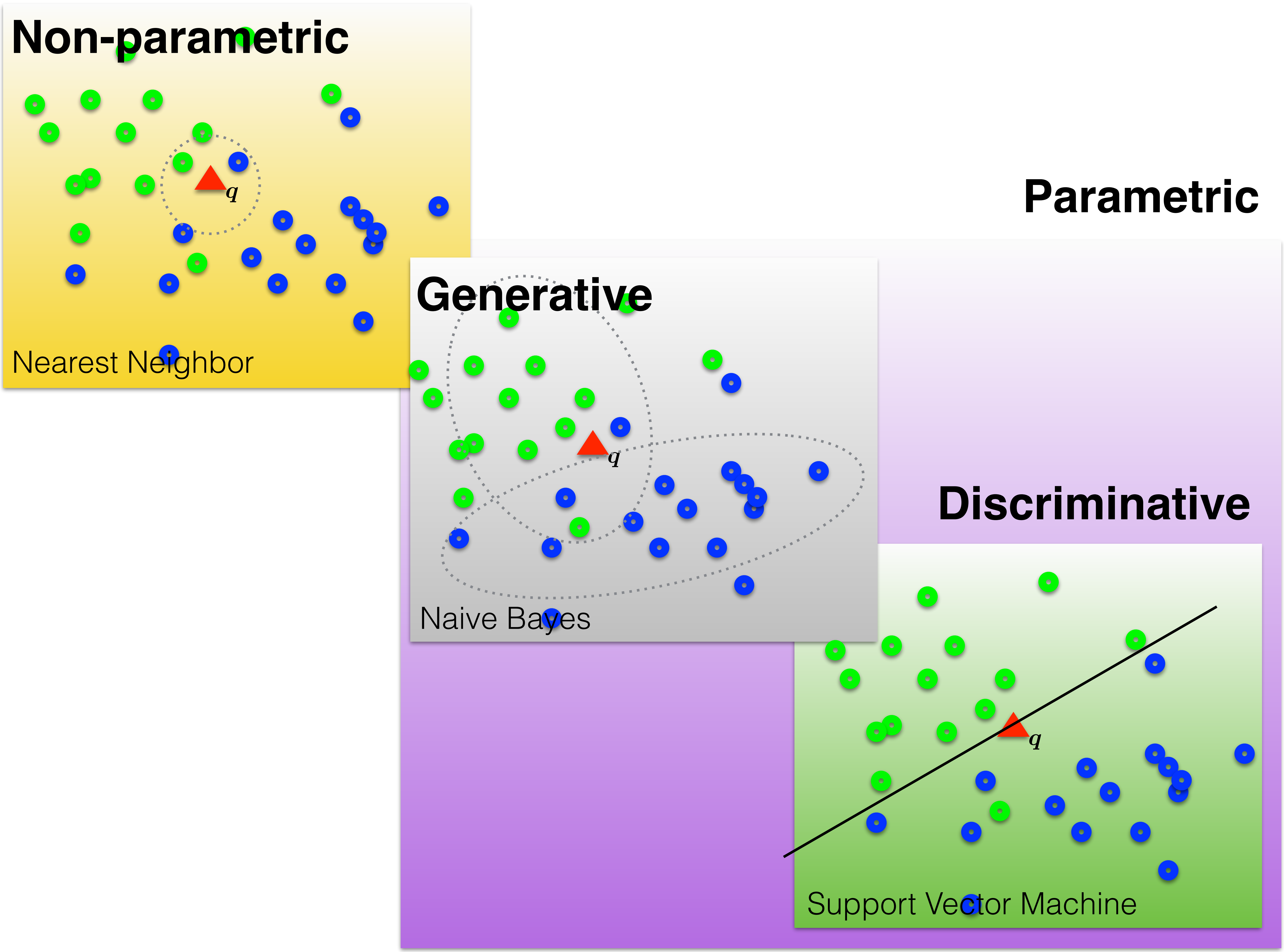
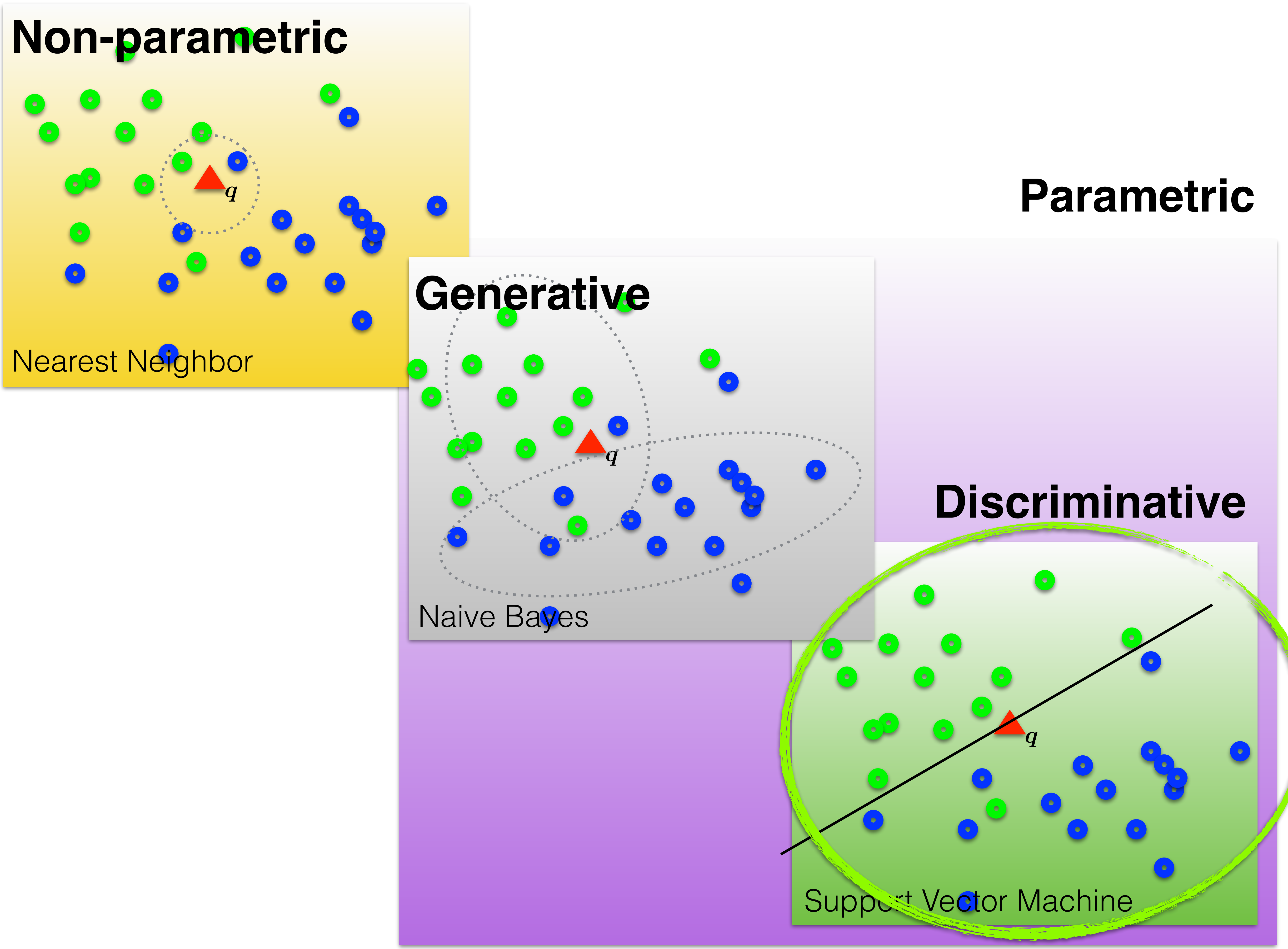
Support Vector Machine

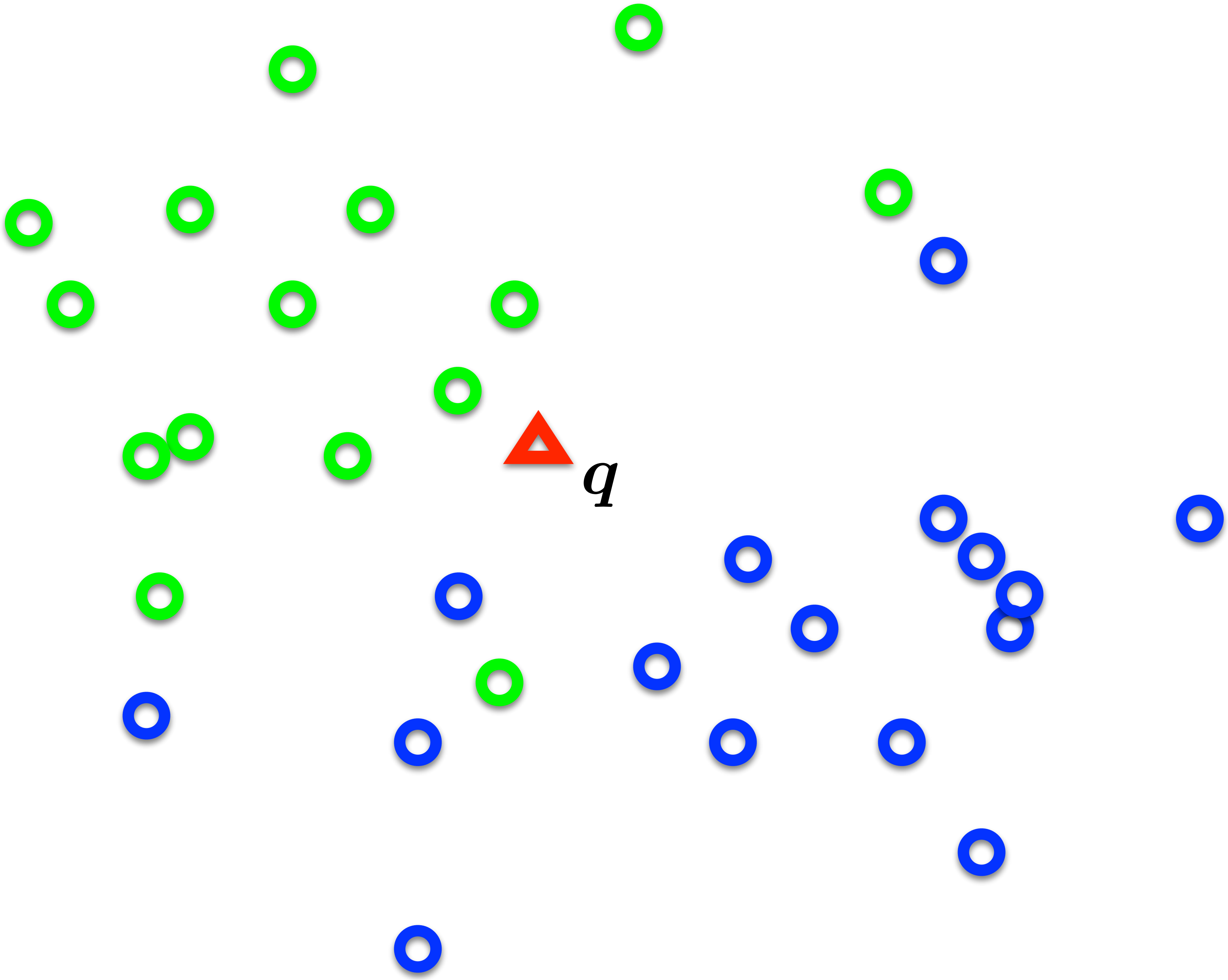


Non-parametric
Nearest Neighbor
Parametric
Generative
Naive Bayes
Discriminative
Support Vector Machine
q
q
q



