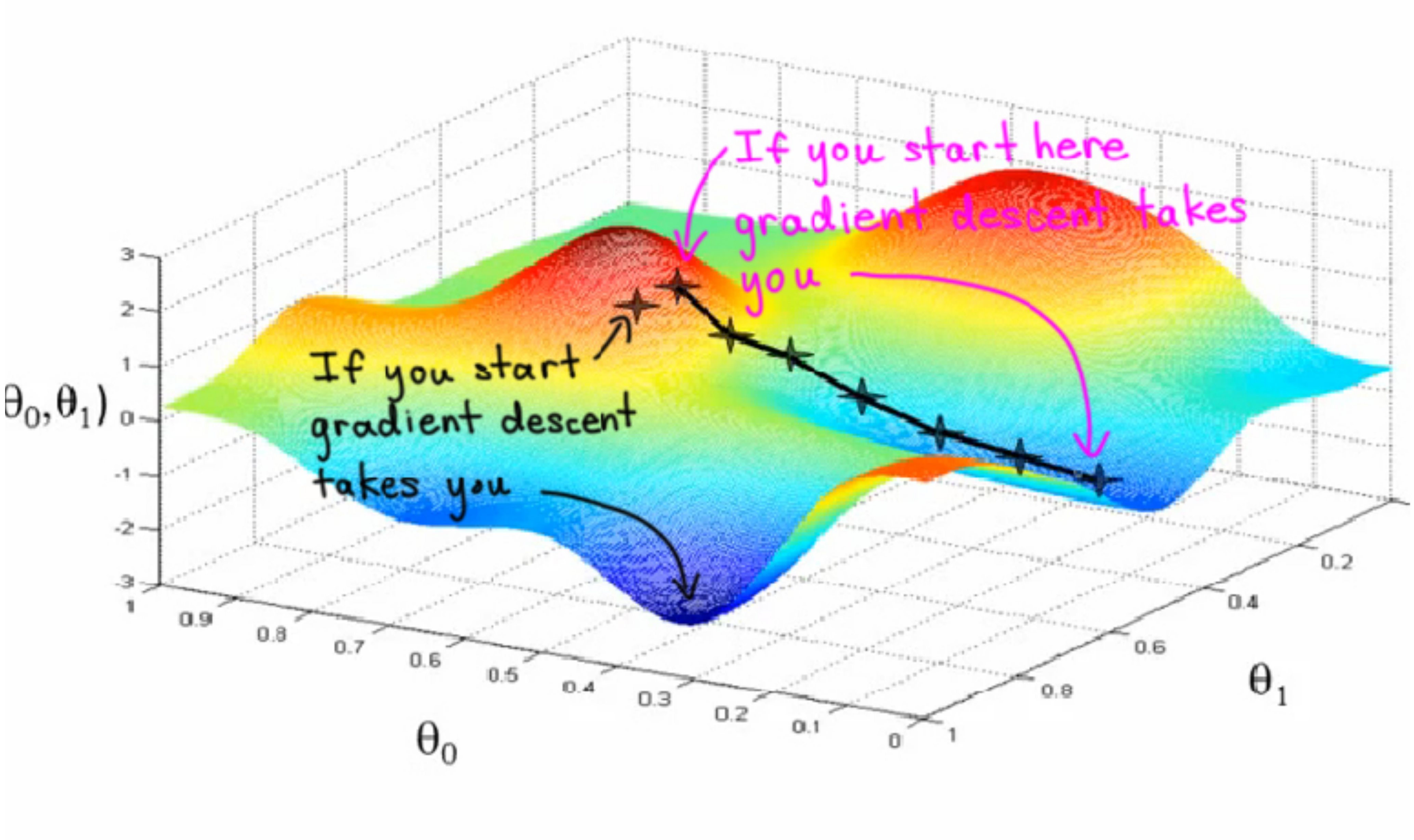


Visualizing Gradient Descent

