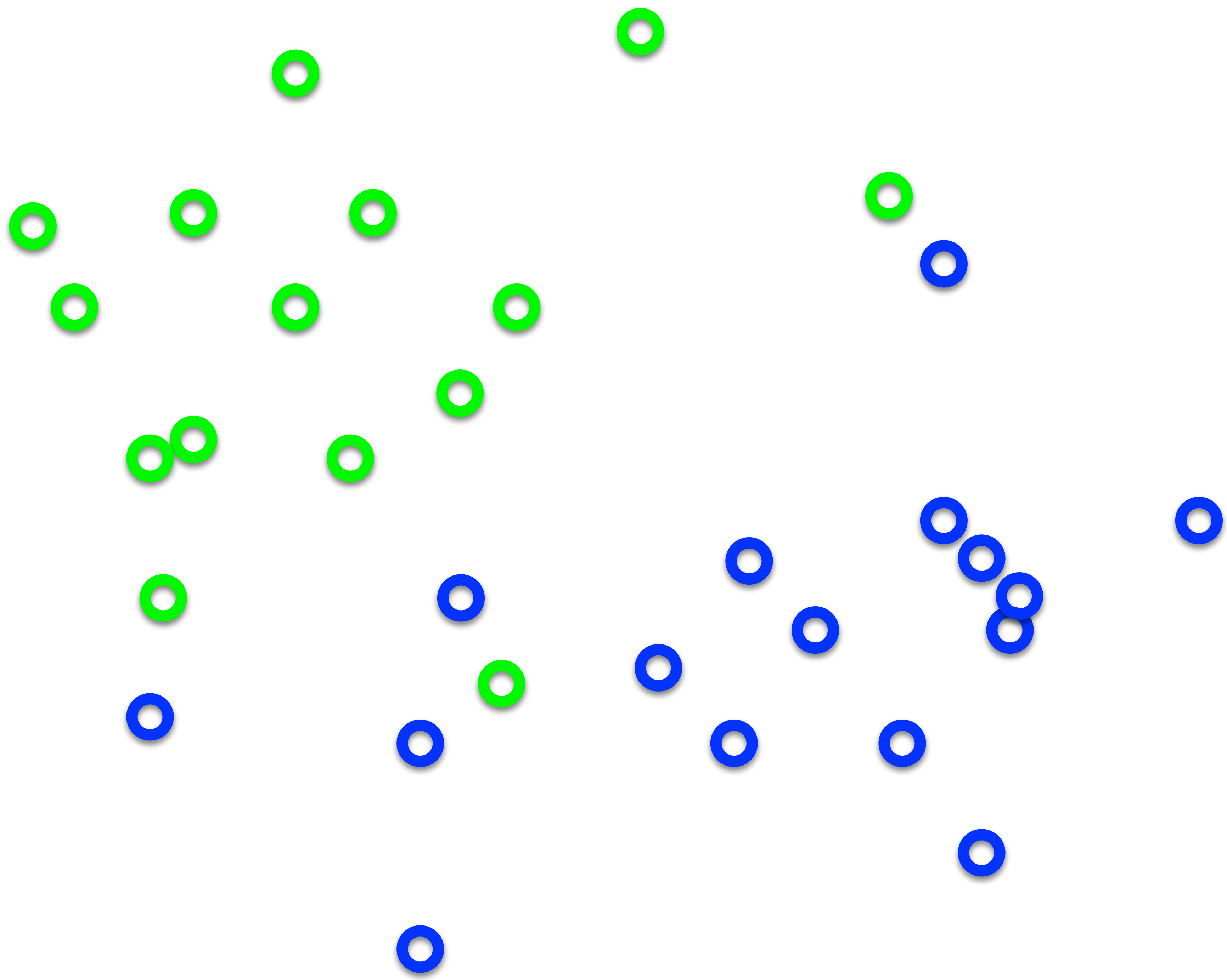
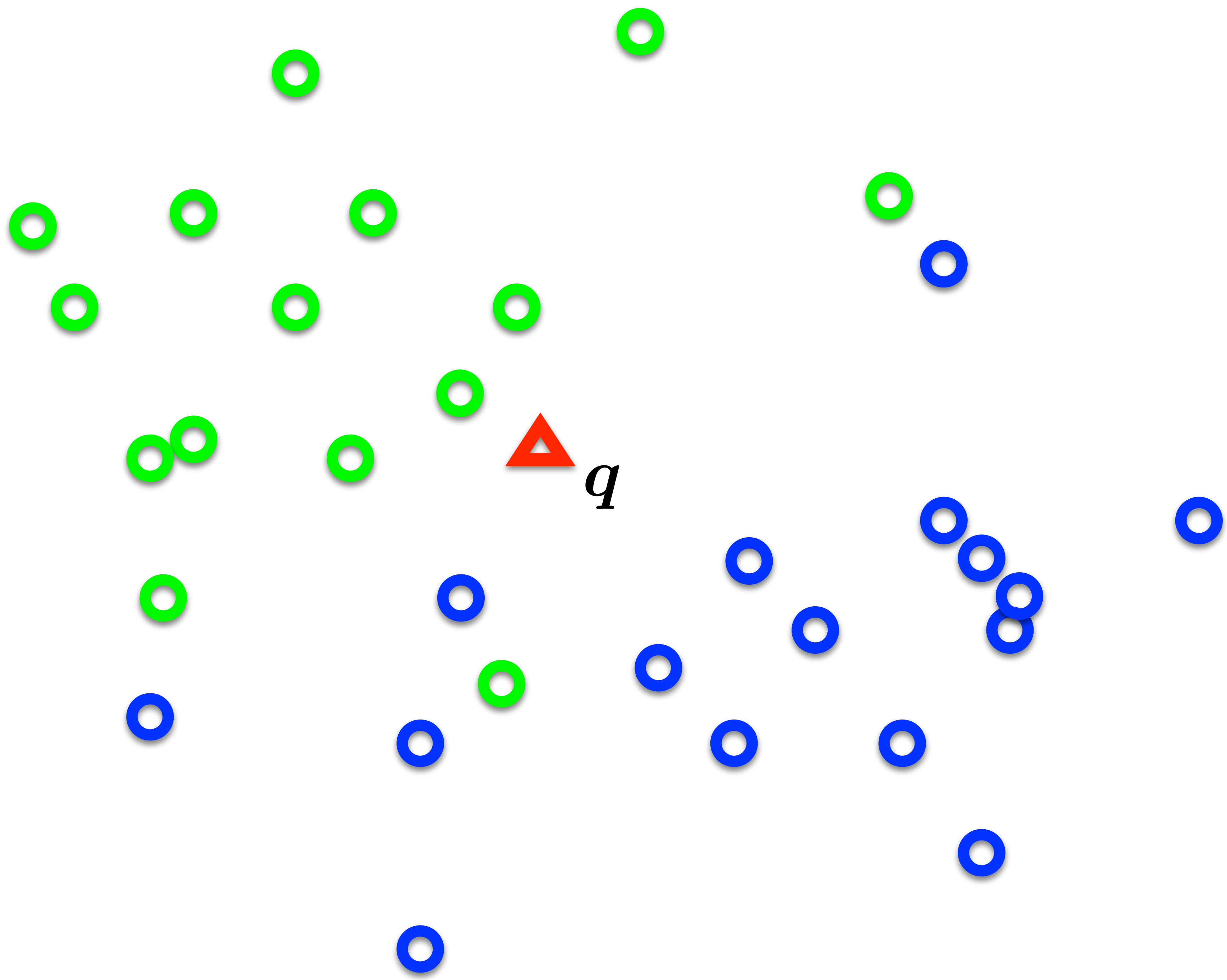
Distribution of data from two classes

