

In[1]:=

```
(* Report And Work Data Generation GLOBALS *)
repFile = "/Users/gediz/Desktop/Report.pdf";
datFile = "/Users/gediz/Desktop/Data.dat";
LogOut = ConstantArray[0, 100];
outCnt = 1;
TraditionallyFormattedOutput[x_, str_] :=
  (LogOut[[outCnt]] = Text[TraditionalForm[{str, x // MatrixForm} // MatrixForm]];
   outCnt++; LogOut[[outCnt - 1]]);
TFO = TraditionallyFormattedOutput;

TFO[
  "Mathematica Built-in Function Test using a problem which has a known solution
  - Meireovitch // Fundamentals of Vibration
  //Two Degree Of Freedom Systems//Example 5.2//Page 221", "CONTENT"]

  Example 5.2. Consider the simplified model of an automobile shown in Fig. 5.5, let the
  parameters have the values  $m = 1,500$  kg,  $I_C = 2,000$  kg m2,  $k_1 = 36,000$  kg/m,  $k_2 =$ 
  40,000 kg/m,  $a = 1.3$  m and  $b = 1.7$  m, calculate the natural modes of the system and
  write an expression for the response.

  TFO[
    
$$\begin{bmatrix} 1,500 & 0 \\ 0 & 2,000 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 76,000 & 21,200 \\ 21,200 & 176,440 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (a)$$

    But, free vibration is harmonic, so that by analogy with Eqs. (5.23) and (5.31) we can write
    
$$x(t) = X \cos(\omega t - \phi), \quad \theta(t) = \Theta \cos(\omega t - \phi) \quad (b)$$

    , "Question"]

  "Equations Of Motion in Matrix Form"]
```

$$\mathbf{x}(t) = C_1 \cos(6.857811t - \phi_1) \begin{bmatrix} 1 \\ -0.257341 \end{bmatrix} + C_2 \cos(9.584211t - \phi_2) \begin{bmatrix} 1 \\ 2.914417 \end{bmatrix} \quad (I)$$

where the amplitudes C_1 , C_2 and the phase angles ϕ_1 , ϕ_2 must be determined from the initial displacements $x(0)$, $\theta(0)$ and the initial velocities $\dot{x}(0)$, $\dot{\theta}(0)$, as will be shown in Sec. 5.4.

TFO [

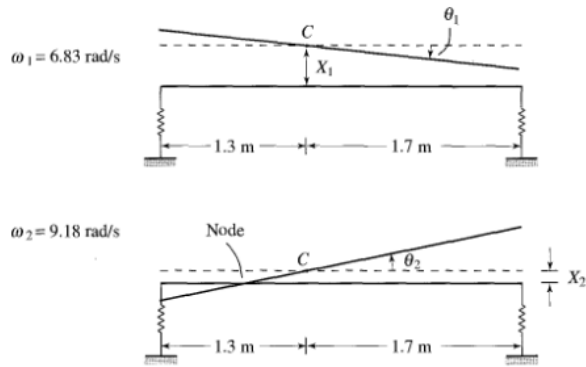


FIGURE 5.8
Natural modes for the automobile model of Fig. 5.5

, "Results"]

```
M = N[{{1500, 0}, {0, 2000}}];
K = N[{{76 000, 21 200}, {21 200, 176 440}}];

TFO[M, "Mass Matrix"]
TFO[K, "Stiffness Matrix"]

eigsys = Eigensystem[{K, M}];
"EigSys Transpose ve Reverse veriyor yani dogal frekanslari sirali vermiyor ve
ozvektorleri de transpose sekilde veriyor !"
eigs = Reverse[eigsys[[1]]];
eigvecs = Transpose[Reverse[eigsys[[2]]]];
Ωm = Table[ωᵢ = Sqrt[eigs[[i]]], {i, 1, 2}];
Um = eigvecs;

TFO[Ωm, "EigenFrequencies"]
TFO[Um, "EigenVectors"]
```

CONTENT

Out[7]=

Mathematica Built-in Function Test using a problem which has a known solution

– Meireovitch // Fundamentals of Vibration // Two Degree Of Freedom Systems // Example 5.2 // Page 221

Question

Out[8]=

Example 5.2. Consider the simplified model of an automobile shown in Fig. 5.5, let the parameters have the values $m = 1,500$ kg, $I_C = 2,000$ kg m², $k_1 = 36,000$ kg/m, $k_2 = 40,000$ kg/m, $a = 1.3$ m and $b = 1.7$ m, calculate the natural modes of the system and write an expression for the response.

