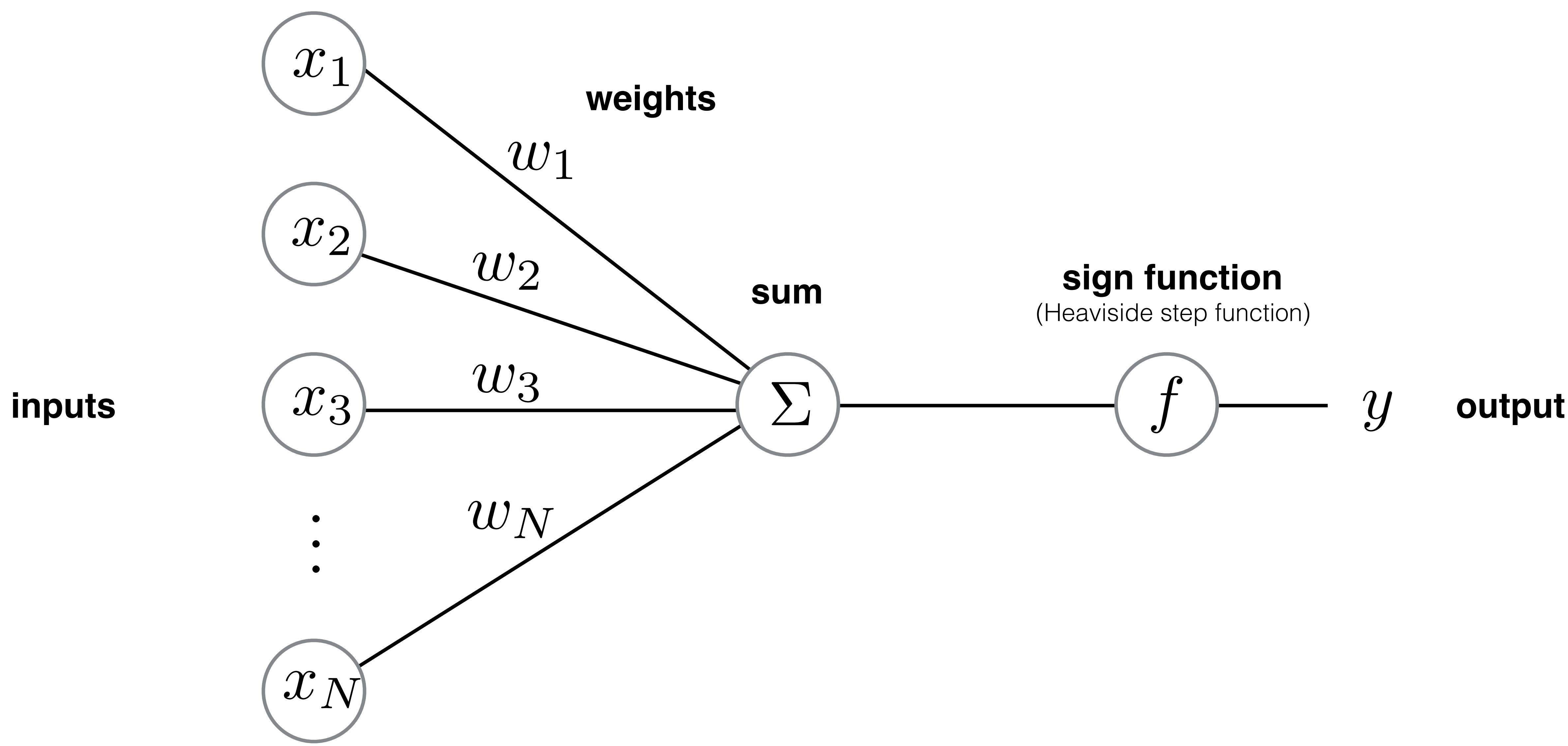
The Perceptron



How do you learn the parameters of a perceptron?

Let's skip the theory for now and just see the algorithm…

function PERCEPTRON ALGORITHM

$$\boldsymbol{w}^{(0)} \leftarrow \mathbf{0}$$

for $t = 1, \ldots, T$ **do**

$$\textsc{Receive}(\boldsymbol{x}^{(t)}) \qquad \boldsymbol{x} \in \{0, 1\}^N$$

N-d binary vector

$$\hat{y}_A^{(t)} = \mathbf{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle \right)$$

sign of zero is +1

Classification result

$$\textsc{Receive}(y^t) \qquad y \in \{1, -1\}$$

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$

Update the parameters

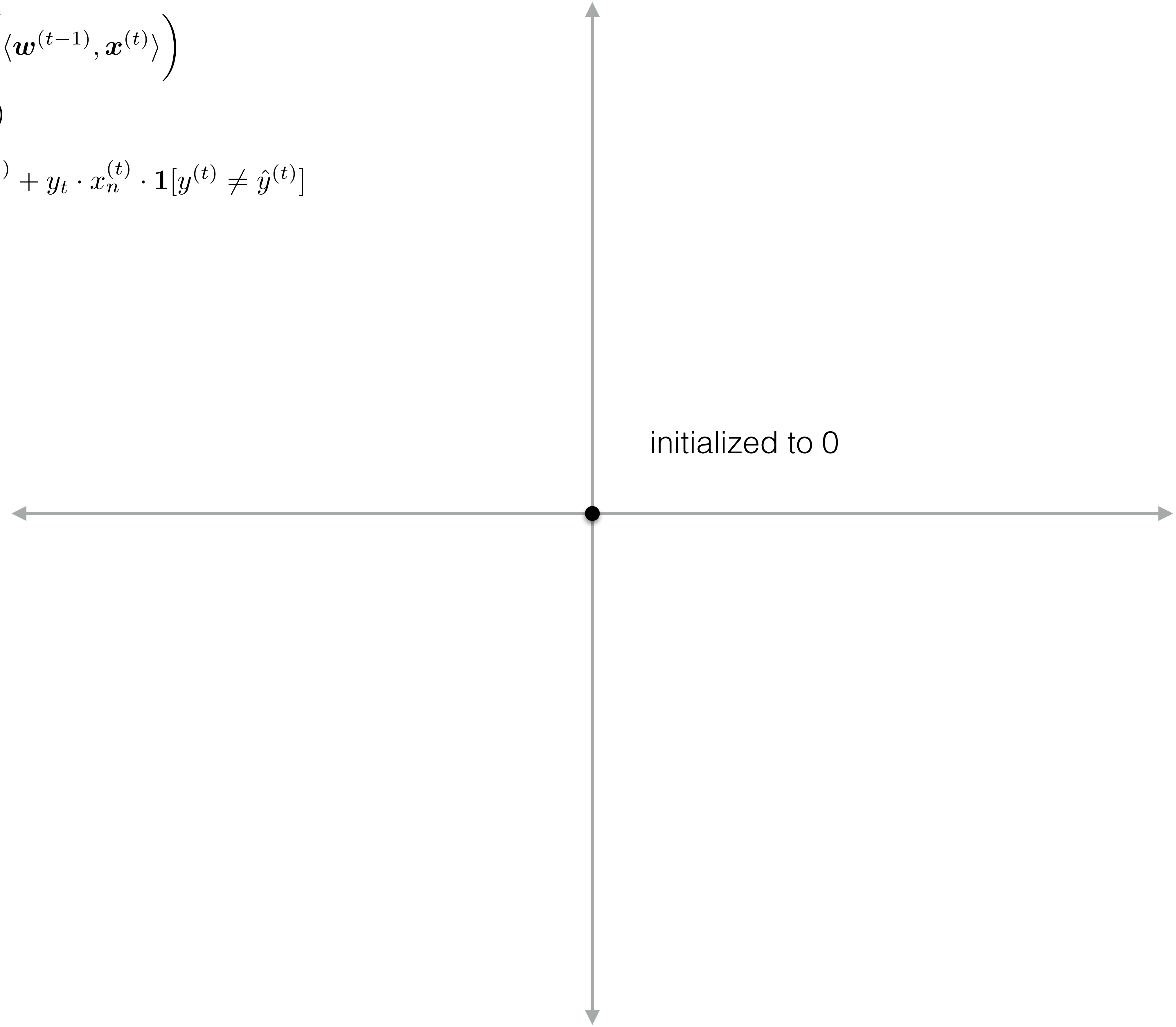
What does this look like visually…

$\textsc{Receive}(\boldsymbol{x}^{(t)})$

$\hat{y}_A^{(t)} = \operatorname{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\right)$