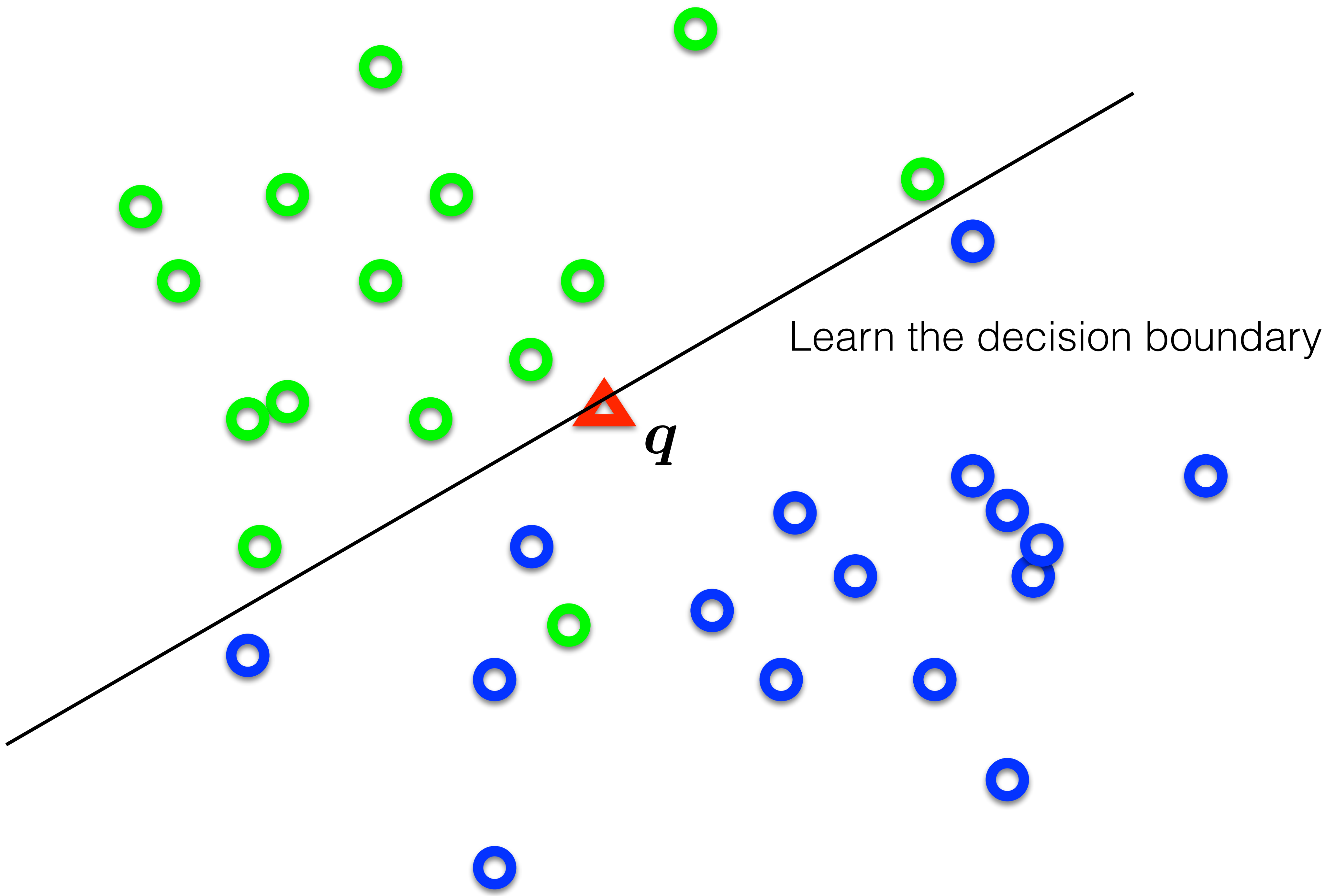


Distribution of data from two classes



Which class does q belong too?