Out[9]=

Equations Of Motion in Matrix Form

$$\begin{bmatrix} 1,500 & 0 \\ 0 & 2,000 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 76,000 & 21,200 \\ 21,200 & 176,440 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (a)$$

But, free vibration is harmonic, so that by analogy with Eqs. (5.23) and (5.31) we can write

$$x(t) = X \cos(\omega t - \phi), \quad \theta(t) = \Theta \cos(\omega t - \phi) \quad (b)$$

Out[10]=

Results

$$\mathbf{x}(t) = C_1 \cos(6.85781t - \phi_1) \begin{bmatrix} 1 \\ -0.257341 \end{bmatrix} + C_2 \cos(9.58421t - \phi_2) \begin{bmatrix} 1 \\ 2.914417 \end{bmatrix} \quad (I)$$

where the amplitudes C_1 , C_2 and the phase angles ϕ_1 , ϕ_2 must be determined from the initial displacements $x(0)$, $\theta(0)$ and the initial velocities $\dot{x}(0)$, $\dot{\theta}(0)$, as will be shown in Sec. 5.4.

FIGURE 5.8
Natural modes for the automobile model of Fig. 5.5

Out[13]=

$$\begin{pmatrix} \text{Mass Matrix} \\ \begin{pmatrix} 1500. & 0. \\ 0. & 2000. \end{pmatrix} \end{pmatrix}$$

Out[14]=

$$\begin{pmatrix} \text{Stifness Matrix} \\ \begin{pmatrix} 76\,000. & 21\,200. \\ 21\,200. & 176\,440. \end{pmatrix} \end{pmatrix}$$

Out[16]=

EigSys Transpose ve Reverse veriyor yani dogal frekanslari sirali vermiyor ve ozvektorleri de transpose sekilde veriyor !

Out[21]=

$$\begin{pmatrix} \text{EigenFrequencies} \\ \begin{pmatrix} 6.85781 \\ 9.58421 \end{pmatrix} \end{pmatrix}$$

Out[22]=

$$\begin{pmatrix} \text{EigenVectors} \\ \begin{pmatrix} -0.968447 & 0.324548 \\ 0.249221 & 0.945869 \end{pmatrix} \end{pmatrix}$$

```
In[23]:= (* Ortogonalite Degerlendirmesi *)
Ur = Um[[1 ;; 2, 1]];
Us = Um[[1 ;; 2, 2]];
Ur.K.Us;
TFO[Ur.K.Us, "Checking Orthogonality"]
```

```
Out[26]= ( Checking Orthogonality )
          3.63798 × 10-12
```

```
In[27]:= (* Kutle Normalizasyon *)
Ur2 = Ur / Sqrt[Ur.M.Ur];
Ur2.M.Ur2;
Us2 = Us / Sqrt[Us.M.Us];
Us2.M.Us2;
Um2 = Transpose[{Ur2, Us2}];
TFO[Um2, "Mass Normalized EigenVectors"]
```

```
Out[32]= ( Mass Normalized EigenVectors )
          ( -0.0247503  0.0073546 )
          (  0.00636927  0.0214344 )
```

```
In[33]:= (* Birim Normalizasyon *)
Ur3 = Ur / Ur[[1]];
Us3 = Us / Us[[1]];
Um3 = Transpose[{Ur3, Us3}];
TFO[Um3, "Unit Normalized EigenVectors"]
```

```
Out[36]= ( Unit Normalized EigenVectors )
          (  1.          1.          )
          ( -0.257341  2.91442 )
```

```
In[43]:= Table[LogOut[[i]], {i, 1, outCnt}] // MatrixForm
Export[repFile, % // MatrixForm];
```

