

**COT 5600 Quantum Computing
Spring 2019**

Homework 3

Out: Wed 04/08

Due: Wed 04/19

Problem 1 (Quantum Fourier transform)

Let $N = 2^n$, $[N] = \{0, \dots, N-1\}$, and $\omega = e^{2\pi i/N}$ be an N th root of unity. The Quantum Fourier transform F_N of size N is

$$F_N = \frac{1}{\sqrt{N}} \sum_{k, \ell \in [N]} \omega^{k \cdot \ell} |k\rangle \langle \ell|.$$

Show that F_N is unitary.

Problem 2 (Quantum Phase estimation)

Let $\varphi \in [0, 1)$ be arbitrary and

$$|\varphi\rangle = \bigotimes_{k=n-1, \dots, 0} \frac{1}{\sqrt{2}} (|0\rangle + \exp(2\pi i 2^k \varphi) |1\rangle) .$$

Create a Python notebook that lets you compute and plot the probabilities for measuring $x \in \{0, 1\}^n$ when the state is

$$F_N^\dagger |\varphi\rangle$$

for different N and φ . The plot should look similar to the plots on the slides depicting the different probability distributions. Do not forget about the bit-reversal that we talked about in class.