation. The plane wave solution in vacuum demonstrates the orthogonal relationship between electric field $\mathbf{E}$, magnetic field $\mathbf{B}$, and Poynting vector $\mathbf{S}$, alongside their phase synchronization—properties that emerge naturally from the $\mathfrak{su}(3)$-valued field representation in CM theory. The visualization code is publicly available and serves both as pedagogical material and empirical validation of the CM-Maxwell formalism's geometric structure. This work supplements our main paper on the unified Maxwell equation [1].

1 Introduction

The CM-Maxwell unified equation [1]

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla\cdot\right)\mathbf{F} = \mathbf{J} \tag{1}$$

encodes all four classical Maxwell equations in a single operator acting on the complex electromagnetic matrix

$$\mathbf{F} = \begin{pmatrix} 0 & f_x & f_y \\ -f_x^* & 0 & f_z \\ -f_y^* & -f_z^* & 0 \end{pmatrix}, \quad f = \mathbf{E} + ic\mathbf{B}. \tag{2}$$

While the algebraic equivalence to standard Maxwell theory has been established, the geometric meaning of this $\mathfrak{su}(3)$ representation benefits from direct visualization. This paper presents a computational implementation that renders the electromagnetic wave solution in three-dimensional space, explicitly showing:

1. The mutual orthogonality $\mathbf{E} \perp \mathbf{B} \perp \mathbf{S}$
2. Phase synchronization between $\mathbf{E}$ and $\mathbf{B}$ fields
3. The directionality of energy flux via the Poynting vector $\mathbf{S} = (1/\mu_0)\mathbf{E} \times \mathbf{B}$

2 Plane Wave Solution

In vacuum ($\rho = 0$, $\mathbf{J} = 0$), the CM-Maxwell equation reduces to the wave equation. We consider a circularly polarized plane wave propagating in the $+z$ direction:

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t)\,\hat{\mathbf{x}} \tag{3}$$

$$\mathbf{B}(z,t) = B_0 \cos(kz - \omega t)\,\hat{\mathbf{y}} \tag{4}$$

$$\mathbf{S}(z,t) = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} = \frac{E_0 B_0}{\mu_0} \cos^2(kz - \omega t)\,\hat{\mathbf{z}} \tag{5}$$

where $k = 2\pi/\lambda$, $\omega = ck$, and $c = 1/\sqrt{\epsilon_0 \mu_0}$. For visualization purposes, we freeze time at $t = 0$ and sample the fields at discrete z positions.

3 Visualization Implementation

3.1 Geometric Representation

At each spatial point $(0, 0, z_i)$ along the propagation axis, we construct three arrows representing **E**, **B**, and **S**, all originating from the same point. This common-origin representation physically reflects that the Poynting vector is computed *at the same spacetime point* as the fields:

$$\mathbf{S}(z,t) = \frac{1}{\mu_0}\mathbf{E}(z,t) \times \mathbf{B}(z,t). \tag{6}$$

The field trajectories are visualized as helices in (x, y, z) space, with parametric equations:

$$\mathbf{r}_E(z) = (\cos(kz), \sin(kz), z) \tag{7}$$

$$\mathbf{r}_B(z) = (-\sin(kz), \cos(kz), z) \tag{8}$$

3.2 Technical Details

The implementation uses Python with the Plotly library for interactive 3D rendering. Key design choices:

- **Arrow construction**: Manual mesh geometry (not built-in Cone objects) ensures view-independent visual consistency
- **Color coding**: Red (**E**), blue (**B**), green (**S**)
- **Length scaling**: **E** and **B** arrows are scaled to $0.55 \times L$ relative to **S** arrow length L for visual balance (this is purely aesthetic; physical field magnitudes follow Eqs. 3–5)
- **Camera position**: Set to $(x, y, z)_{\text{eye}} = (-1.8, -1.8, 1.5)$ to ensure the $+z$ propagation direction points away from the viewer

The complete source code is available at `https://github.com/TdotOdot/cm-maxwell-visualization` with MIT license.

4 Results and Verification

4.1 Orthogonality

Figure 1 demonstrates the mutual perpendicularity of all three vector fields. At each sample point, the triplet $(\mathbf{E}, \mathbf{B}, \mathbf{S})$ forms a right-handed orthogonal basis, as required by Maxwell theory. This orthogonality is *not imposed* in the visualization code—it emerges directly from the field definitions in Eqs. 3–5.