$\textsc{Receive}(y^t)$

$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$



$\boxed{\textsc{Receive}(\boldsymbol{x}^{(t)})}$

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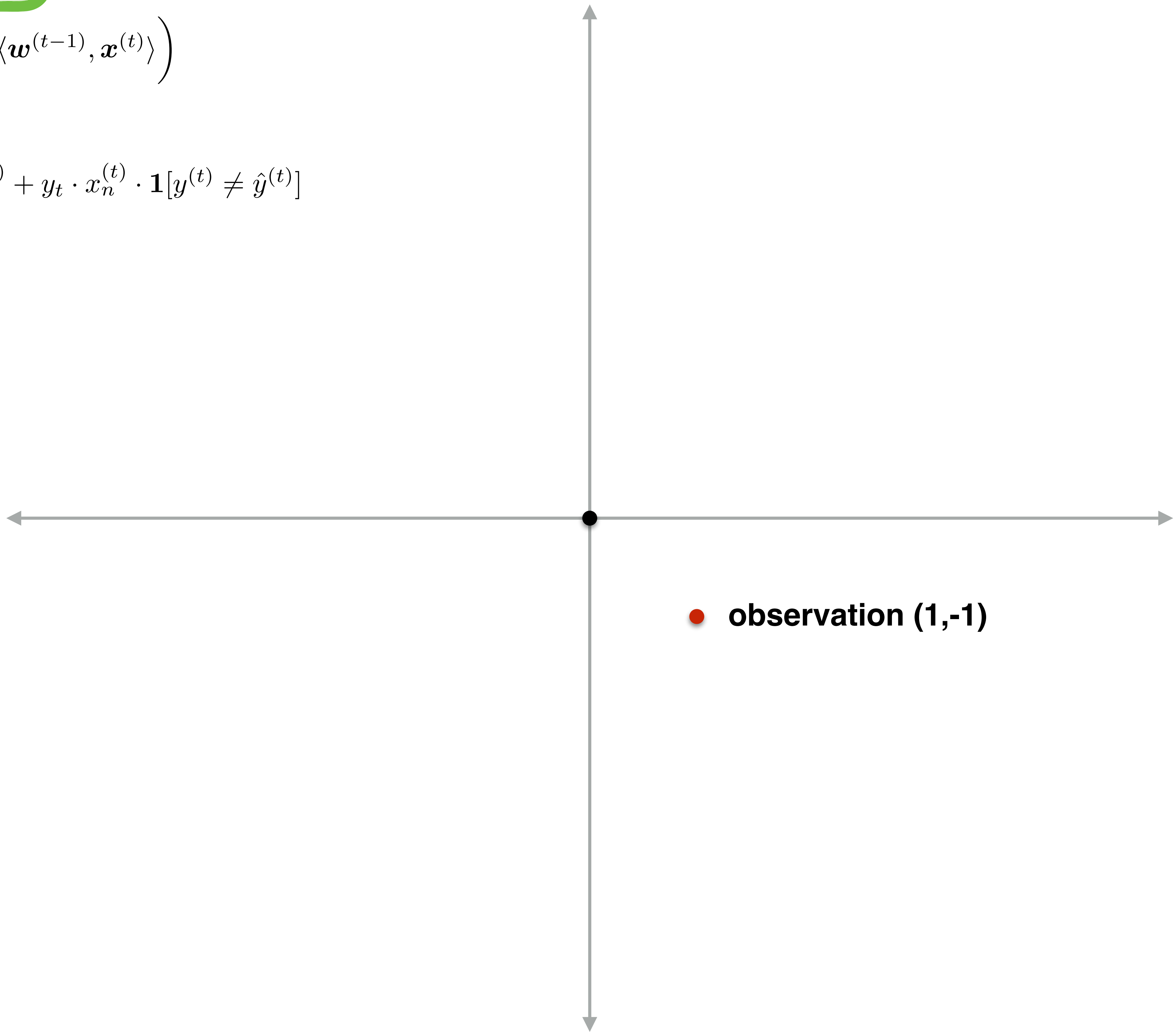
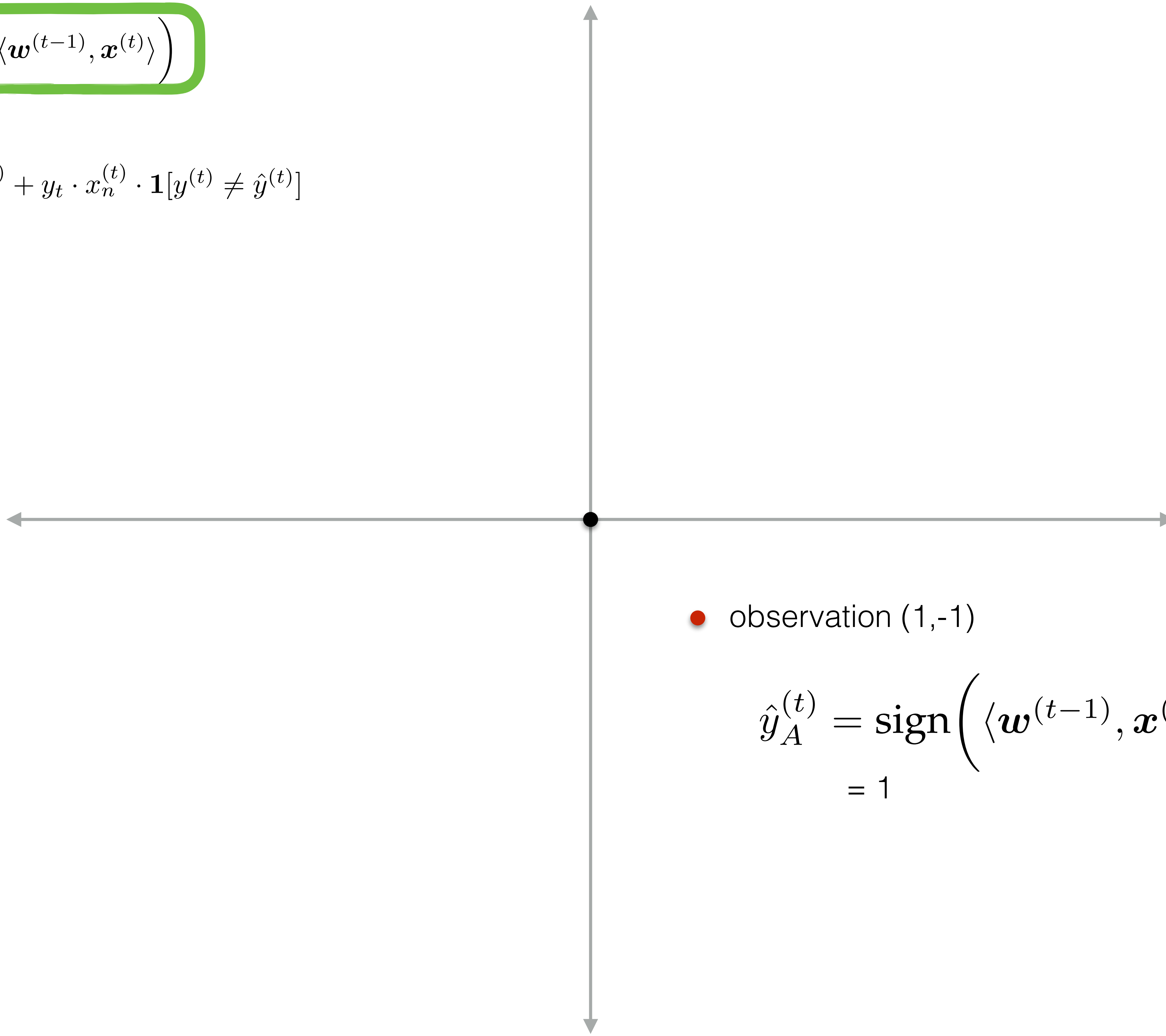


$\textsc{Receive}(\boldsymbol{x}^{(t)})$

$$\boxed{\hat{y}_A^{(t)} = \operatorname{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\right)}$$

$\textsc{Receive}(y^t)$

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$



$$\hat{y}_A^{(t)} = \operatorname{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\right)$$

= 1

$\textsc{Receive}(\boldsymbol{x}^{(t)})$

$$\hat{y}_A^{(t)} = \text{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\right)$$

