

"Question"

Example 5.2. Consider the simplified model of an automobile shown in Fig. 5.5, let the parameters have the values $m = 1,500$ kg, $I_C = 2,000$ kg m², $k_1 = 36,000$ kg/m, $k_2 = 40,000$ kg/m, $a = 1.3$ m and $b = 1.7$ m, calculate the natural modes of the system and write an expression for the response.

"Equations Of Motion in Matrix Form"

$$\begin{bmatrix} 1,500 & 0 \\ 0 & 2,000 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 76,000 & 21,200 \\ 21,200 & 176,440 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (a)$$

But, free vibration is harmonic, so that by analogy with Eqs. (5.23) and (5.31) we can write

$$x(t) = X \cos(\omega t - \phi), \quad \theta(t) = \Theta \cos(\omega t - \phi) \quad (b)$$

"Results"

$$\mathbf{x}(t) = C_1 \cos(6.85781t - \phi_1) \begin{bmatrix} 1 \\ -0.257341 \end{bmatrix} + C_2 \cos(9.58421t - \phi_2) \begin{bmatrix} 1 \\ 2.914417 \end{bmatrix} \quad (l)$$

where the amplitudes C_1 , C_2 and the phase angles ϕ_1 , ϕ_2 must be determined from the initial displacements $x(0)$, $\theta(0)$ and the initial velocities $\dot{x}(0)$, $\dot{\theta}(0)$, as will be shown in Sec. 5.4.

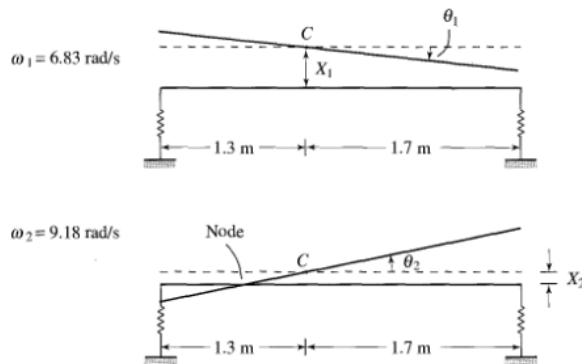


FIGURE 5.8
Natural modes for the automobile model of Fig. 5.5

"Mass Matrix"

$$\begin{pmatrix} 1500. & 0. \\ 0. & 2000. \end{pmatrix}$$

"Stiffness Matrix"

$$\begin{pmatrix} 76\,000. & 21\,200. \\ 21\,200. & 176\,440. \end{pmatrix}$$

"EigenFrequencies"

$$\{6.85781, 9.58421\}$$

"EigenVectors"

$$\begin{pmatrix} -0.968447 & 0.324548 \\ 0.249221 & 0.945869 \end{pmatrix}$$

"Checking Orthogonality"

$$3.63798 \times 10^{-12}$$

"Mass Nomalization Formula"

$$U_{S2} = \frac{U_S}{\sqrt{U_S \cdot M \cdot U_S}}$$

"Mass Normalized EigenVectors"

$$\begin{pmatrix} -0.0247503 & 0.0073546 \\ 0.00636927 & 0.0214344 \end{pmatrix}$$

"Unit Normalized EigenVectors"

$$\begin{pmatrix} 1. & 1. \\ -0.257341 & 2.91442 \end{pmatrix}$$