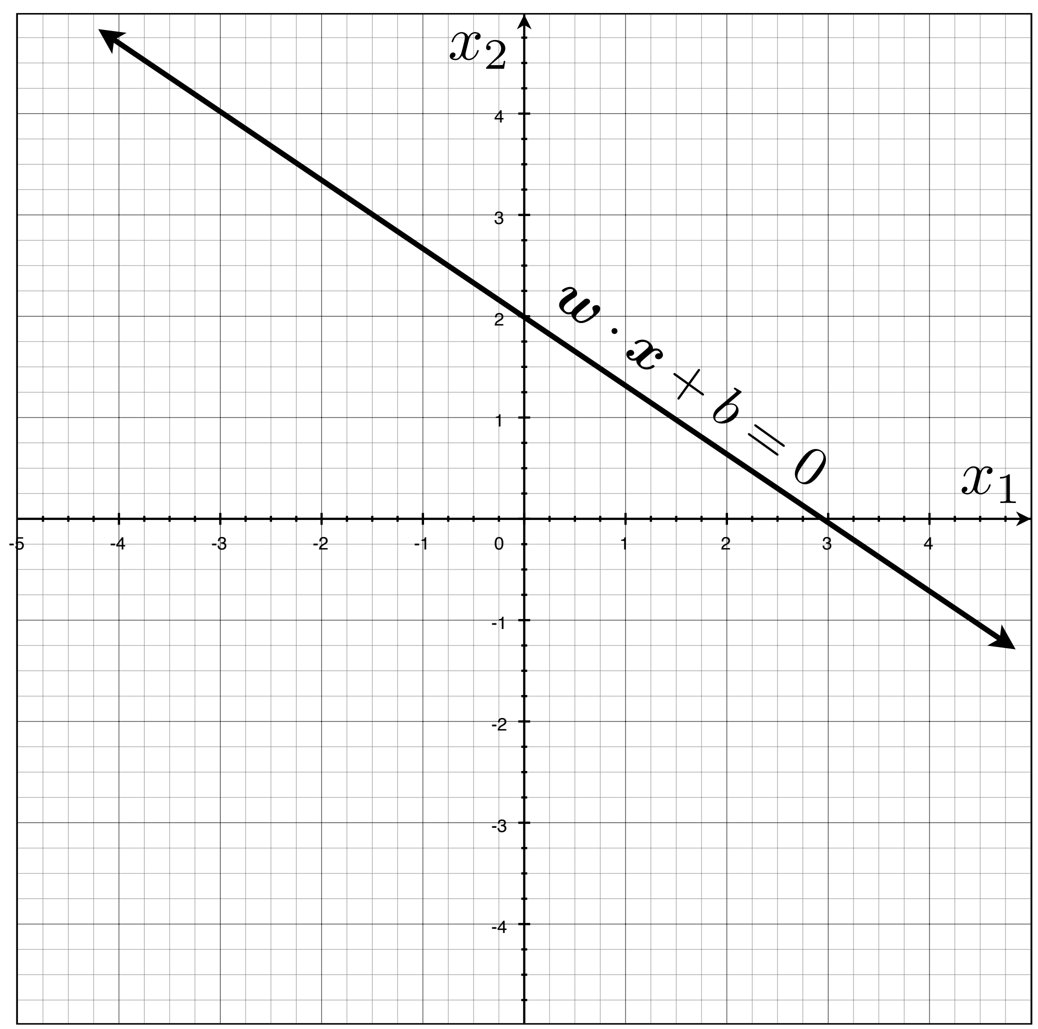
Distribution of data from two classes



First we need to understand hyperplanes…

Hyperplanes (lines) in 2D

$$w_1 x_1 + w_2 x_2 + b = 0$$



a line can be written as
dot product plus a bias

$$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$$

$$\boldsymbol{w} \in \mathcal{R}^2$$

another version, add a weight 1
and push the bias inside

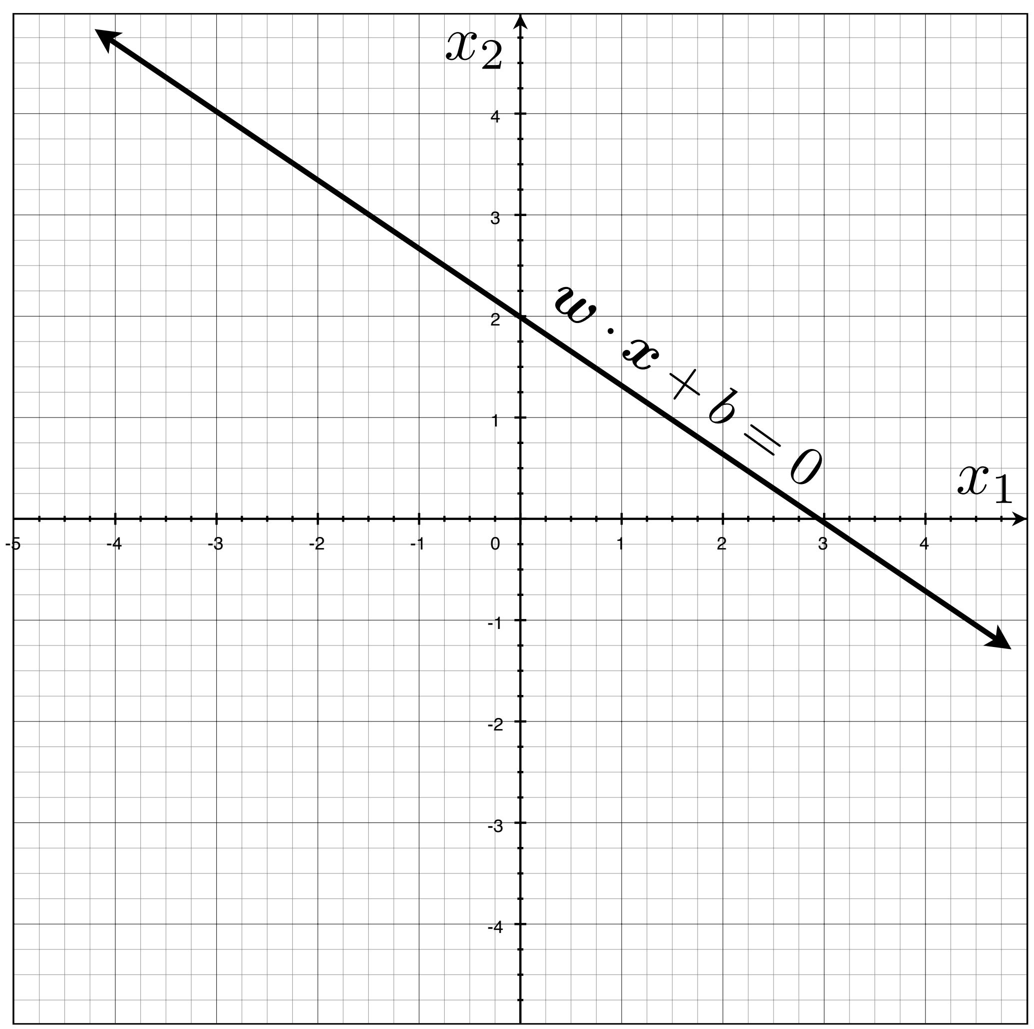
$$\boldsymbol{w} \cdot \boldsymbol{x} = 0$$

$$\boldsymbol{w} \in \mathcal{R}^3$$

Hyperplanes (lines) in 2D

$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ (offset/bias outside) $\boldsymbol{w} \cdot \boldsymbol{x} = 0$ (offset/bias inside)