$\textsc{Receive}(y^t)$

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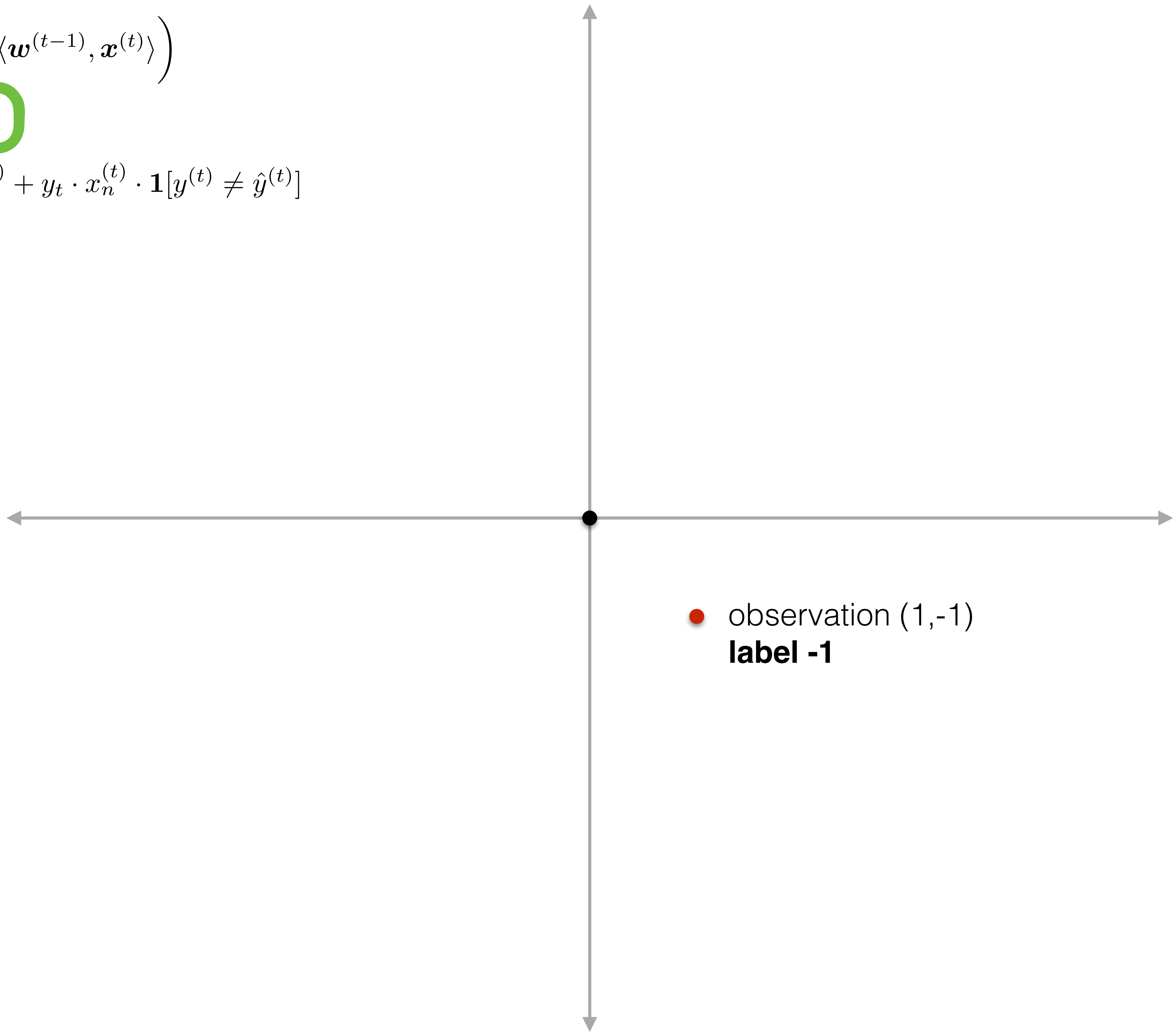

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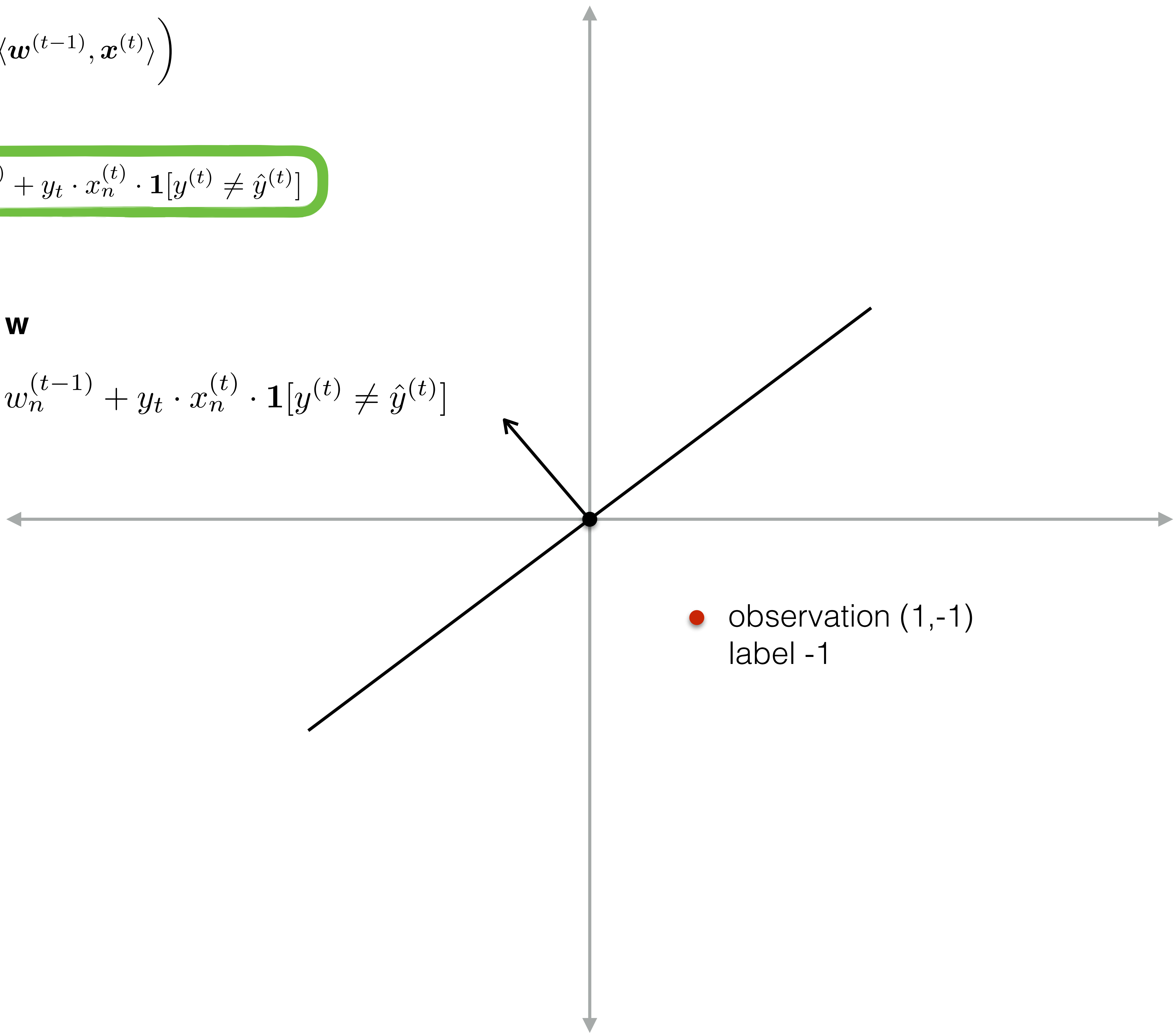
update w

