Filtering vs Convolution

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Filters we have learned so far …

The 'Box' filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Gaussian filter

$$\frac{1}{16} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Sobel filter

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

Laplace filter

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

Filtering vs Convolution

output filter image

filtering

(cross-correlation)

$h = g \otimes f$

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

What's the difference?

convolution

$h = g \star f$

$$h[m,n] = \sum_{k,j} g[k,l] f[m-k, n-l]$$

Credit: Steve Seitz

Filtering vs Convolution

output filter image

filtering
(cross-correlation)

$h = g \otimes f$

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

filter flipped vertically
and horizontally

convolution

$h = g \star f$

$$h[m,n] = \sum_{k,j} g[k,l] f[m-k, n-l]$$

Filtering vs Convolution

output filter image

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(cross-correlation)

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$$h = g \star f$$

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Suppose g is a Gaussian filter.
How does convolution differ from filtering?

Recall...

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Commutative

"can move stuff around"

$$a \star b = b \star a \; .$$

Associative

"can regroup things"

$$(((a \star b_1) \star b_2) \star b_3) = a \star (b_1 \star b_2 \star b_3)$$

Distributes over addition

"can take things through parenthesis"

$$a \star (b + c) = (a \star b) + (a \star c)$$

Scalars factor out

$$\lambda a \star b = a \star \lambda b = \lambda (a \star b)$$

$$\boxed{\text{Derivative Theorem of Convolution} \quad \frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x} h) \star f}$$

can precompute this

Derivative Theorem of Convolution

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

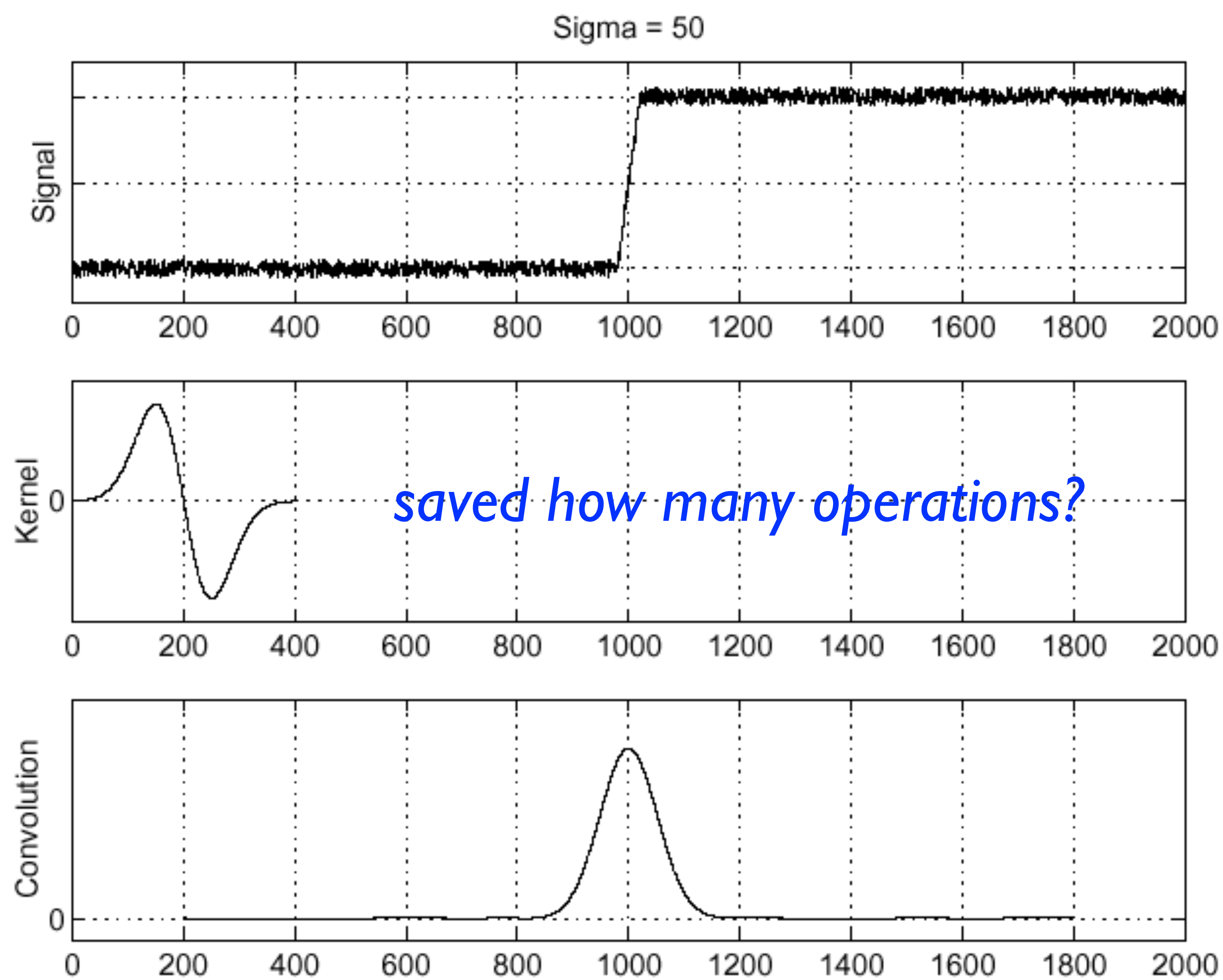
Input

Derivative of Gaussian

$\frac{\partial}{\partial x}h$

Output

$(\frac{\partial}{\partial x}h) \star f$



Recall ...

Input

Gaussian

Smoothed input

Derivative

Output