MDOF HESAP AKIŞI

$\Rightarrow m$ – mass array $\rightarrow$ $M \rightarrow$ mass matrix

$\Rightarrow k$ – stiffness array $\rightarrow$ $K \rightarrow$ stiffness matrix

$$x(K-\lambda M)=0 \rightarrow U,\ [\omega_i]$$

$$E_{TOP} = \frac{1}{2}\dot{q}^T M \dot{q} + \frac{1}{2} q^T K q$$

$$\boxed{\begin{array}{l} L = T - V = \frac{1}{2}\left(\dot{q}^T M \dot{q} - q^T K q\right) \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = M\ddot{q} + Kq = 0 \end{array}}$$

$$q = U\eta$$

$$U_m = \frac{U}{\sqrt{U^T M U}} \Rightarrow \text{mass normalization}$$

Verification

$$U_m^T K U_m = [\omega_i^2]$$

$$U_m^T M U_m = I$$

$$E_{\substack{kinetik \\ sattelites}} = \left\{ \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2}(k_i - k_1)^2 q_2^2 + \dots \right\} -$$

$$q = q_h + q_p$$

$$q_h = \sum_{r=1}^{n} \left[U_m^T M q(0) \cos \omega_r t + \frac{1}{\omega_r} U_m^T M \dot{q}(0) \sin \omega_r t \right] U_m$$ * Homojen Çözüm

$$q_p = \sum_{r=1}^{n} \left[\frac{U_m^T}{\omega_r} \int_0^t Q(t-\tau) \sin \omega_r \tau \, d\tau \right] U_m$$ Partiküler Çözüm

$$E_{MODAL} = \dot{\eta}^2 + [\omega_i^2]\eta$$

Excitation Force $Q(t) = Q_0 \cos \omega t$

$$E_{ERROR} = E_{TOT} - E_{IMP} - E_{QIN}$$

$$E_{IMP} = \frac{1}{2}\left(\dot{q}(0)^T M \dot{q}(0) + q(0)^T K q(0) \right)$$

$$E_Q = \int Q \dot{q}(t) \, dt$$ constant için $E_{IN_Q}(t) = 0$ olmalı

TEORİ

$E_{pot} = \frac{1}{2}\underline{x}^T \underline{K}\, \underline{x}$

$E_{kin} = \frac{1}{2}\dot{x}^T M \dot{x}$

$E = E_{pot} + E_{kin} = \frac{1}{2}\left[\dot{x}^T M \dot{x} + x^T K x\right]$

$L = T - V = \frac{1}{2}\left[\dot{x}^T M \dot{x} - x^T K x\right]$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = Q_{(i)}^{nc} \longrightarrow M\ddot{x} + Kx = Q \rightarrow x(K - \lambda M) = 0$$

$\lambda = \omega^2 \rightarrow [\cdot\omega_i^2\cdot]$

$\eta = U^{-1} \cdot x$ → modal disp

$\tilde{K} = U^T K U$ → modal stiffness

$\tilde{M} = U^T M U$ → modal Mass

$N(t) = U^T Q(t)$ → modal Forces

$M\ddot{x} + Kx = Q$

$\eta(-\lambda U^T M U + U^T K U) = U^T Q$

$\eta(-\underset{[\cdot\omega_i^2\cdot]}{\lambda} \tilde{M} + \tilde{K}) = \tilde{N} \rightarrow \eta\left(-[\cdot\omega_i^2\cdot]\underset{I}{\tilde{M}} + \underset{[\cdot\omega_i^2\cdot]}{\tilde{K}}\right) = \tilde{N}$

Aynı sekilde Emodal

$E_{Top} \in D[x] \rightarrow E_{modal} \in D[\eta]$ ‖ $E_A = \frac{1}{2}\eta^2(-U^T M U [\cdot\omega_{i}^2\cdot] + U^T K U) \rightarrow \frac{1}{2}\eta\left(-[\omega_i^2]\underset{I}{\tilde{M}} + \underset{[\cdot\omega_i^2\cdot]}{\tilde{K}}\right)$

$E_{modal} = \frac{1}{2}\left(\dot{\eta}^2 + [\cdot\omega_i^2\cdot]\,\eta^2\right)$ → burdan ayrık (diagonalize edilmiş sistemde

$E_{i_{modal}} = \frac{1}{2}\left(\dot{\eta}_i^2 + \omega_i^2 \eta_i\right)$ → i. moddaki enerji

$E_{modal_i} = \frac{1}{2} C_i^2 \omega_i^2$

ANALİTİK ÇÖZÜM:

$$\eta_r(t) = C_r \cos(\omega_r t - \varphi_r) = \underbrace{\eta_r(0)}_{{}^aC_r} \cos(\omega_r t) + \underbrace{\frac{\dot{\eta}_r(0)}{\omega_r}}_{{}^bC_r} \sin \omega_r t$$

Modal initial Conditions

$C_r = \sqrt{{}^aC_r^2 + {}^bC_r^2}$

$\varphi_r = \arctan\left(\frac{-{}^bC_r}{{}^aC_r}\right)$

$(U^T M U)$ → eğer mass normalize etmezsen α^2 veriyor.

böylece $q(t) = \sum_{r=1}^{n}\left(u_r^T M q(0) \cos \omega_r t + \frac{1}{\omega_r} u_r^T M \dot{q}(0) \sin \omega_r t\right) u_r$