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$



$\textsc{Receive}(\boldsymbol{x}^{(t)})$

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$\textsc{Receive}(y^t)$

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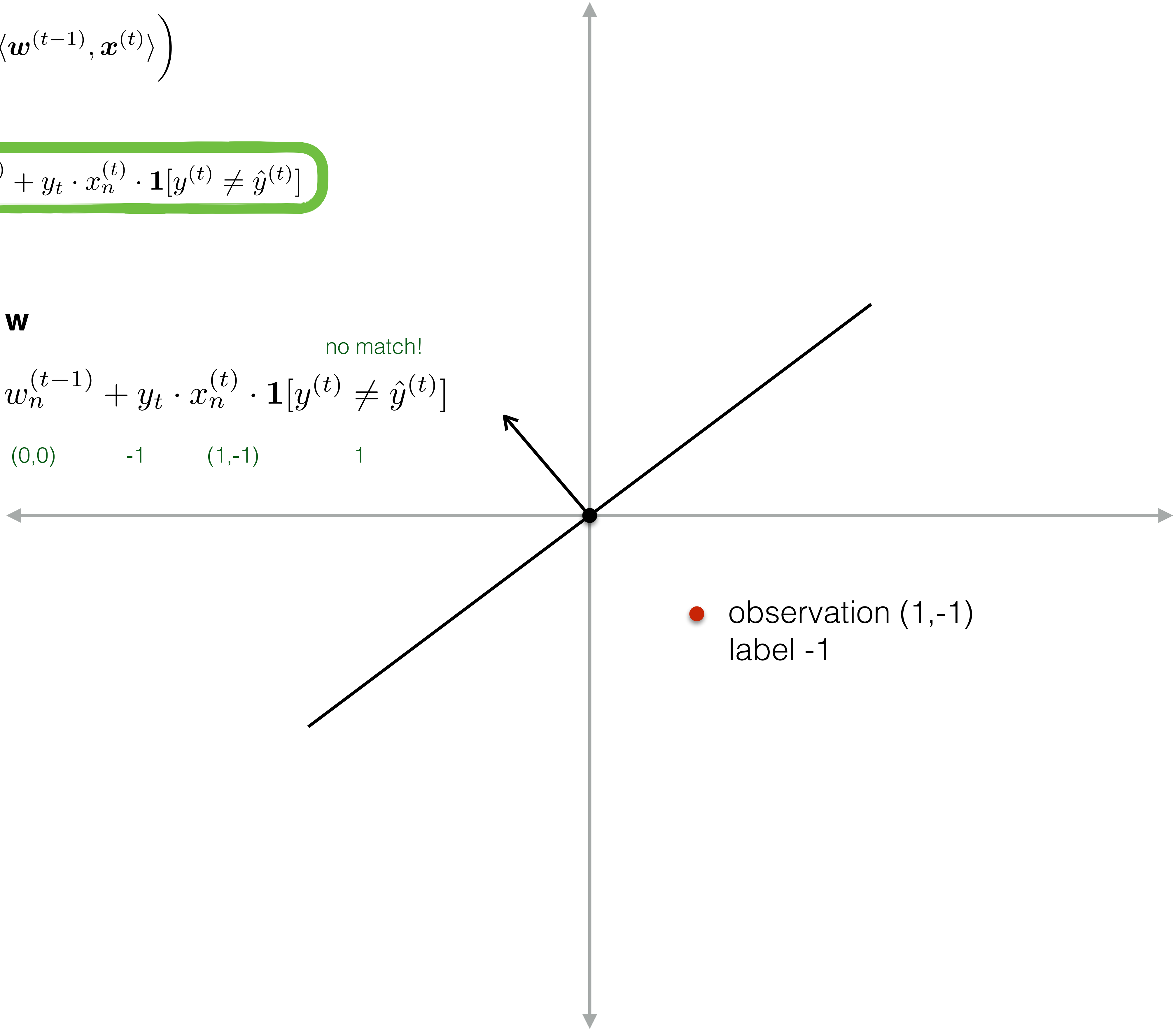
update w

no match!

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$

(-1,1) (0,0) -1 (1,-1) 1



$\textsc{Receive}(\boldsymbol{x}^{(t)})$

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$\textsc{Receive}(y^t)$

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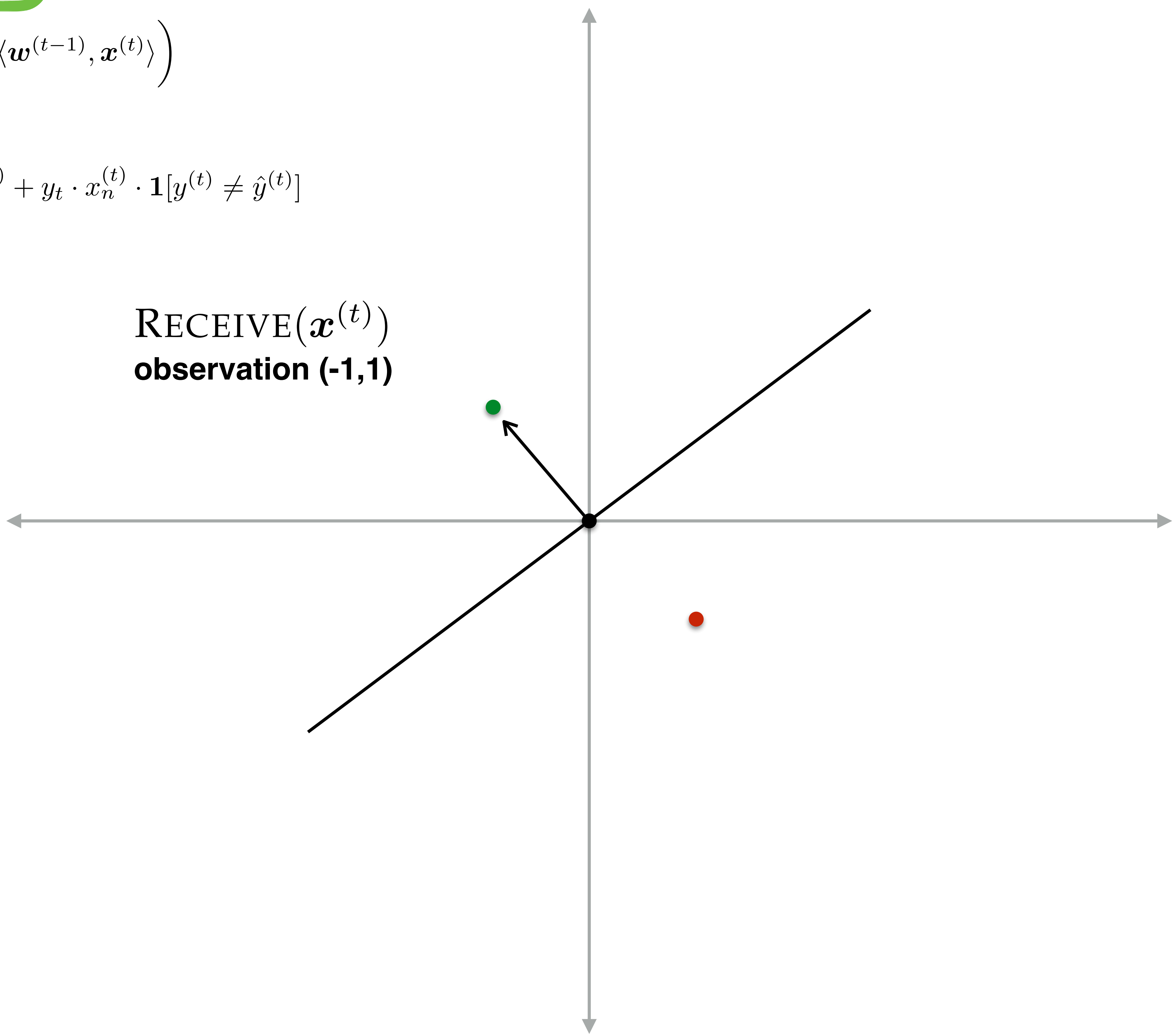
(-1,1)



Receive($\boldsymbol{x}^{(t)}$)

