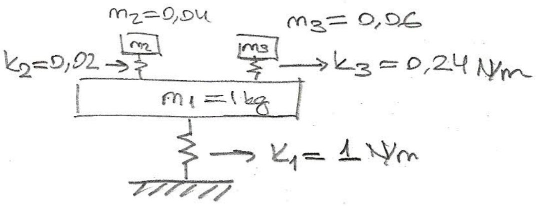
Elle yapılanları kontrol



In[1]:= $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.06 \end{pmatrix}$ $\underline{M}$

Out[1]= {{1, 0, 0}, {0, 0.04, 0}, {0, 0, 0.06}}

In[2]:= $K = \begin{pmatrix} 1.26 & -0.02 & -0.24 \\ -0.02 & 0.02 & 0 \\ -0.24 & 0 & 0.24 \end{pmatrix}$ $\underline{K}$

Out[2]= {{1.26, -0.02, -0.24}, {-0.02, 0.02, 0}, {-0.24, 0, 0.24}}

In[3]:=

```
Inverse[M] // MatrixForm
```

Out[3]//MatrixForm=

$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 25. & 0. \\ 0. & 0. & 16.6667 \end{pmatrix}$ M^{-1} ~~K~~

In[4]:=

```
Inverse[M].K // MatrixForm
```

Out[4]//MatrixForm=

$\begin{pmatrix} 1.26 & -0.02 & -0.24 \\ -0.5 & 0.5 & 0. \\ -4. & 0. & 4. \end{pmatrix}$ $M^{-1}K$

In[5]:= $\text{Inverse[M].K} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $(M^{-1}K - \lambda)$ Eigenvalue prob.

```
% // MatrixForm
```

Out[6]//MatrixForm=

$\begin{pmatrix} 1.26-\lambda & -0.02 & -0.24 \\ -0.5 & 0.5-\lambda & 0. \\ -4. & 0. & 4.-\lambda \end{pmatrix}$ $\longrightarrow (M^{-1}K - \lambda)$

In[7]:=

```
Det[%]
Solve[% == 0]
ωn = λ /. %
```

Out[7]= $2. - 6.7\lambda + 5.76\lambda^2 - \lambda^3$ $\longrightarrow$ karakteristik denk $\det|(M^{-1}K - \lambda)|$

Out[8]= {{λ → 0.480275}, {λ → 0.96517}, {λ → 4.31455}} $\longrightarrow \lambda_{1,2,3}$

Out[9]= {0.480275, 0.96517, 4.31455} $\longrightarrow \{\omega_i^2\}$

* ÖRNEK PROBLEM ÜZERİNDEN DECOUPLING MODAL TRANSFORMASYON-MODAL ENERJİ KAVRAMI 'nın sonuca bağlanma çalışması.

In[10]:=
```
esys = Eigensystem[Inverse[M].K];
U = Transpose[Reverse[esys[[2]]]];
ω2e = Reverse[esys[[1]]];
ω2 = {{%[[1]], 0, 0}, {0, %[[2]], 0}, {0, 0, %[[3]]}};
% // MatrixForm
U // MatrixForm
Um = U;
Table[
  Um[[All, i]] = U[[All, i]] / Sqrt[ U[[All, i]].M.U[[All, i]] ]
  , {i, 1, 3}];
Um // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} 0.480275 & 0 & 0 \\ 0 & 0.96517 & 0 \\ 0 & 0 & 4.31455 \end{pmatrix} \longrightarrow [\omega_i^2]$$

Out[15]//MatrixForm=

$$\begin{pmatrix} -0.0393794 & 0.506853 & -0.0783925 \\ -0.998222 & -0.544804 & 0.0102754 \\ -0.0447528 & 0.668048 & 0.99687 \end{pmatrix} \longrightarrow U \text{ (özvektörler)}$$

Out[18]//MatrixForm=

$$\begin{pmatrix} -0.193239 & 0.932324 & -0.305665 \\ -4.89838 & -1.00213 & 0.0400656 \\ -0.219607 & 1.22883 & 3.88695 \end{pmatrix} \longrightarrow \text{Kütle normalize özvektörler } U_m$$

