Generalized Hough Transform

Computer Vision

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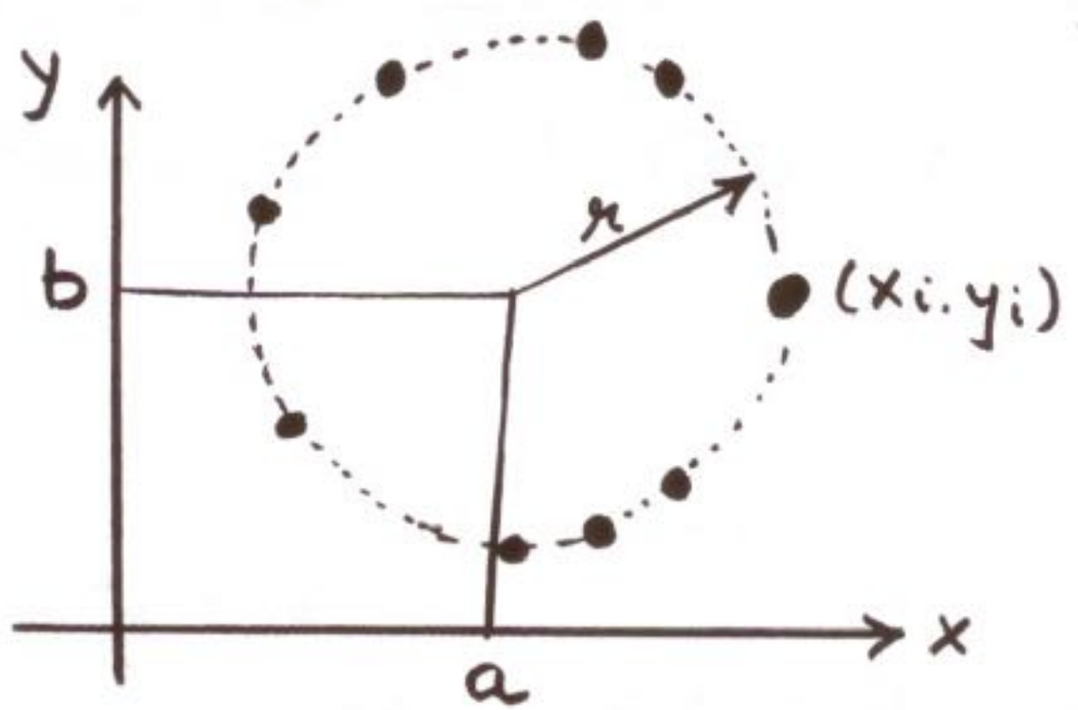
Hough Circles

Finding Circles by Hough Transform

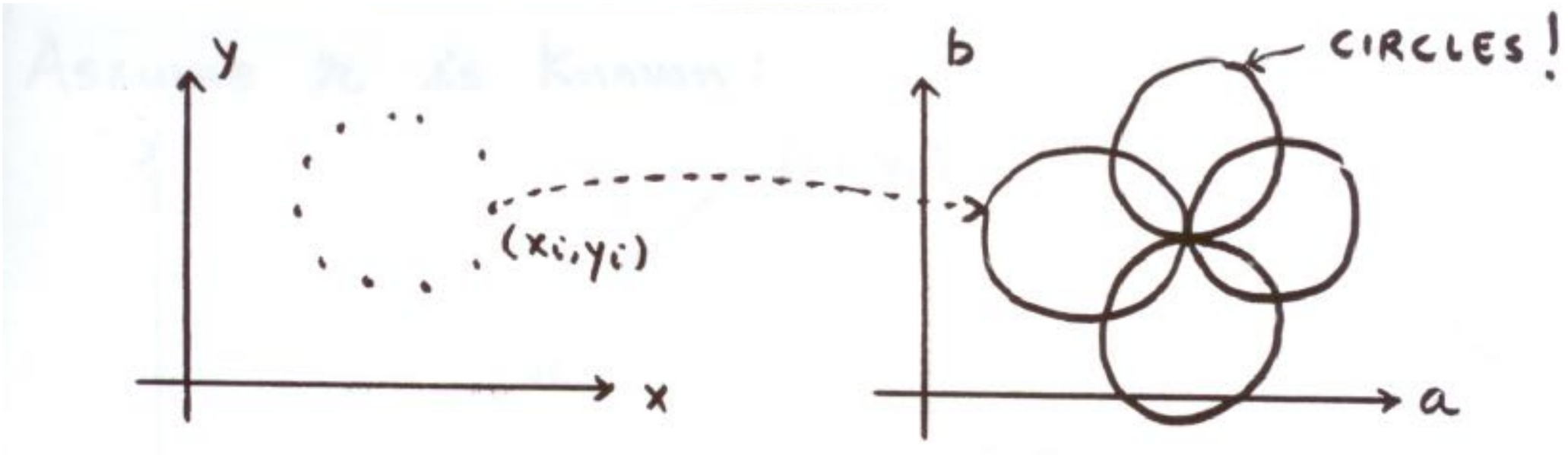
Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)



Accumulator Array $A(a,b)$



parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

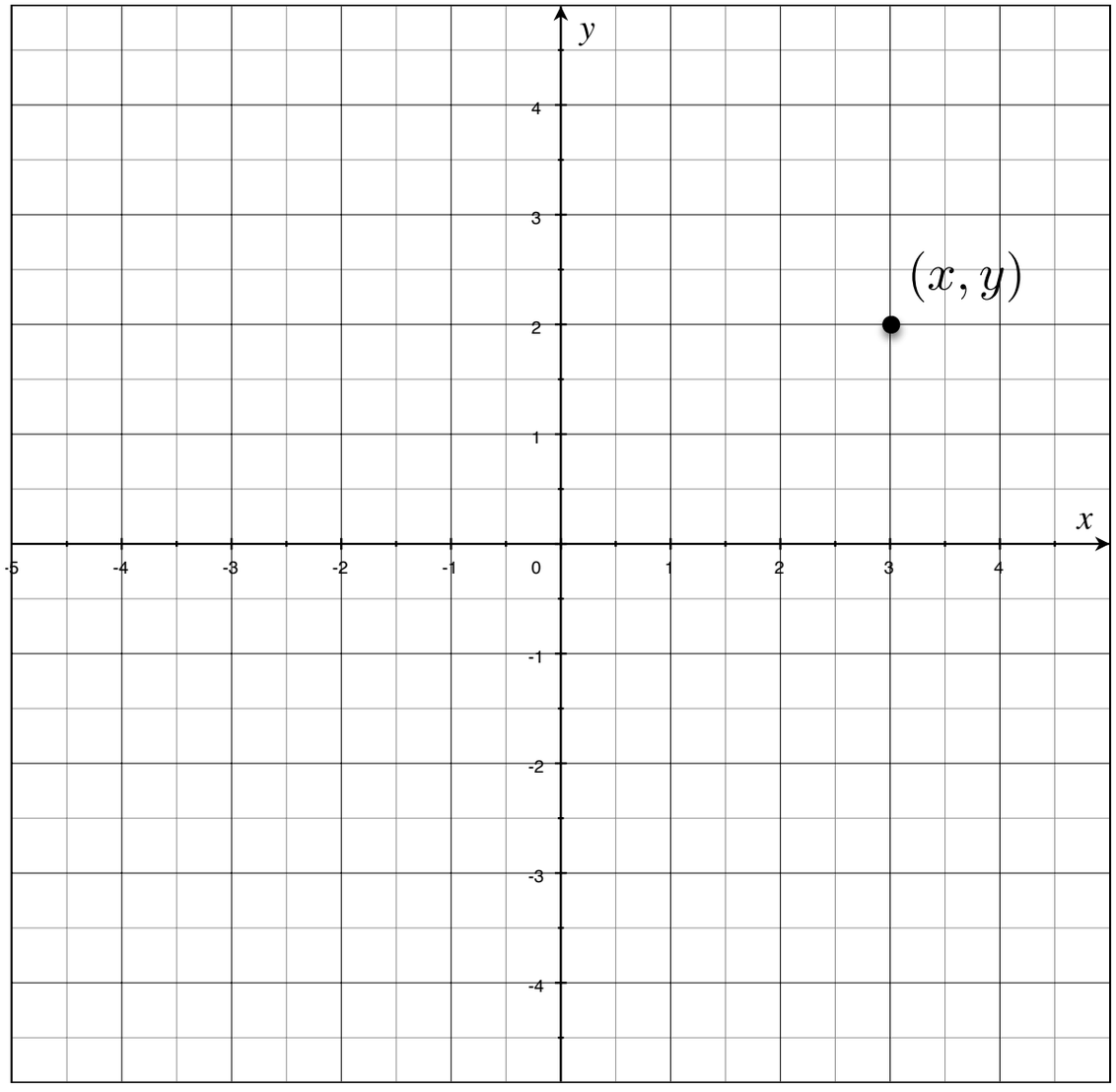
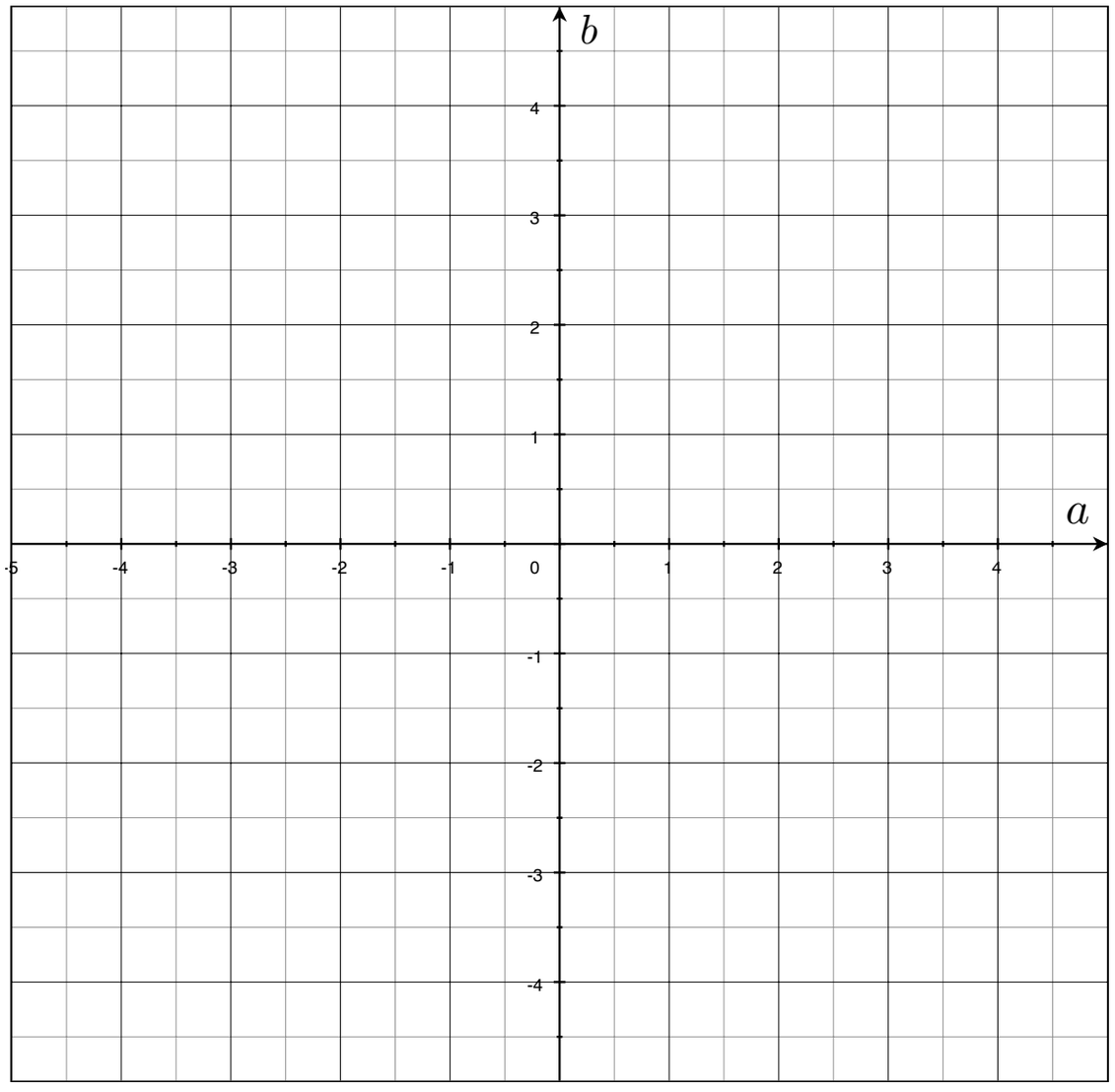