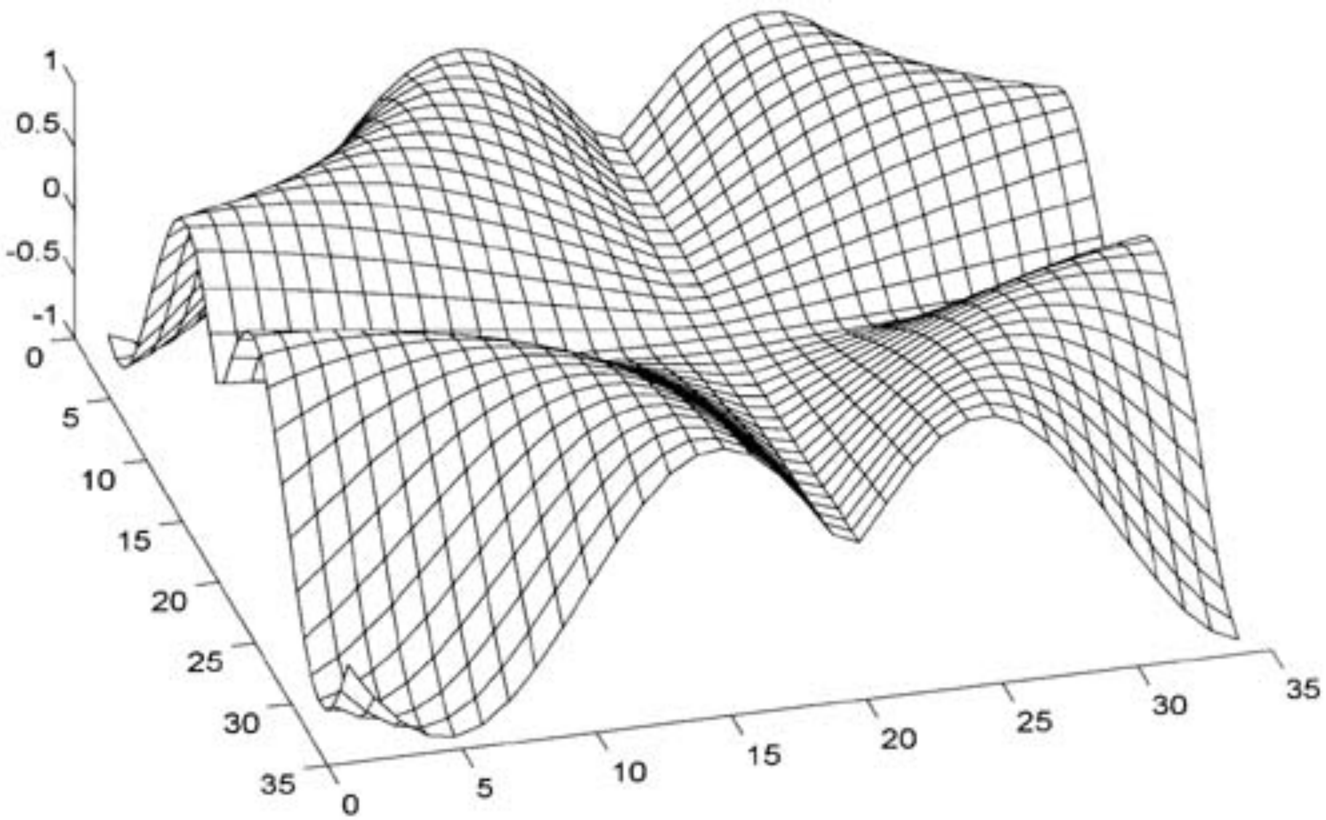
Computer Vision

Carnegie Mellon University (Kris Kitani)