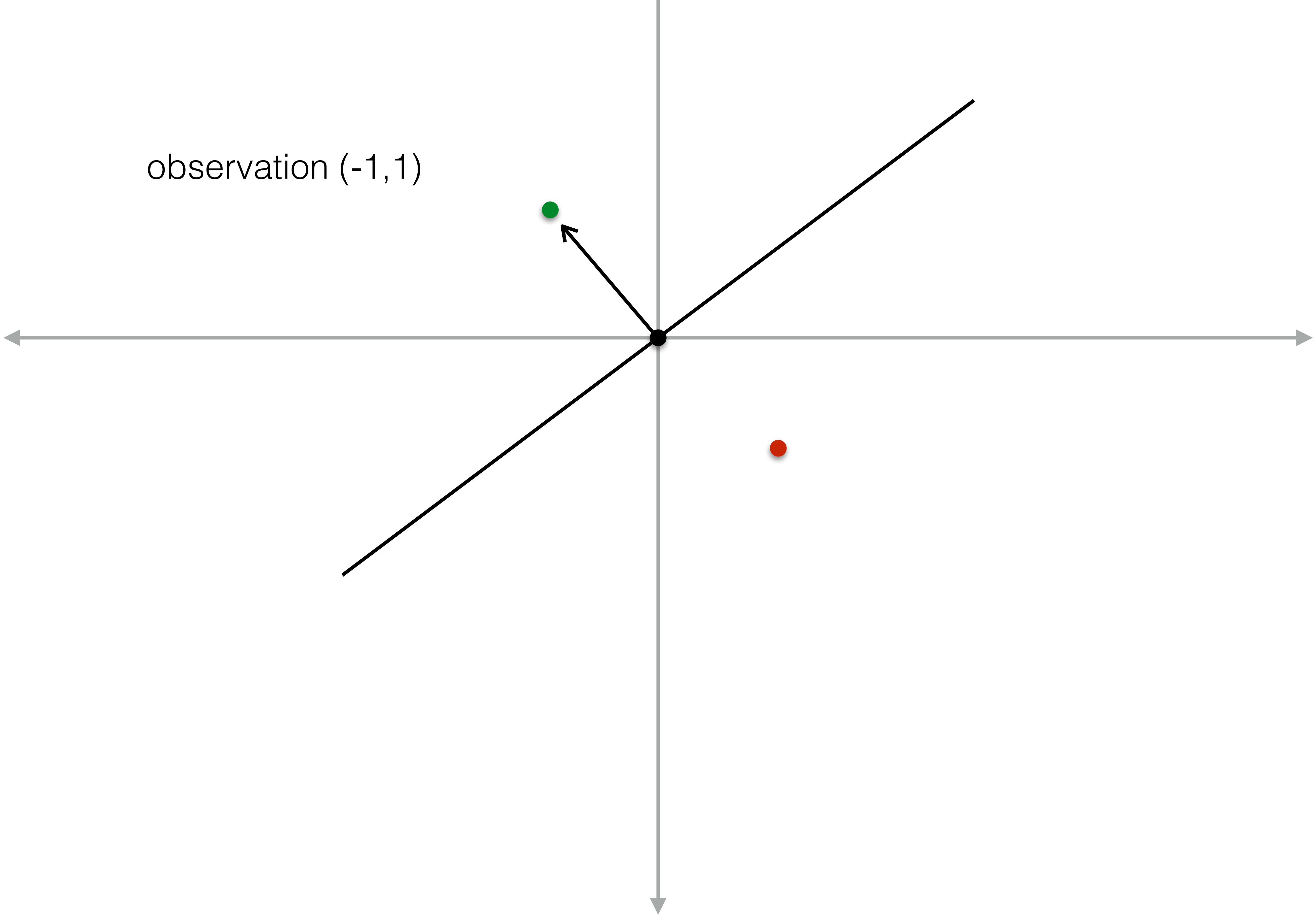
$$\boxed{\hat{y}_A^{(t)} = \text{sign}\Big(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle\Big)}$$

Receive(y^t)

$$w_n^{(t)} = \underset{(-1,1)}{w_n^{(t-1)}} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$

$$\hat{y}_A^{(t)} = \text{sign}\Big(\langle \underset{(-1,1)}{\boldsymbol{w}^{(t-1)}}, \underset{(-1,1)}{\boldsymbol{x}^{(t)}} \rangle\Big)$$

$= 1$



$\textsc{Receive}(\boldsymbol{x}^{(t)})$

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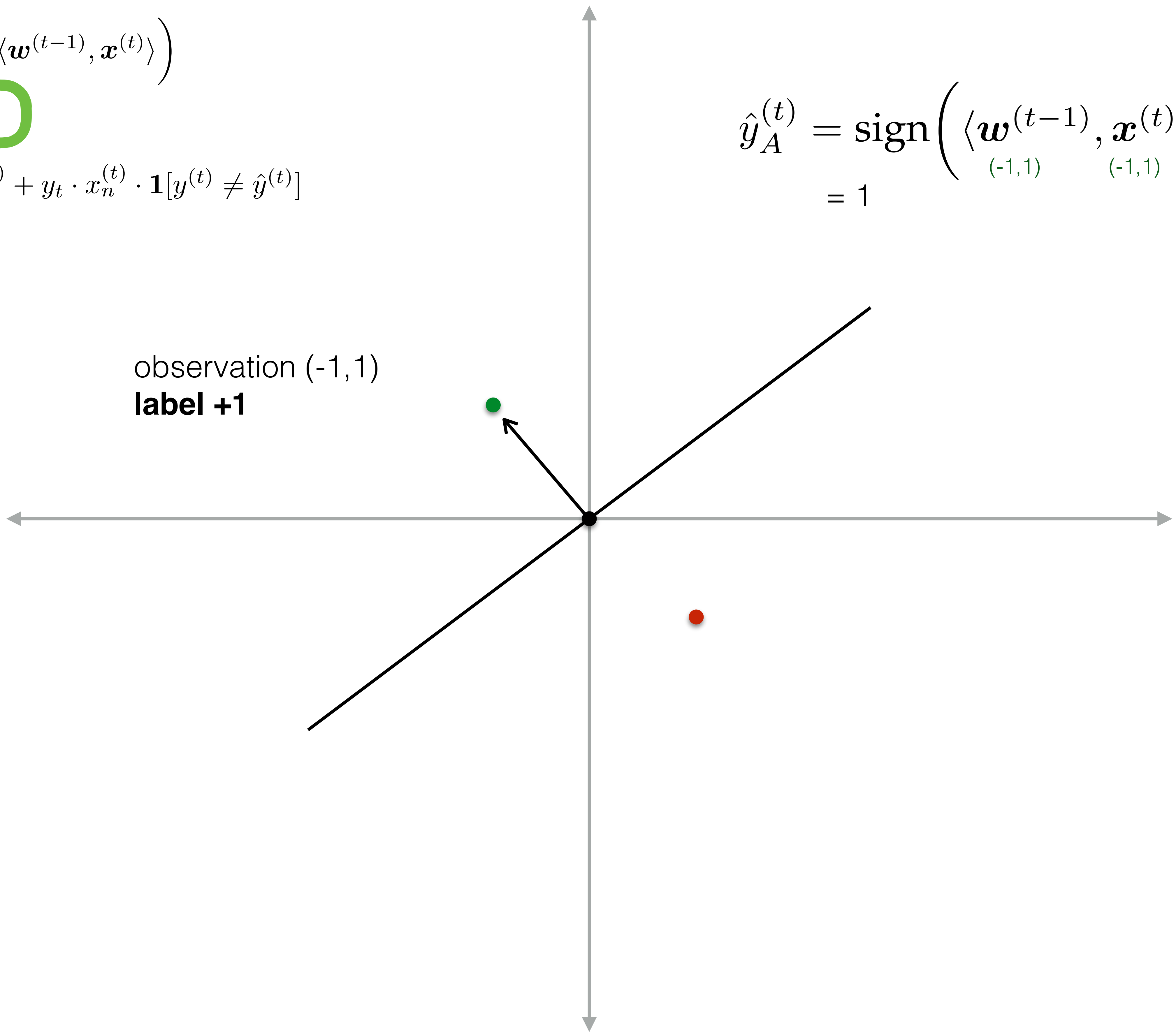

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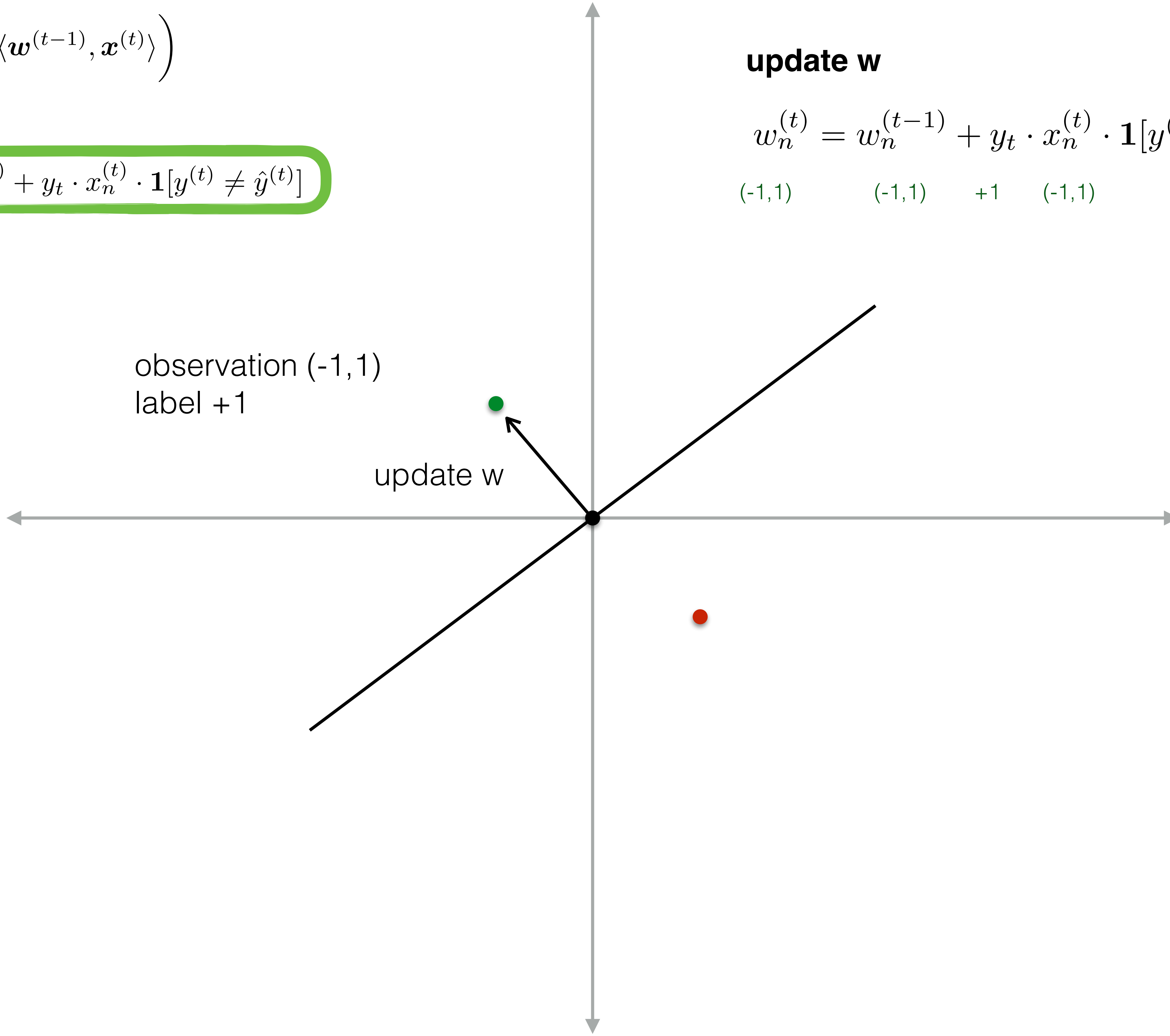
update w