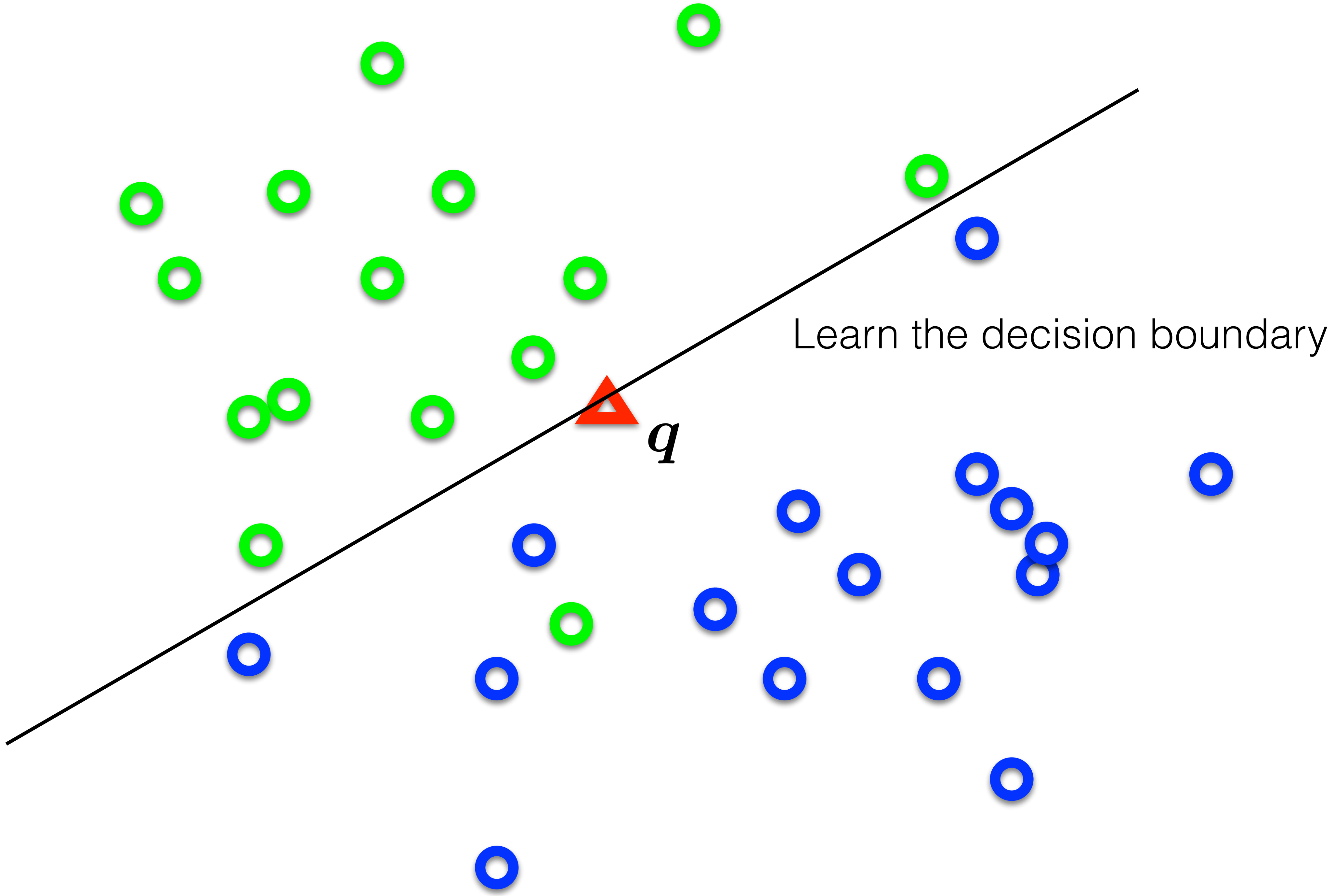
Non-parametric
q
Nearest Neighbor
Parametric
Generative
q
Naive Bayes
Discriminative
q
Support Vector Machine

Distribution of data from two classes

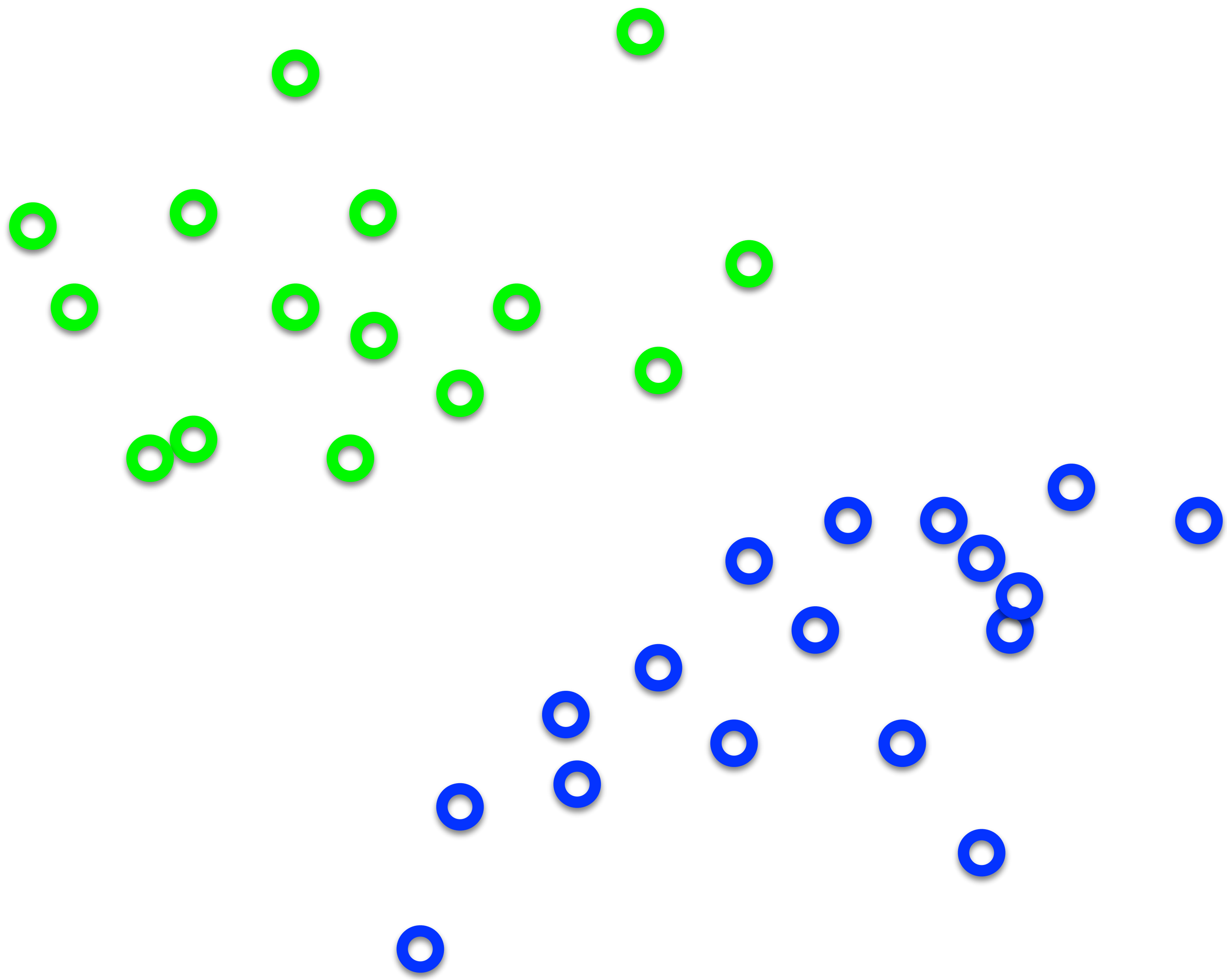


Which class does q belong too?

Distribution of data from two classes



What's the best **w**?



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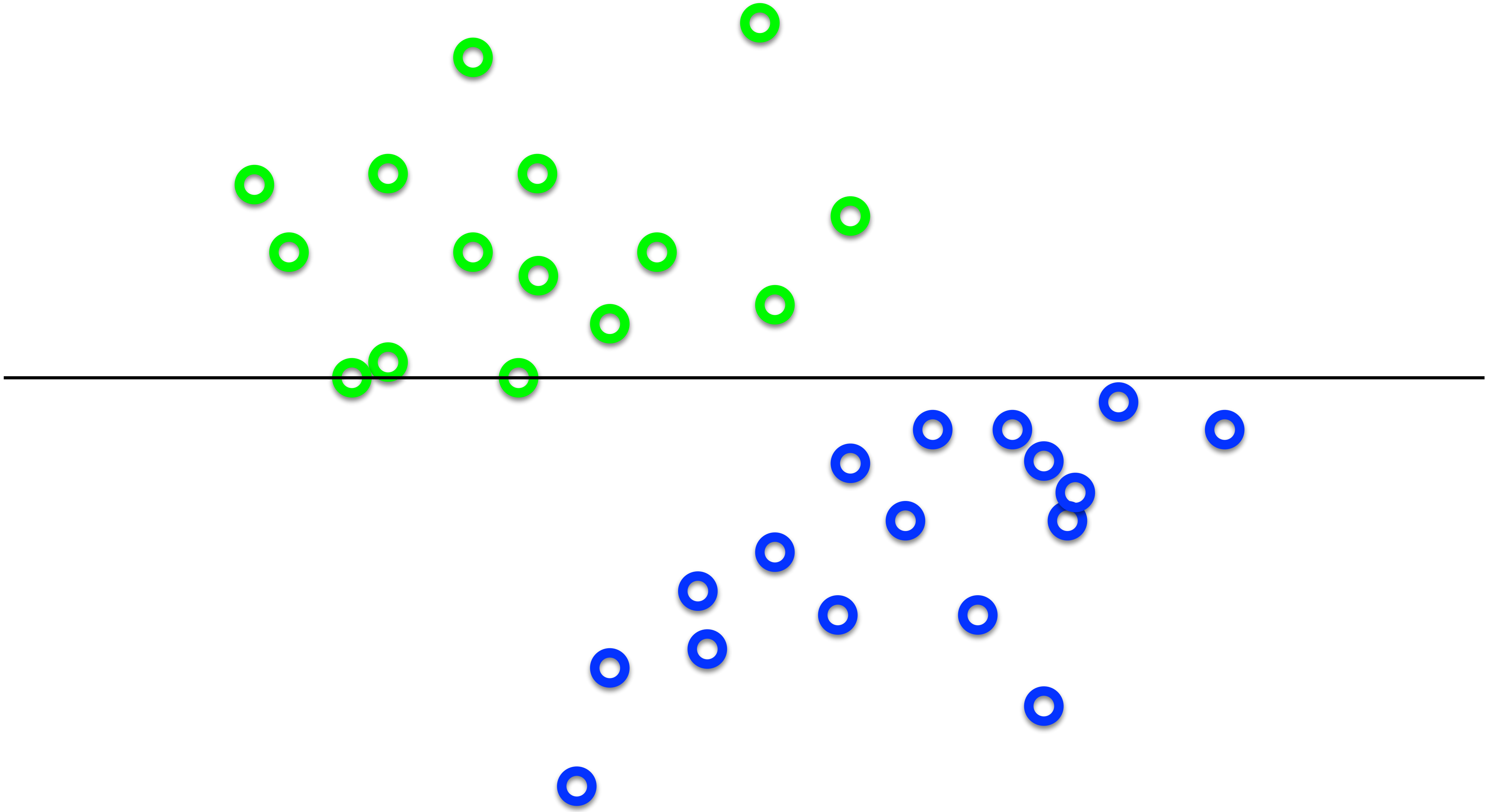

