LATENT VARIABLES IN STATISTICS AND THEIR APPLICATION TO A PROBLEM OF RATING ESTIMATION

Experimental study

A.M. Andronov, A. A.Ressin and K. Pilyushonoka

30 мая 2014 г.

Data volume

Total in Netflix Prize(1) Dataset:

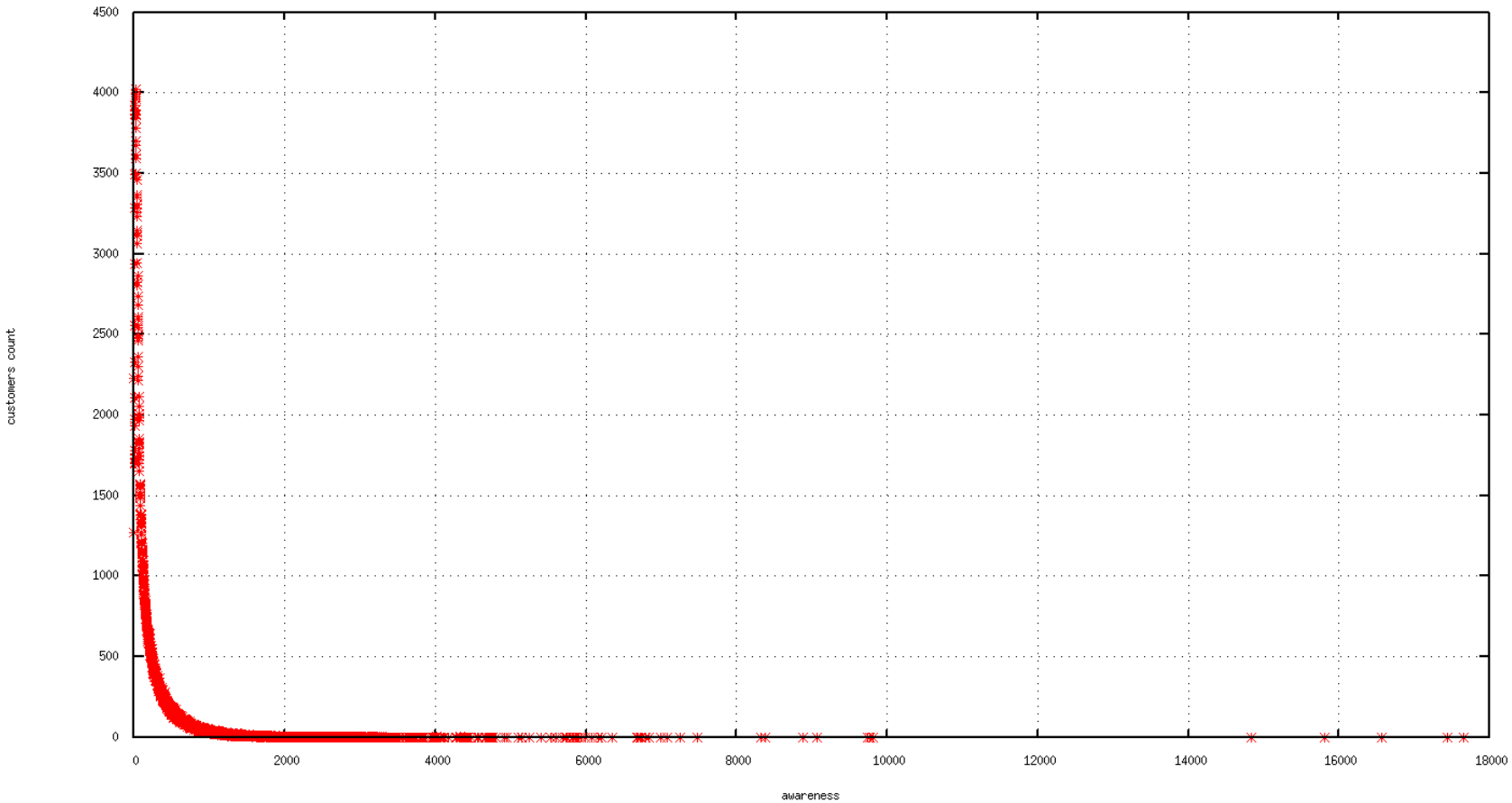
- movies – 17,770;
- customers – 480,189;
- cross-scores – 100,480,507.