```
Mm = Round[Transpose[U].M.U, 0.00000000000001];
% // MatrixForm
Km = Round[Transpose[U].K.U, 0.00000000000001];
% // MatrixForm
```

Nodal Transformasyona uğramış Kütle ve Yay matrisleri

Out[20]//MatrixForm=

$$\begin{pmatrix} 0.0415288 & 0 & 0 \\ 0 & 0.29555 & 0 \\ 0 & 0 & 0.0657746 \end{pmatrix} \longrightarrow M' = M_m = U^T M U$$

Out[22]//MatrixForm=

$$\begin{pmatrix} 0.0199452 & 0 & 0 \\ 0 & 0.285256 & 0 \\ 0 & 0 & 0.283788 \end{pmatrix} \longrightarrow K' = K_m = U^T K U$$

Diagonalize edilmiş (Uncoupling)

In[26]:=
```
{Km // MatrixForm, ω2 // MatrixForm, Mm // MatrixForm}
```

Out[26]=

$$\left\{ \begin{pmatrix} 0.0199452 & 0 & 0 \\ 0 & 0.285256 & 0 \\ 0 & 0 & 0.283788 \end{pmatrix} \rightarrow K_m, \begin{pmatrix} 0.480275 & 0 & 0 \\ 0 & 0.96517 & 0 \\ 0 & 0 & 4.31455 \end{pmatrix}, \begin{pmatrix} 0.0415288 & 0 & 0 \\ 0 & 0.29555 & 0 \\ 0 & 0 & 0.0657746 \end{pmatrix} \right\} \rightarrow M_m$$

In[27]:=
```
Km - ω2.Mm // MatrixForm
Round[Det[%], .000000000000001] === 0
```

$\lambda = \omega^2$

Out[27]//MatrixForm=

$$\begin{pmatrix} 2.76515\times10^{-15} & 0. & 0. \\ 0. & 3.33067\times10^{-15} & 0. \\ 0. & 0. & 2.10942\times10^{-15} \end{pmatrix} \longrightarrow \underline{K_m - \lambda M_m \approx 0}$$

Out[28]= True ← $K_m - \lambda M_m \overset{?}{=} 0$

In[29]:=
```
True