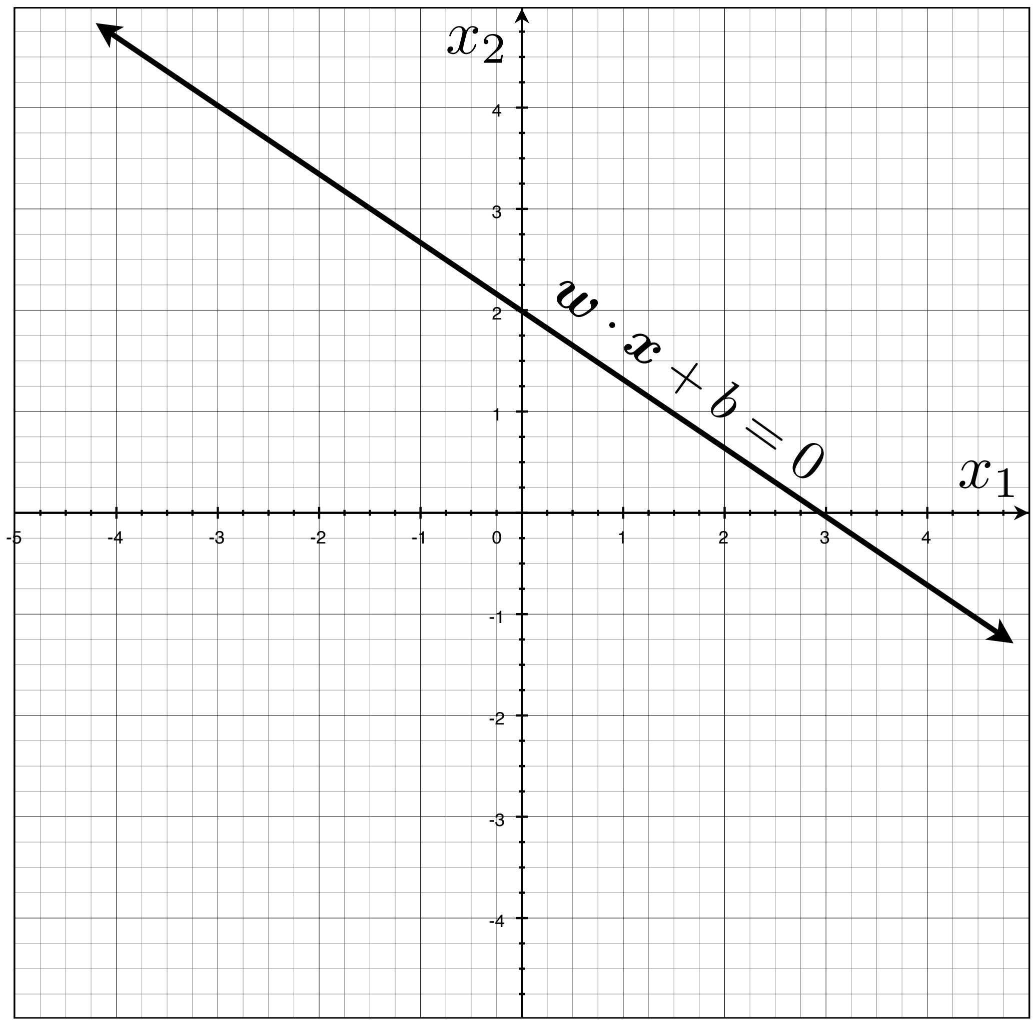
$$w_1 x_1 + w_2 x_2 + b = 0$$



Hyperplanes (lines) in 2D

$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ (offset/bias outside) $\boldsymbol{w} \cdot \boldsymbol{x} = 0$ (offset/bias inside)

$$w_1 x_1 + w_2 x_2 + b = 0$$



Important property:

Free to choose any normalization of w

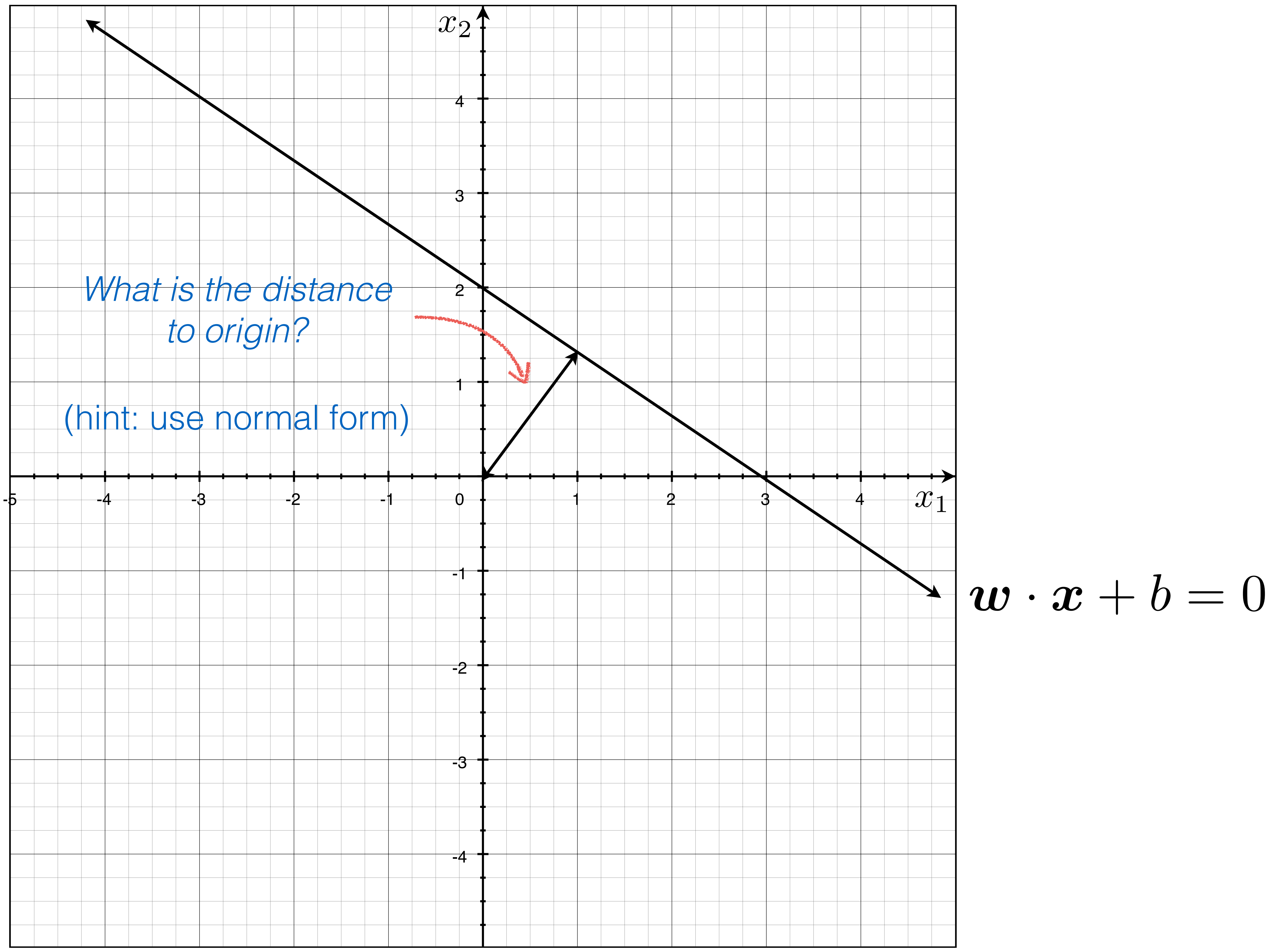
The line

$$w_1 x_1 + w_2 x_2 + b = 0$$

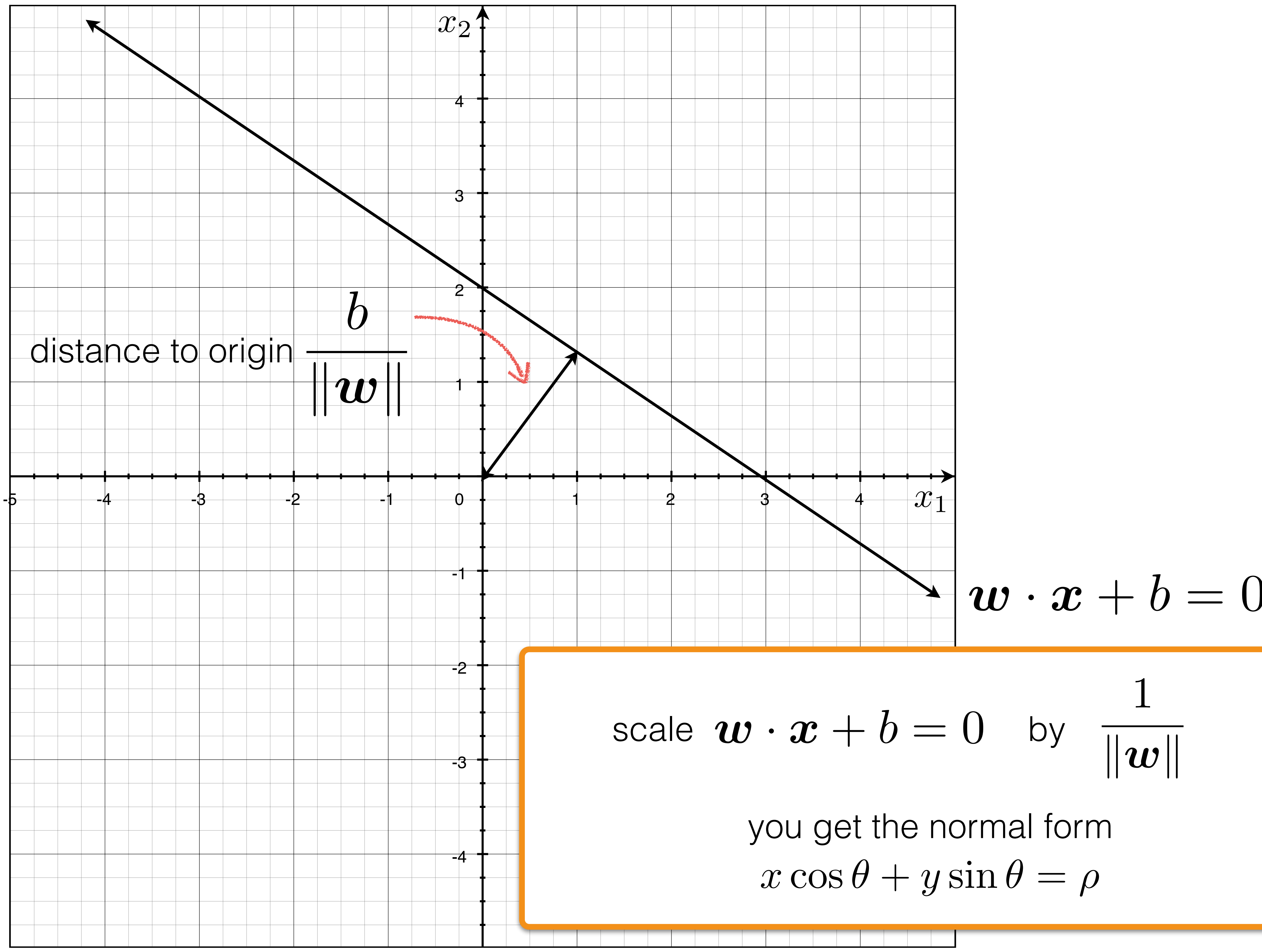
and the line

$$\lambda(w_1 x_1 + w_2 x_2 + b) = 0$$

define the same line



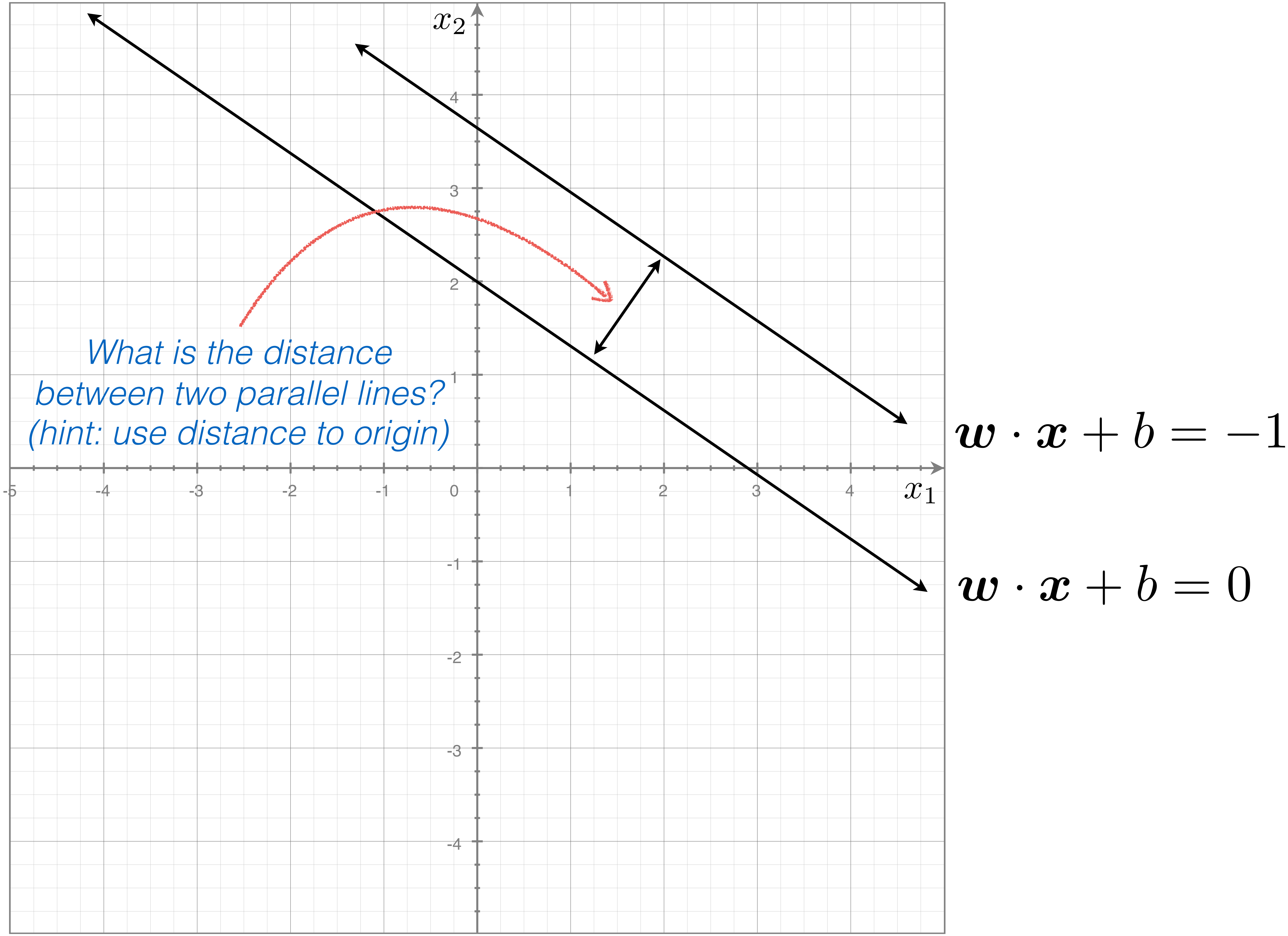