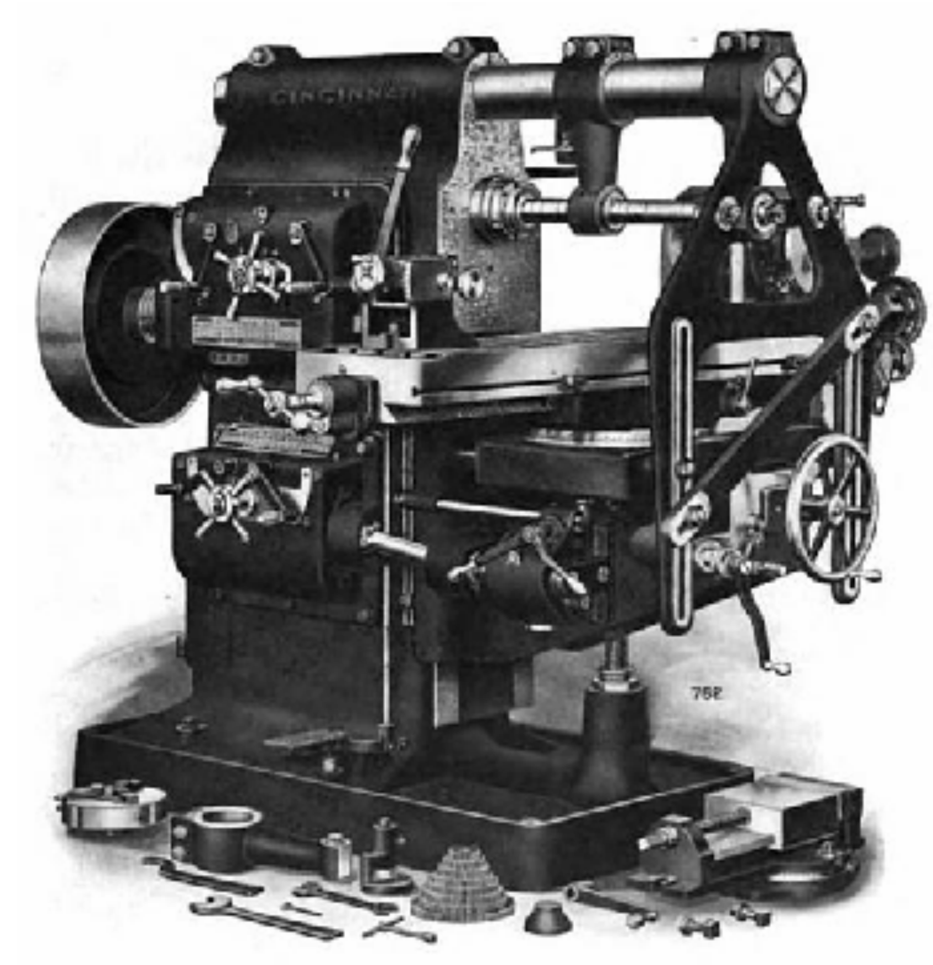
(partial) derivatives

tell us how much one variable affects another

Two ways to think about them:

