

**COT 5600 Quantum Computing**  
**Spring 2019**  
**Homework 2**

**Problem 1** (Hadamard test)

Implement in Python a simulation of the Hadamard test for arbitrary unitaries  $U \in \mathbb{C}^{2 \times 2}$  and arbitrary quantum tests  $|\psi\rangle \in \mathbb{C}^2$ . (Do not just code up the formula from slides. The application of each quantum gate has to be simulated.)

Determine the probabilities for  $U = X$  (bit flip) and  $|+\rangle$  and  $|-\rangle$  using your code. Also, plot the probabilities for  $U = \text{diag}(1, e^{2\pi i \varphi})$  for  $\varphi = [0, 1)$  and  $|\psi\rangle = |1\rangle$ .

**Problem 2** (SWAP test)

Implement in Python a simulation of the SWAP test for arbitrary  $|\psi_1\rangle, |\psi_2\rangle \in \mathbb{C}^2$ .

Determine the probabilities for  $|\psi_1\rangle = |\psi_2\rangle = |0\rangle$  and  $|\psi_1\rangle = |0\rangle$  and  $|\psi_2\rangle = |1\rangle$ .

Plot the probabilities for  $|\psi_1\rangle = |0\rangle$  and  $|\psi_2\rangle = \sin(2\pi\theta)|0\rangle + e^{2\pi i\varphi}\cos(2\pi\theta)|1\rangle$  for different  $\theta, \varphi \in [0, 1)$ .