What is the distance to origin?
(hint: use normal form)
x_1
x_2
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$

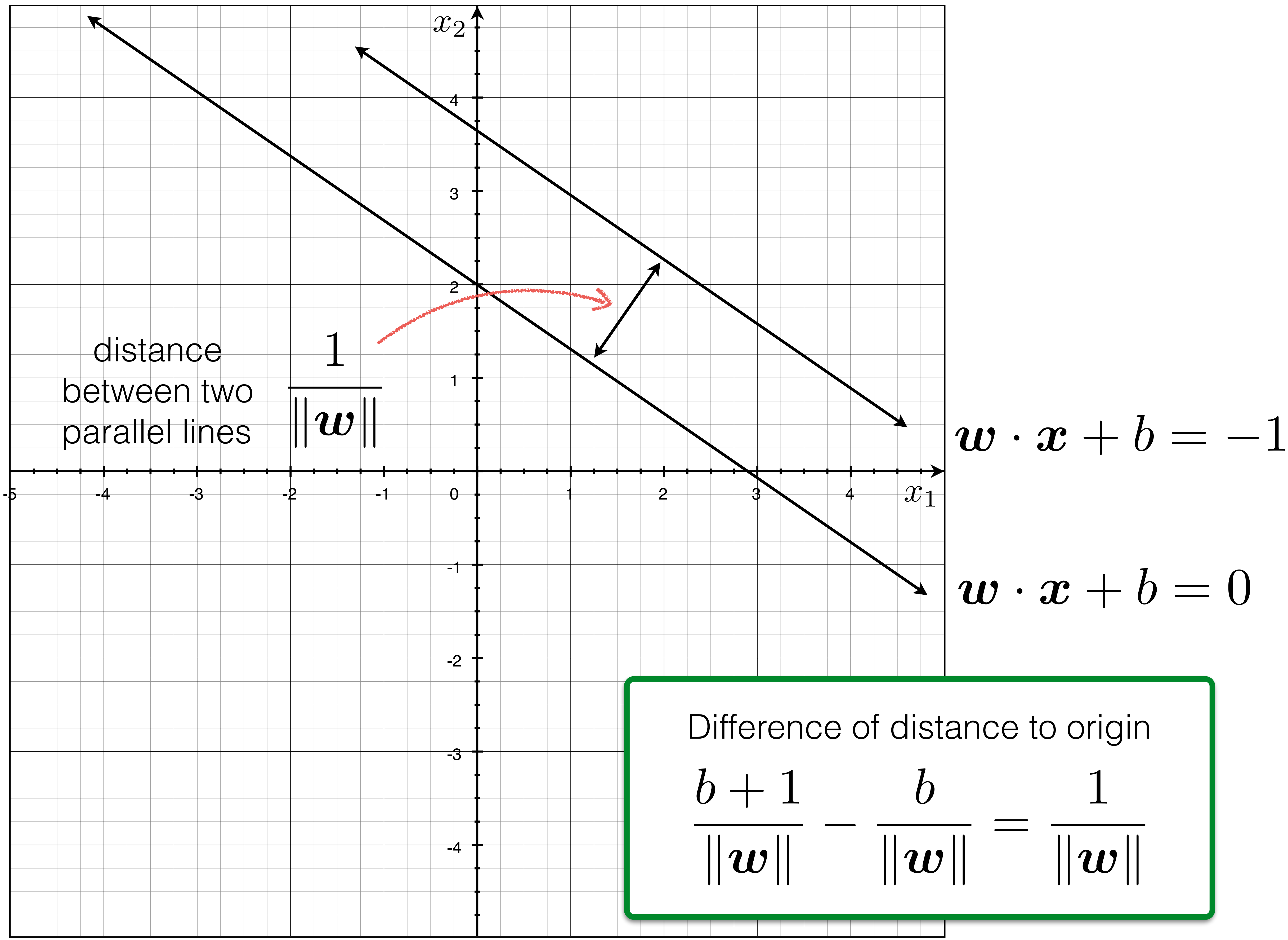


scale $\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ by $\dfrac{1}{\|\boldsymbol{w}\|}$