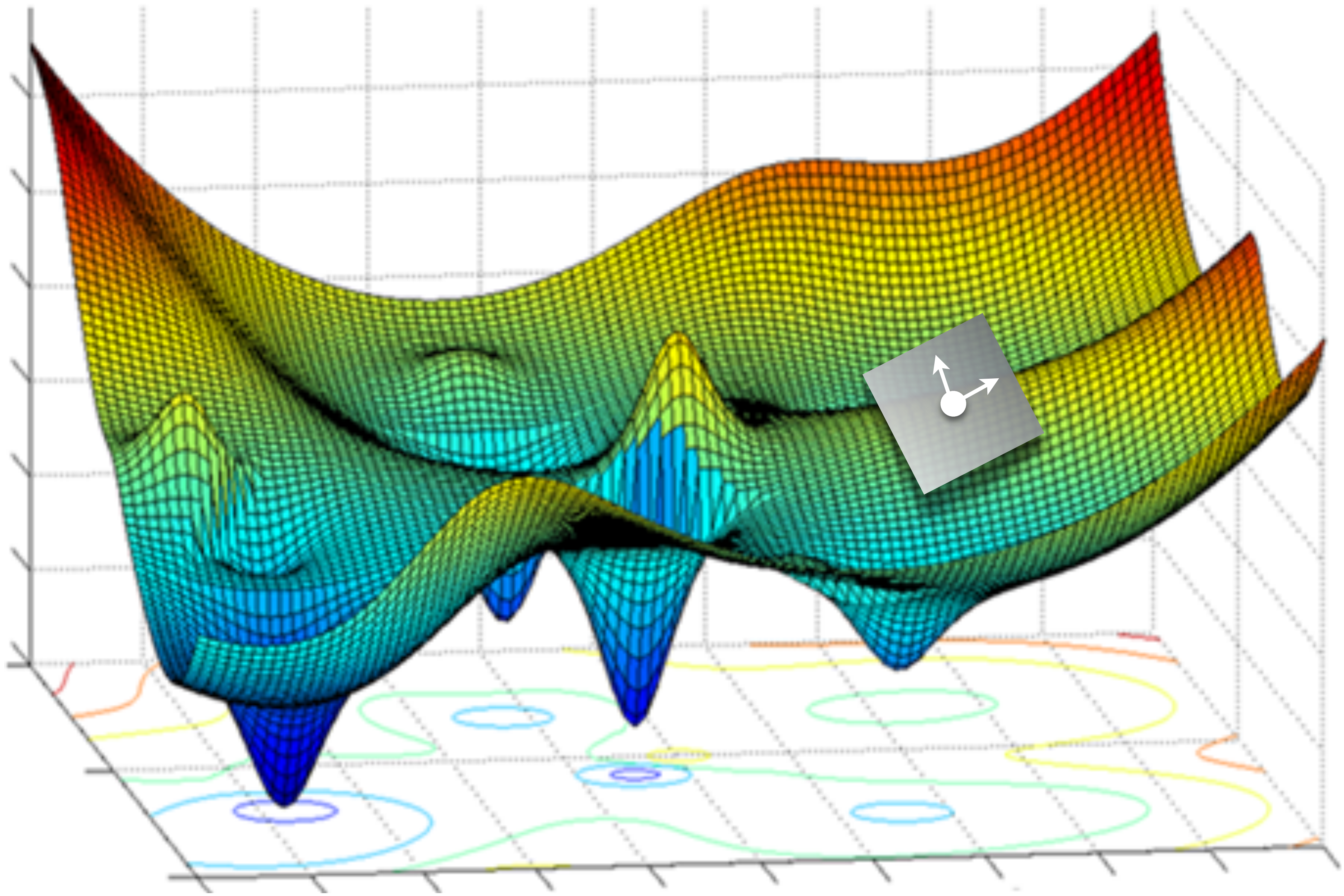


Slope of a function



Knobs on a machine

1. Slope of a function:



$$\frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} = \left[\frac{\partial f(\boldsymbol{x})}{\partial x}, \frac{\partial f(\boldsymbol{x})}{\partial y}\right]$$

describes the slope around a point

2. Knobs on a machine:

input
$\boldsymbol{x}$

output
$f(\boldsymbol{x}; \boldsymbol{w})$

2. Knobs on a machine:

input
$\boldsymbol{x}$

output
$f(\boldsymbol{x};\boldsymbol{w})$

describes how each 'knob' affects the output

$$\frac{\partial f(x)}{\partial w_1} \qquad \frac{\partial f(x)}{\partial w_2} \qquad \frac{\partial f(x)}{\partial w_3}$$