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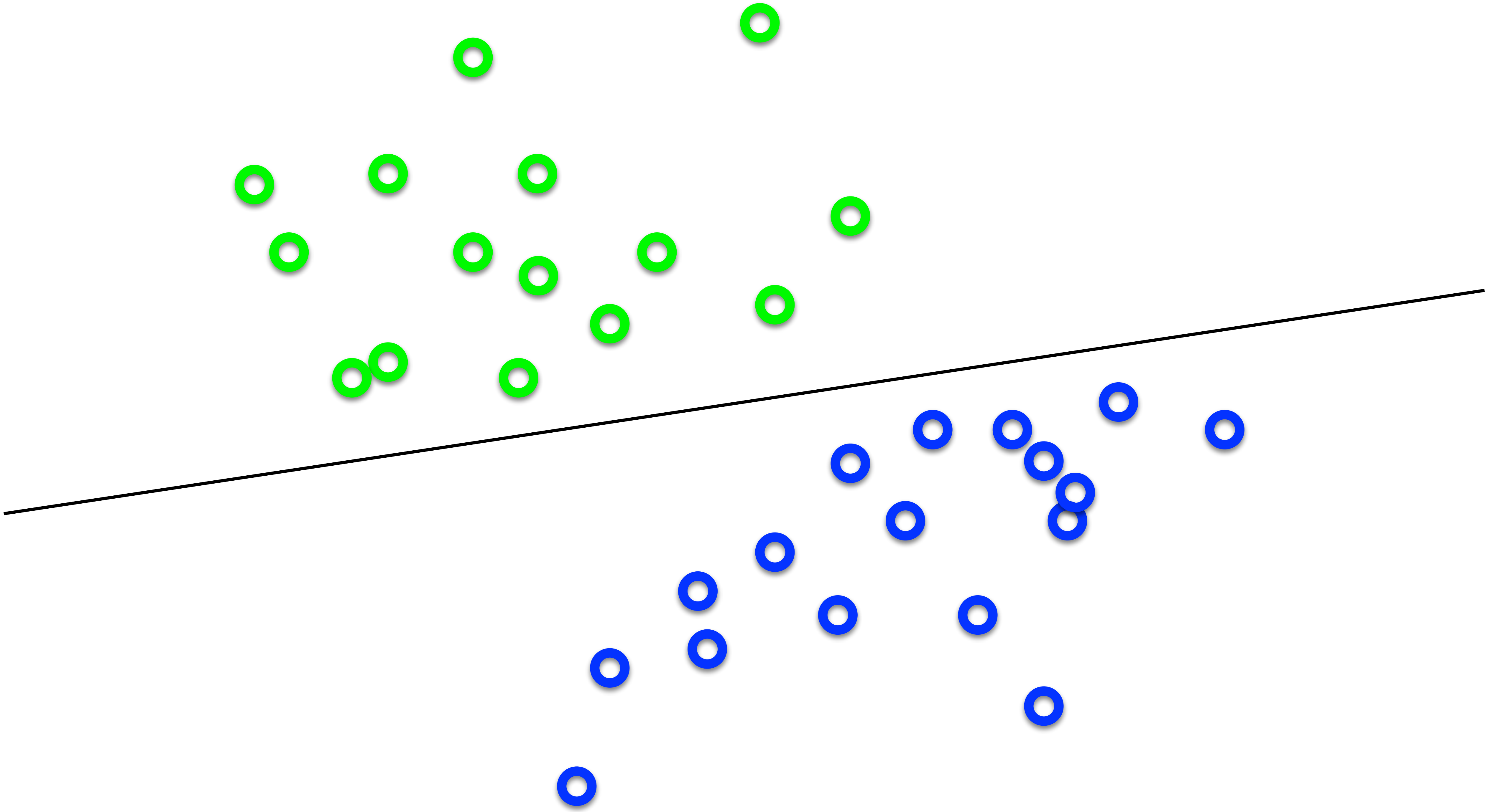

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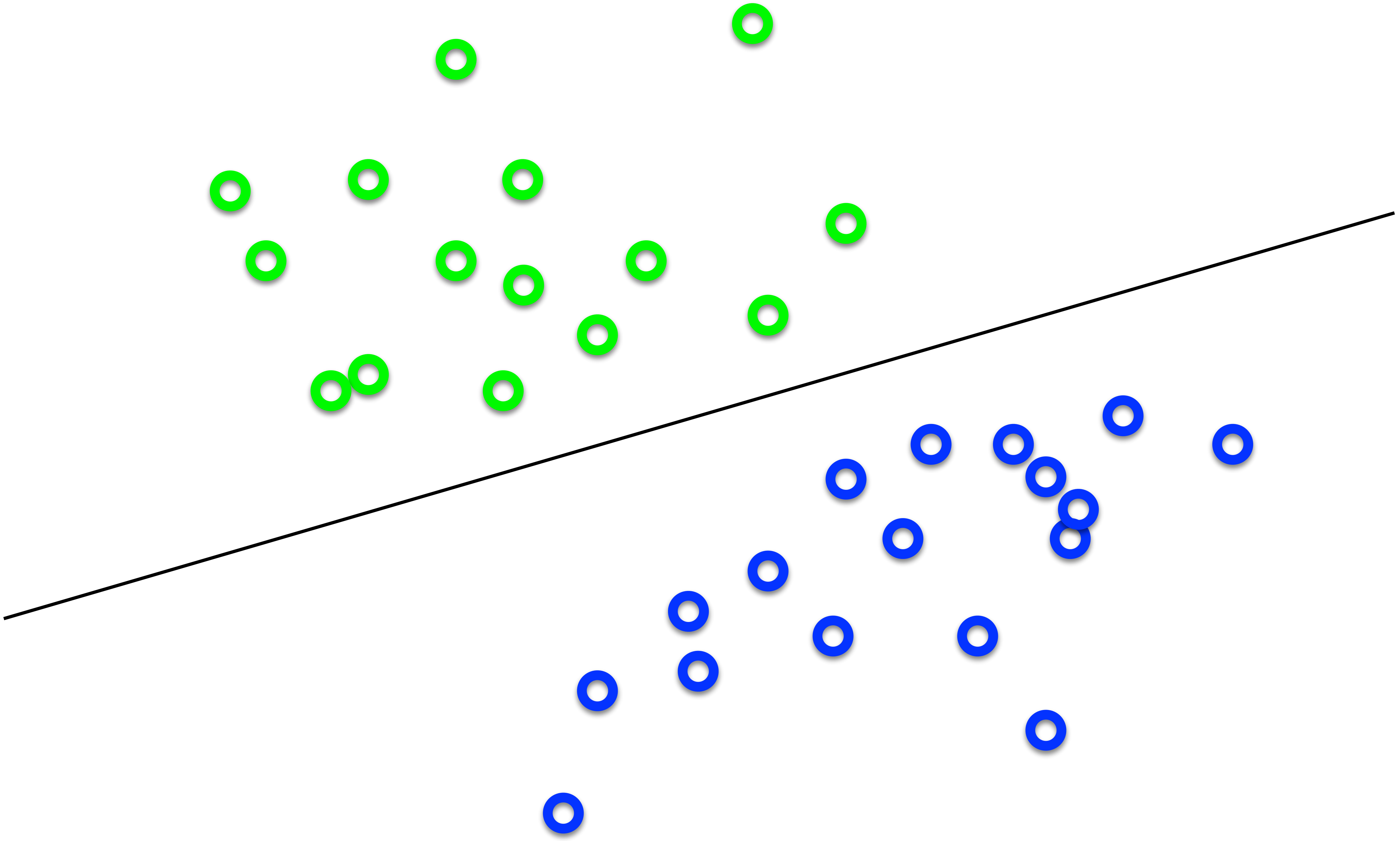