you get the normal form

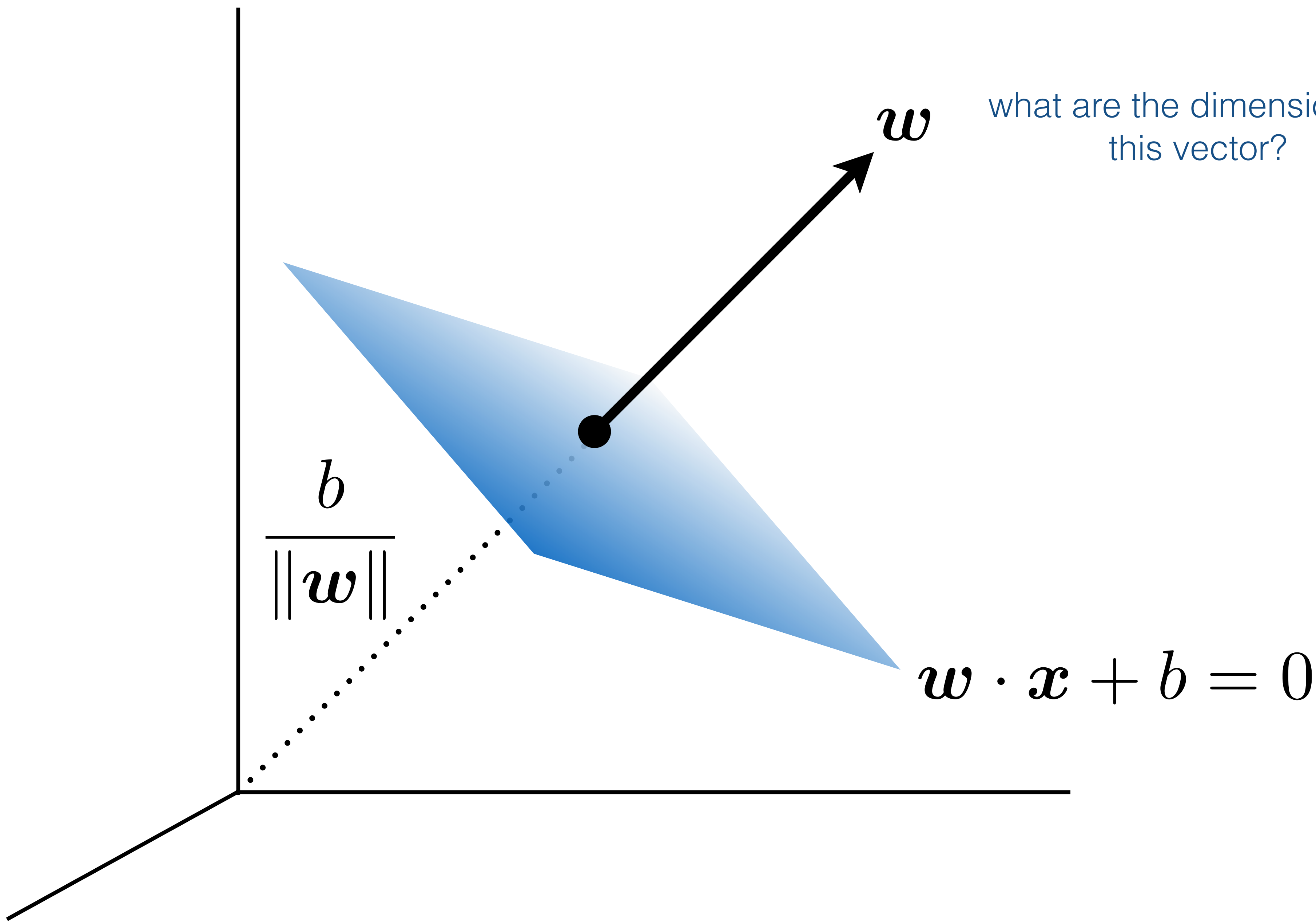
$$x \cos\theta + y \sin\theta = \rho$$


What is the distance
between two parallel lines?
(hint: use distance to origin)
x_1
x_2
$\boldsymbol{w} \cdot \boldsymbol{x} + b = -1$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$


x_2
x_1
-5
-4
-3
-2
-1
0
1
2
3
4
distance between two parallel lines
$\frac{1}{\|\boldsymbol{w}\|}$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = -1$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$
Difference of distance to origin
$\frac{b+1}{\|\boldsymbol{w}\|} - \frac{b}{\|\boldsymbol{w}\|} = \frac{1}{\|\boldsymbol{w}\|}$