2. Knobs on a machine:

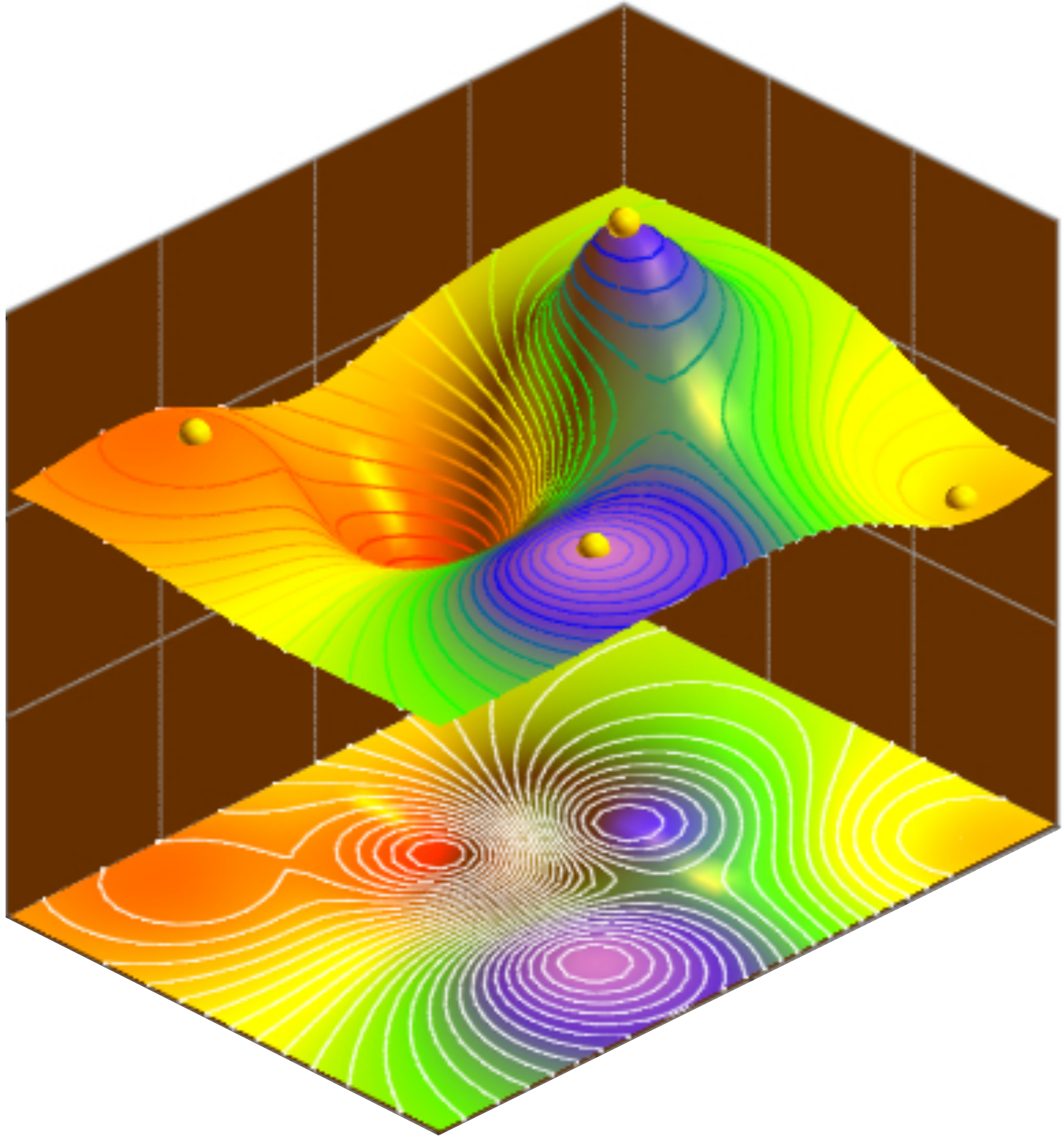
input
$\boldsymbol{x}$

output
$f(\boldsymbol{x};\boldsymbol{w})$

describes how each
'knob' affects the
output

$$\frac{\partial f(x)}{\partial w_1} \qquad \frac{\partial f(x)}{\partial w_2} \qquad \frac{\partial f(x)}{\partial w_3}$$