"Aynisinin Mass Normalizationla"
Mm = Round[Transpose[Um].M.Um, 0.00000000000001];
% // MatrixForm
Km = Round[Transpose[Um].K.Um, 0.00000000000001];
% // MatrixForm
{Km // MatrixForm, ω2 // MatrixForm, Mm // MatrixForm}
Km - ω2.Mm // MatrixForm
Round[Det[%], .000000000000001] === 0
```

Out[29]= True

Out[30]= Aynisinin Mass Normalizationla

Out[32]//MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 \\ 0 & 1. & 0 \\ 0 & 0 & 1. \end{pmatrix} \longrightarrow \tilde{M} = U_m^T M U_m \longrightarrow$$ Kütle normalize ve diagonalize edilmiş modal M

Out[34]//MatrixForm=

$$\begin{pmatrix} 0.480275 & 0 & 0 \\ 0 & 0.96517 & 0 \\ 0 & 0 & 4.31455 \end{pmatrix} \longrightarrow \tilde{K} = U_m^T K U_m \longrightarrow$$ Kütle normalize ve diag... moda K

Out[35]=

$$\left\{ \begin{pmatrix} 0.480275 & 0 & 0 \\ 0 & 0.96517 & 0 \\ 0 & 0 & 4.31455 \end{pmatrix}, \begin{pmatrix} 0.480275 & 0 & 0 \\ 0 & 0.96517 & 0 \\ 0 & 0 & 4.31455 \end{pmatrix}, \begin{pmatrix} 1. & 0 & 0 \\ 0 & 1. & 0 \\ 0 & 0 & 1. \end{pmatrix} \right\}$$

$\tilde{K} = \lambda = [\omega_i^2]$ $\quad$ $\lambda = [\omega_i^2]$ $\quad$ $\tilde{M} = I$

Out[36]//MatrixForm=

$$\begin{pmatrix} 2.77556 \times 10^{-16} & 0. & 0. \\ 0. & -2.22045 \times 10^{-15} & 0. \\ 0. & 0. & -1.77636 \times 10^{-15} \end{pmatrix} \longrightarrow \tilde{K} - \lambda \tilde{M} = \lambda - \lambda I = 0$$

Out[37]= True $\quad \mathrm{Det}\,|\tilde{K} - \lambda \tilde{M}| \stackrel{?}{=} 0$

In[174]:=
```
"Modal Cozum"
ηm = Table[Ci * Cos[Sqrt[ω2[[i, i]]] * t - ϕi], {i, 1, 3}];
% // MatrixForm
xm = Um.ηm;
% // MatrixForm
x = xm;
1 / 2 * (D[x, t].M.D[x, t] + x.K.x)
Simplify[%]
1 / 2 * (ω2.ηm^2 + D[ηm, t]^2)
Simplify[%]
```

Out[174]= Modal Cozum

Out[176]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[0.693019\,t - \phi_1]\,C_1 \\ \mathrm{Cos}[0.982431\,t - \phi_2]\,C_2 \\ \mathrm{Cos}[2.07715\,t - \phi_3]\,C_3 \end{pmatrix} \longrightarrow$$ Modal koordinatlar $\{\eta_m\}$

Out[178]//MatrixForm=

$$\begin{pmatrix} -0.193239\,\mathrm{Cos}[0.693019\,t - \phi_1]\,C_1 + 0.932324\,\mathrm{Cos}[0.982431\,t - \phi_2]\,C_2 - 0.305665\,\mathrm{Cos}[2.07715\,t - \phi_3] \\ -4.89838\,\mathrm{Cos}[0.693019\,t - \phi_1]\,C_1 - 1.00213\,\mathrm{Cos}[0.982431\,t - \phi_2]\,C_2 + 0.0400656\,\mathrm{Cos}[2.07715\,t - \phi_3] \\ -0.219607\,\mathrm{Cos}[0.693019\,t - \phi_1]\,C_1 + 1.22883\,\mathrm{Cos}[0.982431\,t - \phi_2]\,C_2 + 3.88695\,\mathrm{Cos}[2.07715\,t - \phi_3] \end{pmatrix}$$

$\rightarrow x = U.\eta_m$ (Genelleştirilmiş esas koordinatlar modal koordinatlarda yazılışı)

4

Enerji TOP:

$$E_{TOP}=\frac{1}{2}\left(x^T K x + x'^T M x'\right)$$

Quadratik

Out[180]=
$$\begin{aligned}
&\frac{1}{2}\Big(0.06\,(0.152192\,\mathrm{Sin}[0.693019\,t-\phi_1]\,C_1 - \\
&\quad 1.20724\,\mathrm{Sin}[0.982431\,t-\phi_2]\,C_2 - 8.07379\,\mathrm{Sin}[2.07715\,t-\phi_3]\,C_3)^2 + \\
&\quad 0.04\,(3.39467\,\mathrm{Sin}[0.693019\,t-\phi_1]\,C_1 + 0.984525\,\mathrm{Sin}[0.982431\,t-\phi_2]\,C_2 - \\
&\quad\quad 0.0832223\,\mathrm{Sin}[2.07715\,t-\phi_3]\,C_3)^2 + \\
&\quad (0.133918\,\mathrm{Sin}[0.693019\,t-\phi_1]\,C_1 - 0.915943\,\mathrm{Sin}[0.982431\,t-\phi_2]\,C_2 + \\
&\quad\quad 0.634912\,\mathrm{Sin}[2.07715\,t-\phi_3]\,C_3)^2 + \\
&\quad (-4.89838\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 - 1.00213\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + \\
&\quad\quad 0.0400656\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) \\
&\quad\ (-0.02\,(-0.193239\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 0.932324\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 - \\
&\quad\quad\quad 0.305665\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) + 0.02\,(-4.89838\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 - \\
&\quad\quad\quad 1.00213\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + 0.0400656\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3)) + \\