Data volume analysis

- Histogram of customer awareness of movies
  - X – user awareness: count of different movies watched by a single customer;
  - Y – count of users with given awareness.
- Histogram of movies popularities
  - X – movie popularity: count of different customers that watched a single movie;
  - Y – count of movies with given popularity.

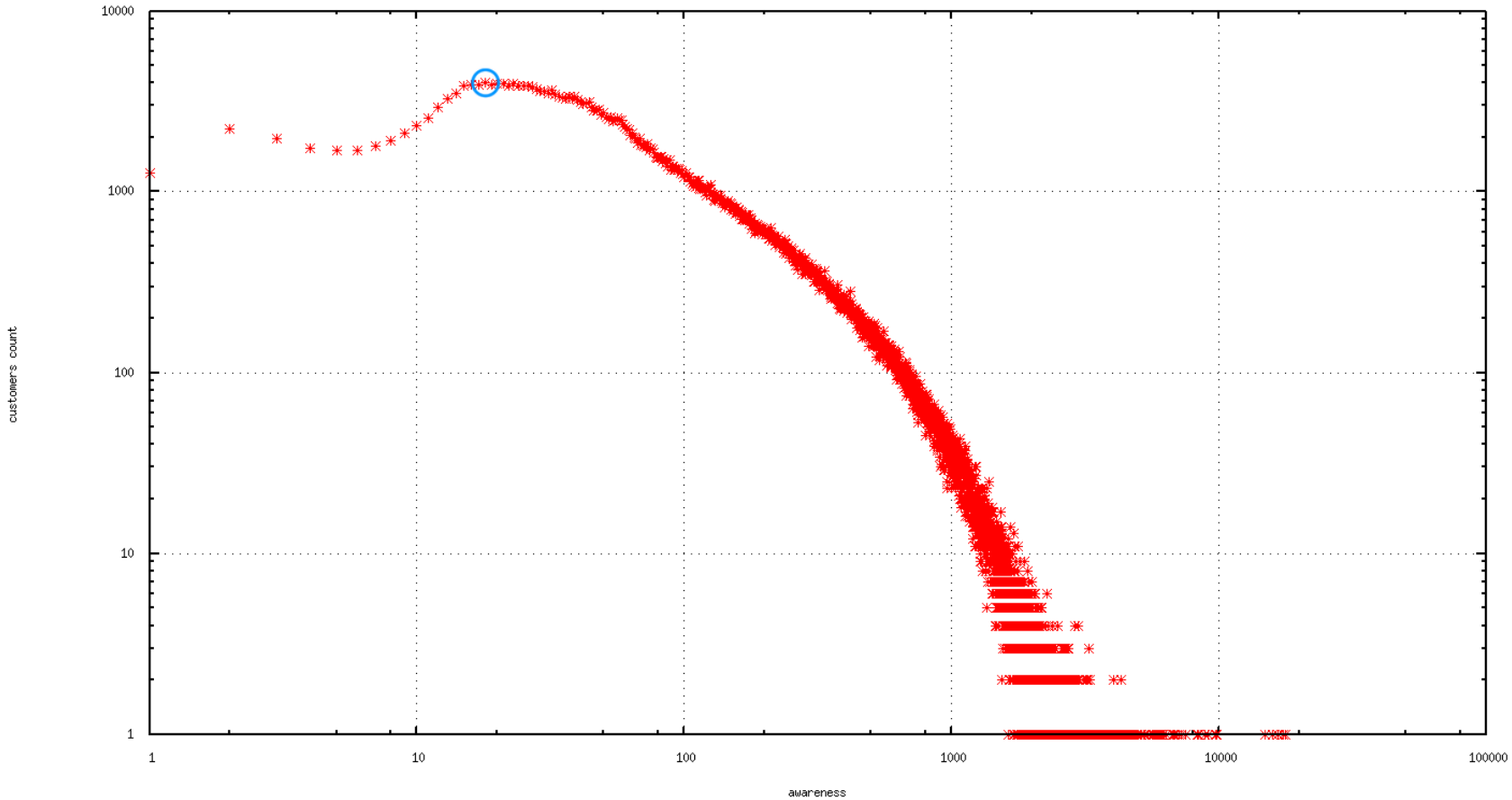
Data volume analysis

Users awareness



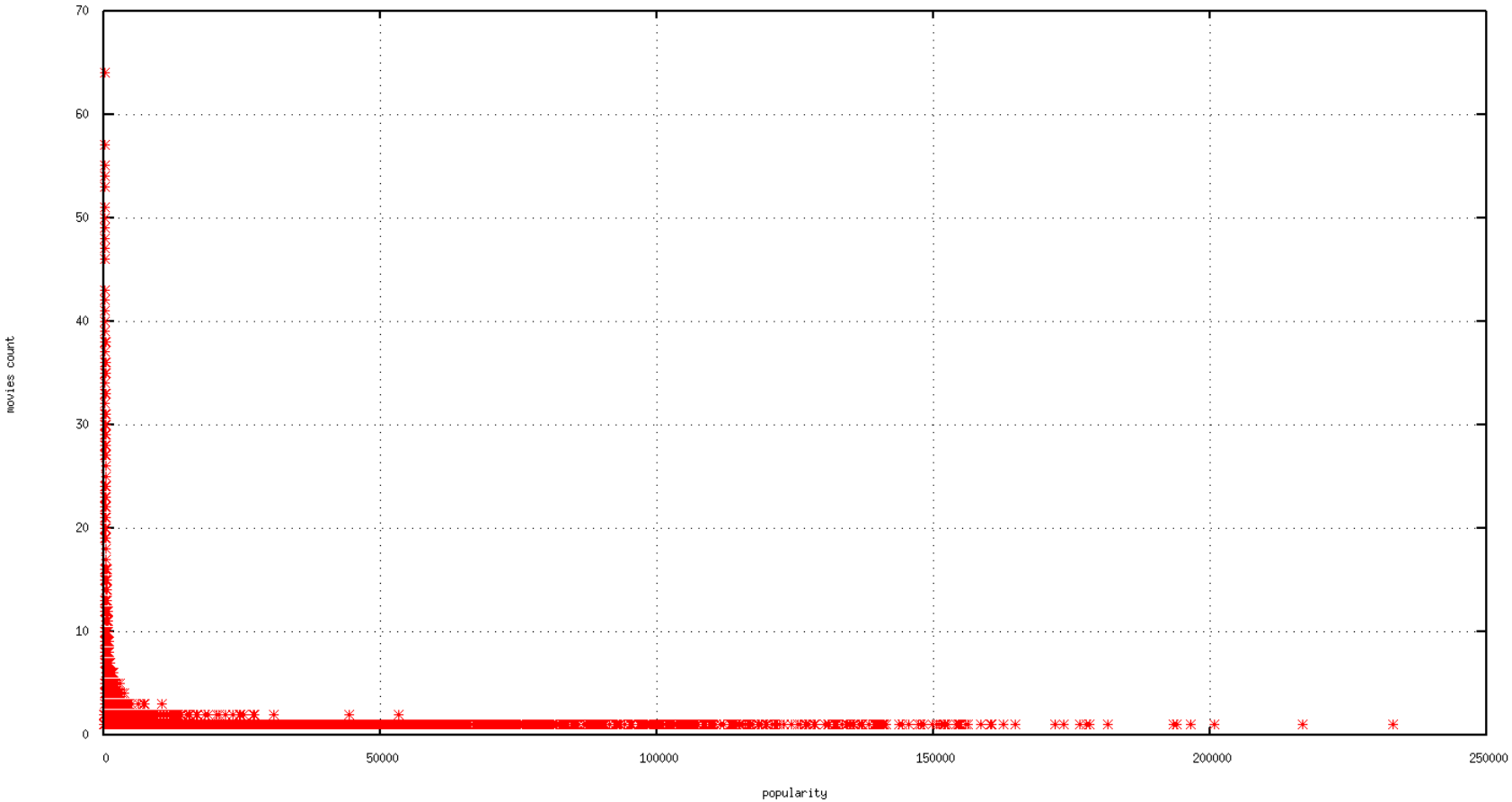
Data volume analysis

Users awareness (log-scale)