Image space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

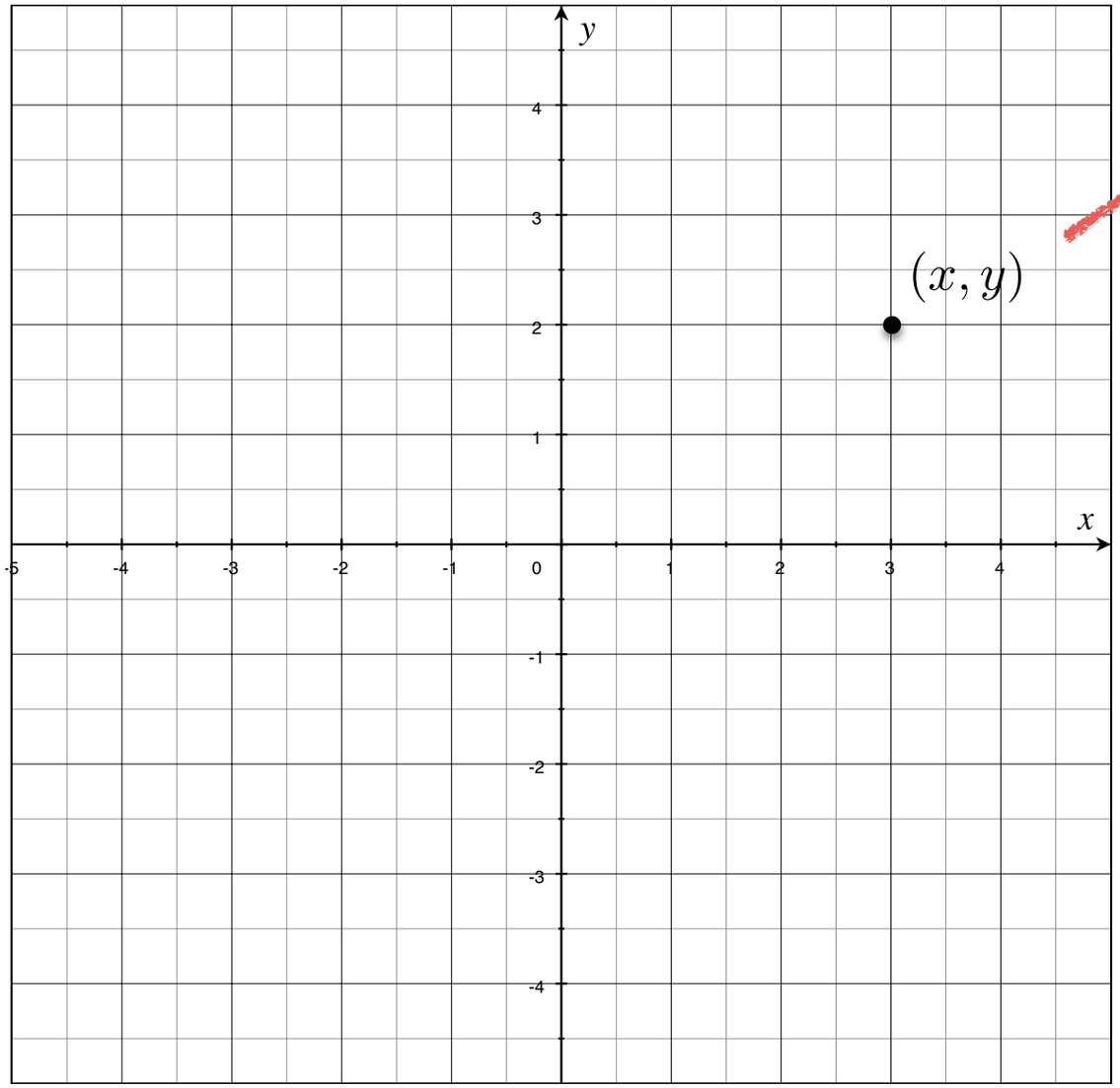


Parameter space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



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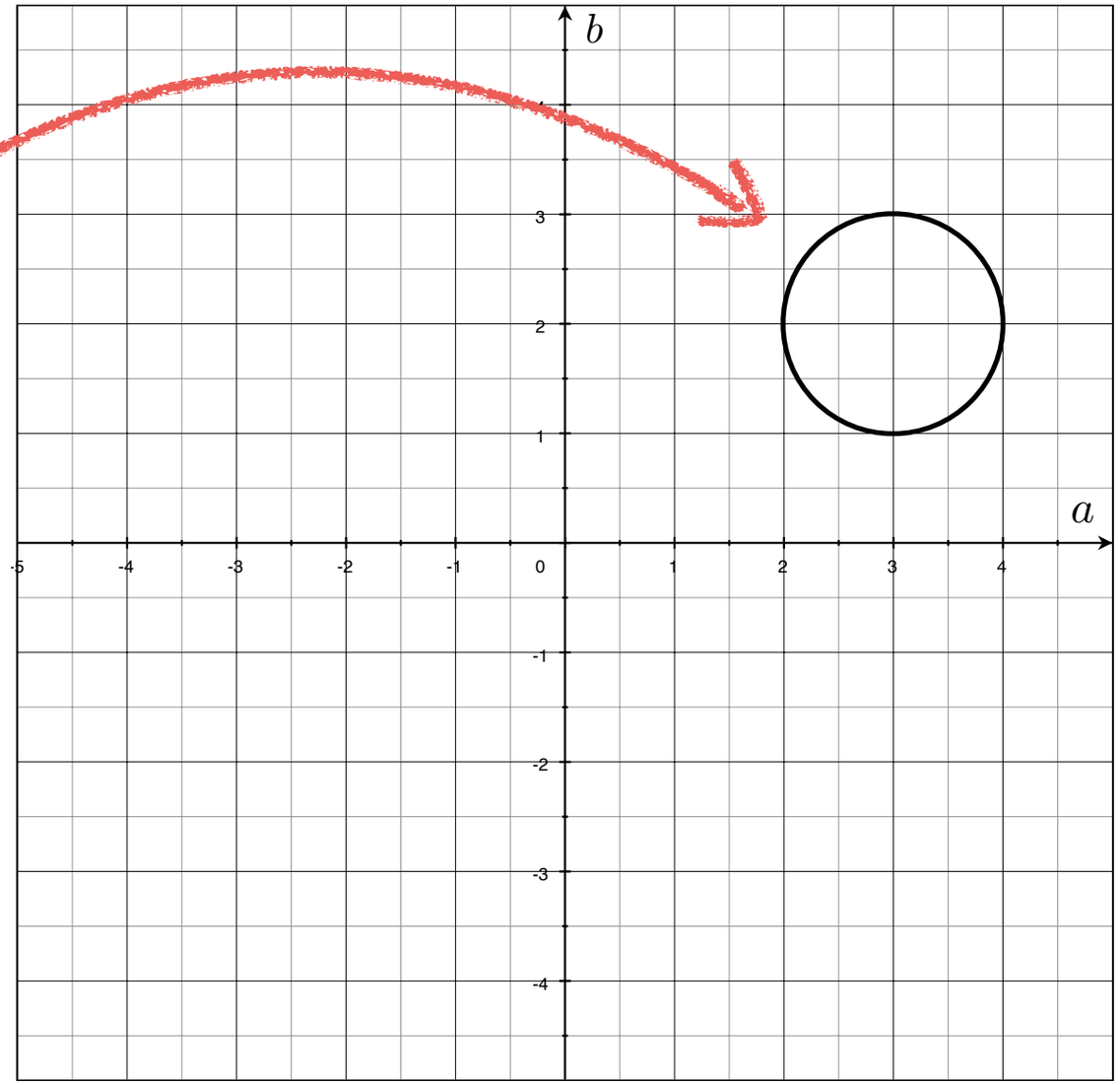

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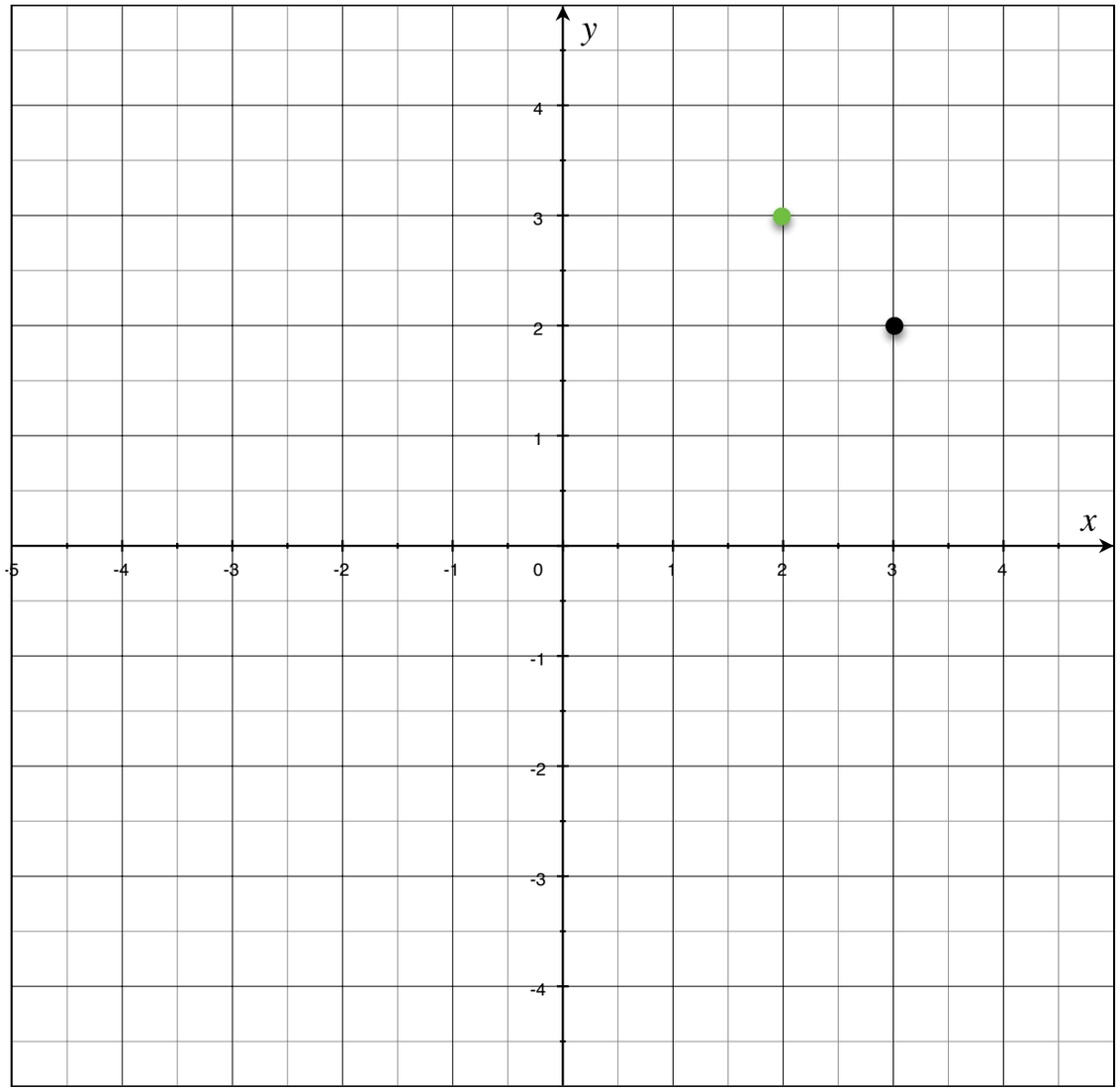

