How to Train Your Perceptron

Computer Vision

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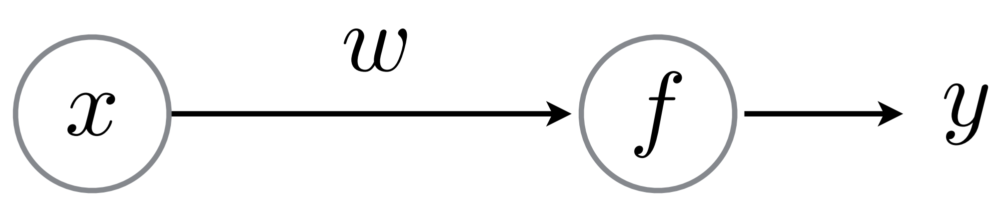
Let’s start easy

world's smallest perceptron!



$$y = wx$$

(a.k.a. line equation, linear regression)

Learning a Perceptron

Given a set of samples and a Perceptron

$$\{x_i, y_i\}$$

$$y = f_{\mathrm{PER}}(x; w)$$

Estimate the parameters of the Perceptron

$$w$$

Given training data:

x	y
10	10.1
2	1.9
3.5	3.4
1	1.1

What do you think the weight parameter is?

$$y = wx$$

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What do you think the weight parameter is?

$$y = wx$$

not so obvious as the network gets more complicated so we use …

An Incremental Learning Strategy

(gradient descent)

Given several examples

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

and a perceptron

$$\hat{y} = wx$$

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Modify weight w such that $\hat{y}$ gets **'closer'** to y

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perceptron parameter **perceptron output** **true label**

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what does this mean?