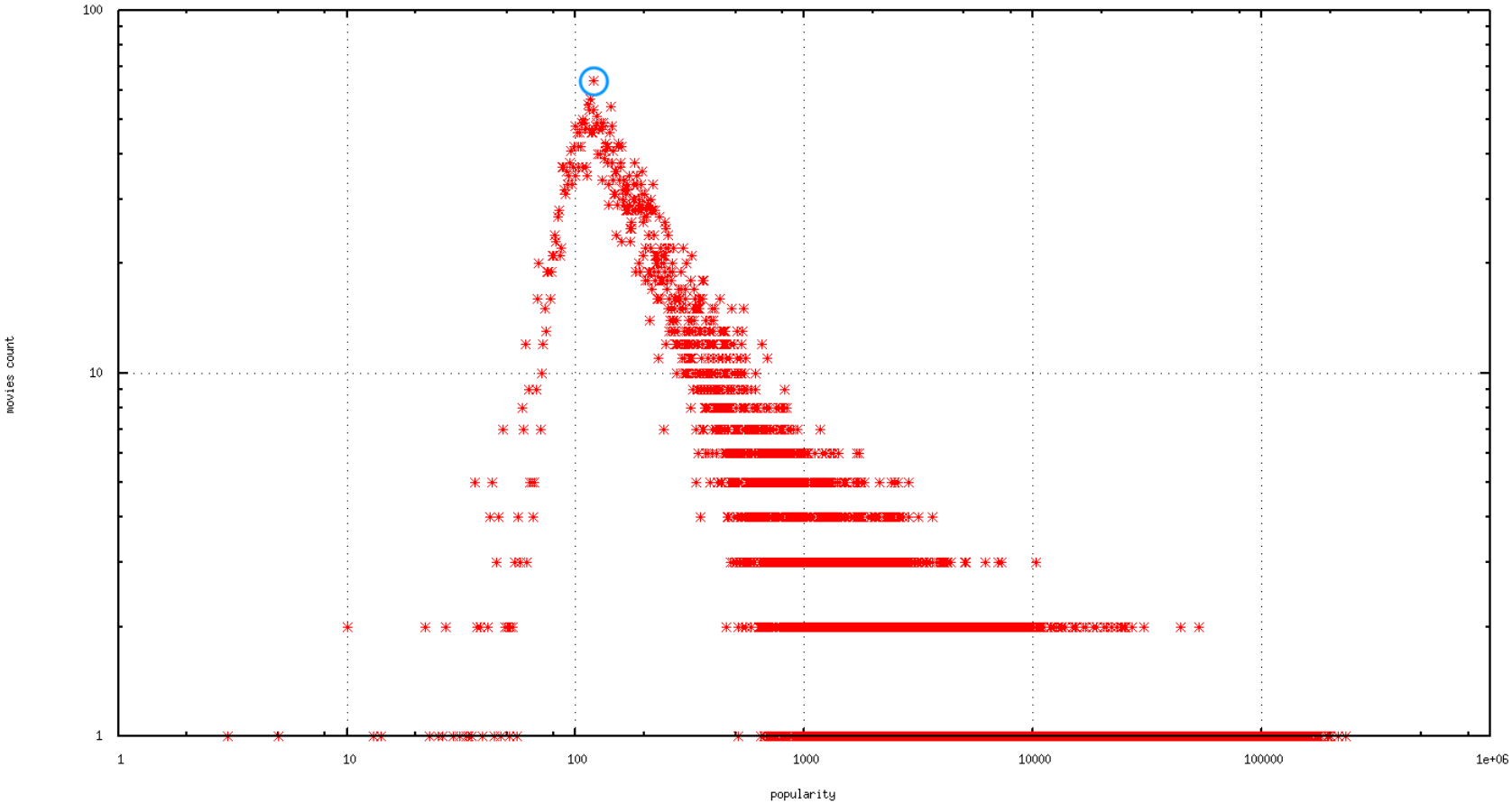
Data volume analysis

Movies popularity



Data volume analysis

Movies popularity (log-scale)