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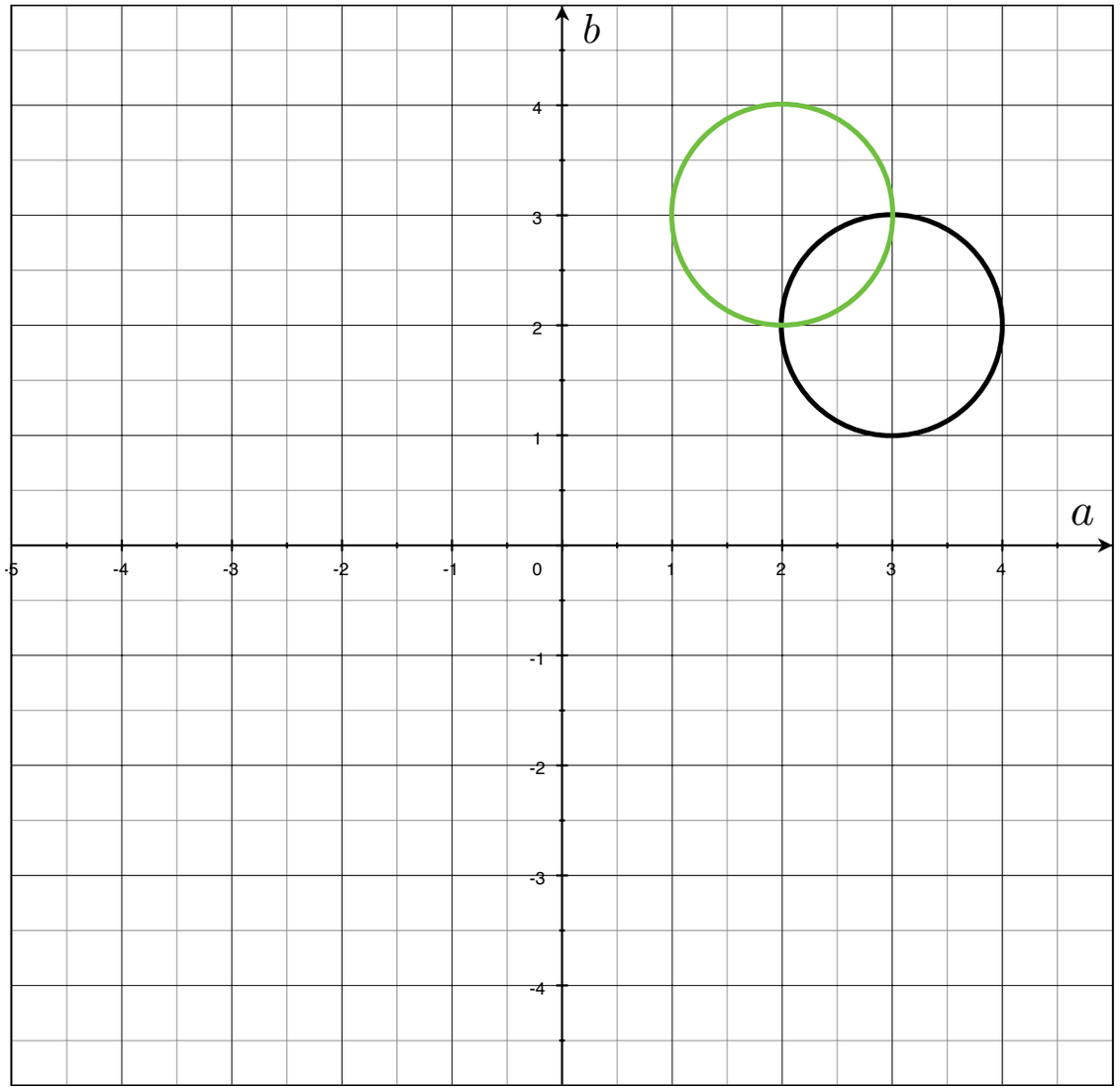

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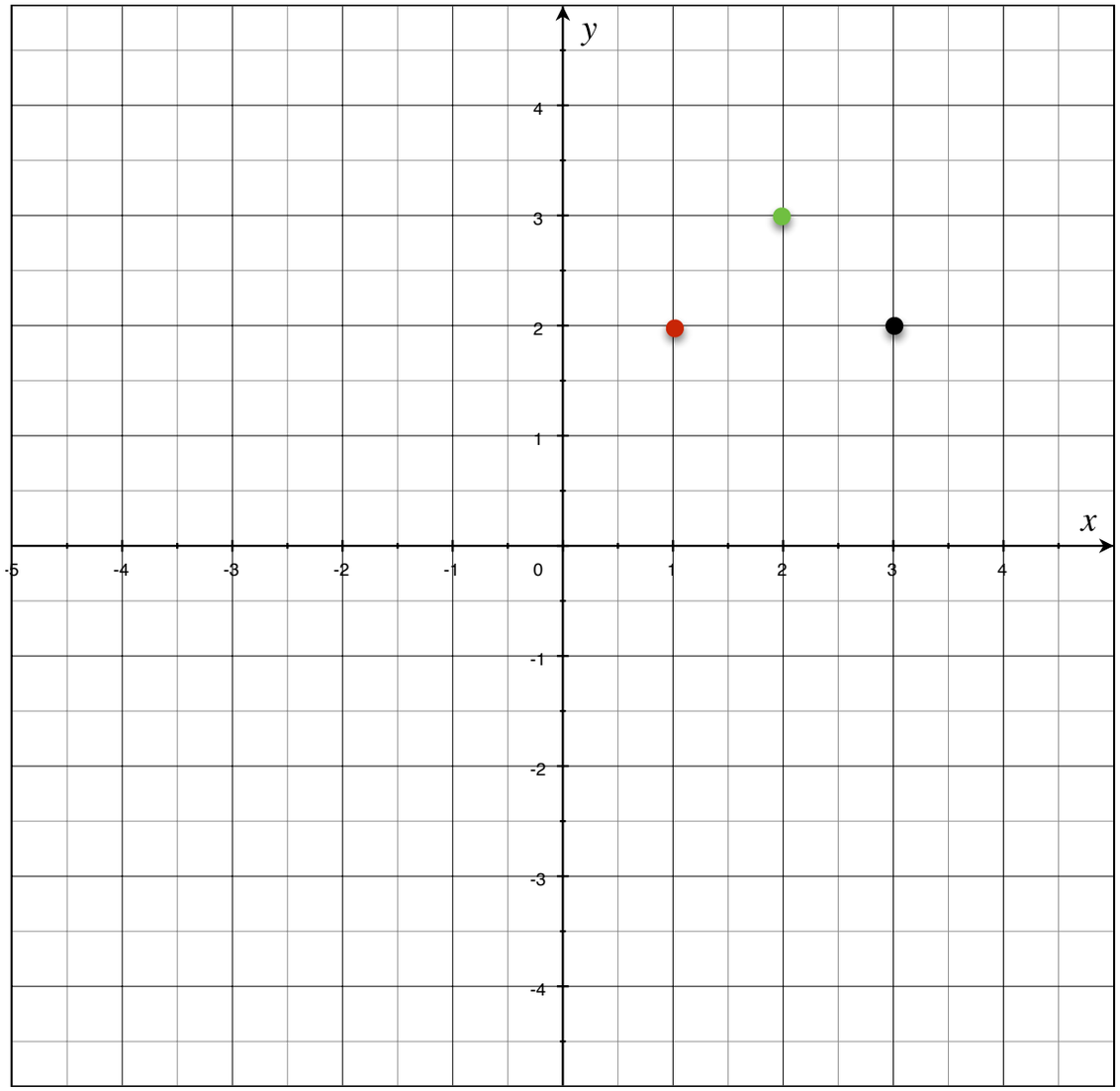

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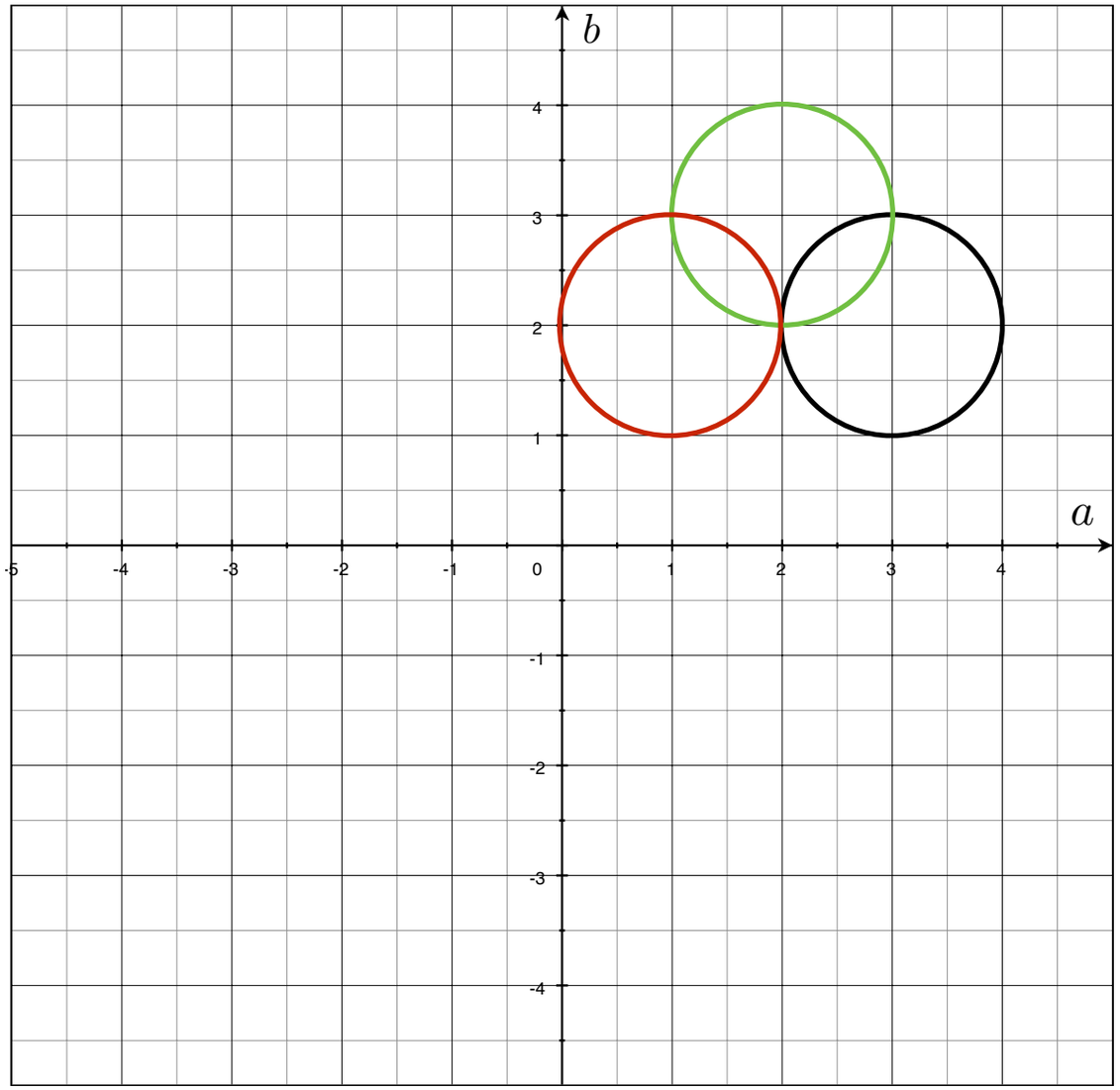

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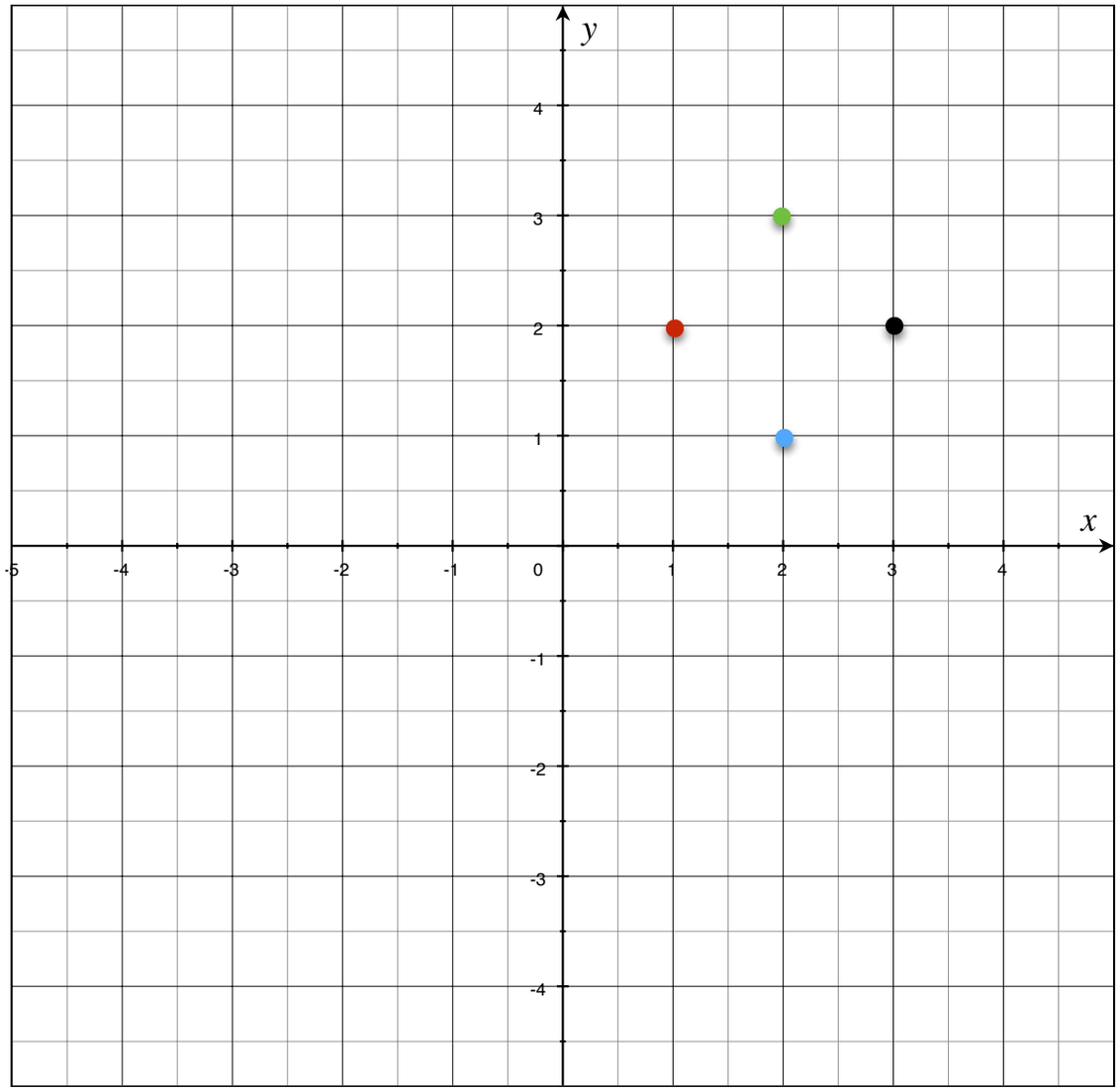

