MA615 Assignment 2 Part 1

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Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$

```r
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = T)
O <- matrix(rep(0,9), nrow=3, byrow=T)
sum(A %*% A %*% A == O) / length(A) == 1 #Check to see if A to the third is identical to the zero matri
```

```
## [1] TRUE
```

(b) Replace the third column of A by the sum of the second and third columns

---

First, produce A

```r
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

Then, add the columns 2 and 3 and assign the sum to the third column

```r
A[,3] <- A[,2] + A[,3]

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

---

2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \cdots & \cdots & \cdots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```r
#first create B
B <- matrix(rep(c(10,-10, 10), length.out = 15), nrow=5, ncol=3, byrow = T)
B
```

```
##      [,1] [,2] [,3]
## [1,]   10  -10   10
## [2,]   10  -10   10
## [3,]   10  -10   10
## [4,]   10  -10   10
## [5,]   10  -10   10
```

```r
#now calculate B.T@B
out <- crossprod(B,B)
out
```

```
##      [,1] [,2] [,3]
## [1,]  500 -500  500
## [2,] -500  500 -500
## [3,]  500 -500  500
```

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

Now, create the 6 x 6 matix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is matE, a 6x6 matrix of 0's followed by row(matE) and col(matE)

```r
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)

# Note what the functions row() and col() do
row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    2    2    2    2    2    2
## [3,]    3    3    3    3    3    3
## [4,]    4    4    4    4    4    4
## [5,]    5    5    5    5    5    5
## [6,]    6    6    6    6    6    6
```

```r
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2    3    4    5    6
## [2,]    1    2    3    4    5    6
## [3,]    1    2    3    4    5    6
## [4,]    1    2    3    4    5    6
## [5,]    1    2    3    4    5    6
## [6,]    1    2    3    4    5    6
```

```r
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
## [6,]    5    4    3    2    1    0
```

```r
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

---

4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

Use outer() a little more to make sure you get it.

```r
B <- outer(a,a, "*")
B
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    1    2    3    4
## [3,]    0    2    4    6    8
## [4,]    0    3    6    9   12
## [5,]    0    4    8   12   16
```

```r
# and
b <- 5:10
C <- outer(a,b,"+")
C
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    5    6    7    8    9   10
## [2,]    6    7    8    9   10   11
## [3,]    7    8    9   10   11   12
## [4,]    8    9   10   11   12   13
## [5,]    9   10   11   12   13   14
```

```r
# and finally  --  make sure you check the values.
D <- outer(b,a, "%%")
D
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    1    2    1
## [2,]   NA    0    0    0    2
## [3,]   NA    0    1    1    3
## [4,]   NA    0    0    2    0
## [5,]   NA    0    1    0    1
## [6,]   NA    0    0    1    2
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```r
a <- 0:4
A <- outer(a,a, "+")
A <- A %% length(a) #take sum modularly to generate the effect of increasing by 1 going down columns
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
#this is the same thing essentially with a different range
a  <-  0:9
A <- outer(a,a, "+")
A <- A %% length(a)
A
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
```

```
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```r
#this is the same thing essentially with a different range
a  <-  0:8
A <- outer(a,a, "+")
A <- A %% length(a)
#Now the matrix here is essentially the correct matrix, but columns 2 to 9 are in the wrong order
A[,2:9] <- A[,9:2]
A
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation Ax = y.

$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 7$
$2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 = -1$
$3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = -3$
$4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 = 5$
$5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 = 17$

```r
#first create the matrix A
A <-  matrix(rep(1:5, length.out=25), nrow=5, byrow = T)
A <-  abs(row(A) - col(A)) + 1
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    3    4    5
## [2,]    2    1    2    3    4
```

```
## [3,]    3    2    1    2    3
## [4,]    4    3    2    1    2
## [5,]    5    4    3    2    1
```

```
#x is the matrix we want to solve for, lets create y
y <- matrix(c(7,-1,-3,5,17), nrow=5, byrow = T)
y
```

```
##      [,1]
## [1,]    7
## [2,]   -1
## [3,]   -3
## [4,]    5
## [5,]   17
```

```
#now solve
x <- solve(A,y)
x #this is the answer
```

```
##      [,1]
## [1,]   -2
## [2,]    3
## [3,]    5
## [4,]    2
## [5,]   -4
```