match!

$$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$$

(-1,1) (-1,1) +1 (-1,1) 0



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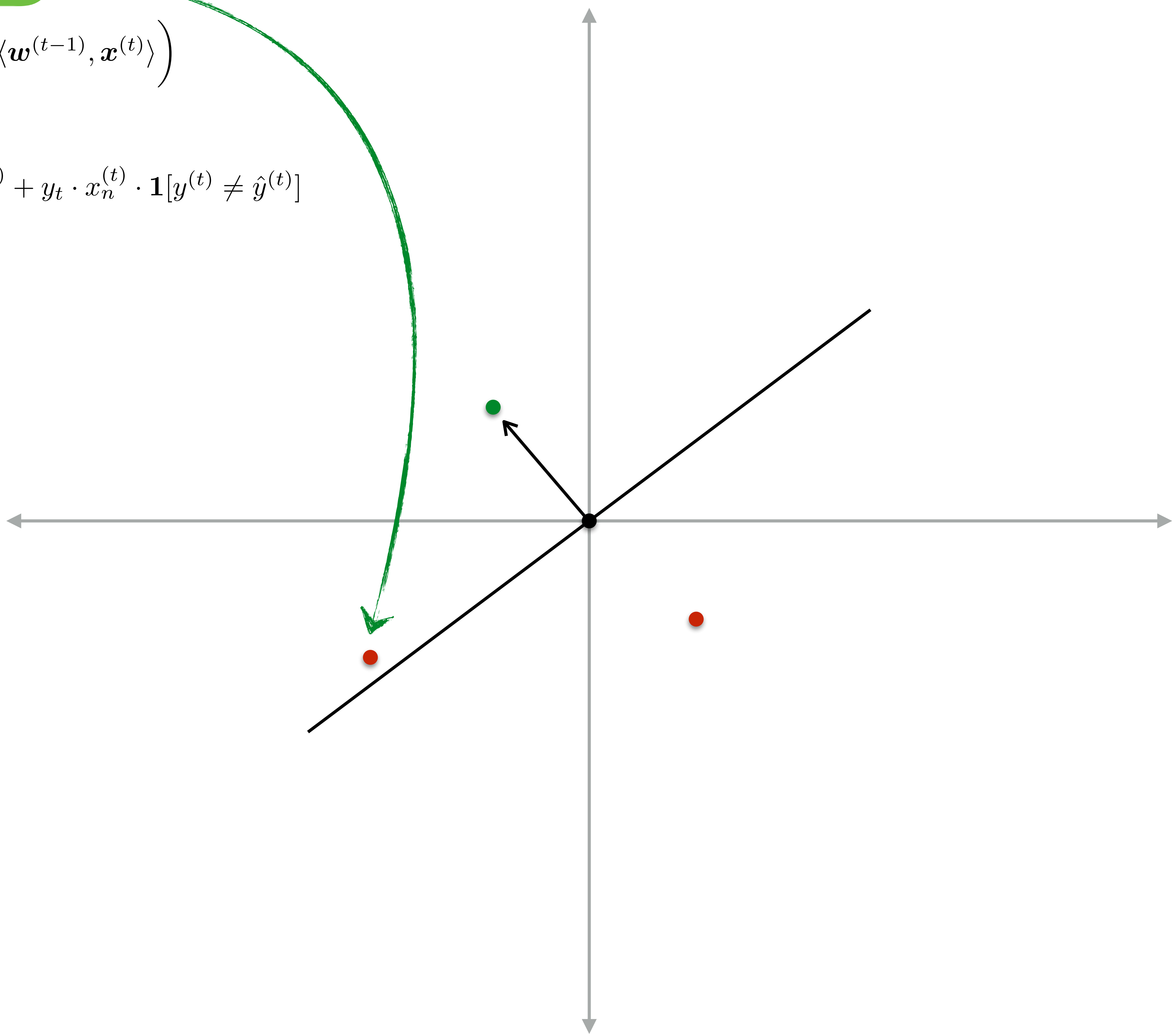

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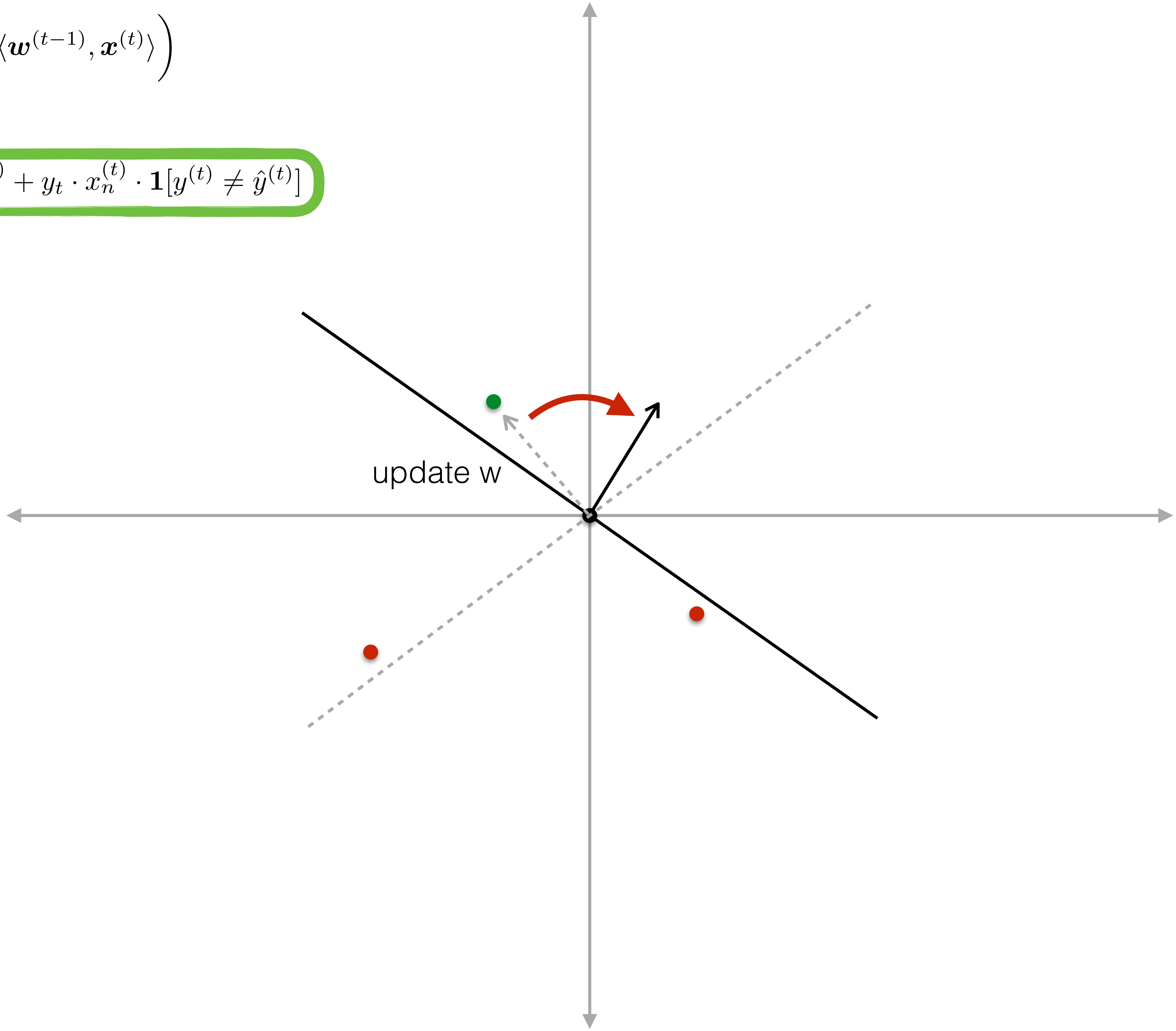