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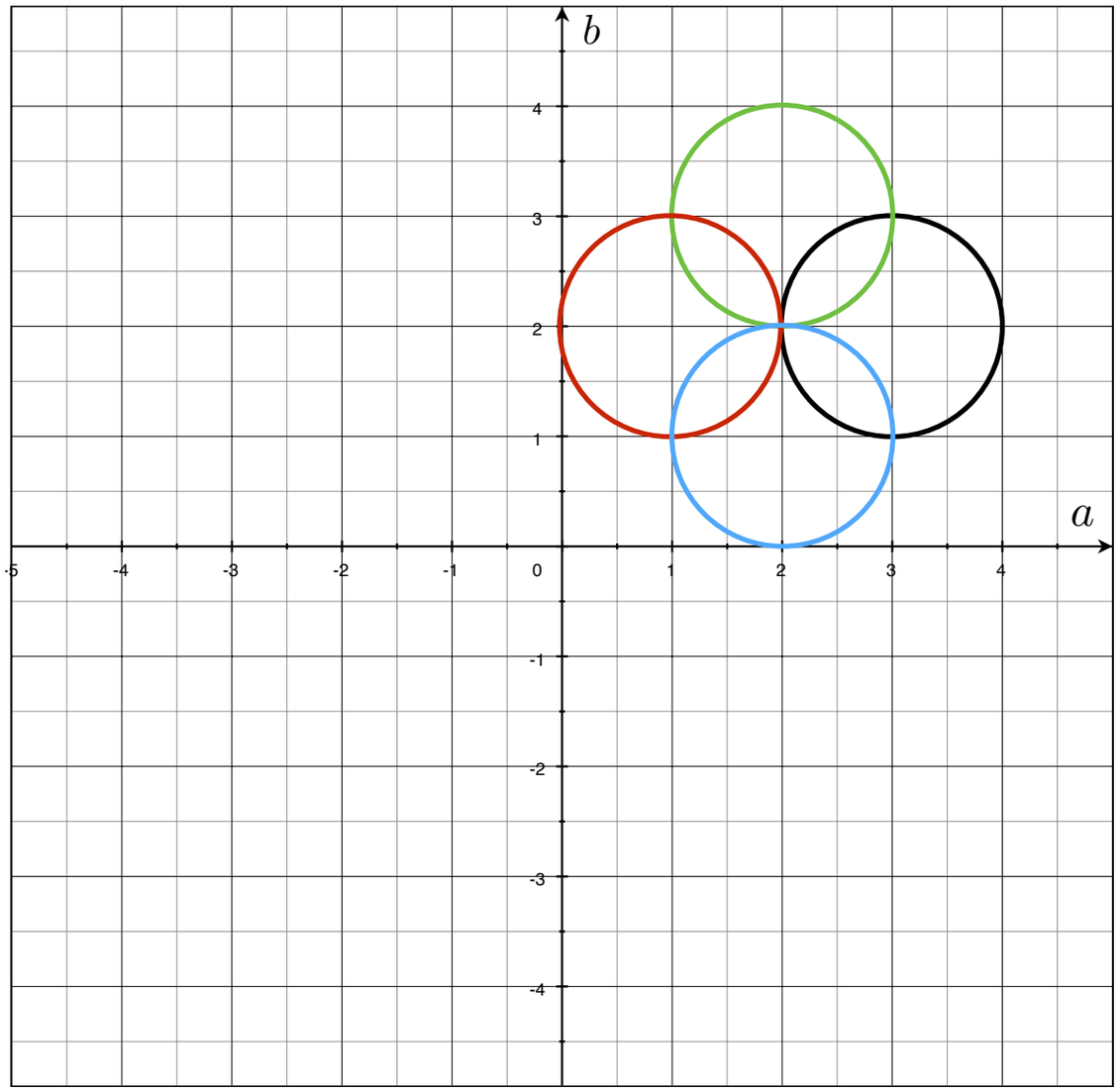
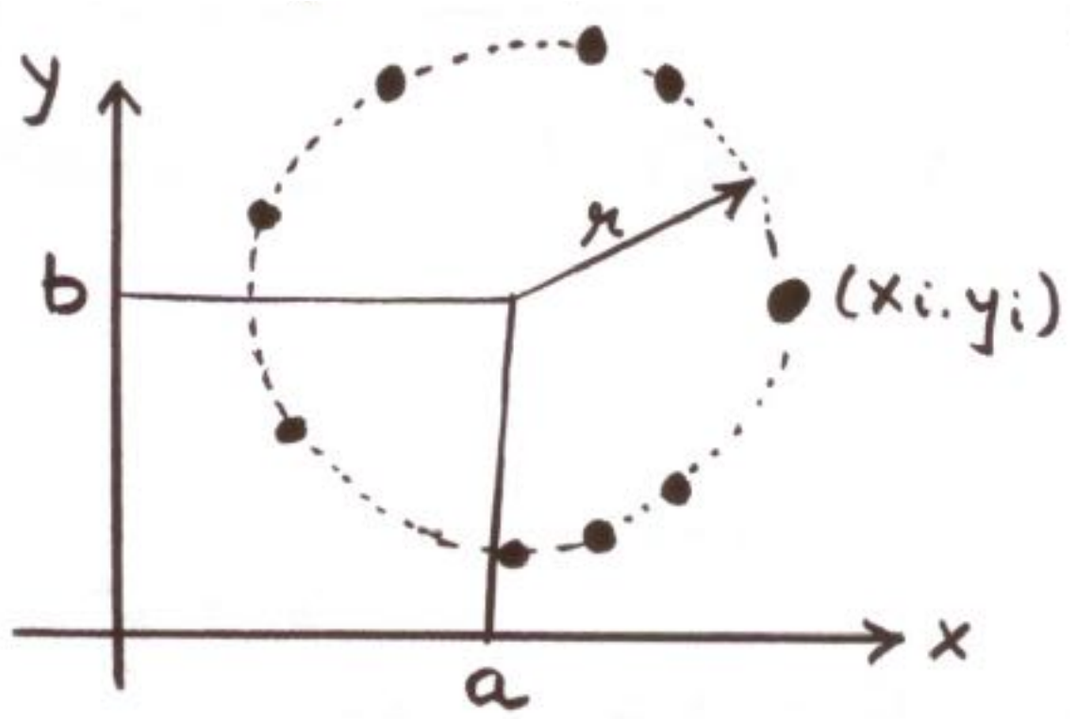


Finding Circles by Hough Transform

Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$



If radius is not known: 3D Hough Space!

Use Accumulator array $A(a, b, r)$

What is the surface in the hough space?

Finding Circles by Hough Transform

Equation of Circle:

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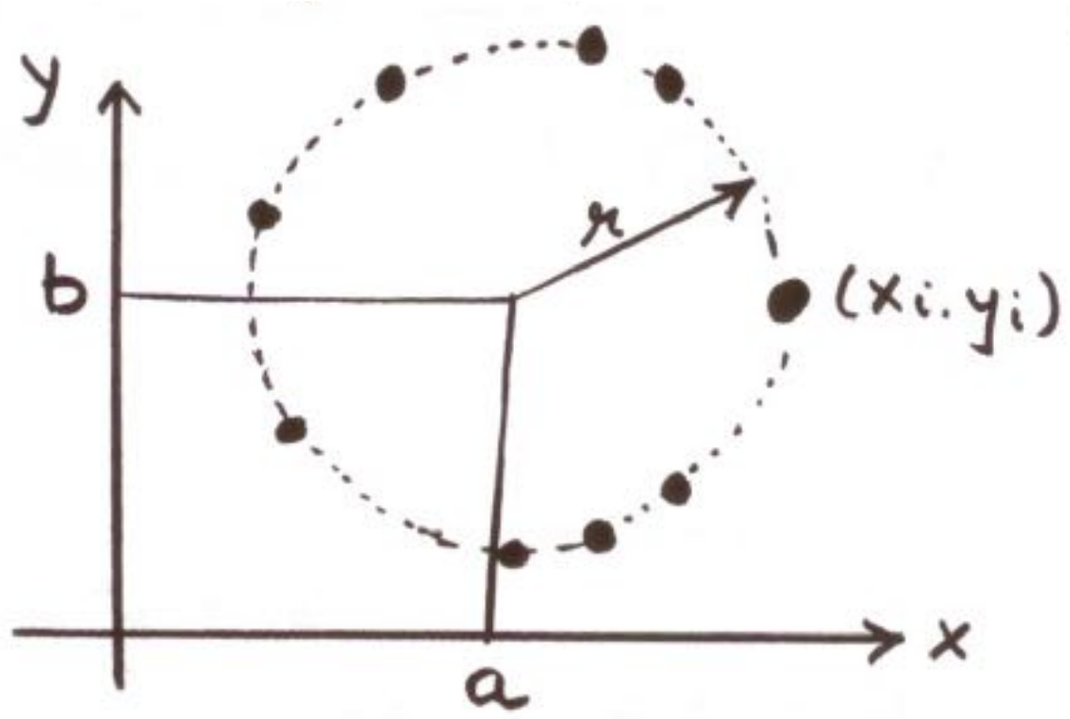

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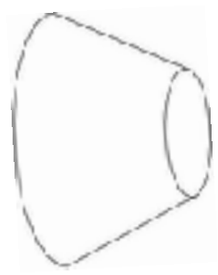
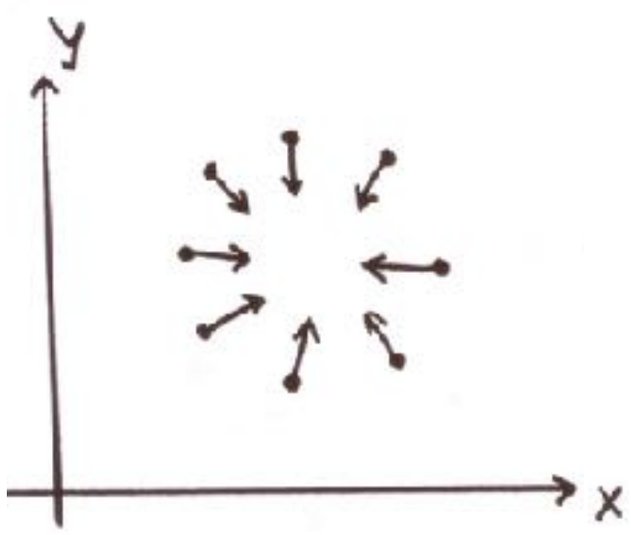


Using Gradient Information

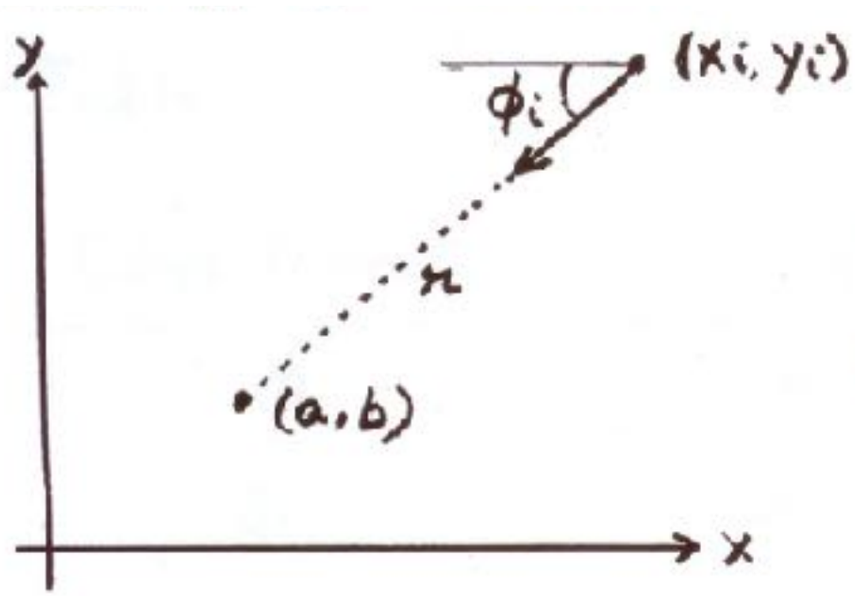
Gradient information can save lot of computation:

Edge Location (x_i, y_i)

Edge Direction ϕ_i



Assume radius is known:



$$a = x - r\cos\phi$$

$$b = y - r\sin\phi$$

Need to increment only one point in accumulator!!