true label

Before diving into gradient descent, we need to understand …

Loss Function

defines what is means to be

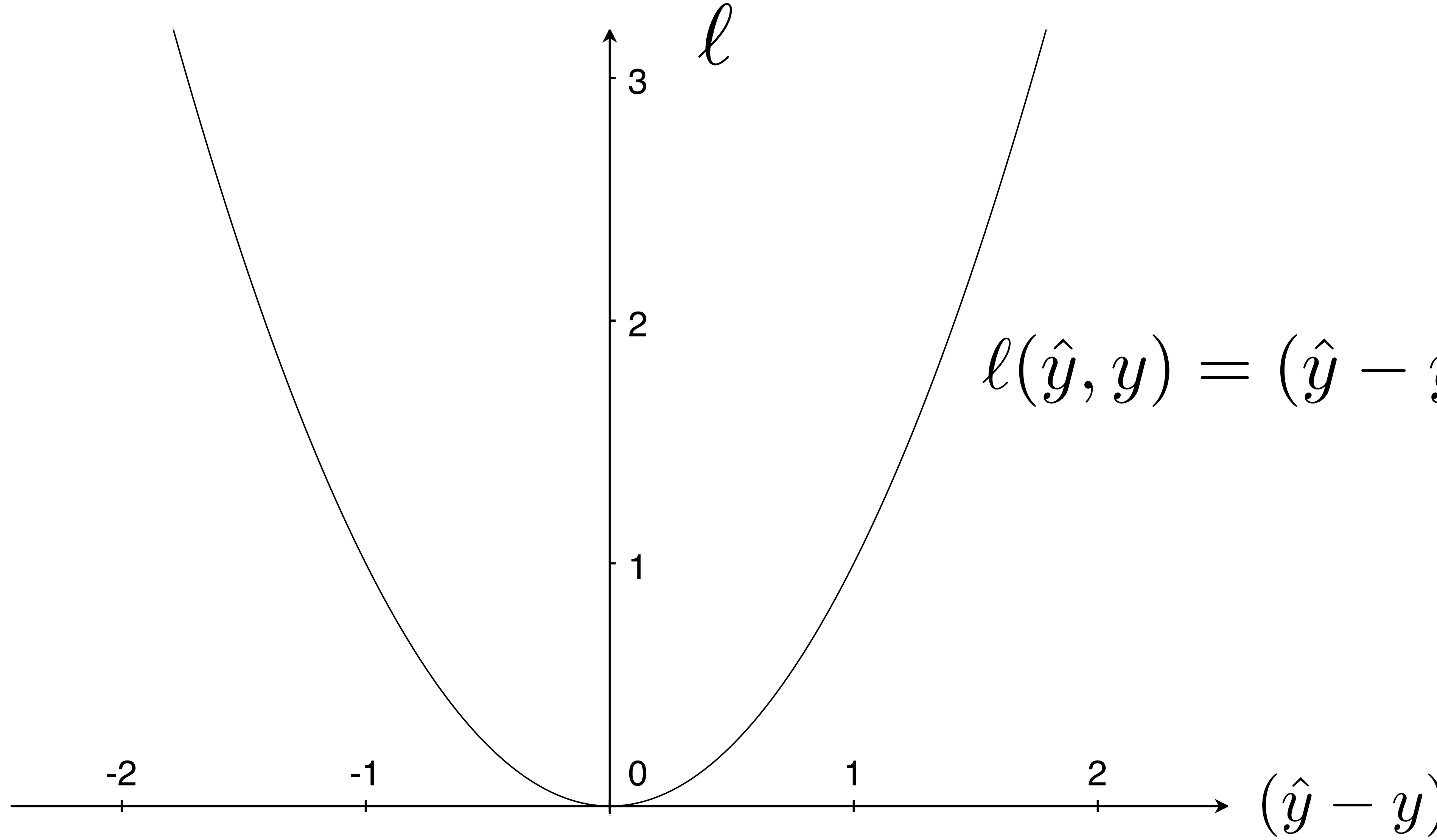
close to the true solution

YOU get to chose the loss function!

(some are better than others depending on what you want to do)

Squared Error (L2)

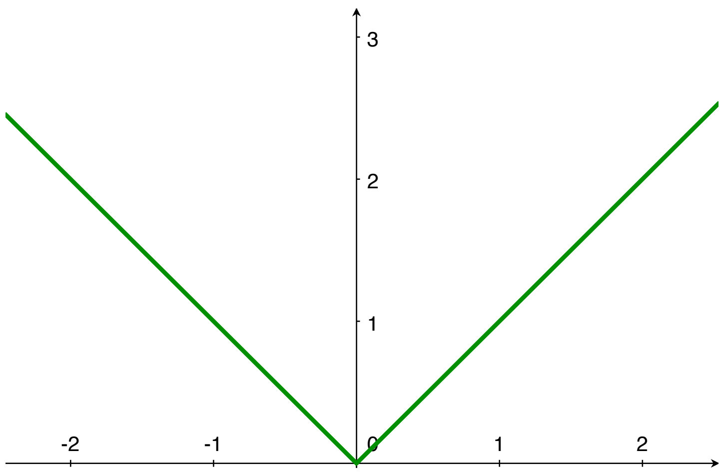
(a popular loss function)