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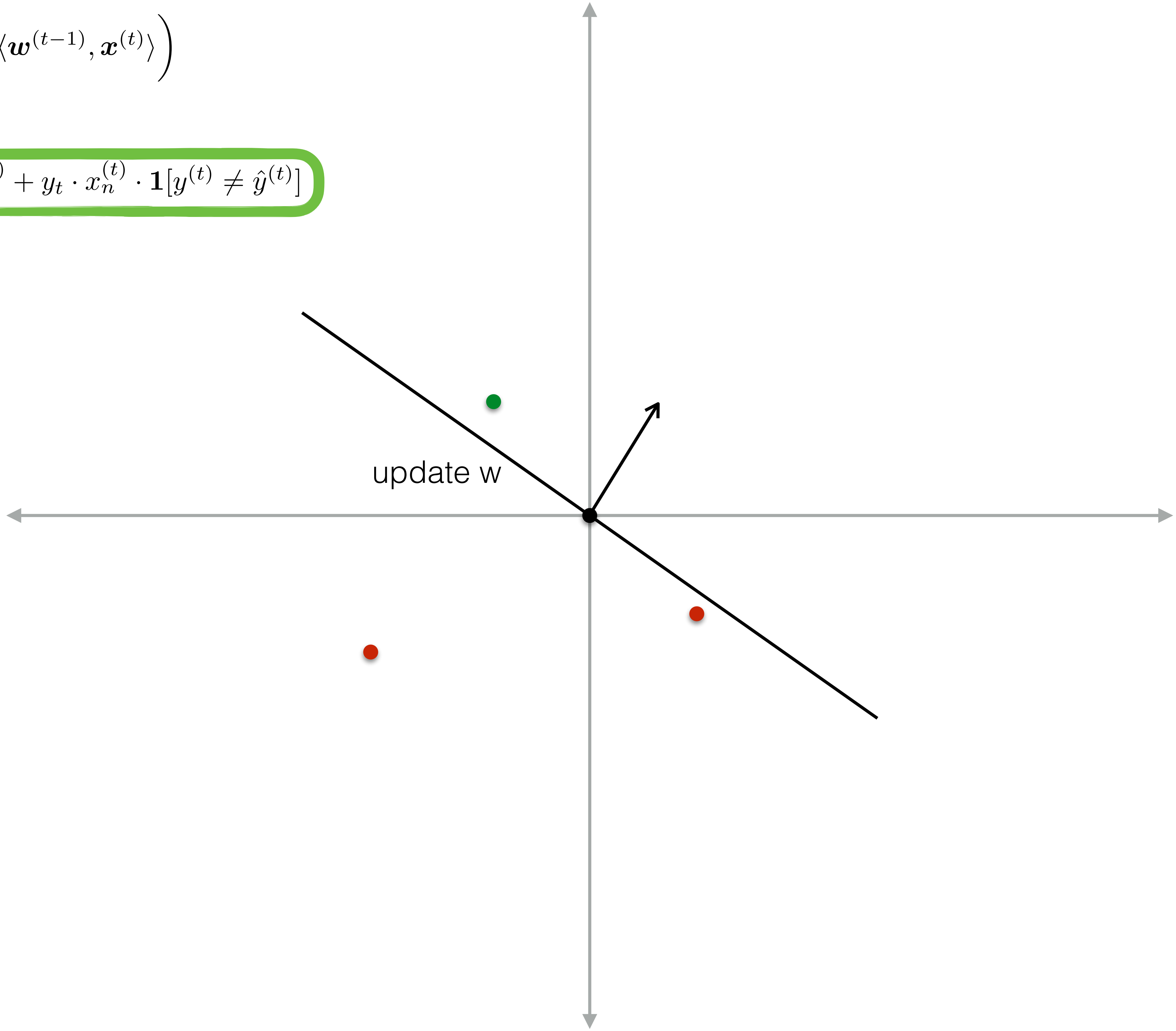

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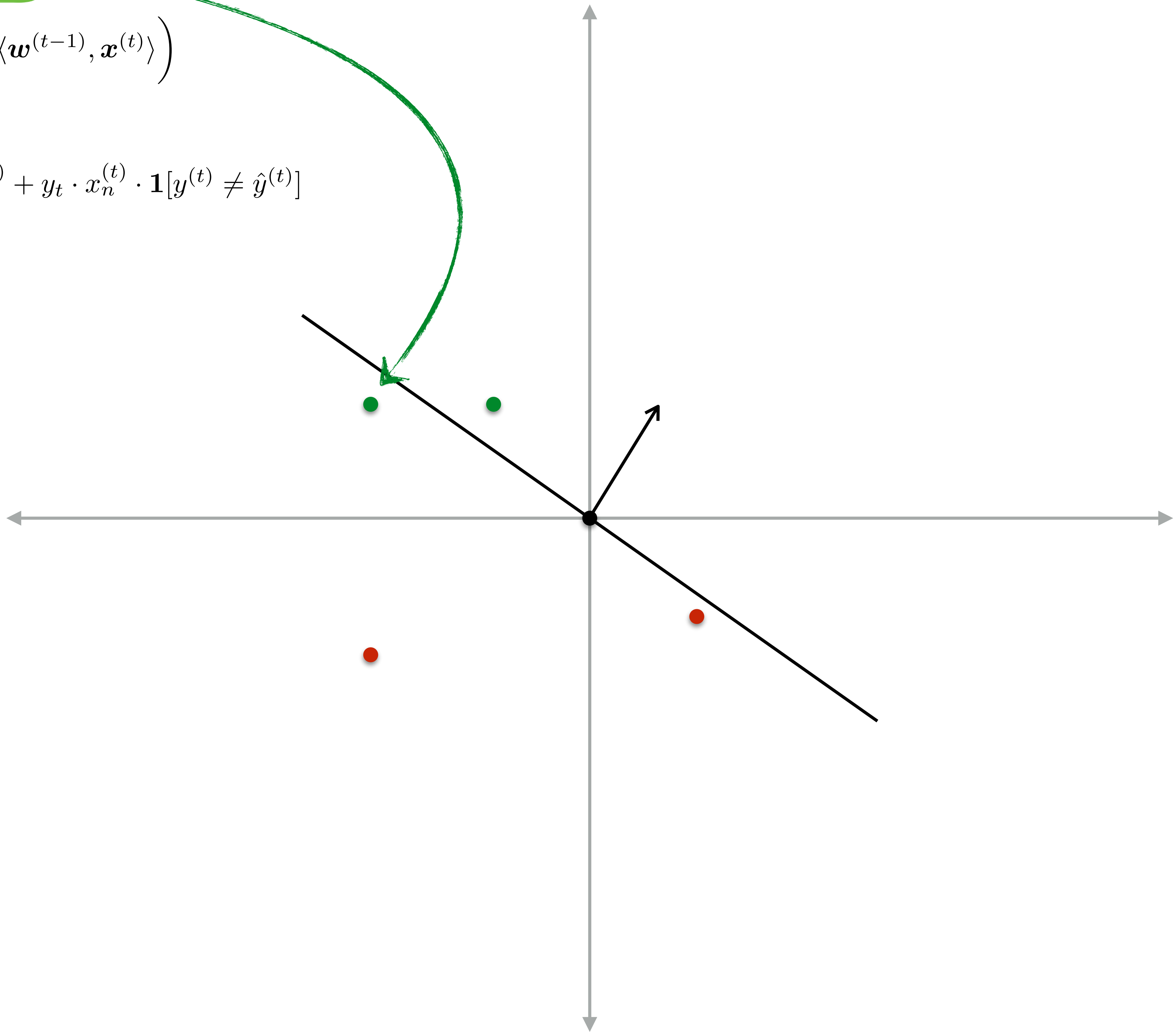