Out[43]//MatrixForm=

```
( ( CONTENT
  ( Mathematica Built-in Function Test using a problem which has a known solution
    - Meireovitch // Fundamentals of Vibration //Two Degree Of Freedom Systems//Example 5.2//Page 221 )
  ( Question ) ) )
```

Example 5.2. Consider the simplified model of an automobile shown in Fig. 5.5, let the parameters have the values $m = 1,500$ kg, $I_C = 2,000$ kg m², $k_1 = 36,000$ kg/m, $k_2 = 40,000$ kg/m, $a = 1.3$ m and $b = 1.7$ m, calculate the natural modes of the system and write an expression for the response.

Equations Of Motion in Matrix Form

$$\begin{bmatrix} 1,500 & 0 \\ 0 & 2,000 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 76,000 & 21,200 \\ 21,200 & 176,440 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (a)$$

But, free vibration is harmonic, so that by analogy with Eqs. (5.23) and (5.31) we can write

$$x(t) = X \cos(\omega t - \phi), \quad \theta(t) = \Theta \cos(\omega t - \phi) \quad (b)$$

Results

$$\begin{aligned} \mathbf{x}(t) = & C_1 \cos(6.857811t - \phi_1) \begin{bmatrix} 1 \\ -0.257341 \end{bmatrix} \\ & + C_2 \cos(9.584211t - \phi_2) \begin{bmatrix} 1 \\ 2.914417 \end{bmatrix} \end{aligned} \quad (l)$$

where the amplitudes C_1 , C_2 and the phase angles ϕ_1 , ϕ_2 must be determined from the initial displacements $x(0)$, $\theta(0)$ and the initial velocities $\dot{x}(0)$, $\dot{\theta}(0)$, as will be shown in Sec. 5.4.

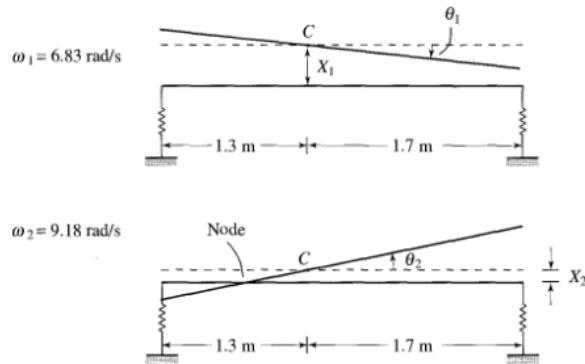


FIGURE 5.8
Natural modes for the automobile model of Fig. 5.5

$$\left(\begin{array}{c} \text{Mass Matrix} \\ \left(\begin{array}{cc} 1500. & 0. \\ 0. & 2000. \end{array} \right) \end{array} \right)$$

$$\left(\begin{array}{c} \text{Stiffness Matrix} \\ \left(\begin{array}{cc} 76\,000. & 21\,200. \\ 21\,200. & 176\,440. \end{array} \right) \end{array} \right)$$

$$\left(\begin{array}{c} \text{EigenFrequencies} \\ \left(\begin{array}{c} 6.85781 \\ 9.58421 \end{array} \right) \end{array} \right)$$

$$\left(\begin{array}{c} \text{EigenVectors} \\ \left(\begin{array}{cc} -0.968447 & 0.324548 \\ 0.249221 & 0.945869 \end{array} \right) \end{array} \right)$$

$$\left(\begin{array}{c} \text{Checking Orthogonality} \\ 3.63798 \times 10^{-12} \end{array} \right)$$

$$\left(\begin{array}{c} \text{Mass Normalized EigenVectors} \\ \left(\begin{array}{cc} -0.0247503 & 0.0073546 \\ 0.00636927 & 0.0214344 \end{array} \right) \\ \text{Unit Normalized EigenVectors} \\ \left(\begin{array}{cc} 1. & 1. \\ -0.257341 & 2.91442 \end{array} \right) \end{array} \right)$$

0