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

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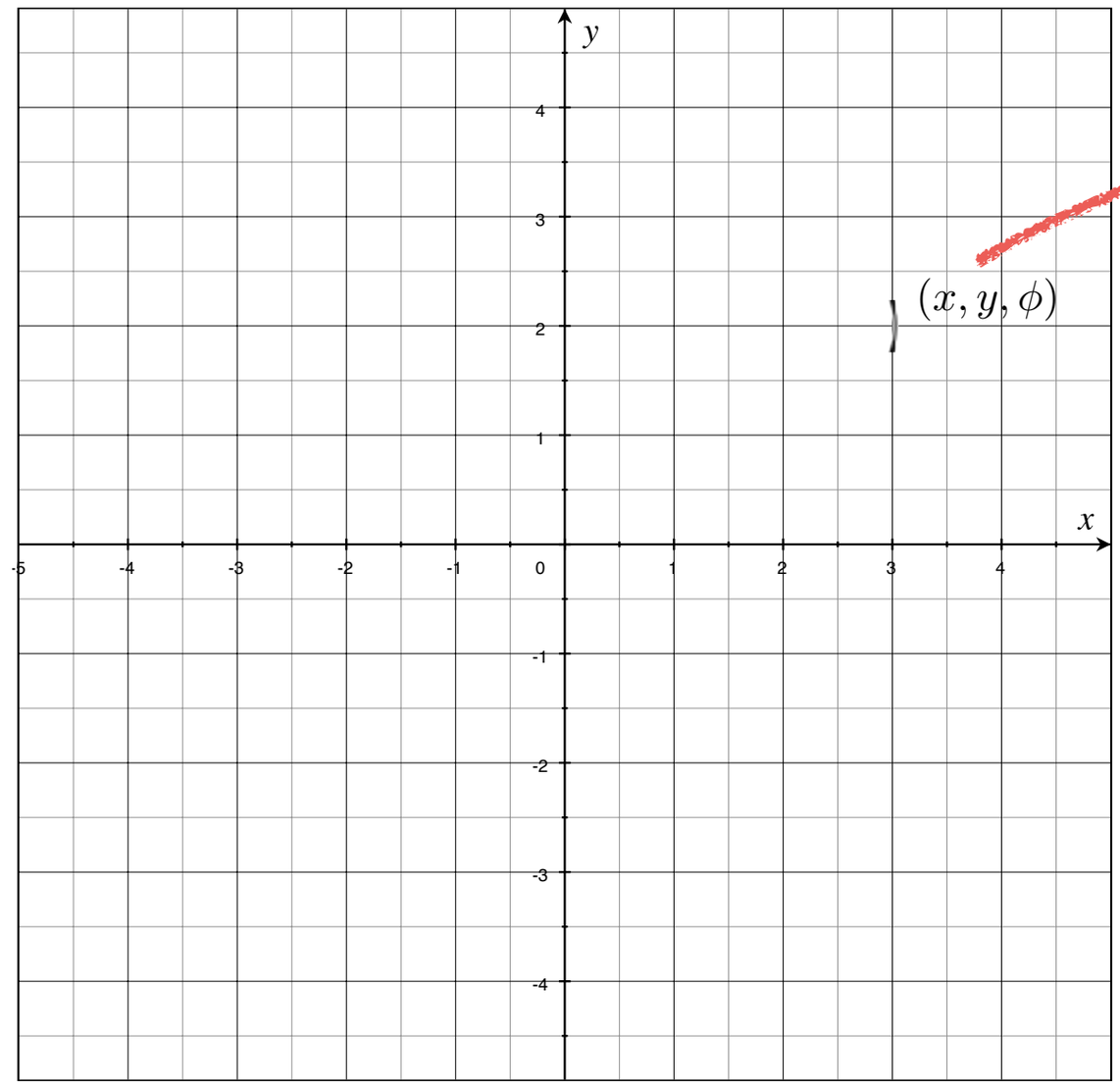
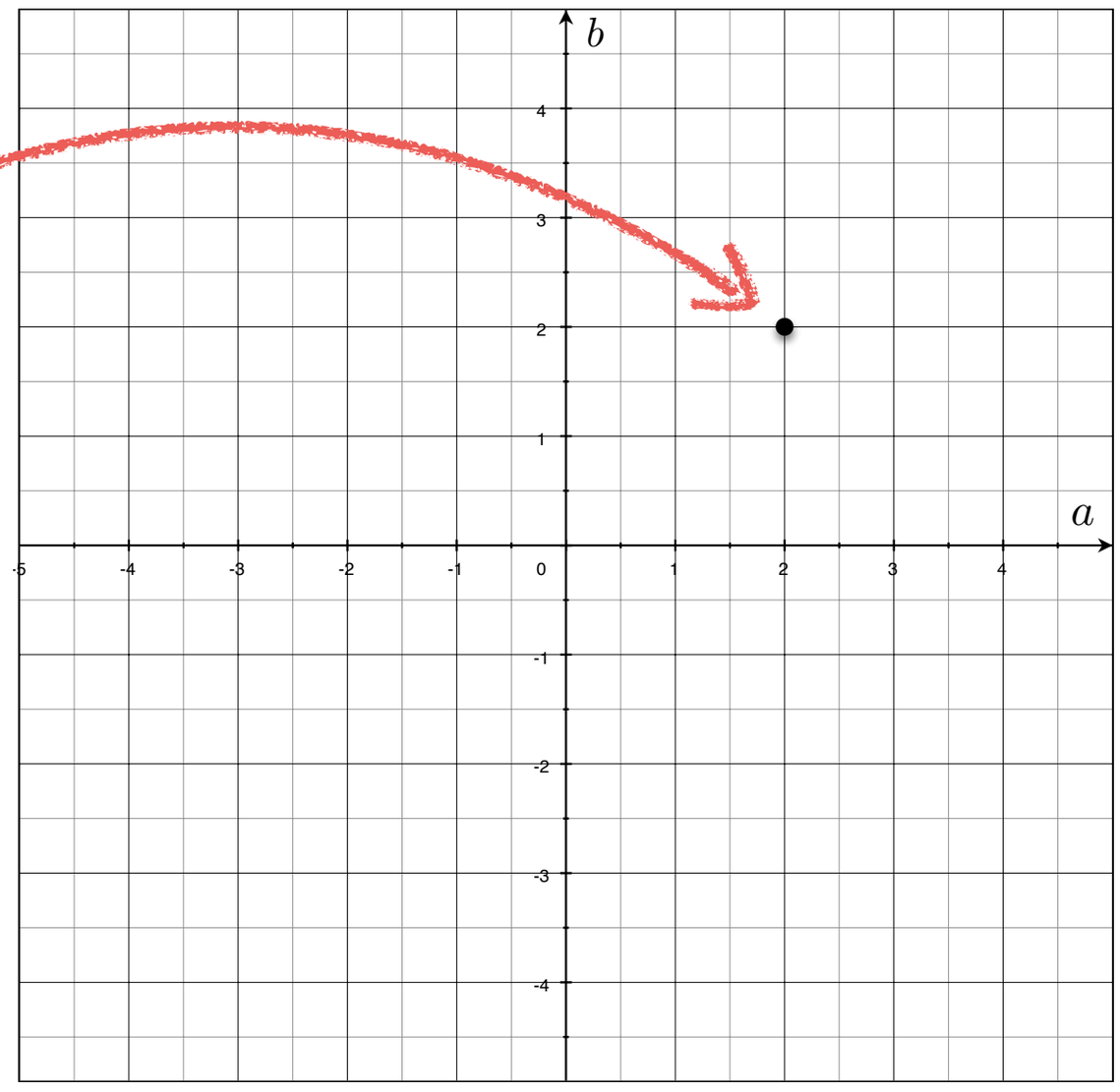


(x, y, ϕ)



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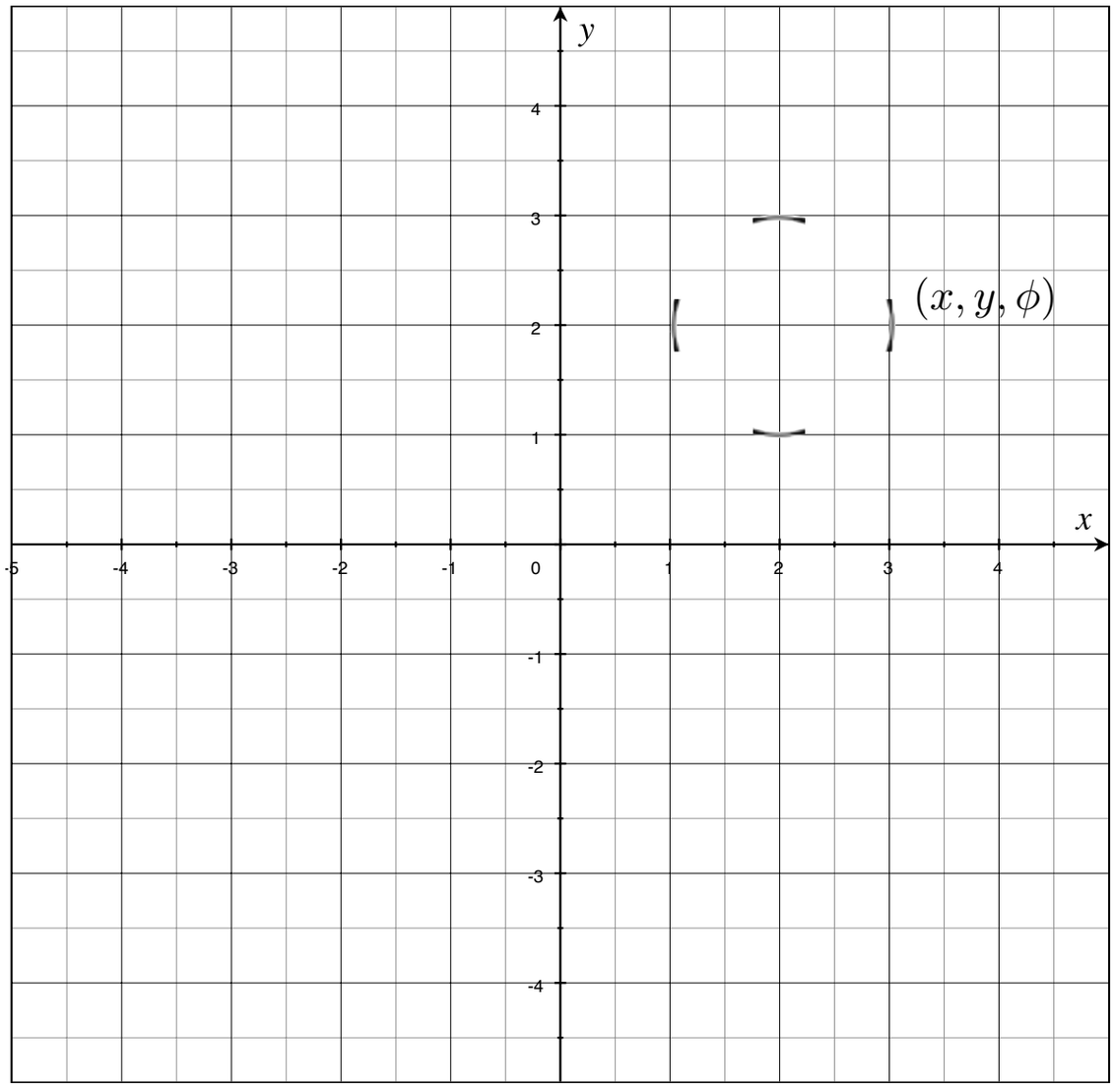