Now we can go to 3D …

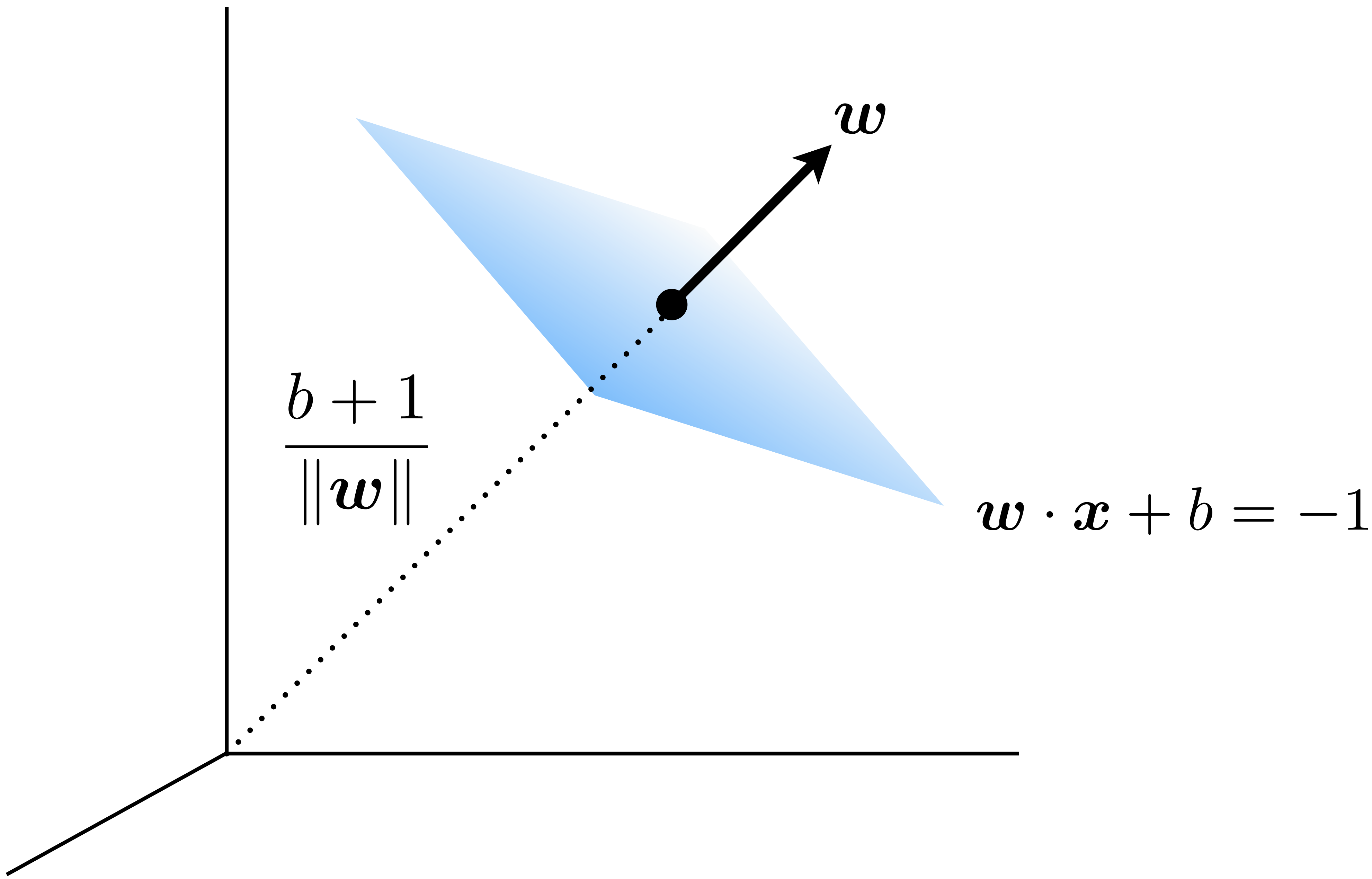
Hyperplanes (planes) in 3D



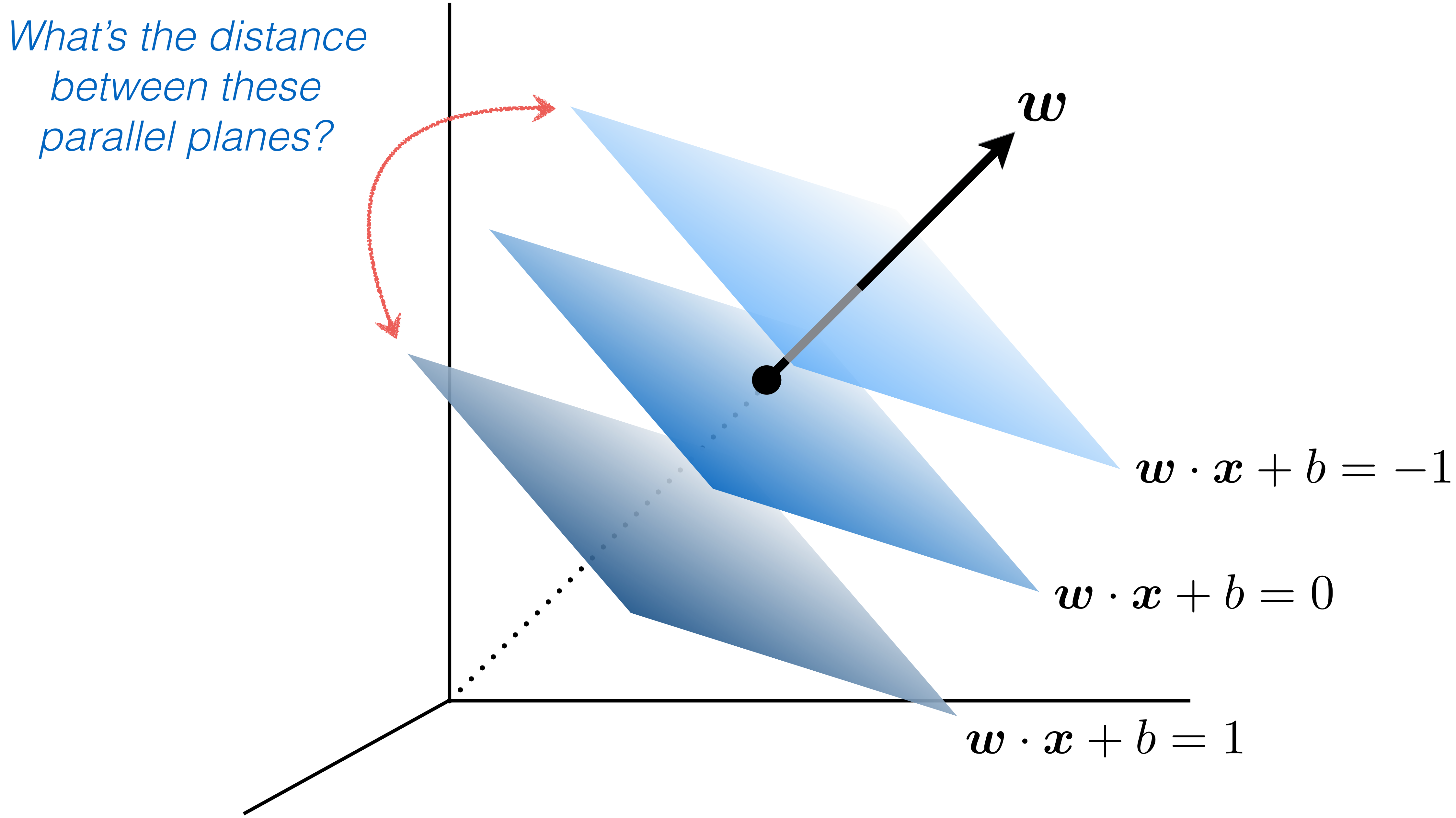
what are the dimensions of this vector?

*What happens if you change **b**?*

Hyperplanes (planes) in 3D



Hyperplanes (planes) in 3D



Hyperplanes (planes) in 3D