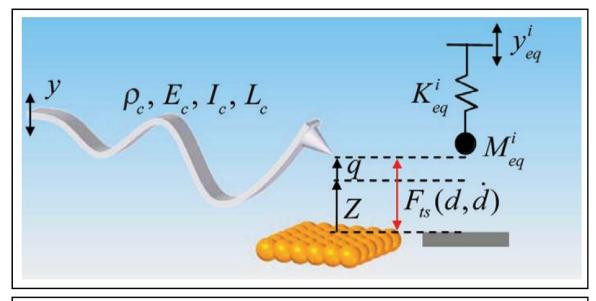
## STUDY: AFM Signle Tip Cantilever Mechanical Point Mass Model Review

Date: 19-10-2010

## MODEL PICKED TO SIMULATE:



$$M^i_{eq}\ddot{q} + (M^i_{eq}\omega_i/Q_i)\dot{q} + K^i_{eq}q = F_{ts} + K^i_{eq}y^i_{eq}$$

INTRODUCTION: This review is performed to decide the mathematical model of single cantilever AFM which will be

## **DETAILS FROM REFERENCES:**

Popular formulas in dynamic AFM for average force  $\bar{F}_{\rm ts}$ , average power dissipation  $\bar{P}_{\rm ts}$ , and frequency shift  $\Delta f$ , for resonant excitation include

$$\bar{F}_{ts} = kA_0/2Q_i\sqrt{1 - (A/A_0)^2},\tag{4}$$

$$\bar{P}_{ts} = k\omega_i/2Q_i(AA_0\sin\phi - A^2),\tag{5}$$

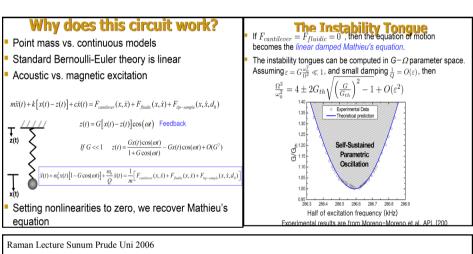
$$\Delta f = \frac{f_0}{\pi k A^2} \int_{-A}^{A} \frac{F_{ts}(Z - q')}{\sqrt{A^2 - q'^2}} dq',$$
 (6)

where Z, A,  $\phi$ ,  $A_0$ , and  $f_0$  are the base-sample separation,

$$M_{\rm eq}^i = m_c/4, \tag{1}$$

$$K_{\rm eq}^i = k_c \alpha_i^4 / 12, \tag{2}$$

$$y_{\text{eq}}^{i} = y(\omega/\omega_{i})^{2} \int_{0}^{L_{c}} \Phi_{i}(x) dx / \int_{0}^{L_{c}} \Phi_{i}^{2}(x) dx,$$
 (3)



Dynamic behavior of the tuning fork AFM probe

The problem is a problem prob

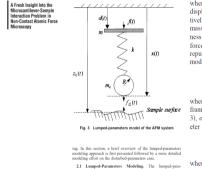
4(b) show the 3D force maps on the vertical cross sectional

plane to the surface including lines X or Y in Fig. 1, respec-

Dynamics of the fluid around the cantilever is governed by the Navier–Stokes equation. For the case where the beam of the cantilever is long in the axis direction and its cross sectional shape changes slowly along the axis, the motion of the cantilever is governed by the following Navier equation:

$$\rho S(z) \frac{\partial^2}{\partial z^2} h(z) + \frac{\partial^2}{\partial z^2} EI(z) \frac{\partial^2}{\partial z^2} h(z) = F^{tiq}(z), \tag{1}$$

where h(z), S(z), and I(z) are the height of the cantilever, the cross sectional area, and the geometrical moment of the beam cross section at the axial coordinate z, respectively. The parameter E in Eq. (1) is Young's modulus. The force from the liquid  $F^{liq}(z)$  in the right hand side of Eq. (1) is solved by the following procedure for any values of the height h(z) and its velocity  $(\partial/\partial t)hz$ . The liquid is approximated in this method to be a two-dimensional system with its motion limited in the cross sectional plane perpendicular to the axis. Then we can easily treat fluid mechanics in the two-dimensional space and the force from the liquids exerted to the cantilever at any of the axial coordinate z is obtained by its pressure and the velocity at the boundary of the beam cross section. From the force  $F^{\text{tiq}}(z)$  thus evaluated, the cantilever oscillation is numerically solved by Eq. (1) with an appropriate boundary condition at the basal point z=0 of the



where d(t) and x(t) denote the base motion and the cantilever tip displacement relative to the fixed base frame coordinate, respectively,  $m \in \Re^1$ ,  $m_e \in \Re^1$ ,  $b \in \Re^1$  denote equivalent base mass, cantilever tip mass, damping coefficient, and spring stiffness coefficient, respectively,  $f(t) \in \Re^1$  represents the controller force input, and  $f_{IL}(t) \in \Re^1$  denotes the van der Waals attraction/repulsion force (i.e., the interaction forces) for lumped-parameters model which is explicitly defined in the following form [13]

$$f_{IL}(t) = \frac{Dk}{(z_0(t) - x(t))^2} - \frac{\sigma^0 Dk}{30(z_0(t) - x(t))^8}$$

where  $z_0(t) \in \Re^1$  represents the distance from the fixed base frame coordinate to the sample (notice,  $Z_0(t) = z_0(t) - x(t)$  in Fig. 3),  $\sigma \in \Re^1$  denotes the molecular diameter, and the model parameter  $D \in \Re^1$  is defined in the following form

$$D = \frac{A_H R}{6k} \tag{3}$$

anoding approach is first presented followed by a more detailed anoding effort on the distinct-formations: one substitution of the state of the sta

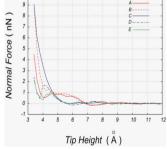


FIG. 2. (Color online) Force curves of mica in water for the (10.0) SWCNT tip. The lateral location of the tip is chosen at above site A, B, C, D or E shown in Fig. 3.

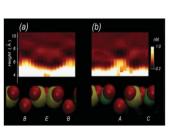


FIG. 4. (Color online) Cross section of the 3D force map over the mica surface in water for the SWCNT tip. Cross sectional plane includes the line X or Y, for the case of (a) and (b), respectively.

Concluding Remarks:

AFM Model selected to simulate is:

Arvind Raman's AFM Model in Equivalent Point-mass models of continuous atomic force microscope probes.