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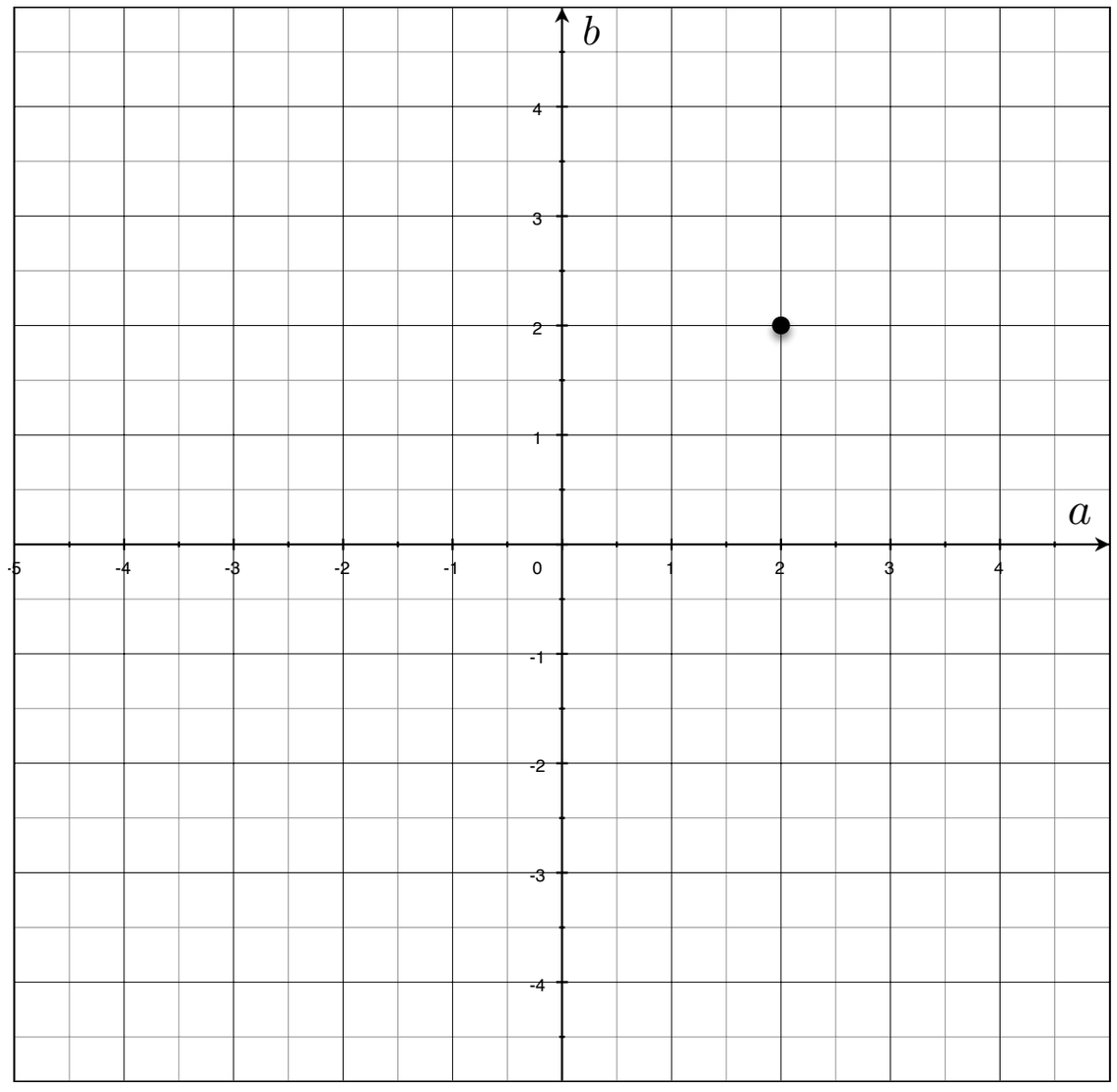
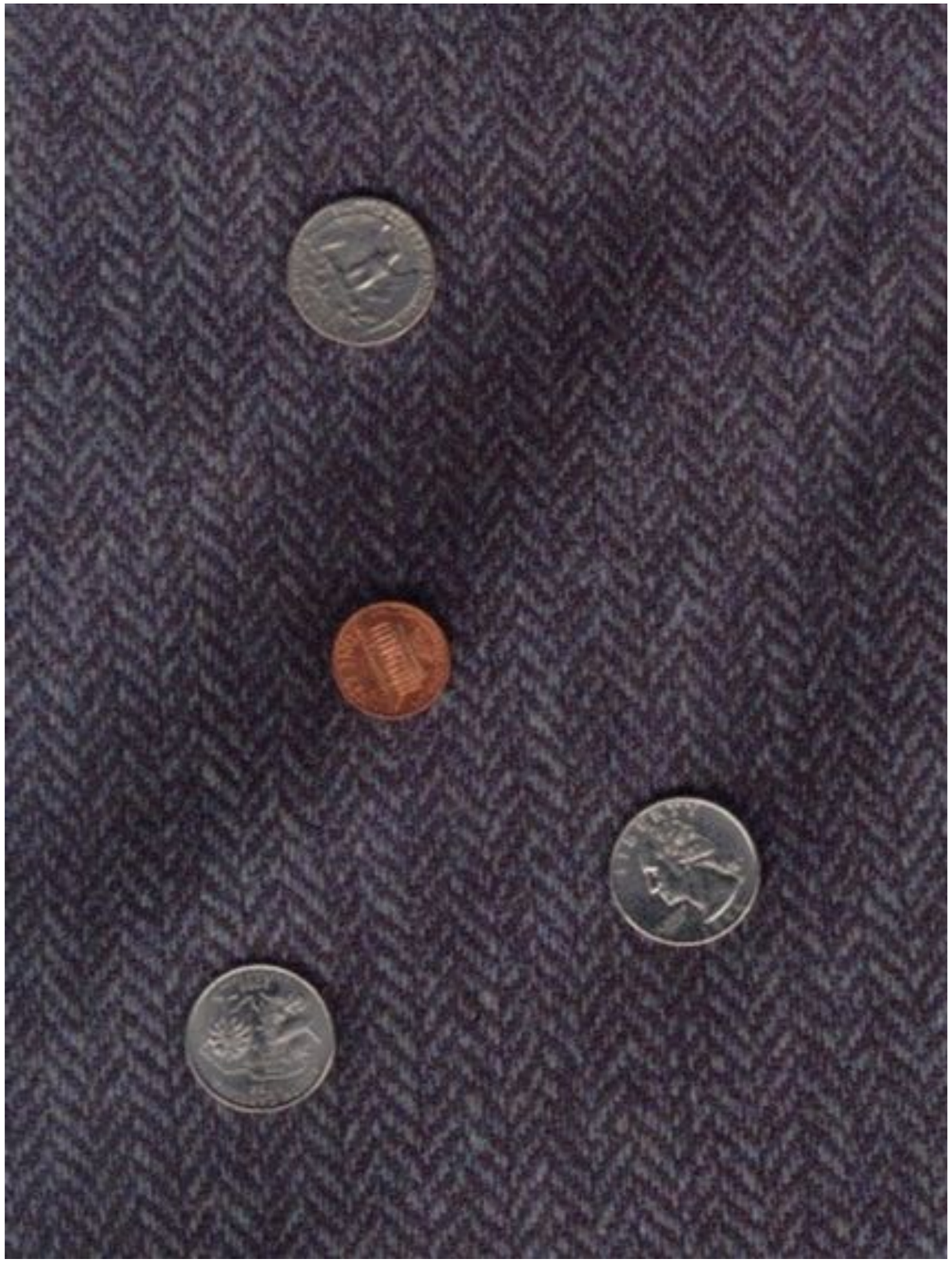
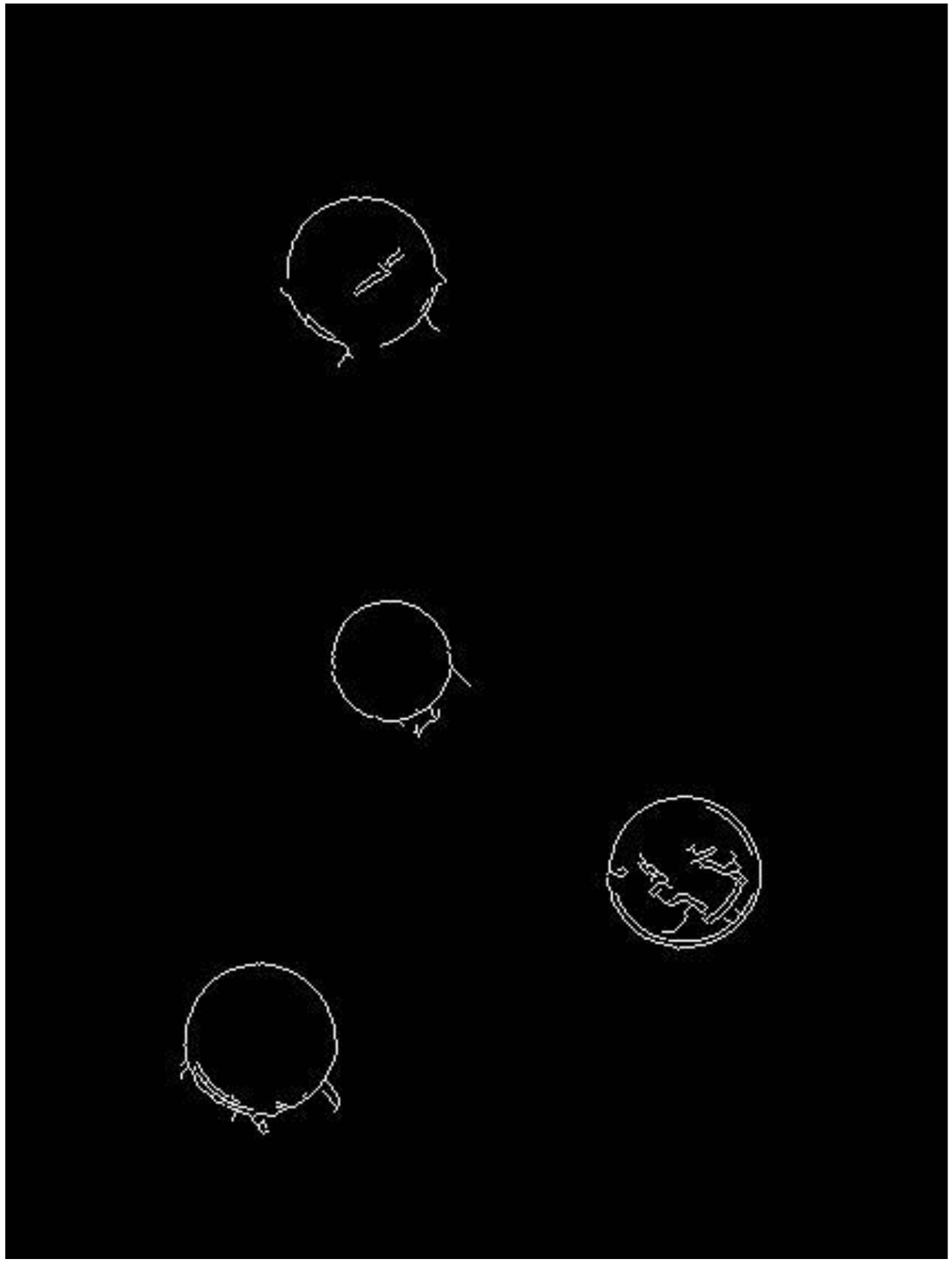