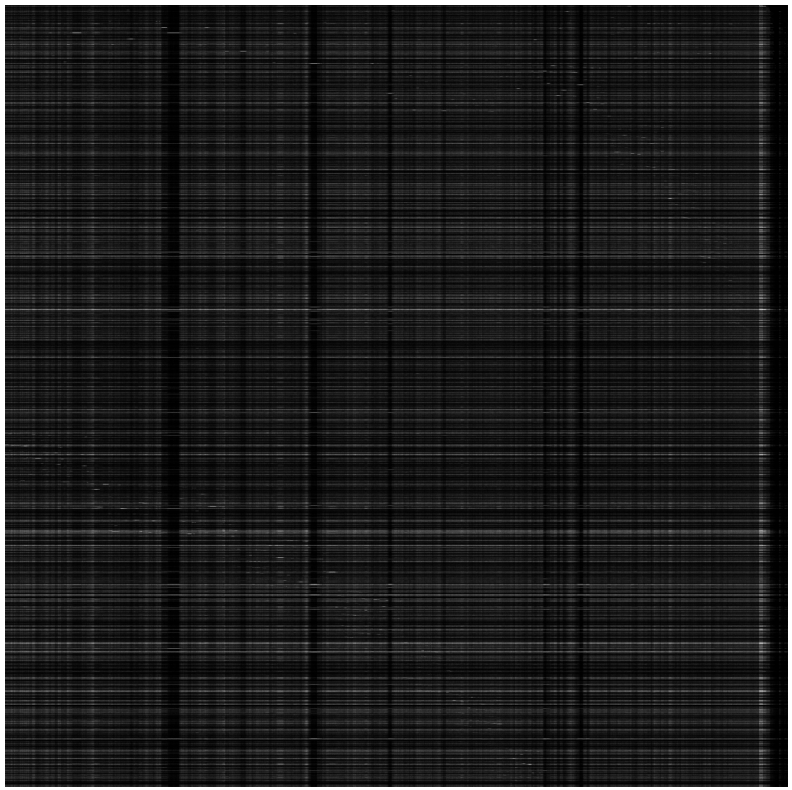
Data volume analysis

Summary

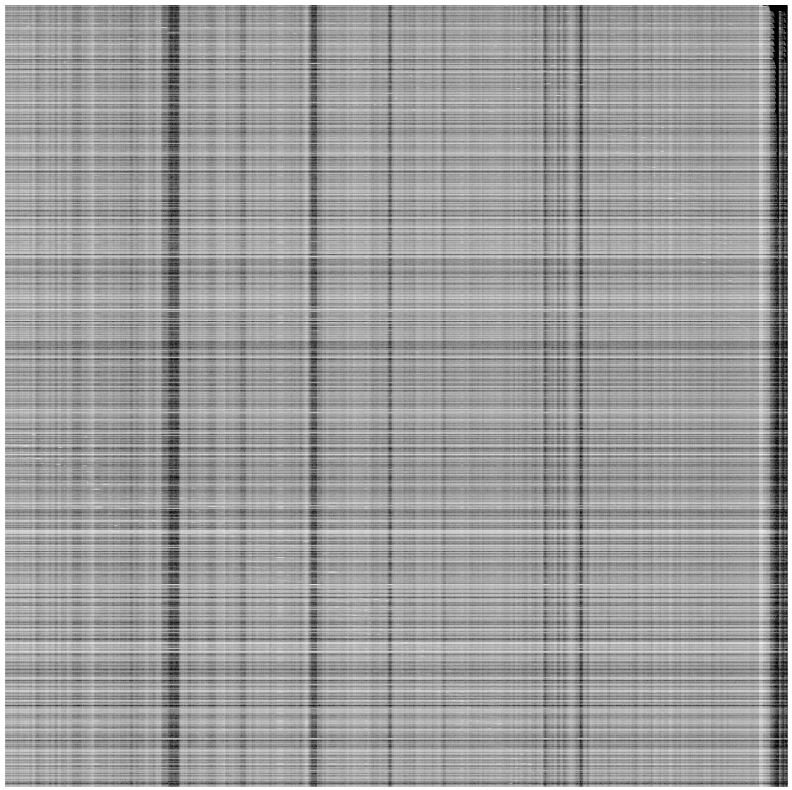
- Awareness:
  - the most frequent (4014 cases) awareness for user is 18;
  - average – 209.25;
  - standard deviation – 302.33.
- Popularity:
  - the most frequent (64 cases) popularity for movie is 120;
  - average – 5654.50;
  - standard deviation – 16909.67.
- Score fill ratio: 1.18%.