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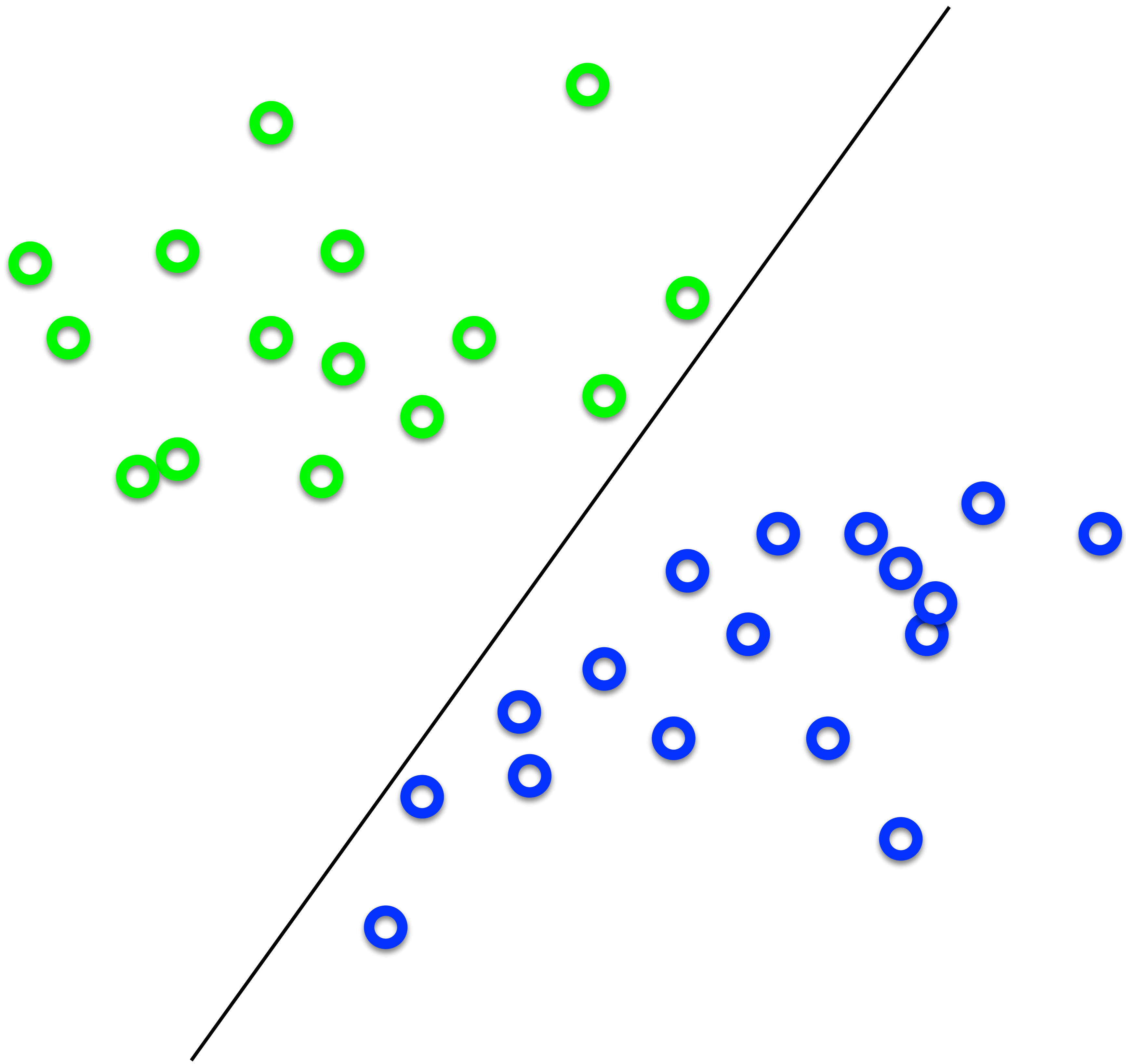
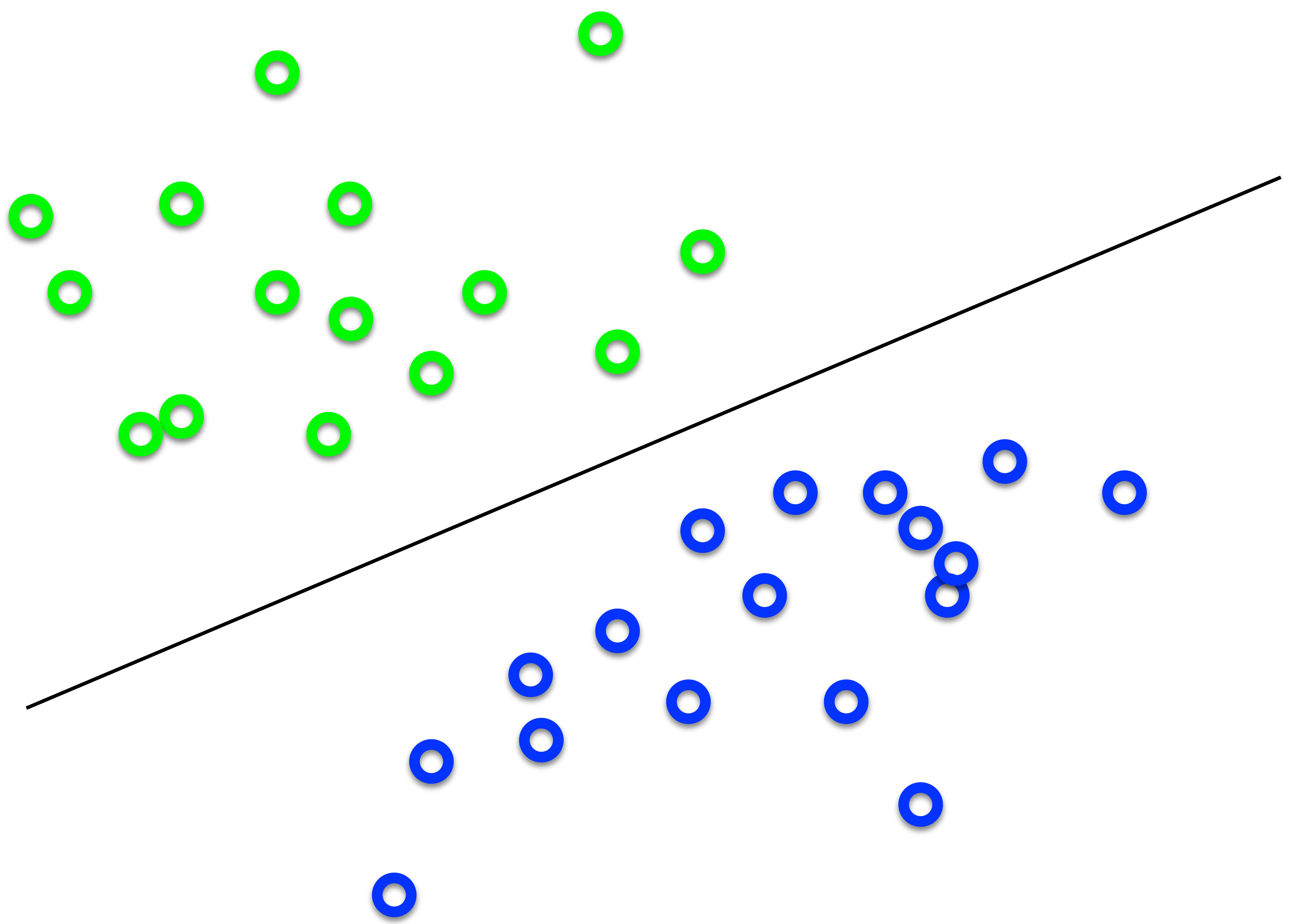
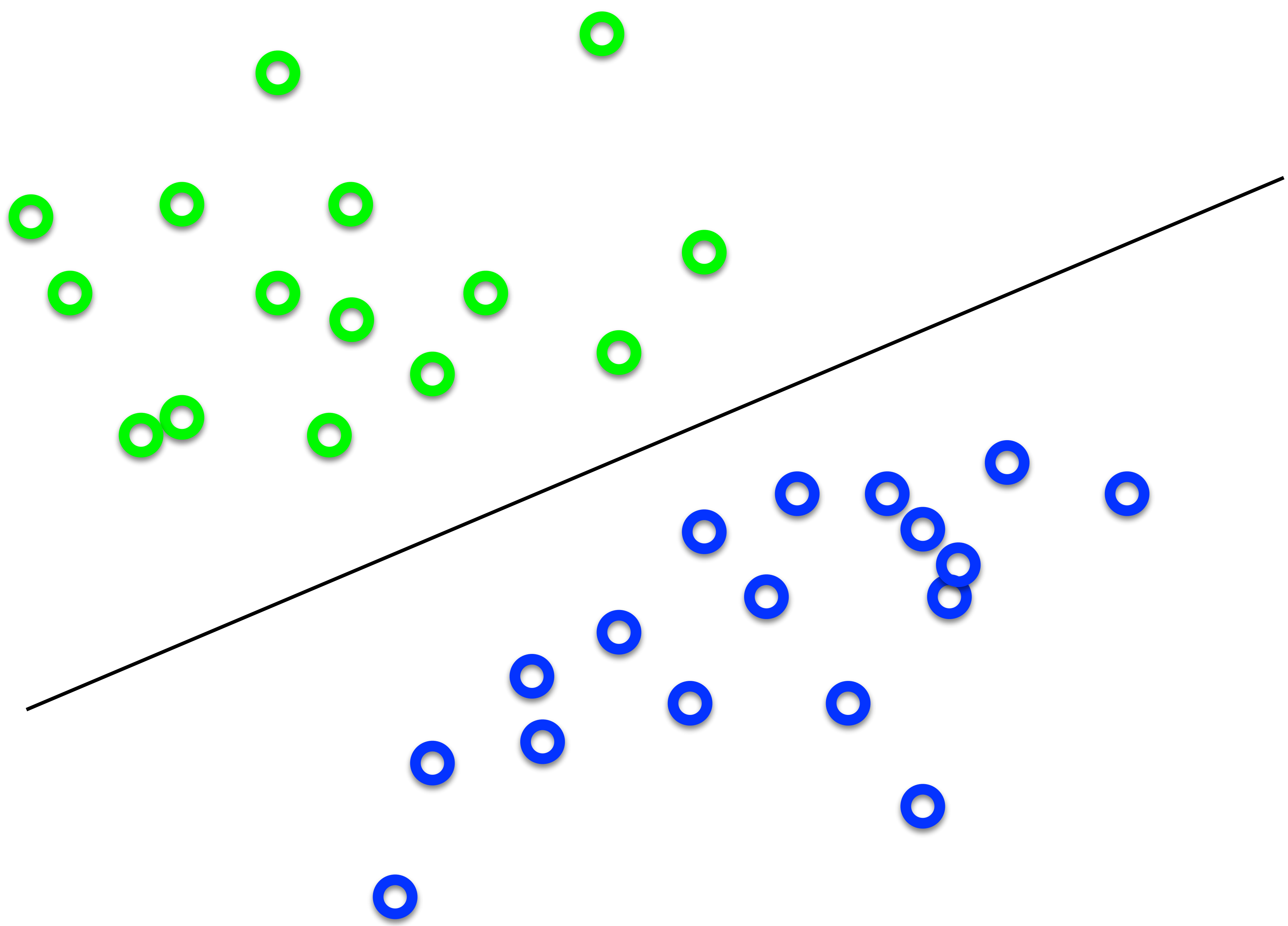


What's the best **w**?



