Out[140]//TraditionalForm=

FORMATTED REPORT 1

SYSTEM ON ENERGY EQUAPARTITION PAPER IS EXPLAINED ON 3 DOF EXAMPLE SYSTEM

PART 1−ANALYTIC

SETTING SYSTEM ATTRIBUTES AND CONSTRUCTION OF MASS AND STIFNESS MATRICES

Mass Matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{pmatrix}$$

Stiffness Matrix

$$\begin{pmatrix} 1.18208 & -0.072908 & -0.109176 \\ -0.072908 & 0.072908 & 0 \\ -0.109176 & 0 & 0.109176 \end{pmatrix}$$

PART 2−ANALYTIC

DEFINING MODAL PLANE MATRICES AND VARIABLES

EigenVectors, Transformation Matrix

$$\begin{pmatrix} -0.303802 & -0.0919473 & -0.105573 \\ -0.797705 & 0.931015 & 0.163593 \\ -0.520934 & -0.35321 & 0.980863 \end{pmatrix}$$

Generalized Coordinates

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Modal Coodrinates

$$\begin{pmatrix} \cos(0.906464\, t - \psi_1)\, C_1 \\ \cos(1.20754\, t - \psi_2)\, C_2 \\ \cos(1.47767\, t - \psi_3)\, C_3 \end{pmatrix}$$

Transition To Modal Coordinates

$$\text{xm} = \text{Um}.\eta\text{m}$$

Displacement Matrix in Modal Plane

$$\begin{pmatrix} -0.303802 \cos(0.906464\, t - \psi_1)\, C_1 - 0.0919473 \cos(1.20754\, t - \psi_2)\, C_2 - 0.105573 \cos(1.47767\, t - \psi_3)\, C_3 \\ -0.797705 \cos(0.906464\, t - \psi_1)\, C_1 + 0.931015 \cos(1.20754\, t - \psi_2)\, C_2 + 0.163593 \cos(1.47767\, t - \psi_3)\, C_3 \\ -0.520934 \cos(0.906464\, t - \psi_1)\, C_1 - 0.35321 \cos(1.20754\, t - \psi_2)\, C_2 + 0.980863 \cos(1.47767\, t - \psi_3)\, C_3 \end{pmatrix}$$

PART 3−ANALYTIC

CALCULATION OF ENERGIES

Kinetic Energy of Master

$$\text{EKinN} = \frac{1}{2} M \left(\frac{\partial \text{xm2}[[1]]}{\partial t}\right)^2$$

Potential Energy of Master

$$\text{EPotN} = \frac{1}{2} K\, \text{xm2}[[1]]^2$$

Kinetic Energy Of Sattelites

$$\text{EKinm} = \text{Table}\left[\frac{1}{2} \text{Mm}[[i,\ i]] \left(\frac{\partial \text{xm2}[[i]]}{\partial t}\right)^2,\ \{i,\ 2,\ n+1\}\right]$$

Potential Energy Of Sattalites

$$\text{EPotm} = \text{Table}\left[\frac{1}{2} \text{Km}[[i,\ i]]\, (\text{xm2}[[i]] - \text{xm2}[[1]])^2,\ \{i,\ 2,\ n+1\}\right]$$

Total Energy by means of summing Total Kin.&Pot. Energy

Etop1 = EKinN + EPotN + Total[EKinm] + Total[EPotm]

Total Energy in Quadratic Form

$$\text{Etop2} = \frac{\text{xm2.Km.xm2}}{2} + \frac{1}{2} \frac{\partial \text{xm2}}{\partial t}.\text{Mm}.\frac{\partial \text{xm2}}{\partial t}$$

SAGLAMA ETOPsum−ETOPQuadratik=0

Simplify[Etop1 − Etop2]

Etop1−Etop2

0

PART 4−ANALYTIC

MODAL ENERGY CALCULATIONS

Total Energy Imparted to Master

$$\text{EtotImp} = \frac{M\, \text{V0}^2}{2}$$

Natural Modes (Frequencies) of System

$$\begin{pmatrix} 0.906464 \\ 1.20754 \\ 1.47767 \end{pmatrix}$$

Modal Energy Form 1

$$\text{EModal} = \text{Table}\left[\frac{1}{2}\left(\omega\text{m}[[i]]^2\, \eta\text{m}[[j]]^2 + \left(\frac{\partial \eta\text{m}[[i]]}{\partial t}\right)^2\right),\ \{i,\ 1,\ n+1\},\ \{j,\ 1,\ n+1\}\right]$$