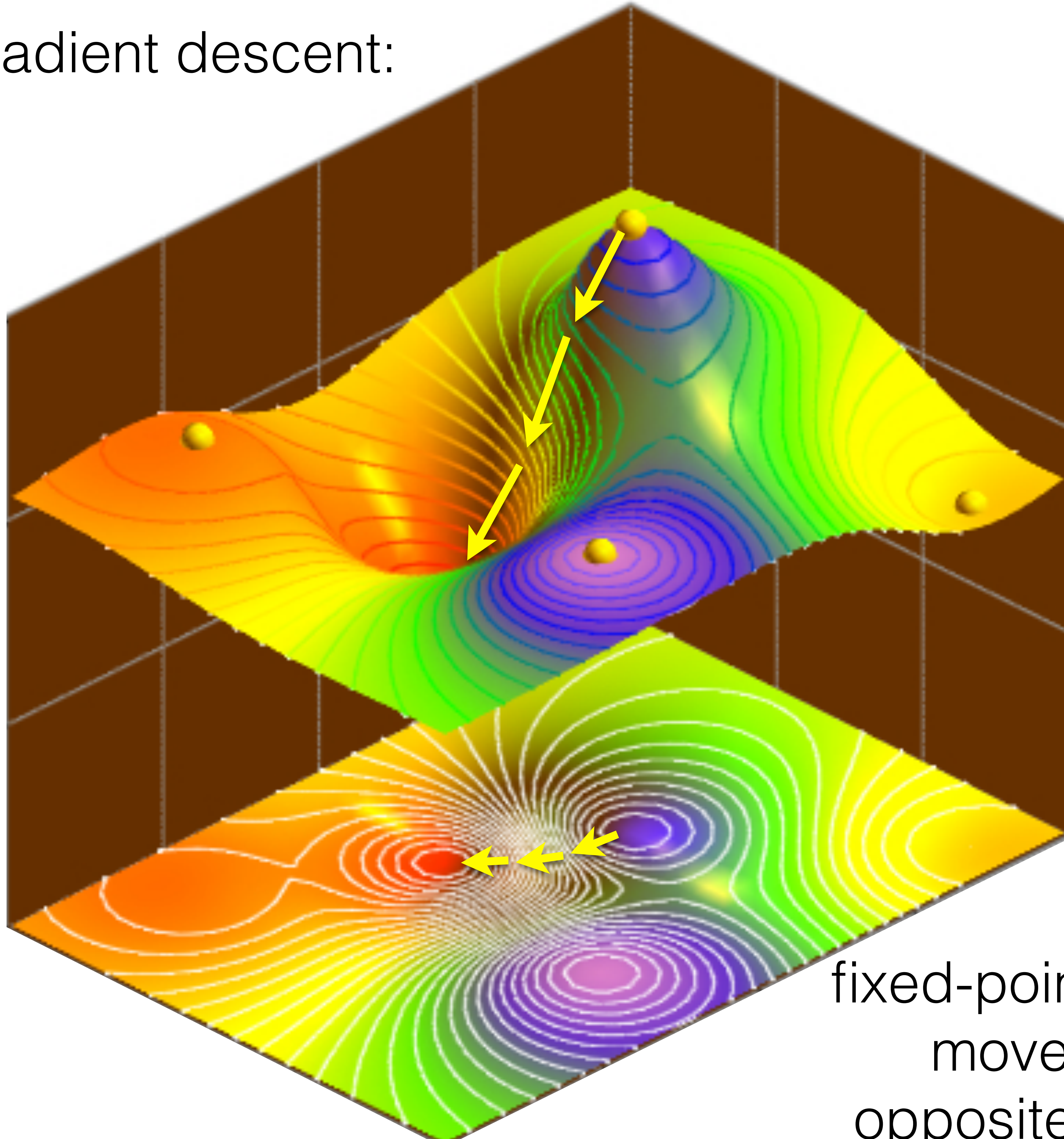
small change in parameter Δw_1 output will change by $\dfrac{\partial f(x)}{\partial w_1}\,\Delta w_1$