Intuitively, the line that is the farthest from all interior points

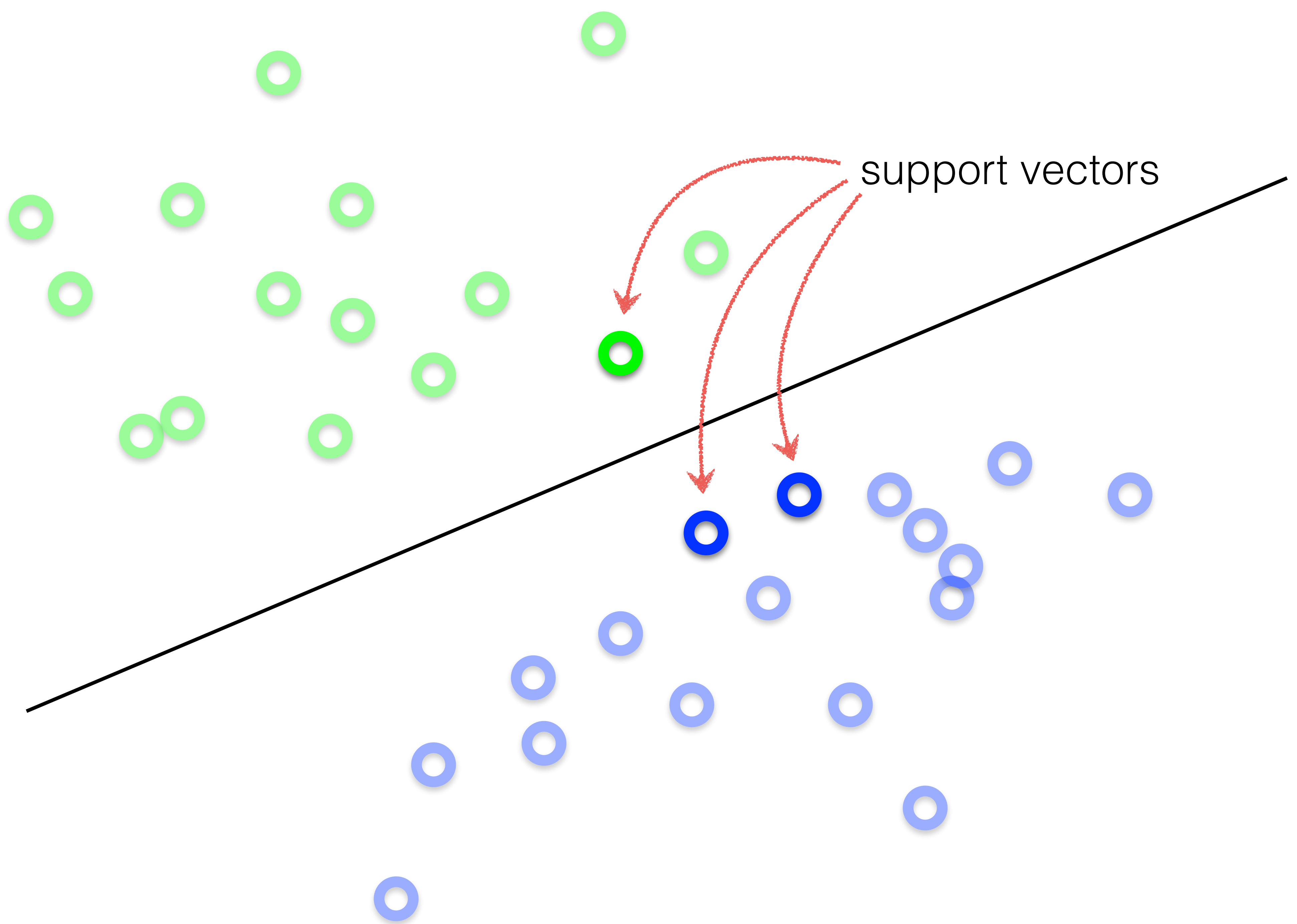
What's the best **w**?



Maximum Margin solution:

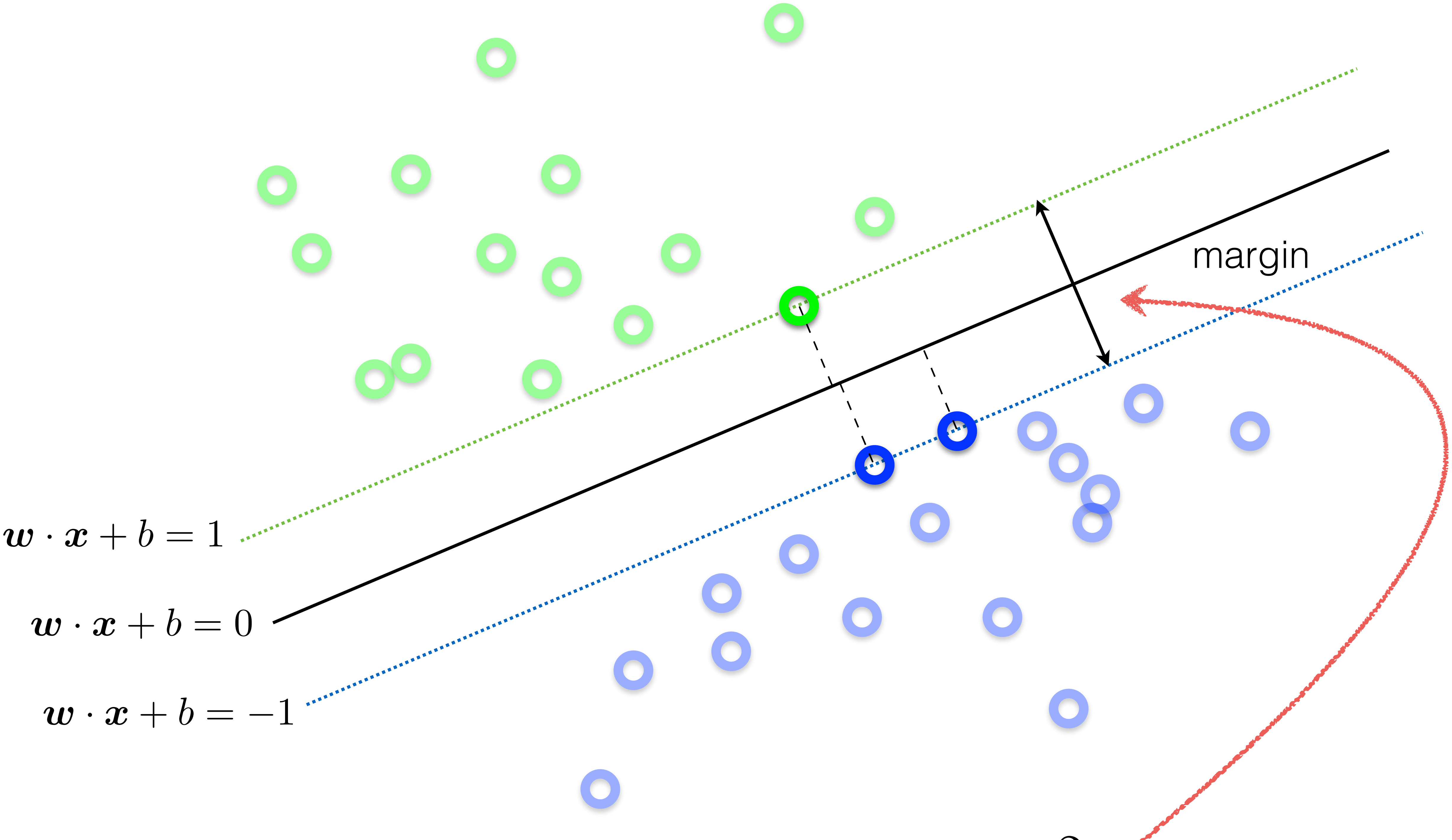
most stable to perturbations of data

What's the best **w**?



Want a hyperplane that is far away from 'inner points'

Find hyperplane **w** such that …



the gap between parallel hyperplanes $\dfrac{2}{\|\boldsymbol{w}\|}$ is maximized

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{array}{l} \geq +1 \;\text{ if }\; y_i = +1 \\ \leq -1 \;\text{ if }\; y_i = -1 \end{array} \text{ for } \; i = 1, \ldots, N$$

What does this constraint mean?

label of the data point

Why is it +1 and -1?

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{array}{l} \geq +1 \quad \text{if} \quad y_i = +1 \\ \leq -1 \quad \text{if} \quad y_i = -1 \end{array} \quad \text{for} \quad i = 1, \ldots, N$$

Equivalently,

Where did the 2 go?

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|$$

$$\text{subject to} \quad y_i(\boldsymbol{w} \cdot \boldsymbol{x}_i + b) \geq 1 \quad \text{for} \quad i = 1, \ldots, N$$

What happened to the labels?

'Primal formulation' of a linear SVM

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|$$

Objective Function

$$\text{subject to } \boxed{y_i(\boldsymbol{w} \cdot \boldsymbol{x}_i + b) \geq 1} \text{ for } \; i = 1, \dots, N$$

Constraints

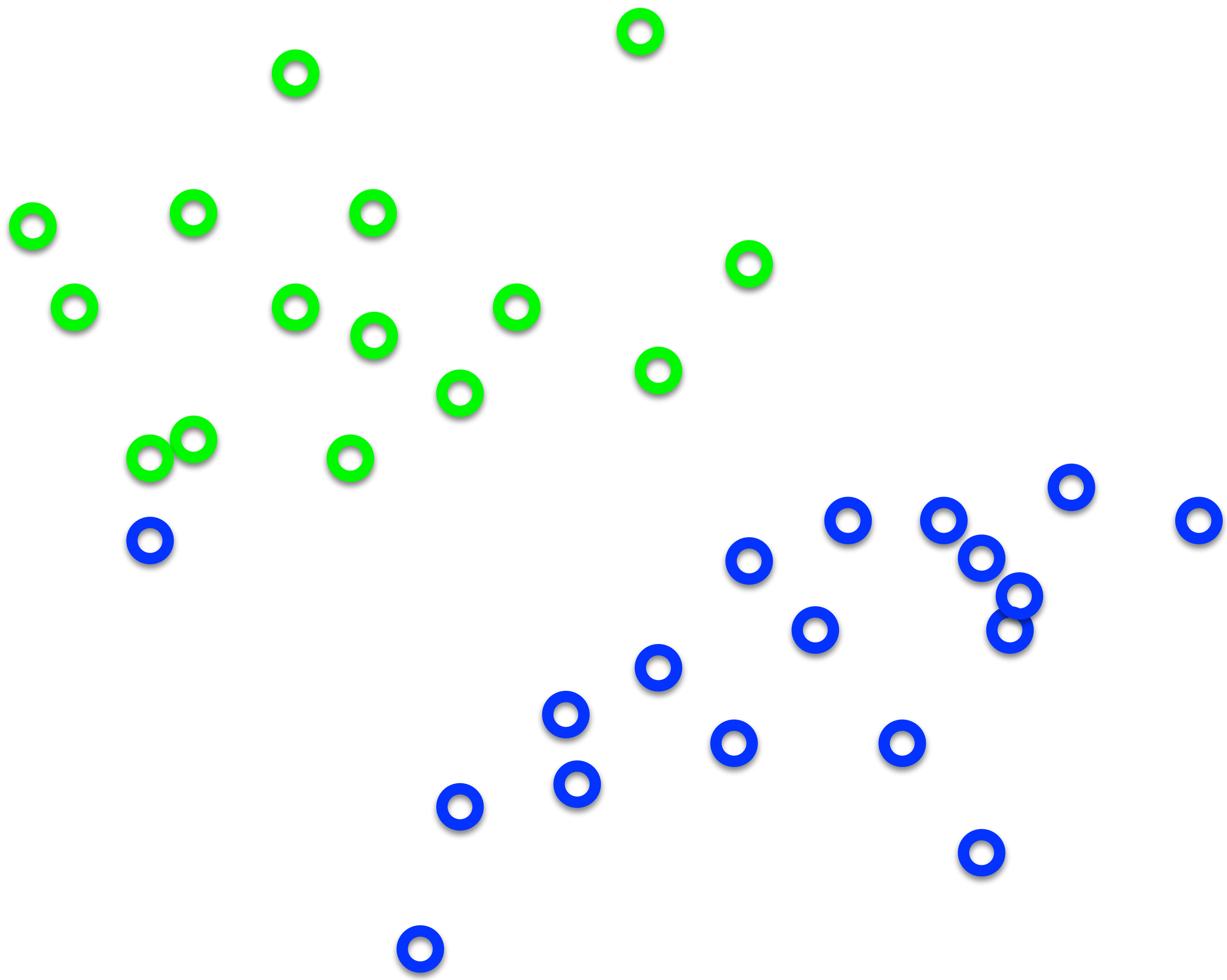
This is a convex quadratic programming (QP) problem