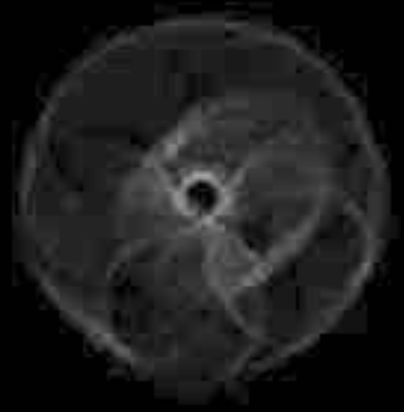




Pennie Hough detector

Quarter Hough detector

Pennie Hough detector



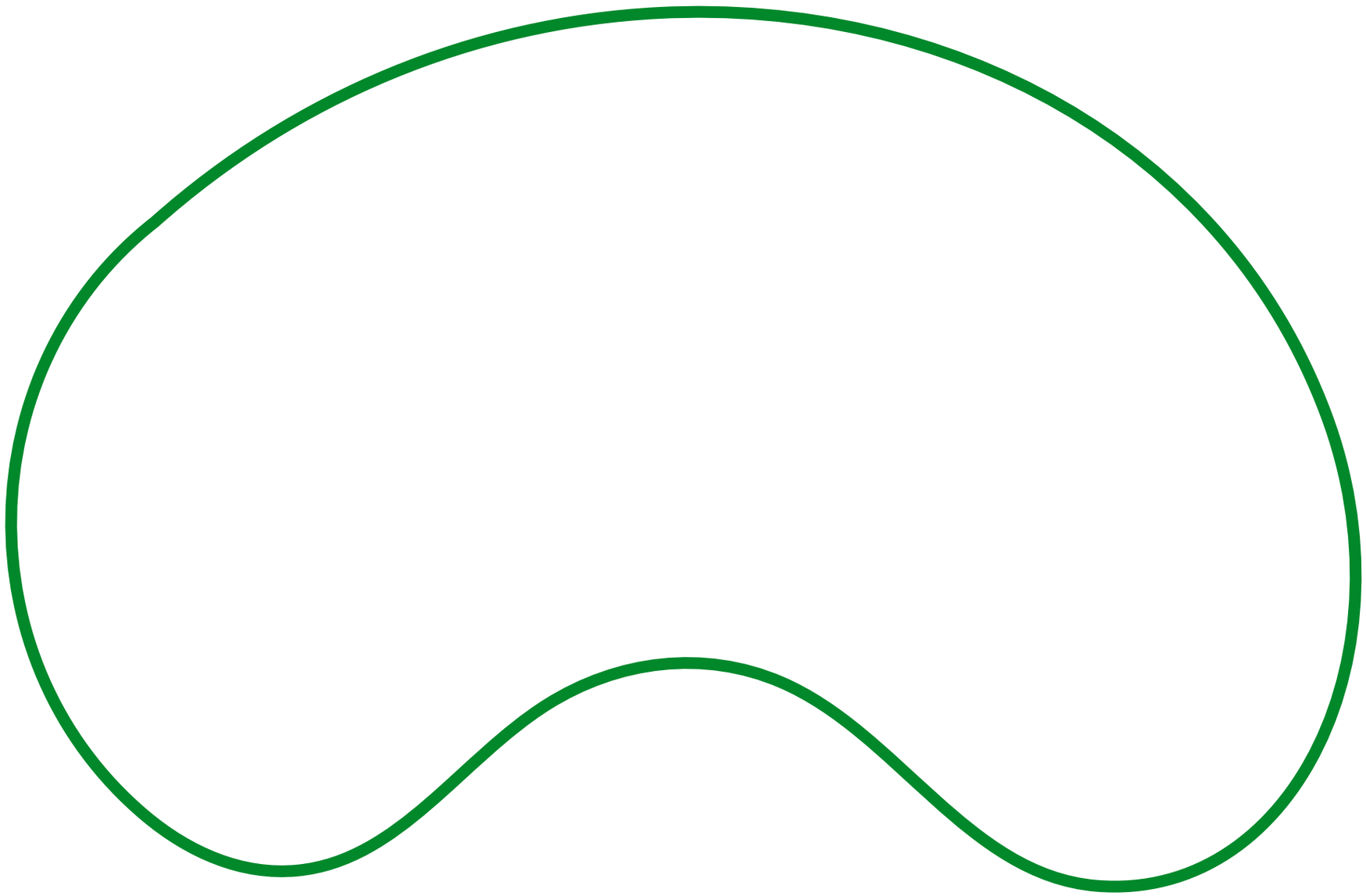
Quarter Hough detector

Can you use Hough Transforms for other objects, beyond lines and circles?