Loss function in 3D

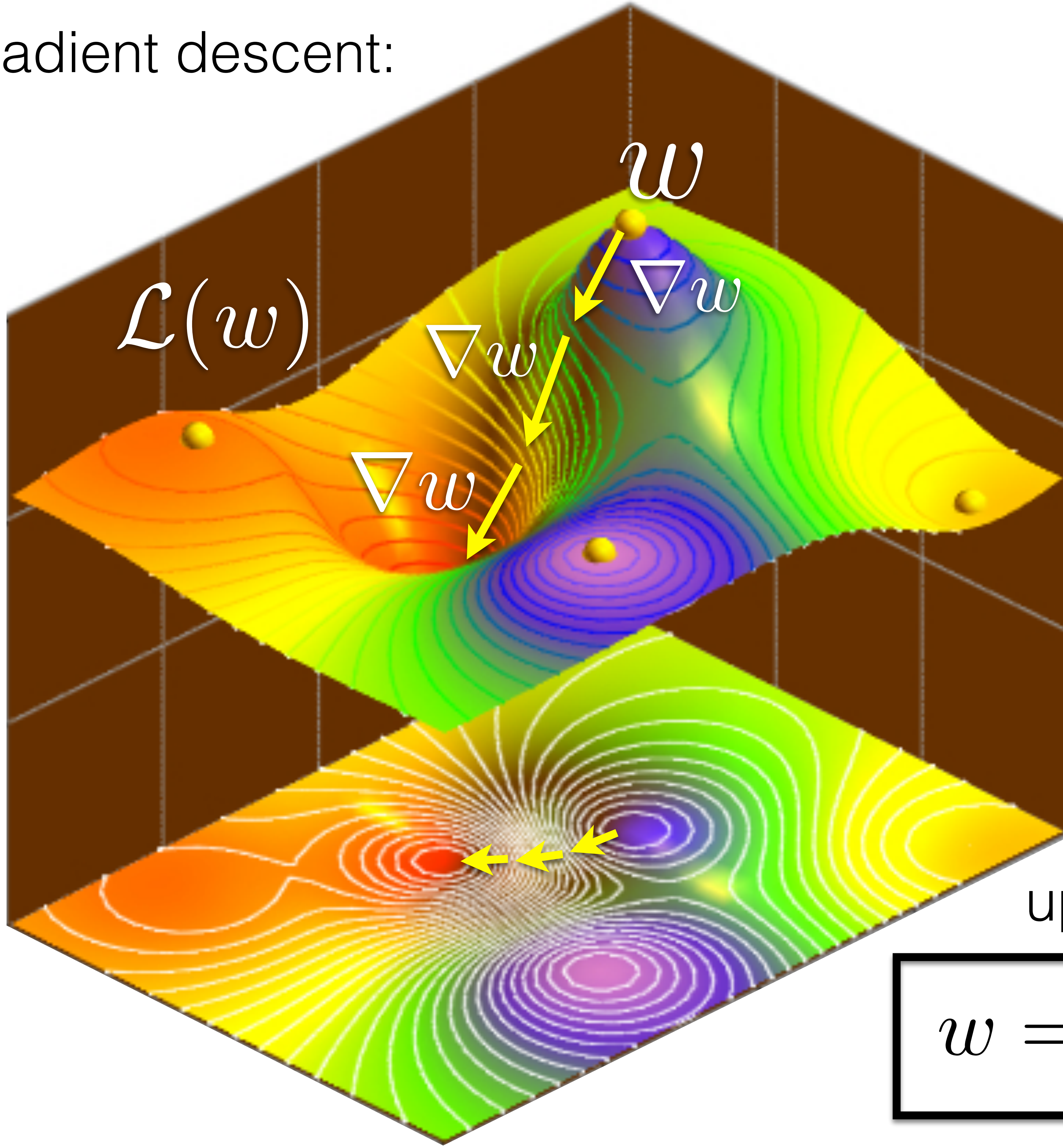
Contour map
visualization of the
same thing

Gradient descent:



Given a
fixed-point on a function,
move in the direction
opposite of the gradient

Gradient descent:



update rule:

$$\boxed{w = w - \nabla w}$$