(a unique solution exists)

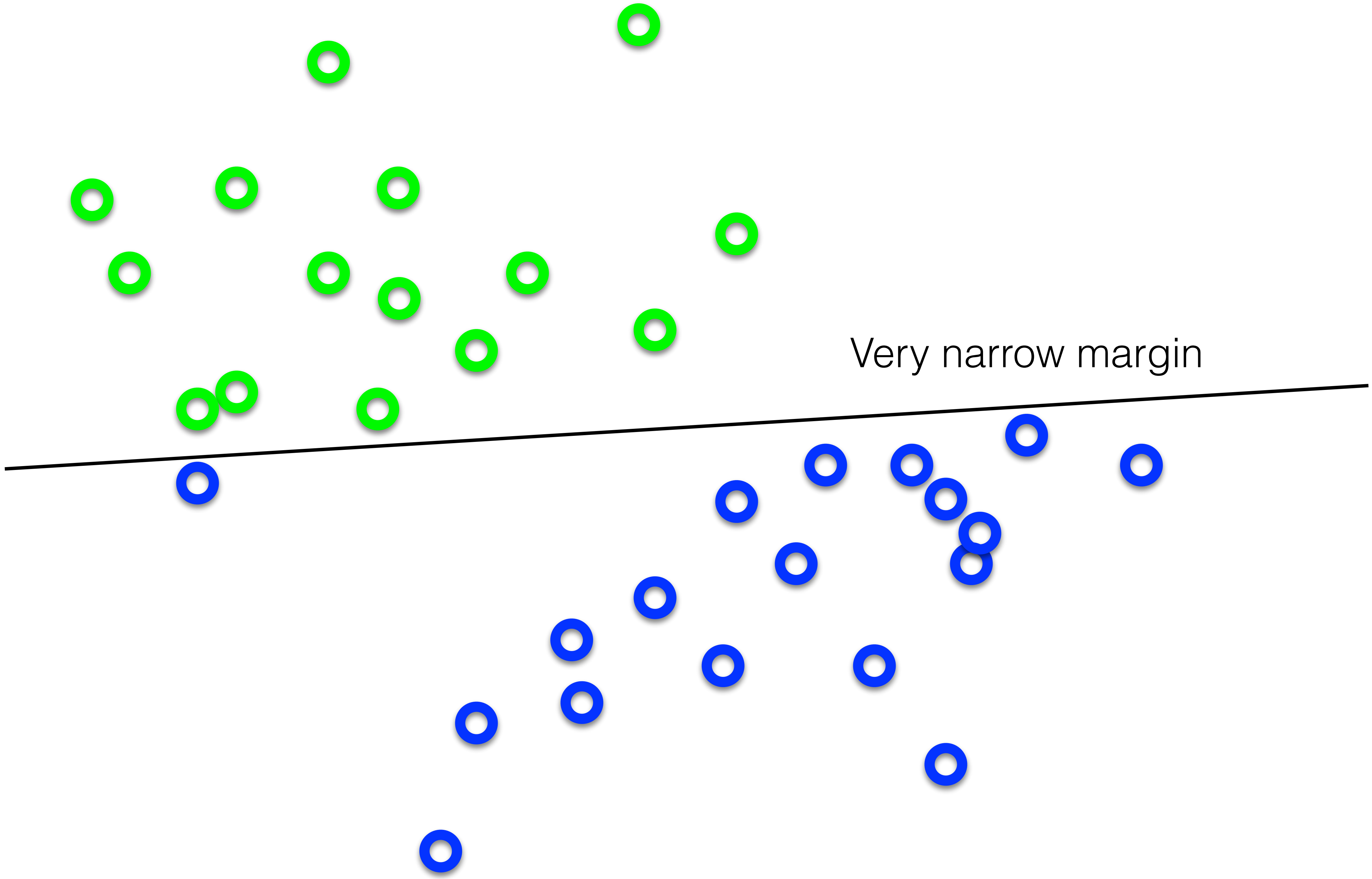
(you can learn more about this in convex optimization)

‘soft’ margin

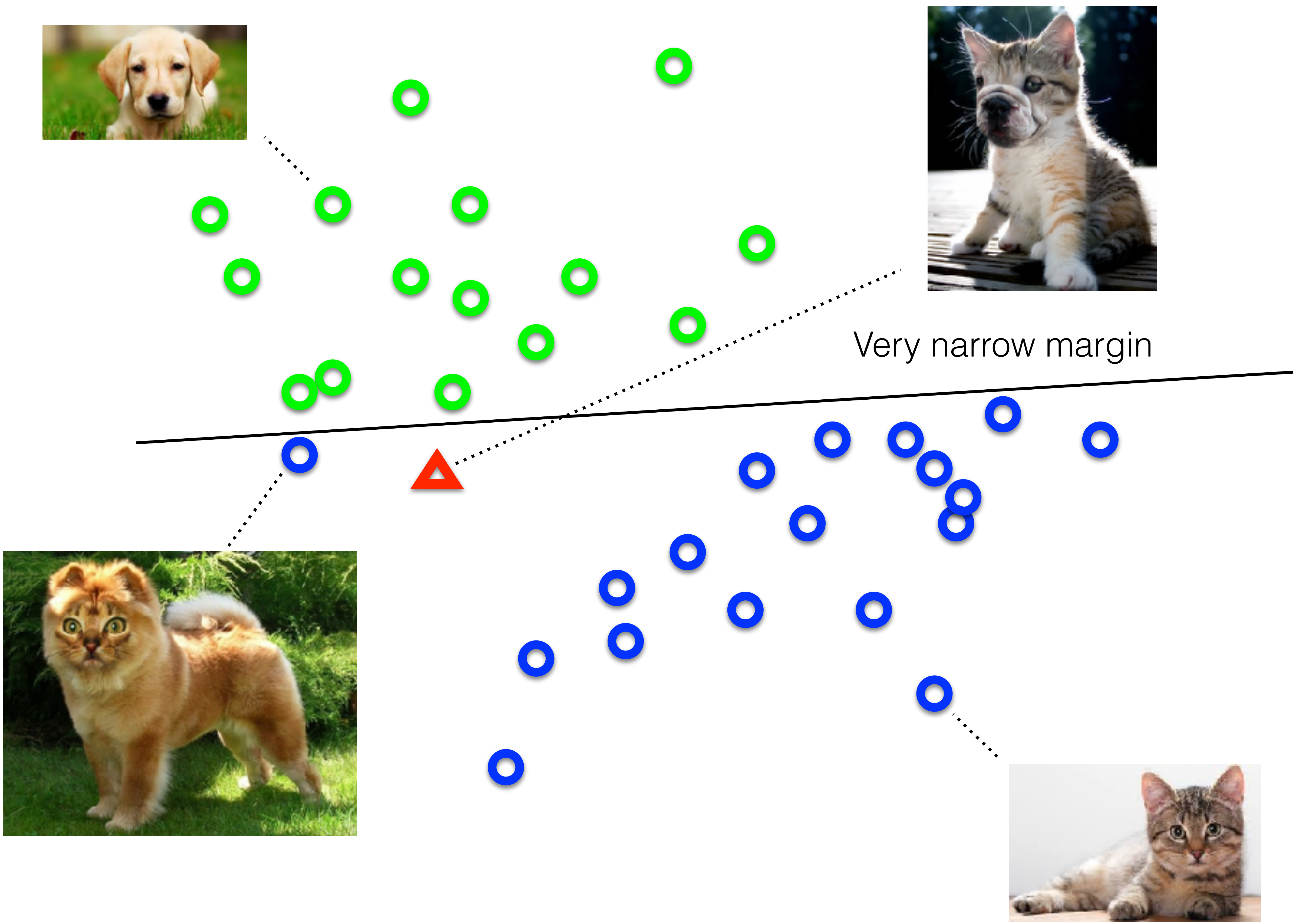
What’s the best **w**?



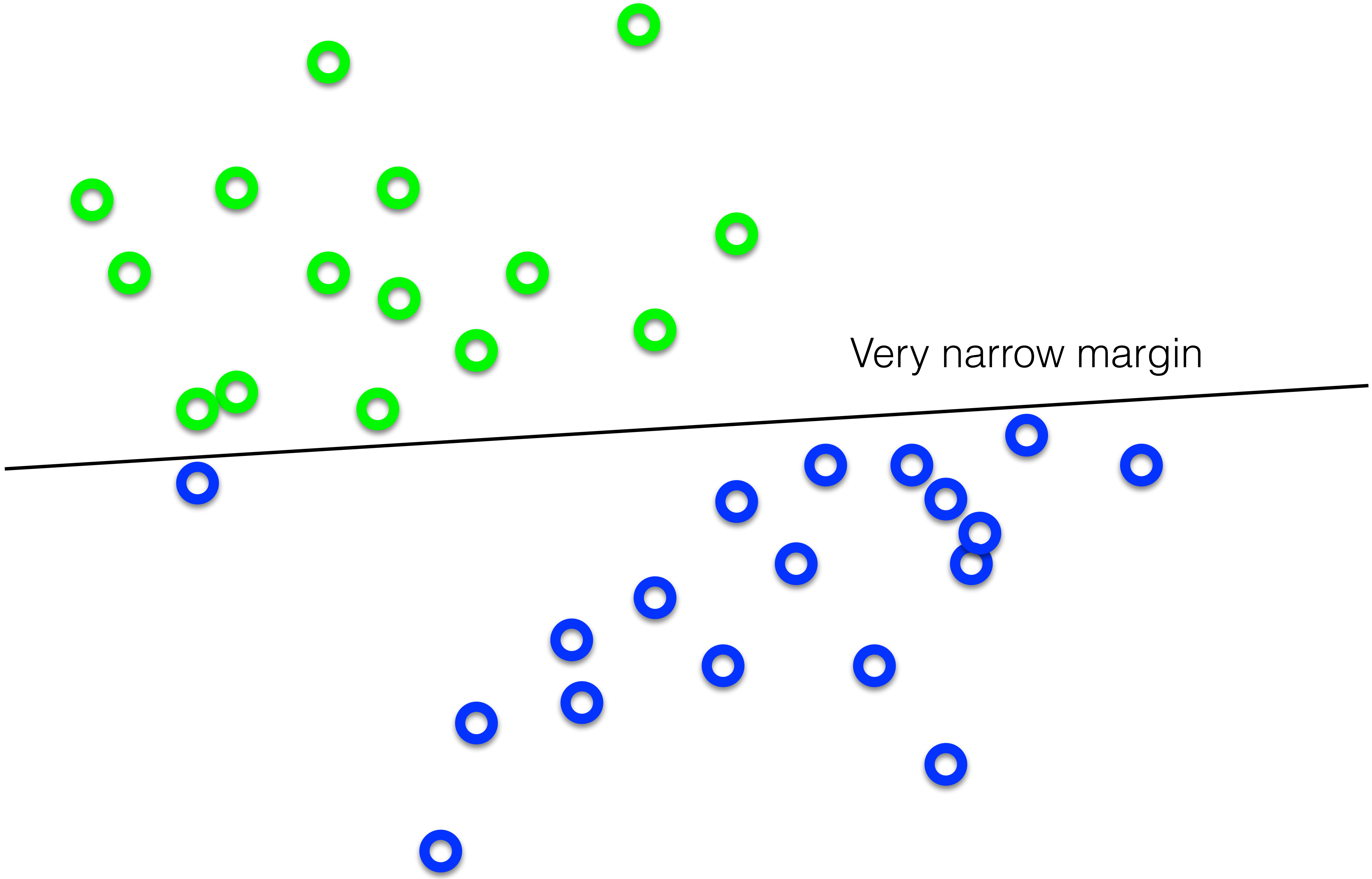
What's the best **w**?