Hough Shapes

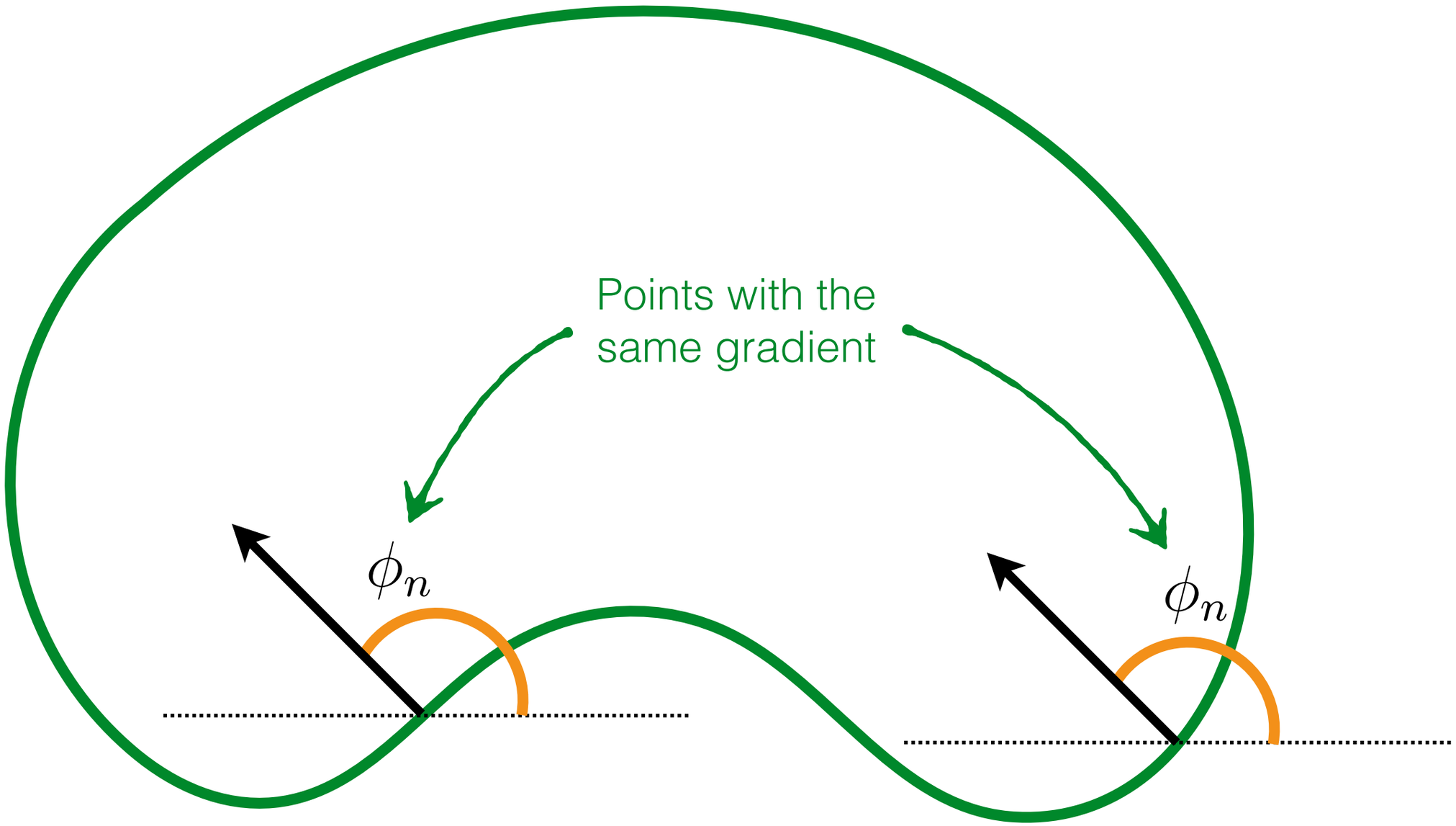
Generalized Hough Transform

Edge Direction	$k = (r, \alpha)$



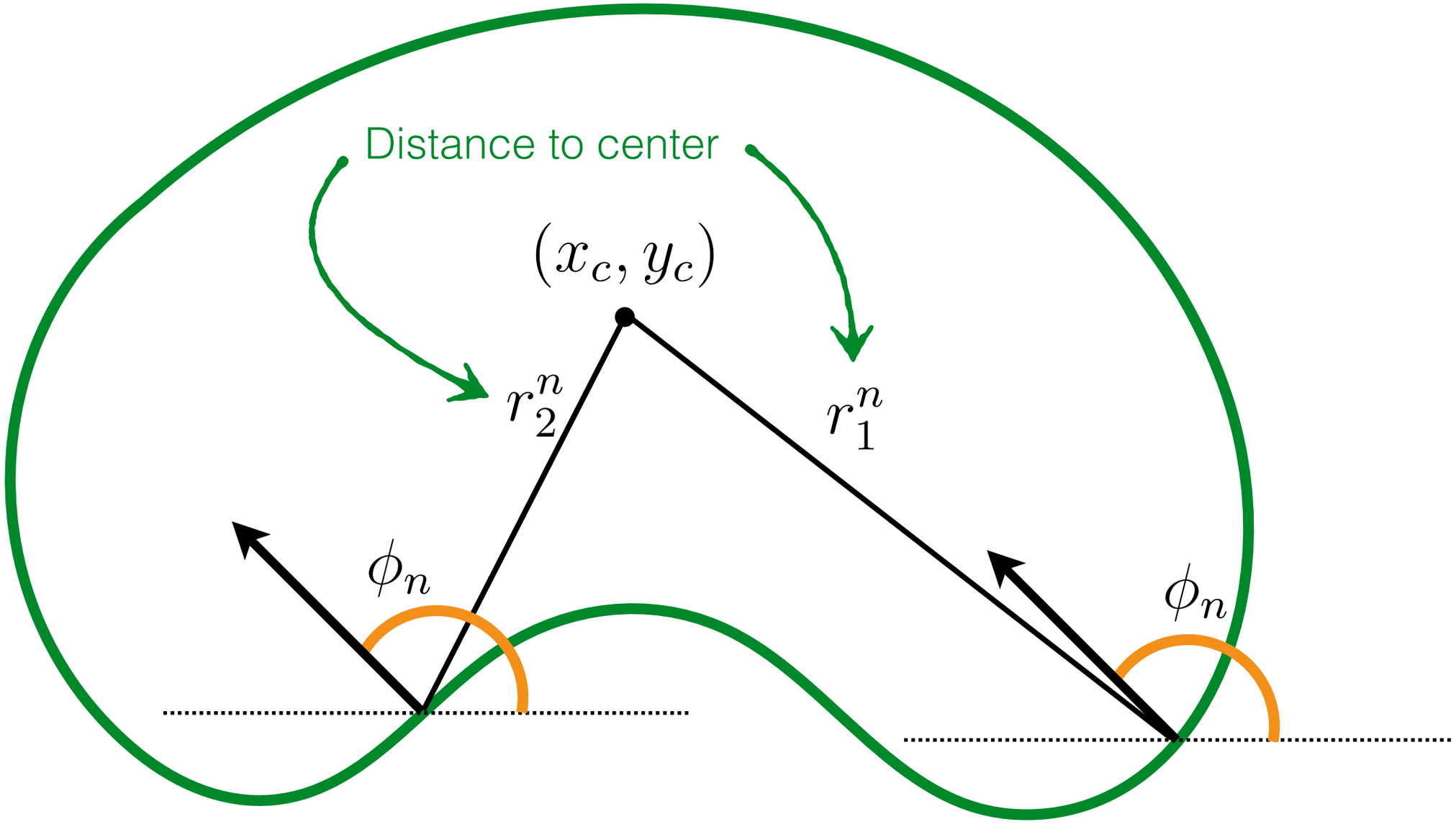
Generalized Hough Transform

Edge Direction	$k = (r, \alpha)$
ϕ_2	



Generalized Hough Transform

Edge Direction	$k = (\ r, \alpha\ )$
ϕ_2	$k_1^2,\ k_2^2$



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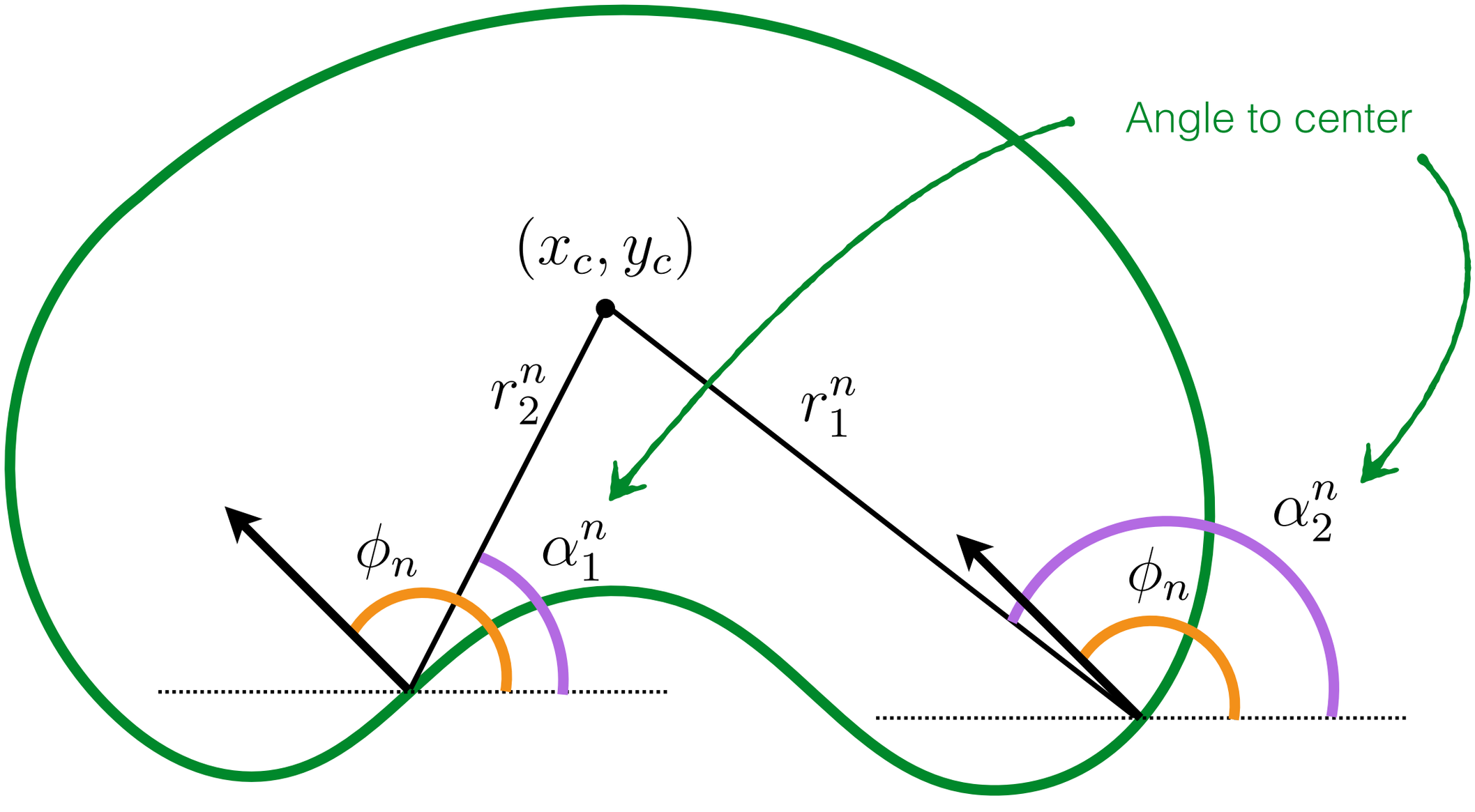
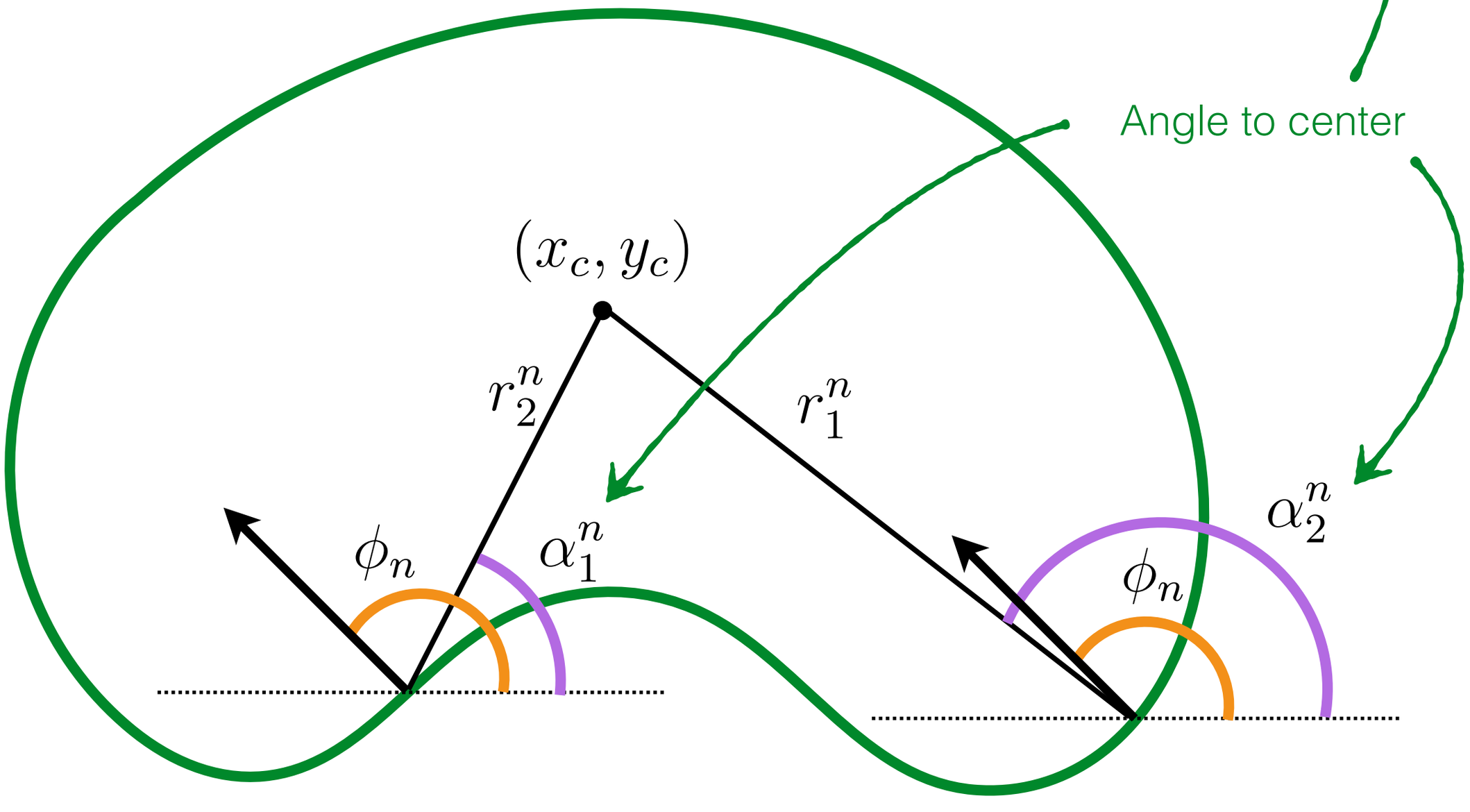


Generalized Hough Transform

Edge Direction	$k = (\ r, \alpha\ )$
ϕ_2	$k_1^2,\ k_2^2$

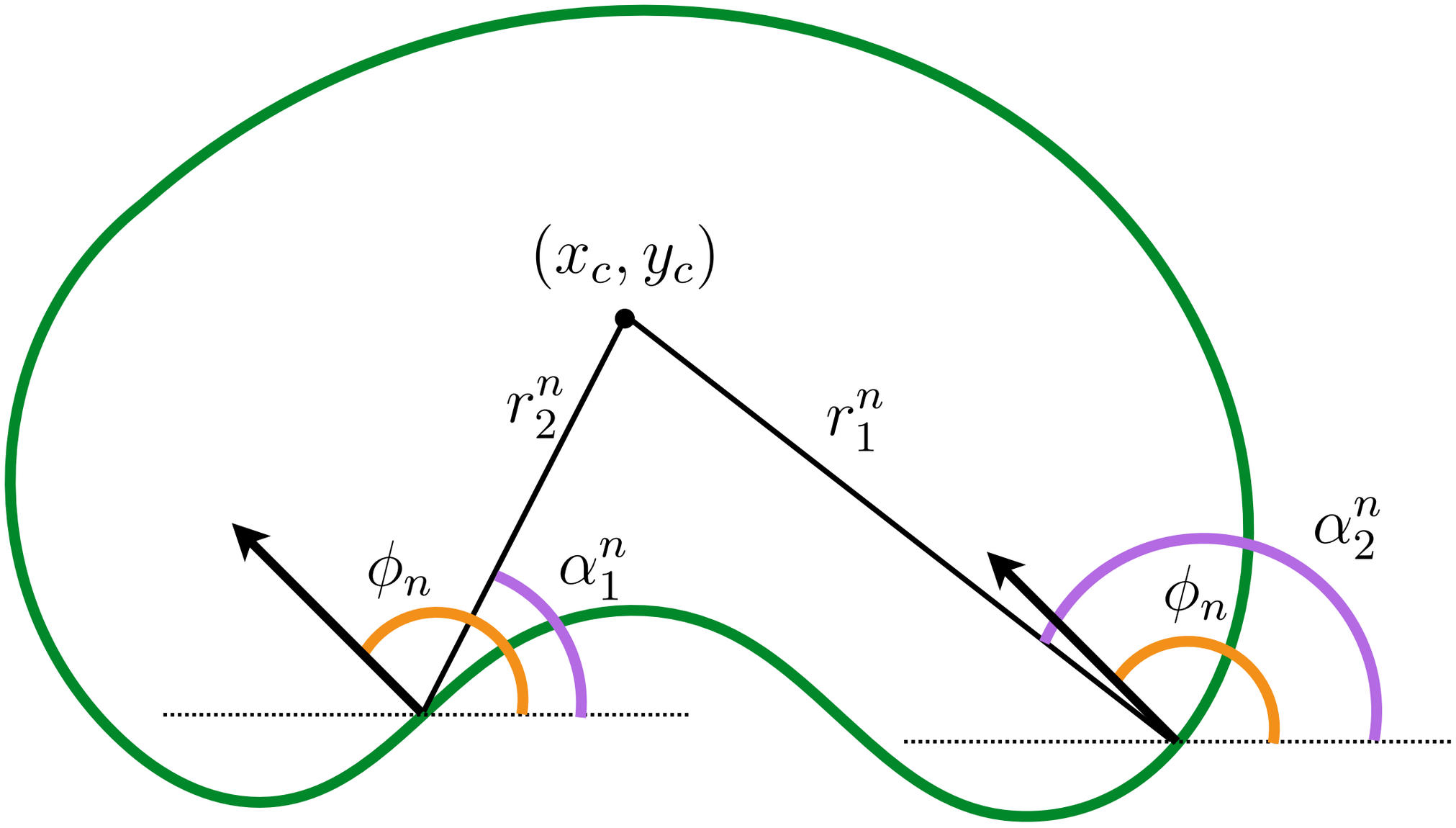
Store in table

Angle to center



Generalized Hough Transform

Edge Direction	$k = (\ r, \alpha\ )$
ϕ_1	$k_1^1,\ k_2^1,\ k_3^1$
ϕ_2	$k_1^2,\ k_2^2$
$\vdots$	$\vdots$
ϕ_N	$k_1^N,\ k_2^N,\ k_3^N,\ k_4^N$



Find Object Center (x_c, y_c) given edges (x_i, y_i, ϕ_i)

Create Accumulator Array $A(x_c, y_c)$

Initialize: $A(x_c, y_c) = 0 \quad \forall (x_c, y_c)$

For each edge point (x_i, y_i, ϕ_i)

For each entry in table, compute:

$$x_c = x_i + r_k^i \cos\alpha_k^i$$

$$y_c = y_i + r_k^i \sin\alpha_k^i$$

Increment Accumulator: $A(x_c, y_c) = A(x_c, y_c) + 1$

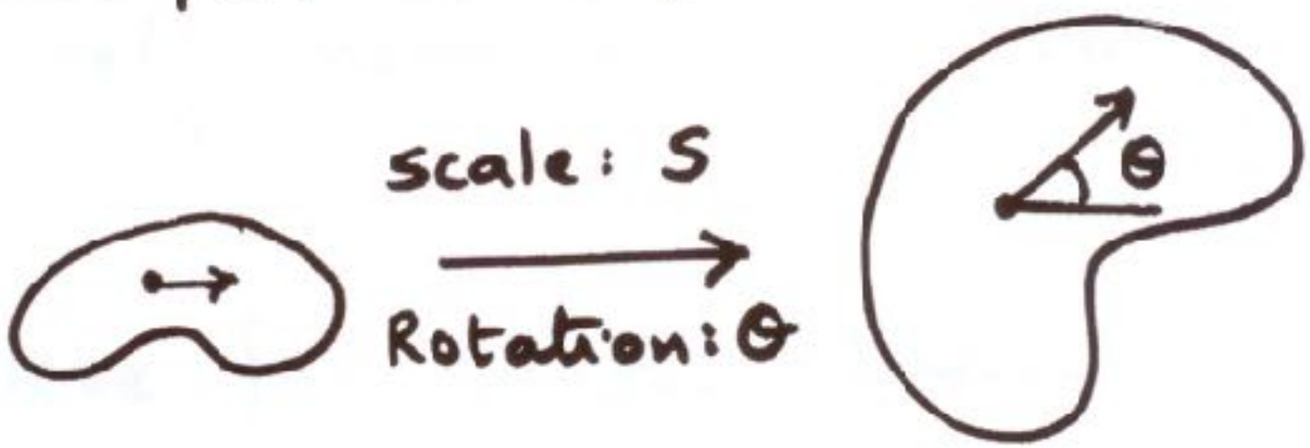
Find Local Maxima in $A(x_c, y_c)$

What happens if you slightly rotate or scale the shape?

Scale & Rotation:

Use Accumulator Array:

$$A[x_c, y_c, S, \theta]$$



Use:

$$x_c = x_i + r^i_k S \cos(\alpha^i_k + \theta)$$

$$y_c = y_i + r^i_k S \sin(\alpha^i_k + \theta)$$

$$A(x_c, y_c, S, \theta) = A(x_c, y_c, S, \theta) + 1 .$$