Data availability

Sparse matrix visual representation: grayscale



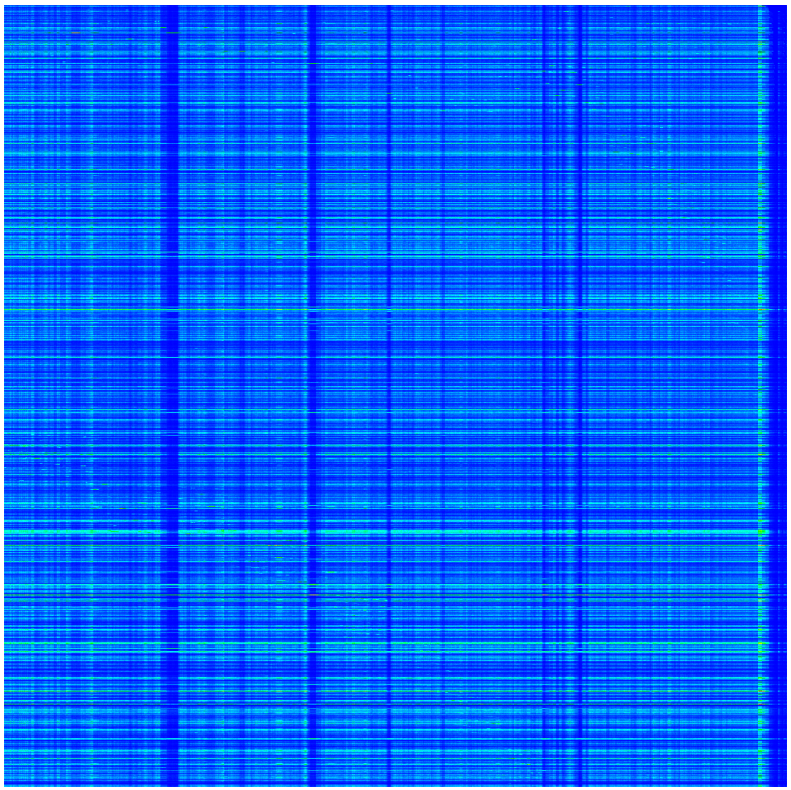
usual



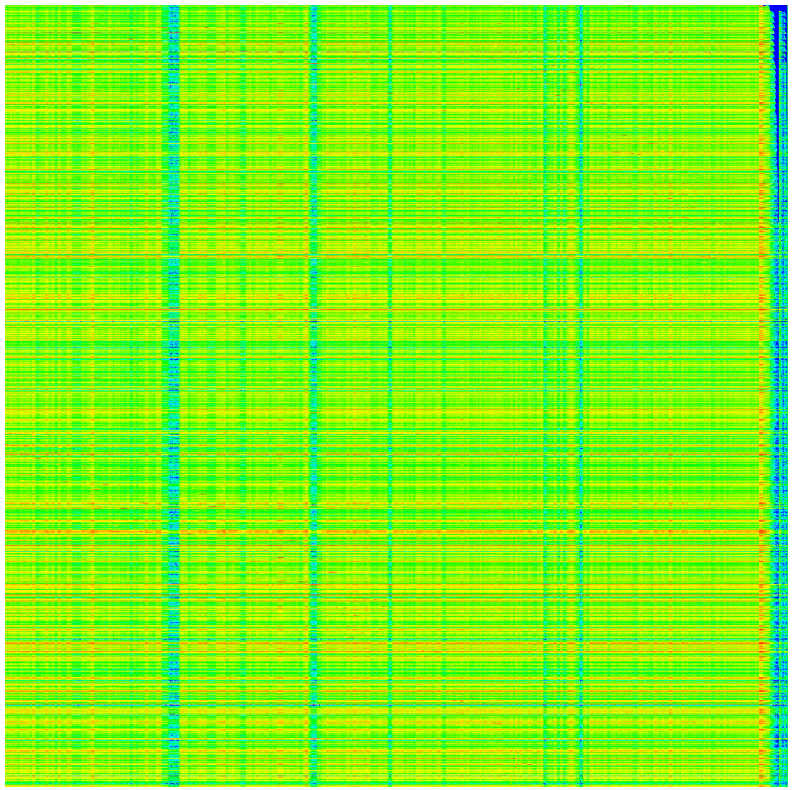
log-scale

Data presence

Sparse matrix visual representation: rainbow



usual



log-scale

Solid submatrix of given sparse matrix

- Let I and J are set of customers and objects indices respectively. The task is to find such $I^* \subseteq I$ and $J^* \subseteq J$, that $\forall i \in I^* \forall j \in J^* \exists y_{ij}$, where y_{ij} is score of object j given by user i and $|I^*||J^*| \to max$.
- The problem is NP-complete, because it is form of famous Clique Problem.
- Need for problem relaxation: maximize submatrix density for given submatrix size $|I^*| \times |J^*|$:

$$|I^*||J^*| - |\{y_{ij} : i \in I^*, j \in J^*, \exists y_{ij}\}| \to min.$$

- The problem still is NP-complete, but now it has heuristical solution.

Heuristical solution

- Let $\mathbf{Q} = \{q_{ij}\}_{n \times m}$ – binary matrix representing data availability.
- Let introduce ϕ_i – authority factors of users, and τ_j – authority factors of movies, such that

$$\phi_i = \frac{\sum_{j=1}^{m} \tau_j q_{ij}}{\sum_{j=1}^{m} q_{ij}}, \text{ and } \tau_j = \frac{\sum_{i=1}^{n} \phi_i q_{ij}}{\sum_{j=1}^{m} q_{ij}}.$$

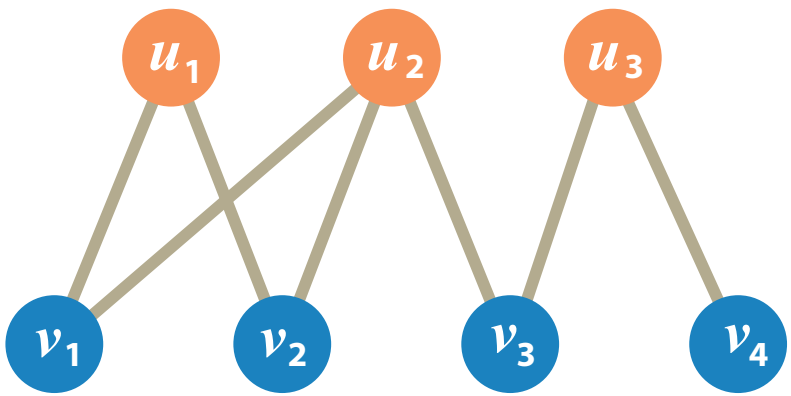
- Take I^* as first $\hat{n}$ indices i from I with maximal ϕ_i, and J^* as first $\hat{m}$ indices j from J with maximal τ_j, i.e.