Separating cats and dogs

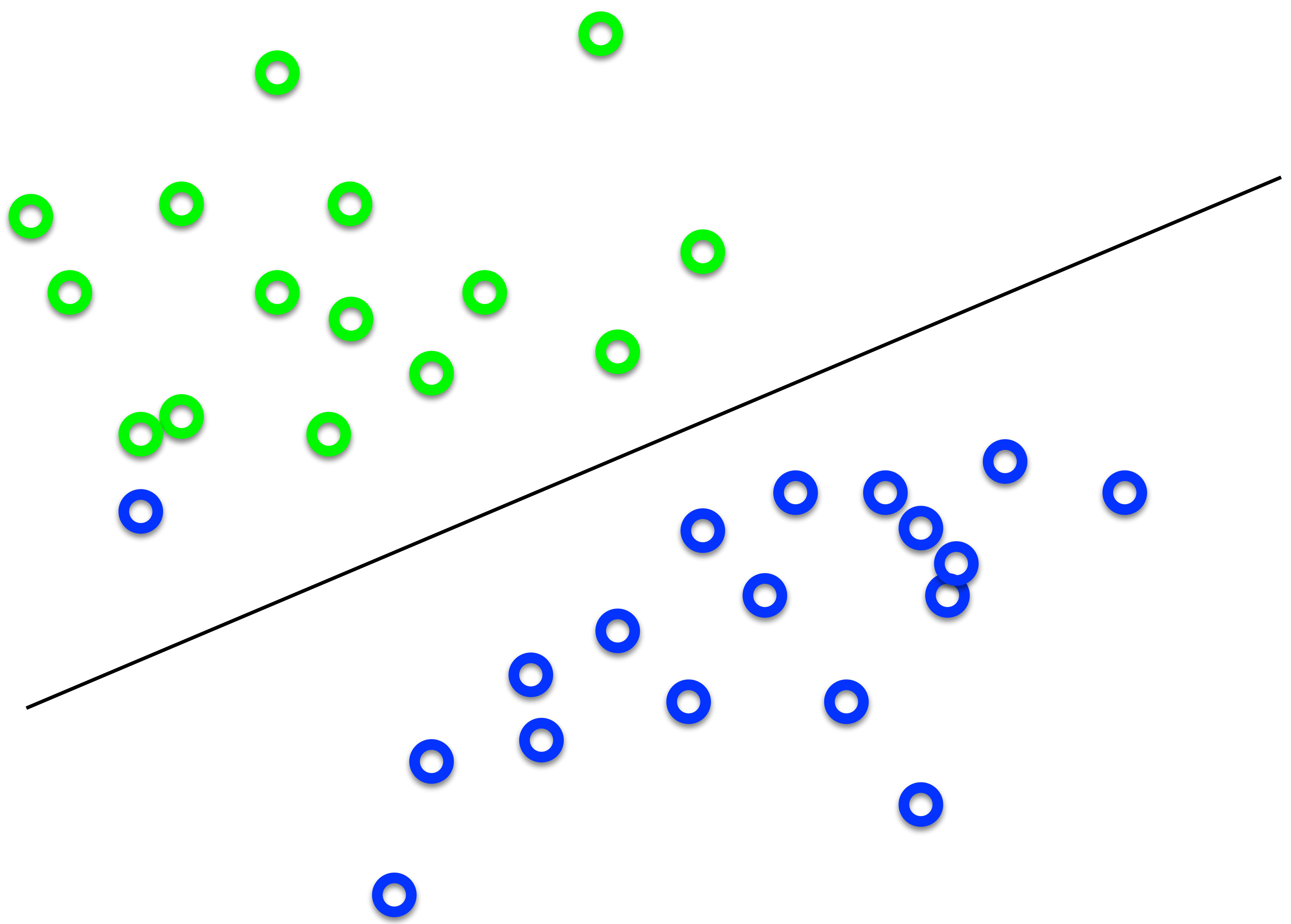


What's the best **w**?



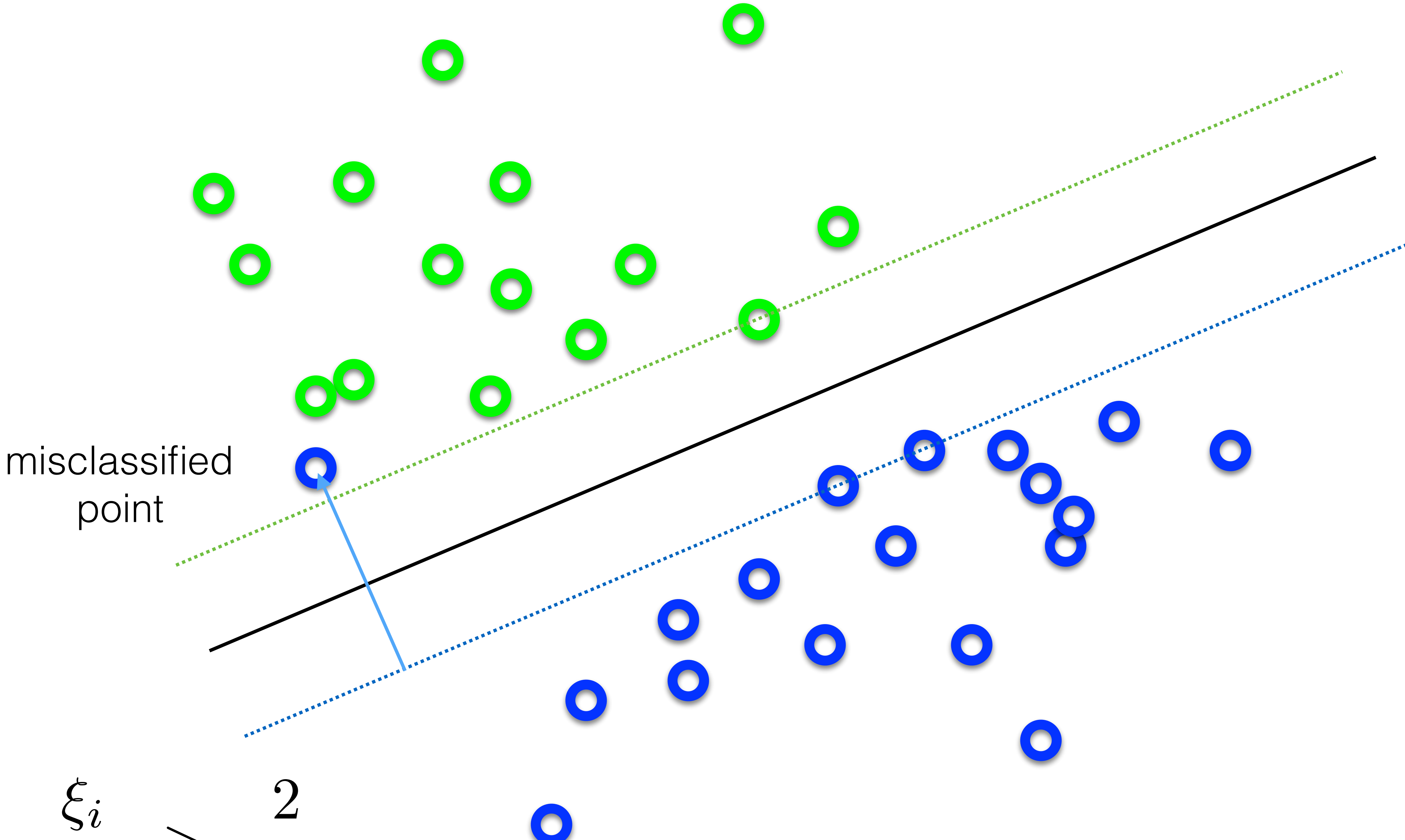
Intuitively, we should allow for some misclassification if we can get more robust classification

What’s the best **w**?



Trade-off between the MARGIN and the MISTAKES
(might be a better solution)

Adding slack variables $\xi_i \geq 0$



$$\frac{\xi_i}{\|\boldsymbol{w}\|} > \frac{2}{\|\boldsymbol{w}\|}$$

‘soft’ margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

for $i = 1, \ldots, N$

‘soft’ margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

$$\text{for} \quad i = 1, \ldots, N$$

The slack variable allows for mistakes,
as long as the inverse margin is minimized.