Modal Energy Form 1 Expanded

$$\begin{pmatrix} 0.410839\, C_1^2 & \frac{1}{2}\left(0.821678 \cos^2(1.20754\, t - \psi_2) + 1.45816 \sin^2(1.20754\, t - \psi_2)\right) C_2^2 & \frac{1}{2}\left(0.821678 \cos^2(1.47767\, t - \psi_3) + 2.18352 \sin^2(1.47767\, t - \psi_3)\right) C_3^2 \\ \frac{1}{2}\left(1.45816 \cos^2(0.906464\, t - \psi_1) + 0.821678 \sin^2(0.906464\, t - \psi_1)\right) C_1^2 & 0.72908\, C_2^2 & \frac{1}{2}\left(1.45816 \cos^2(1.47767\, t - \psi_3) + 2.18352 \sin^2(1.47767\, t - \psi_3)\right) C_3^2 \\ \frac{1}{2}\left(2.18352 \cos^2(0.906464\, t - \psi_1) + 0.821678 \sin^2(0.906464\, t - \psi_1)\right) C_1^2 & \frac{1}{2}\left(2.18352 \cos^2(1.20754\, t - \psi_2) + 1.45816 \sin^2(1.20754\, t - \psi_2)\right) C_2^2 & 1.09176\, C_3^2 \end{pmatrix}$$

Modal Energy Form 3

$$\text{EModal3} = \text{Table}\left[M\, \text{EtotImp}\, \text{Um}[[i,\ j]]^2,\ \{j,\ 1,\ n+1\},\ \{i,\ 1,\ n+1\}\right]$$

Modal Energy Form 3 Expanded

$$\begin{pmatrix} 0.0461479 & 0.318166 & 0.135686 \\ 0.00422716 & 0.433394 & 0.0623785 \\ 0.00557278 & 0.0133814 & 0.481046 \end{pmatrix}$$

numerical values are introduced

Note that Modal Energy Form 2 will be given after

PART5−ANALYTIC

DERIVATION OF EQUATIONS OF MOTION USING LAGRANGIAN MECHANICS (including NonLineer Eq.)

Spring Force For all springs

$$F_{\text{yay}}(\text{x\_},\ \text{k\_}) = k\, x(t)$$

Calculations Inbetween

$$\left\{\text{IFk} = \int F_{\text{yay}}(x,\ k)\, d x(t);\ T = 0.5\, M\, ((x_1)'(t))^2 + 0.5 \sum_{i=2}^{n+1} m_i \left(\frac{\partial x_i(t)}{\partial t}\right)^2;\ \text{V1} = \text{IFk}\ /.\ \{x(t) \to x_1(t),\ k \to K\};\ \text{V2} = \sum_{i=2}^{n+1} (\text{IFk}\ /.\ \{x(t) \to x_i(t) - x_1(t),\ k \to k_i\});\ V = \text{V1} + \text{V2};\right\}$$

Lagrangian Equation

$$L = T - V$$

Euler−Lagrange Function

$$\text{Eqs} = \text{Table}\left[\frac{\partial L}{\partial x_i(t)} - \frac{\partial}{\partial t} \frac{\partial L}{\partial \frac{\partial x_i(t)}{\partial t}} = 0,\ \{i,\ 1,\ n+1\}\right]$$

Equations of Motions

$$\begin{pmatrix} -x_1(t) + 0.072908\, (x_2(t) - x_1(t)) + 0.109176\, (x_3(t) - x_1(t)) - 1.\, (x_1)''(t) = 0 \\ -0.072908\, (x_2(t) - x_1(t)) - 0.05\, (x_2)''(t) = 0 \\ -0.109176\, (x_3(t) - x_1(t)) - 0.05\, (x_3)''(t) = 0 \end{pmatrix}$$

Initial Conditions

$$\begin{pmatrix} x_1(0) = 0 \\ x_2(0) = 0 \\ x_3(0) = 0 \\ (x_2)'(0) = 0 \\ (x_3)'(0) = 0 \\ (x_1)'(0) = 1 \end{pmatrix}$$

PART 6

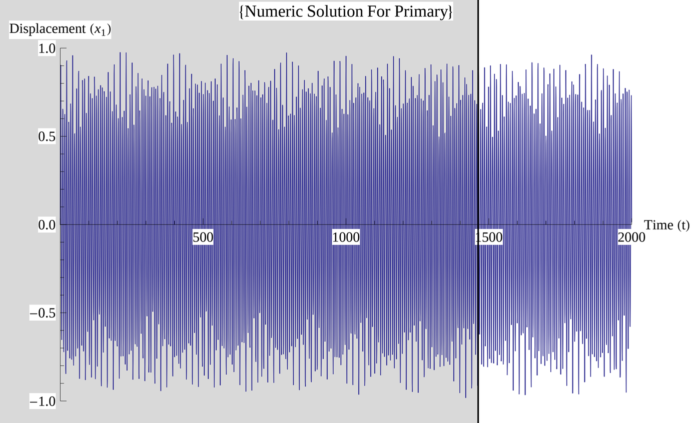
INTRODUCING NUMERICAL VALUES AND CALCULATE SYSTEM NUMERICALLY

Independent Frequencies of Primary And Sattalites

{1, 1.152, 1.40404}

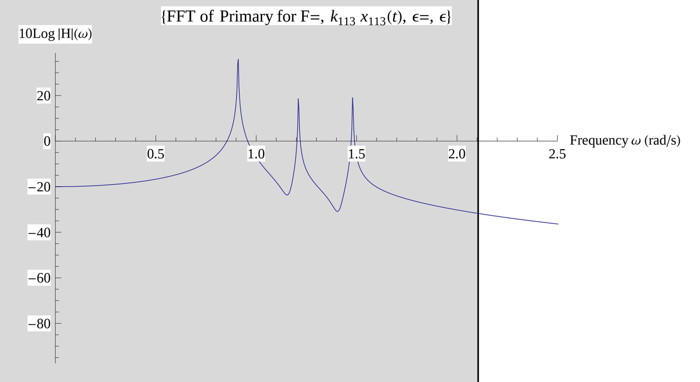
Numerically Solved Displacement Of Primary