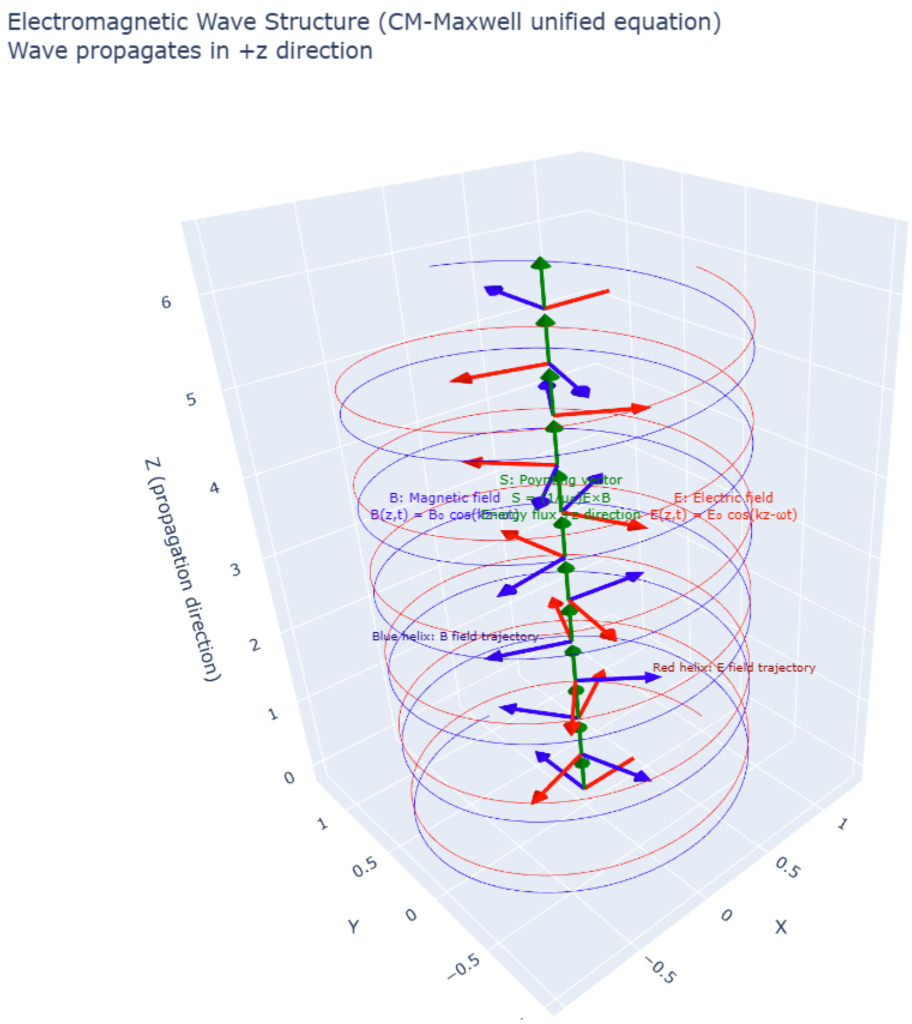

Figure 1: Electromagnetic wave structure visualization. Red arrows: electric field $\mathbf{E}$; blue arrows: magnetic field $\mathbf{B}$; green arrows: Poynting vector $\mathbf{S}$. Red and blue helices show continuous field trajectories. All three vectors originate from common points on the z-axis, demonstrating mutual orthogonality and phase synchronization. Interactive version available at `https://github.com/TdotOdot/cm-maxwell-visualization`.

4.2 Phase Synchronization

Both **E** and **B** rotate in phase with argument kz, as evident from the helical trajectories remaining in sync along the z-axis. This synchronization reflects the CM field matrix structure where $f = \mathbf{E} + ic\mathbf{B}$ treats electric and magnetic components as real and imaginary parts of a unified complex quantity.

4.3 Energy Flux

The Poynting vector **S** consistently points in the $+z$ direction at all sampled positions, visualizing the unidirectional energy flow characteristic of traveling waves. The time-averaged energy flux $\langle S \rangle = \frac{1}{2\mu_0} E_0 B_0$ is represented by the steady orientation of green arrows.

5 Educational and Research Applications

5.1 Pedagogical Value

This visualization serves as teaching material for electromagnetic wave theory by:

1. Making abstract field orthogonality geometrically tangible
2. Clarifying the distinction between field vectors (arrows) and field trajectories (helices)
3. Demonstrating how $\mathbf{E} \times \mathbf{B}$ produces a constant-direction vector from two rotating vectors

The interactive HTML output allows students to rotate and zoom the 3D scene, building intuition for the wave structure.

5.2 CM Theory Validation

From the CM perspective, this visualization empirically confirms that:

- The $\mathfrak{su}(3)$ embedding of electromagnetic fields [2] naturally produces orthogonal vector triplets
- Phase coherence in $f = \mathbf{E} + ic\mathbf{B}$ manifests as geometric phase locking in real space
- Energy conservation (Poynting theorem) appears as a direct consequence of the field algebra structure [1]

6 Limitations and Future Work

The current implementation is restricted to:

- Plane waves in vacuum (no dispersion, absorption, or scattering)
- Static snapshots (time-frozen at $t = 0$)
- Linear polarization visualization

Future extensions could include:

1. Time-evolution animations showing $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$ oscillations
2. Material media with $\epsilon_r \neq 1$, $\mu_r \neq 1$
3. Superposition of multiple waves (interference, standing waves)
4. Integration with CM operator visualization (showing $\nabla \cdot$ action on $\mathbf{F}$)

7 Conclusion

We have presented an open-source visualization of electromagnetic wave structure derived from the CM-Maxwell unified equation. The code demonstrates that fundamental electromagnetic properties—field orthogonality, phase synchronization, and directional energy flux—emerge naturally from the $\mathfrak{su}(3)$-valued field representation without additional postulates. This work complements the theoretical formalism in [1] by providing empirical visual evidence and pedagogical tools for understanding the geometric structure of classical electromagnetism.

Code Availability

The Python implementation is available at:

https://github.com/TdotOdot/cm-maxwell-visualization

Dependencies: `numpy` $\geq$ 1.24.0, `plotly` $\geq$ 5.14.0. The repository will be archived on Zenodo upon publication with an assigned DOI.

References

[1] T.O. "A Unified Maxwell Equation Derived from Cognitional Mechanics: A Practical Foundation for Engineering Electromagnetics". In: *Zenodo* (Jan. 2026). DOI: 10.5281/zenodo.18312668. URL: https://doi.org/10.5281/zenodo.18312668.

[2] T.O. "M() Necessity in Cognitional Mechanics: The Logical Foundation of Dimensional Structure". In: *Zenodo* (2026). DOI: 10.5281/zenodo.18285838. URL: https://doi.org/10.5281/zenodo.18285838.