```
#now check answer
abs(A %*% x - y) < 1e-6
```

```
##      [,1]
## [1,] TRUE
## [2,] TRUE
## [3,] TRUE
## [4,] TRUE
## [5,] TRUE
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
```

Use the matrix you have created to answer these questions:

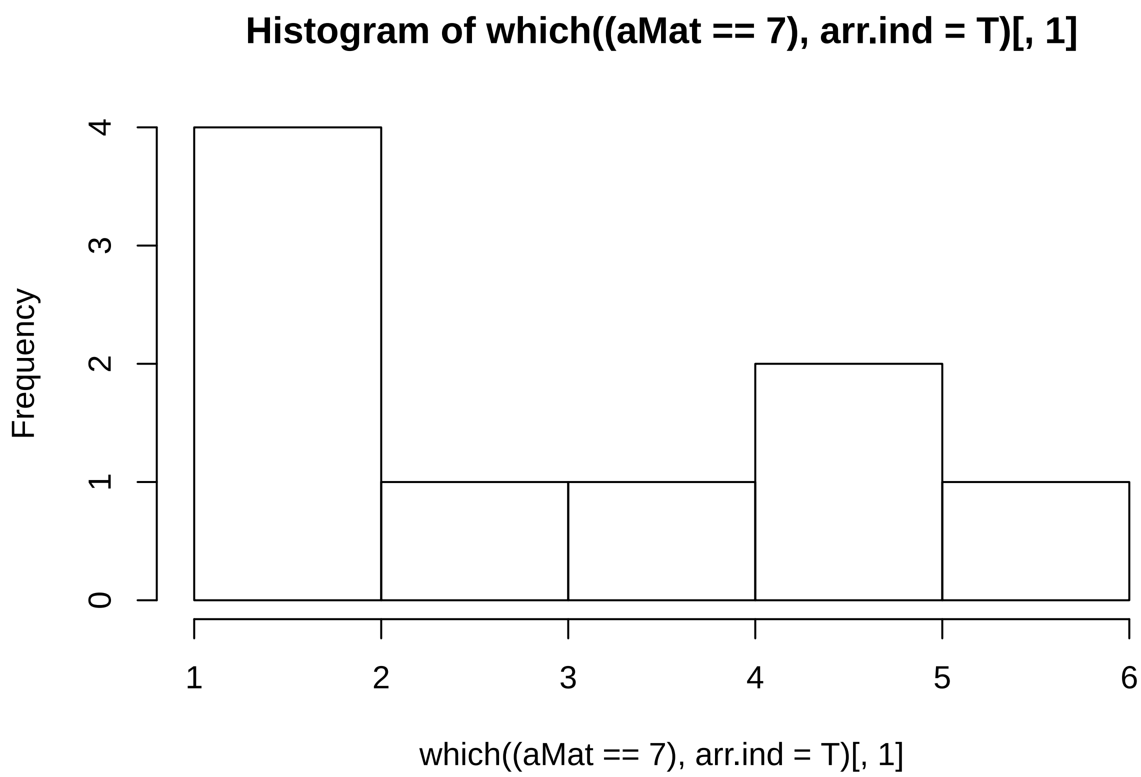
(a) Find the number of entries in each row which are greater than 4.

```
sum(aMat > 4) #we expect this to be around 6/10 * 60 = 36 entries
```

```
## [1] 32
```

(b) Which rows contain exactly two occurrences of the number seven?

```
#answer is row 5
hist(which((aMat == 7), arr.ind = T)[,1]) #histogram illustrating number of 7s in each row
```




```
which(rowSums((aMat == 7)) == 2 ) #sums the rows of the logical vector of locations aMat == 7, gives ro
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater than 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for example, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```
allcolsums <- outer(colSums(aMat), colSums(aMat), "+")
allcolsums
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   36   60   50   51   50   54   49   58   49    38
##  [2,]   60   84   74   75   74   78   73   82   73    62
##  [3,]   50   74   64   65   64   68   63   72   63    52
##  [4,]   51   75   65   66   65   69   64   73   64    53
##  [5,]   50   74   64   65   64   68   63   72   63    52
##  [6,]   54   78   68   69   68   72   67   76   67    56
##  [7,]   49   73   63   64   63   67   62   71   62    51
##  [8,]   58   82   72   73   72   76   71   80   71    60
##  [9,]   49   73   63   64   63   67   62   71   62    51
## [10,]   38   62   52   53   52   56   51   60   51    40
```

```
sol <- which(allcolsums > 75, arr.ind = T)
sol
```

```
##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
```

```
## [5,]    8    6
## [6,]    2    8
## [7,]    6    8
## [8,]    8    8
```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```
allcolsums <- outer(colSums(aMat), colSums(aMat), "+")
allcolsums
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   36   60   50   51   50   54   49   58   49    38
##  [2,]   60   84   74   75   74   78   73   82   73    62
##  [3,]   50   74   64   65   64   68   63   72   63    52
##  [4,]   51   75   65   66   65   69   64   73   64    53
##  [5,]   50   74   64   65   64   68   63   72   63    52
##  [6,]   54   78   68   69   68   72   67   76   67    56
##  [7,]   49   73   63   64   63   67   62   71   62    51
##  [8,]   58   82   72   73   72   76   71   80   71    60
##  [9,]   49   73   63   64   63   67   62   71   62    51
## [10,]   38   62   52   53   52   56   51   60   51    40
```

```
sol <- allcolsums > 75
sol[lower.tri(sol)] <- F #remove below diagonal
diag(sol) <- F #remove the diagonal
sol <- which(sol, arr.ind = T)
sol
```

```
##      row col
## [1,]   2   6
## [2,]   2   8
## [3,]   6   8
```

8. Calculate

(a) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```
sum((1:20)^4) * sum(1/(3+(1:5)))
```

```
## [1] 639215.3
```

```
# or

sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```
i <-1:20
j <- 1:5
sum(i^4/(3+i*j))
```

```
## [1] 15384.2
```

(c) $\sum_{i=1}^{10}\sum_{j=1}^{i}\frac{i^4}{(3+ij)}$

```
i <-1:10
ij  <-  outer(i,i,"*") + 3
ij[upper.tri(ij)] <- Inf #because it wont count in the sum of the next step
sum(i^4/(ij))
```

```
## [1] 6944.743
```