$\text{RECEIVE}(\boldsymbol{x}^{(t)})$

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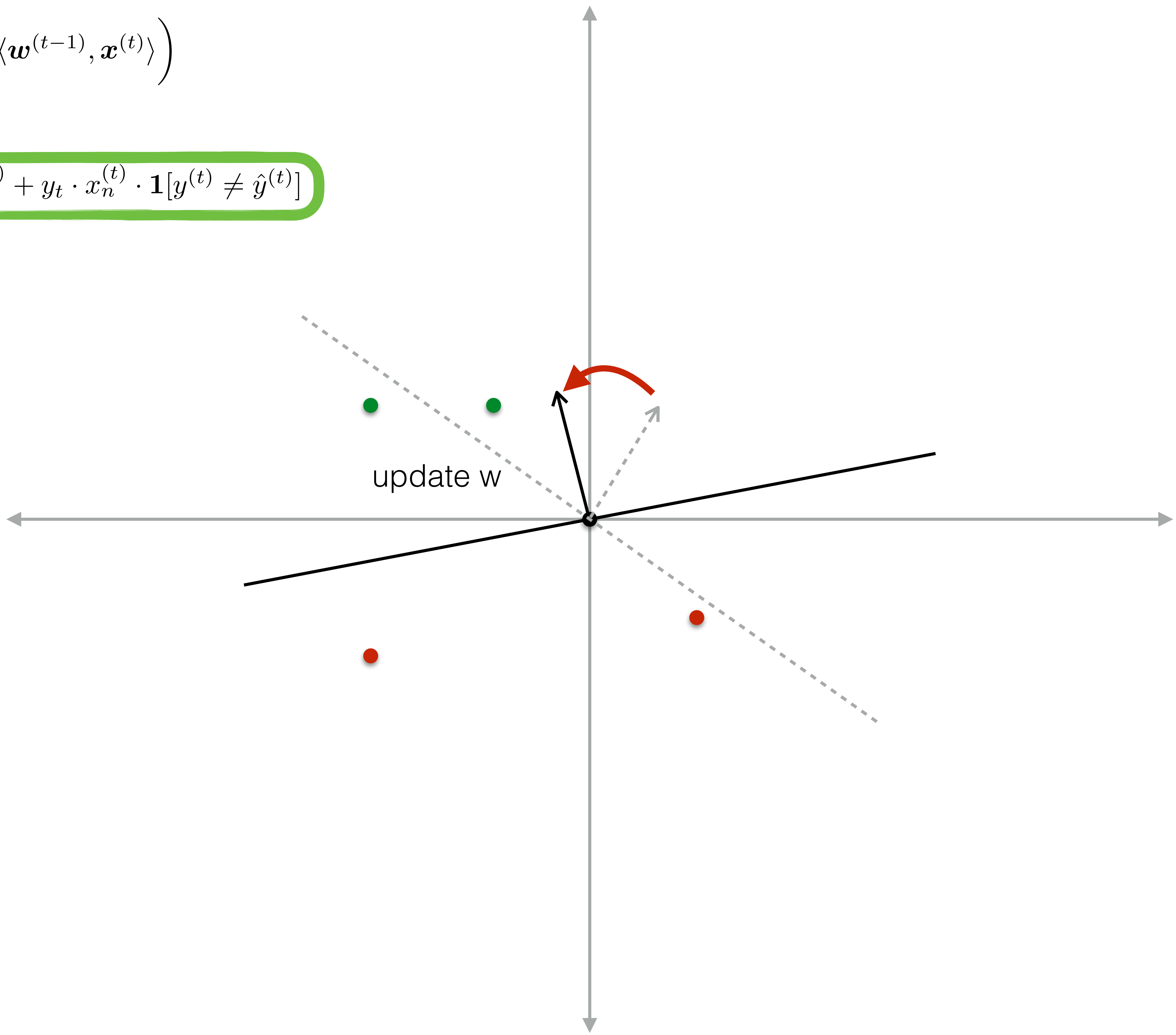

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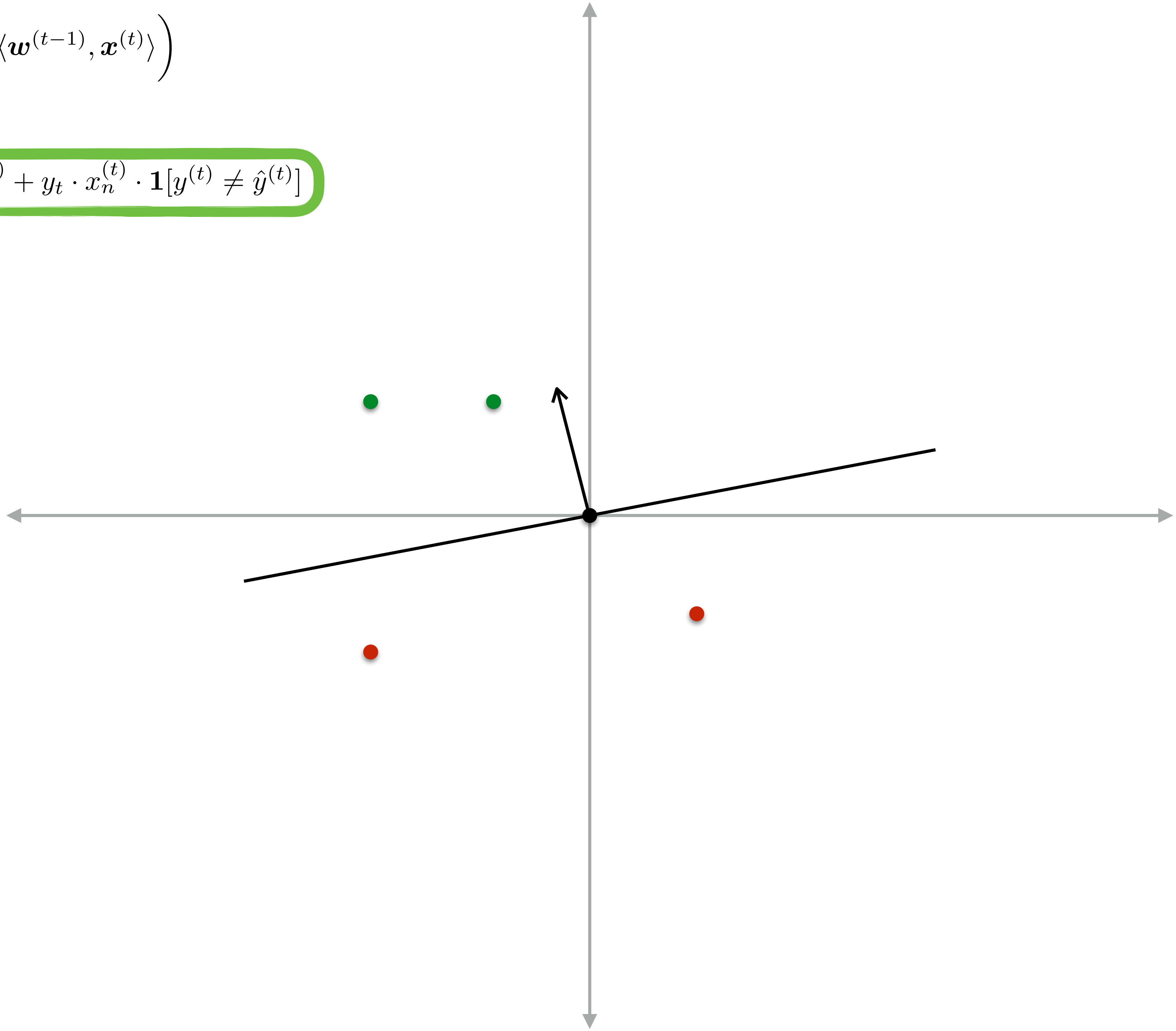

repeat …

function PERCEPTRON ALGORITHM

$\boldsymbol{w}^{(0)} \leftarrow \mathbf{0}$

for $t = 1, \ldots, T$ **do**

RECEIVE$(\boldsymbol{x}^{(t)})$ $\quad \boldsymbol{x} \in \{0,1\}^N$ N-d binary vector

$\hat{y}_A^{(t)} = \mathrm{sign}\left(\langle \boldsymbol{w}^{(t-1)}, \boldsymbol{x}^{(t)} \rangle \right)$ perceptron is just one line of code!

sign of zero is +1

RECEIVE(y^t) $\quad y \in \{1, -1\}$

$w_n^{(t)} = w_n^{(t-1)} + y_t \cdot x_n^{(t)} \cdot \mathbf{1}[y^{(t)} \neq \hat{y}^{(t)}]$

Code to train your perceptron:

$$\textbf{for } \; n = 1 \ldots N$$
$$w = w + (y_n - \hat{y}) x_i;$$

just one line of code!

Theory is coming soon…