{Numeric Solution For Primary}



FFT of Displacement of Primary

{FFT of Primary for F=, $k_{113}\, x_{113}(t)$, ϵ=, ϵ}



PART 7

EigenSystem and Analytic Solution Both using a Symbolic Solver and a Modal Analysis Technique

Modal Transformation Matrix (EigenVectors) using EigenSolution

$$\begin{pmatrix} -0.303802 & -0.0919473 & -0.105573 \\ -0.797705 & 0.931015 & 0.163593 \\ -0.520934 & -0.35321 & 0.980863 \end{pmatrix}$$

Natural Modes (EigenFrequencies) using EigenSolution

{0.906464, 1.20754, 1.47767}

Inverse of EigenVectors

$$\Psi\text{m} = \text{Um}^{-1}$$

Inverse of EigenVectors

{0.906464, 1.20754, 1.47767}

Calculation of Modal Energy Form 2

$$\text{EModal2} = \text{Table}\left[\frac{1}{2} \text{V0}^2\, \Psi\text{m}[[i,\ j]]^2,\ \{j,\ 1,\ n+1\},\ \{i,\ 1,\ n+1\}\right]$$

Modal Energy Form 2

$$\begin{pmatrix} 2.43447 & 1.25522 & 1.51808 \\ 0.041961 & 0.321732 & 0.00911305 \\ 0.0178948 & 0.0463069 & 0.327604 \end{pmatrix}$$

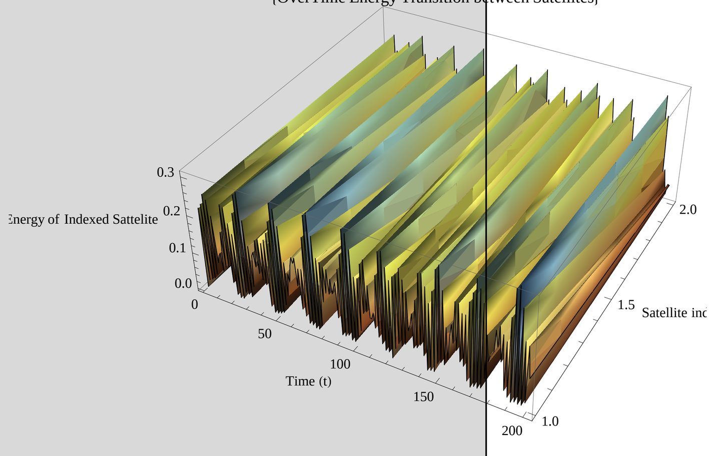
PART X

ENERGY PLOTS and CALCULATIONS AFTER NUMERICAL SOLUTION

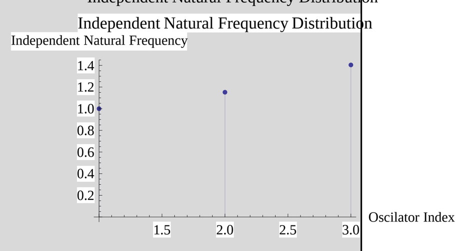
Uydu Kutlelerin Toplam Enerji Gecislerinin Cizdirilmesi

OverTime Energy Transition between Satellites

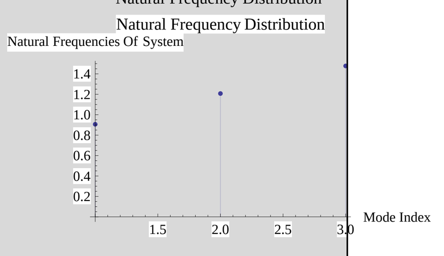
{OverTime Energy Transition between Satellites}



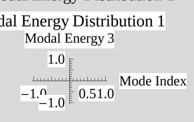
Independent Natural Frequency Distribution



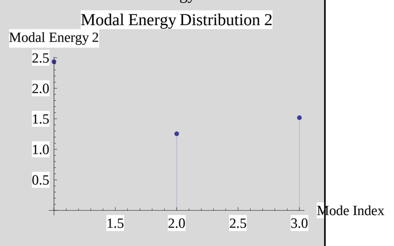
Natural Frequency Distribution



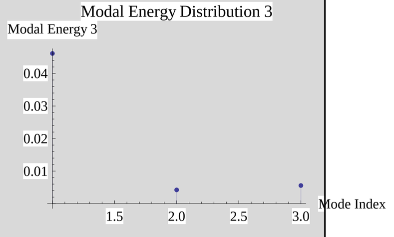
Modal Energy Distribution 1



Modal Energy Distribution 2



Modal Energy Distribution 3



Modal Energy Distribution 3 For Each Mass

{Modal Energy Distribution for each Mass}