$$\ell(\hat{y}, y) = (\hat{y} - y)^2$$

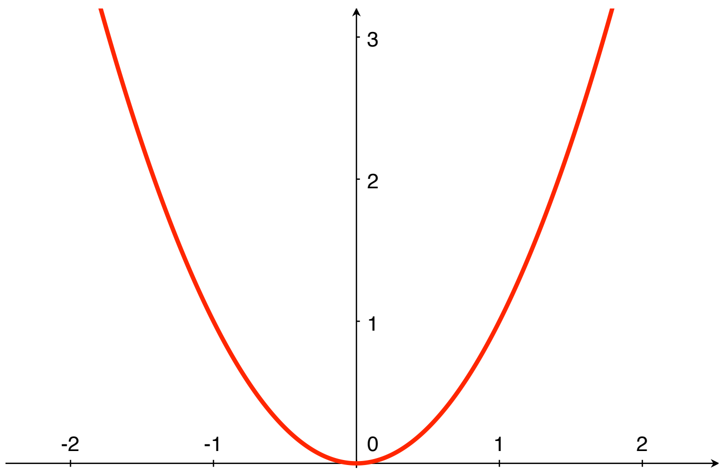
L1 Loss

$$\ell(\hat{y}, y) = |\hat{y} - y|$$



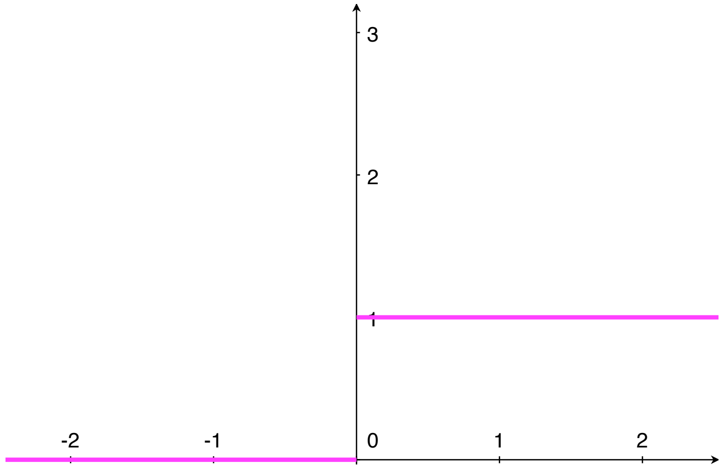
L2 Loss

$$\ell(\hat{y}, y) = (\hat{y} - y)^2$$



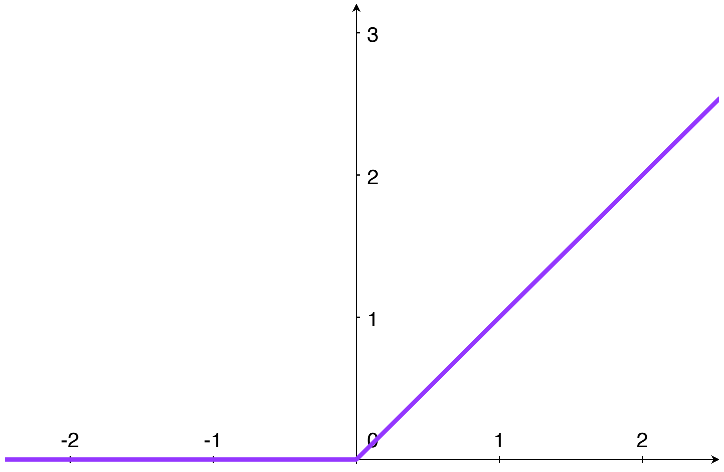
Zero-One Loss

$$\ell(\hat{y}, y) = \mathbf{1}[\hat{y} = y]$$



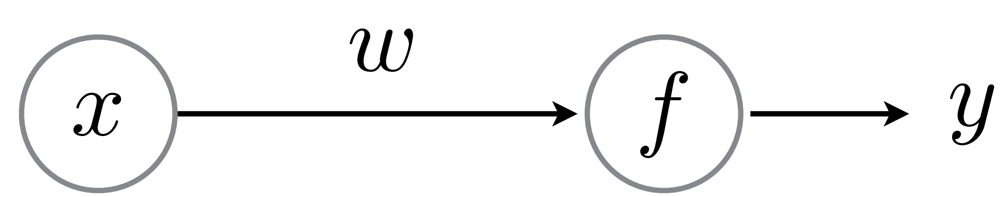
Hinge Loss

$$\ell(\hat{y}, y) = \max(0, 1 - y \cdot \hat{y})$$



back to the…

World's Smallest Perceptron!



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function of **ONE** parameter!

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linear function! $f(x) = wx$

Estimate the parameter of the Perceptron

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Learning Strategy

(gradient descent)

Given several examples

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Let's demystify this process first…

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$$\textbf{for} \;\; n = 1 \ldots N$$
$$w = w + (y_n - \hat{y})x_i;$$

just one line of code!

Let's demystify this process first…

Recall:

Code to train your perceptron:

$$\text{for } n = 1 \ldots N$$
$$w = w + (y_n - \hat{y})x_i;$$

just one line of code!

Now where does this come from?