&\quad (-0.193239\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 0.932324\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 - \\
&\quad\quad 0.305665\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) \\
&\quad\ (1.26\,(-0.193239\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 0.932324\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 - \\
&\quad\quad\quad 0.305665\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) - 0.02\,(-4.89838\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 - \\
&\quad\quad\quad 1.00213\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + 0.0400656\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) - \\
&\quad\quad 0.24\,(-0.219607\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 1.22883\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + \\
&\quad\quad\quad 3.88695\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3)) + \\
&\quad (-0.219607\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 1.22883\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + \\
&\quad\quad 3.88695\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) \\
&\quad\ (-0.24\,(-0.193239\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + 0.932324\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 - \\
&\quad\quad\quad 0.305665\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3) + 0.24\,(-0.219607\,\mathrm{Cos}[0.693019\,t-\phi_1]\,C_1 + \\
&\quad\quad\quad 1.22883\,\mathrm{Cos}[0.982431\,t-\phi_2]\,C_2 + 3.88695\,\mathrm{Cos}[2.07715\,t-\phi_3]\,C_3))\Big)
\end{aligned}$$

0,2401

Out[181]=
$$\begin{aligned}
&(0.240138\,\mathrm{Cos}[0.693019\,t-\phi_1]^2 + 0.240138\,\mathrm{Sin}[0.693019\,t-\phi_1]^2)\,C_1^2 + \\
&(0.482585\,\mathrm{Cos}[0.982431\,t-\phi_2]^2 + 0.482585\,\mathrm{Sin}[0.982431\,t-\phi_2]^2)\,C_2^2 + \\
&(\cancel{-1.22645\times10^{-15}\,\mathrm{Cos}[0.982431\,t-\phi_2]\,\mathrm{Cos}[2.07715\,t-\phi_3]} - \\
&\quad \cancel{1.22125\times10^{-15}\,\mathrm{Sin}[0.982431\,t-\phi_2]\,\mathrm{Sin}[2.07715\,t-\phi_3]})\,C_2\,C_3 + \\
&(2.15728\,\mathrm{Cos}[2.07715\,t-\phi_3]^2 + 2.15728\,\mathrm{Sin}[2.07715\,t-\phi_3]^2)\,C_3^2 + \\
&C_1\,((\cancel{-1.94289\times10^{-16}\,\mathrm{Cos}[0.693019\,t-\phi_1]\,\mathrm{Cos}[0.982431\,t-\phi_2]} - \\
&\quad \cancel{4.71845\times10^{-16}\,\mathrm{Sin}[0.693019\,t-\phi_1]\,\mathrm{Sin}[0.982431\,t-\phi_2]})\,C_2 + \\
&\quad (\cancel{1.38778\times10^{-16}\,\mathrm{Cos}[0.693019\,t-\phi_1]\,\mathrm{Cos}[2.07715\,t-\phi_3]} - \\
&\quad\quad \cancel{2.498\times10^{-16}\,\mathrm{Sin}[0.693019\,t-\phi_1]\,\mathrm{Sin}[2.07715\,t-\phi_3]})\,C_3)
\end{aligned}$$

0

0

0

0

0

0 0

E_{TOP} basitleştirilmiş

$$0{,}240138\,C_1^2 + 0{,}482585\,C_2^2 + 2.15728\,C_3^2$$

Out[182]=
$$\begin{aligned}
\Big\{&\frac{1}{2}\left(0.480275\,\mathrm{Cos}[0.693019\,t-\phi_1]^2\,C_1^2 + 0.480275\,\mathrm{Sin}[0.693019\,t-\phi_1]^2\,C_1^2\right), \\
&\frac{1}{2}\left(0.96517\,\mathrm{Cos}[0.982431\,t-\phi_2]^2\,C_2^2 + 0.96517\,\mathrm{Sin}[0.982431\,t-\phi_2]^2\,C_2^2\right), \\
&\frac{1}{2}\left(4.31455\,\mathrm{Cos}[2.07715\,t-\phi_3]^2\,C_3^2 + 4.31455\,\mathrm{Sin}[2.07715\,t-\phi_3]^2\,C_3^2\right)\Big\}
\end{aligned}$$

$$E_M=\frac{1}{2}\left(w_i^2\,\eta^2+\dot{\eta}^2\right)$$

Out[183]=
$$\left\{0.240138\,C_1^2,\ 0.482585\,C_2^2,\ (2.15728\,\mathrm{Cos}[2.07715\,t-\phi_3]^2 + 2.15728\,\mathrm{Sin}[2.07715\,t-\phi_3]^2)\,C_3^2\right\}$$

Yani $E_{TOP} = E_{Modal}$

$$\Longrightarrow \frac{1}{2}\left(x^T K x + x'^T M x'\right) = \frac{1}{2}\left(w_i^2\,\eta^2+\dot{\eta}^2\right)$$

$$= \frac{1}{2}\left(w_1^2 C_1^2 + w_2^2 C_2^2 + w_3 C_3^2\right)$$

C'ler başlangıç koşullarına bağlı

Peki burda Primary'nin ayrı sattaliklerin ayrı modal Enerjisi gözükmüyor Neden? (Çünkü bu Modal Enerji TOPLAM)