‘soft’ margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

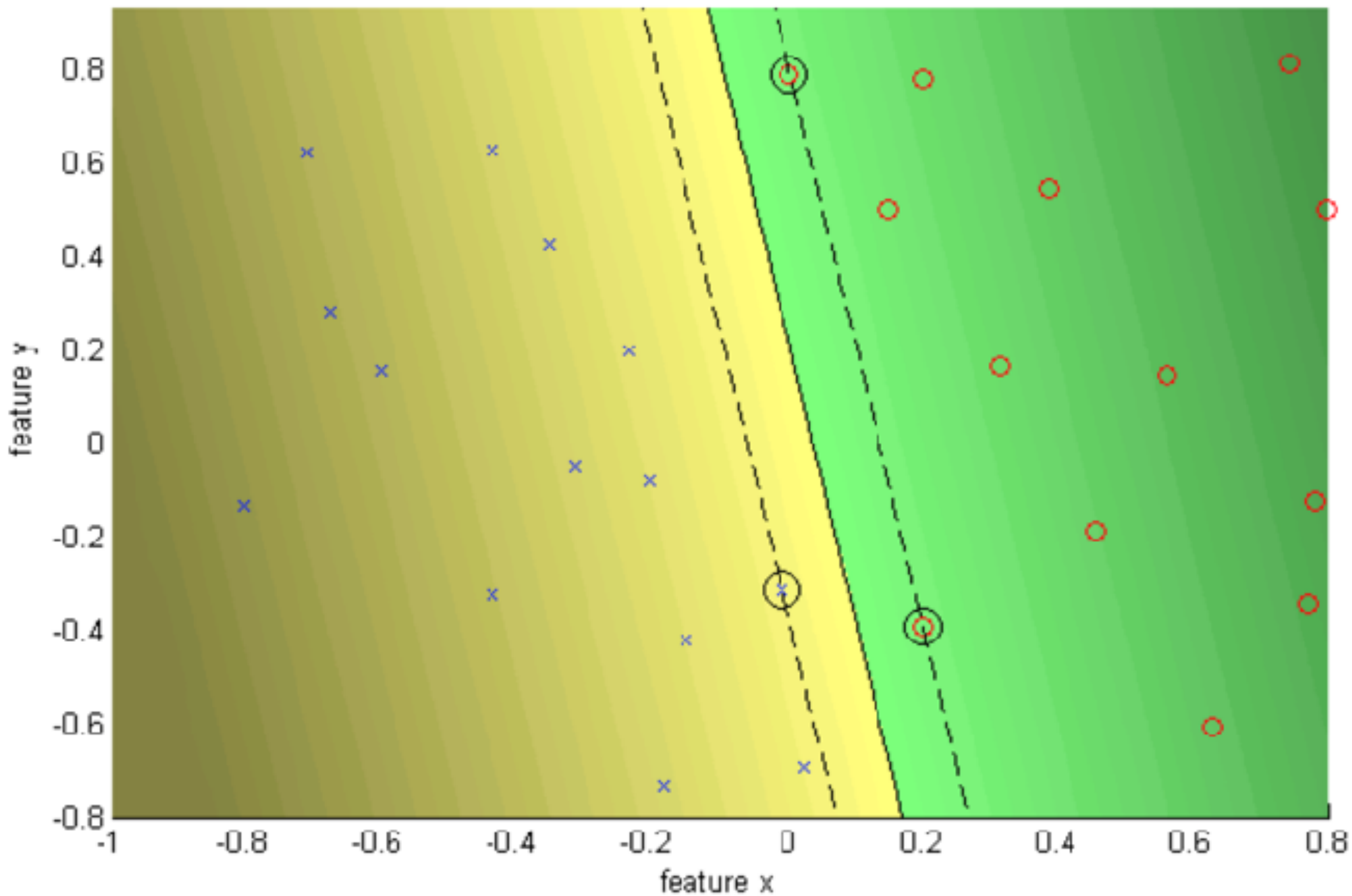
subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

for $i = 1, \ldots, N$

- Every constraint can be satisfied if slack is large
- C is a regularization parameter
  - Small C: ignore constraints (larger margin)
  - Big C: constraints (small margin)
- Still QP problem (unique solution)

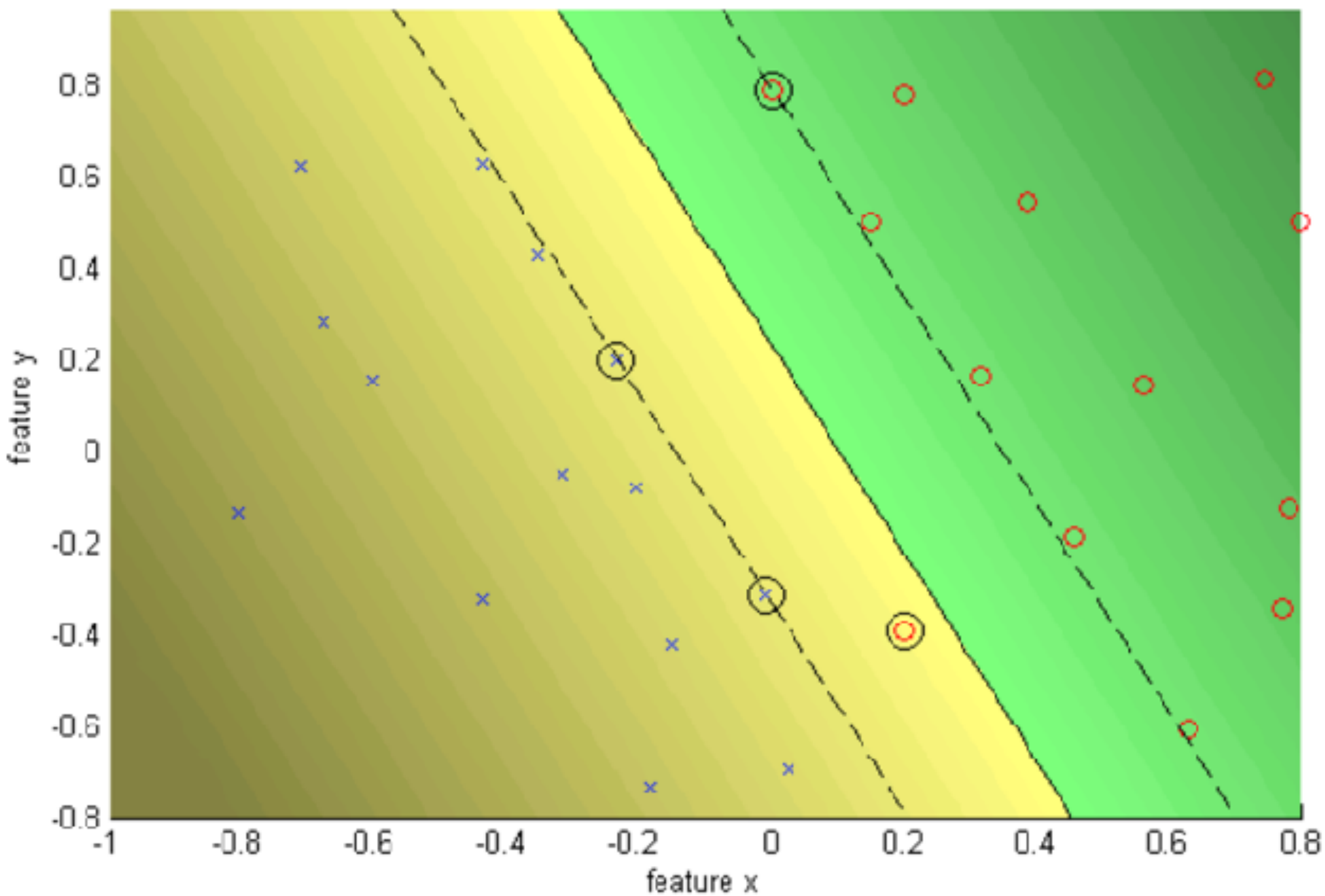
C = Infinity   hard margin



Comment Window

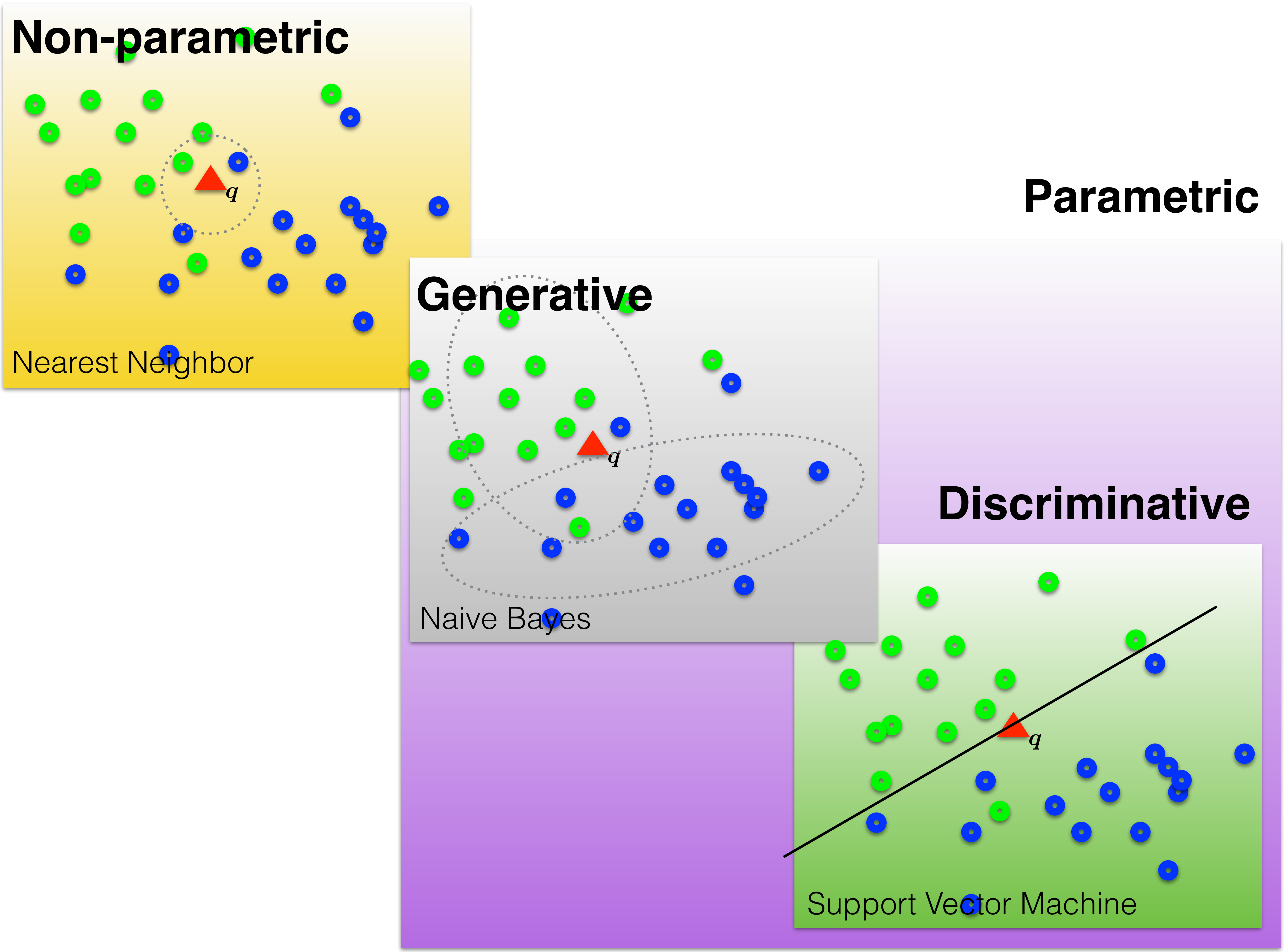
SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: Inf
Kernel evaluations: 971
Number of Support Vectors: 3
Margin: 0.0966
Training error: 0.00%

C = 10   soft margin



Comment Window

SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: 10.0000
Kernel evaluations: 2645
Number of Support Vectors: 4
Margin: 0.2265
Training error: 3.70%


Non-parametric
q
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Support Vector Machine

‘Classical’

Image Classification Pipeline

Input ➔ **encoder** ➔ **decoder** ➔ **output**

image(s)

Extract features

Build descriptor

Classifier

semantic label

Today