$$I^* = \{i_1, \ldots, i_{\hat{n}}\} \text{ where } \phi_{i_1} > \ldots > \phi_{i_{\hat{n}}} > \ldots > \phi_{i_n},$$

$$J^* = \{j_1, \ldots, j_{\hat{n}}\} \text{ where } \tau j_1 > \ldots > \tau j_{\hat{n}} > \ldots > \tau j_n.$$

The idea behind authority factors

We can treat binary matrix $\mathbf{Q}$ as adjacency matrix of some bipartite graph G, which vertices are splitted in two disjoined sets U and V.

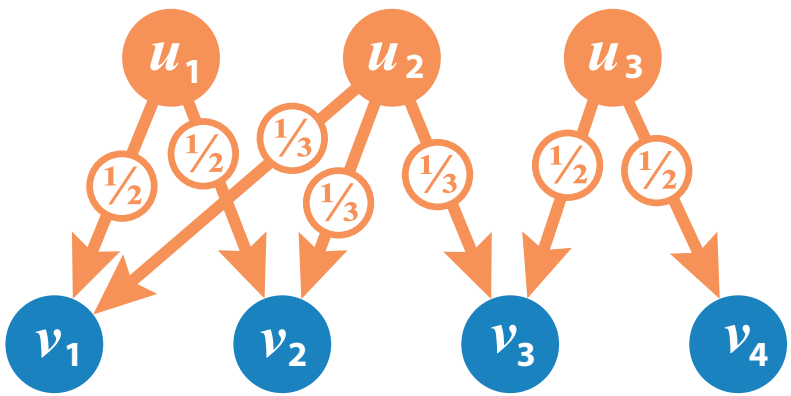


$$\mathbf{Q} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Lets consider random walk process on graph G, with equal probabilities for each outgoing arc. Thus transition from set U to set V can be represented in form of stochastic matrix $\mathbf{Q}_1$ and transition from V to U as $\mathbf{Q}_2^T \ldots$

The idea behind authority factors

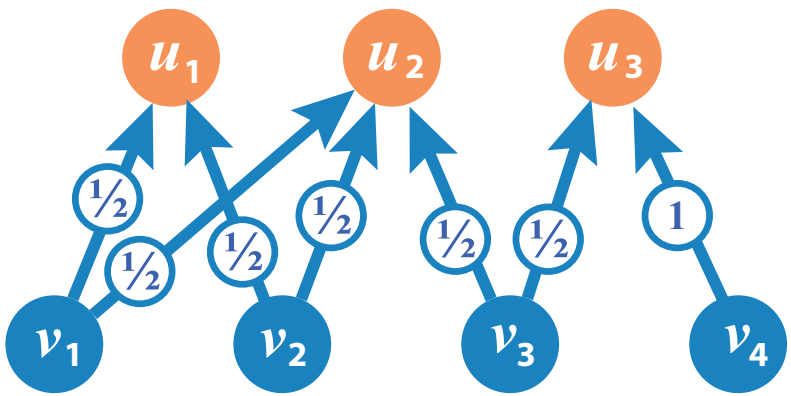
Thus transition from set U to set V can be represented in form of stochastic matrix $\mathbf{Q}_1$ and transition from V to U as $\mathbf{Q}_2^T$



$$\mathbf{Q}_1 = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

The idea behind authority factors

Thus transition from set U to set V can be represented in form of stochastic matrix $\mathbf{Q}_1$ and transition from V to U as $\mathbf{Q}_2^T$



$$\mathbf{Q}_2^T = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

The idea behind authority factors

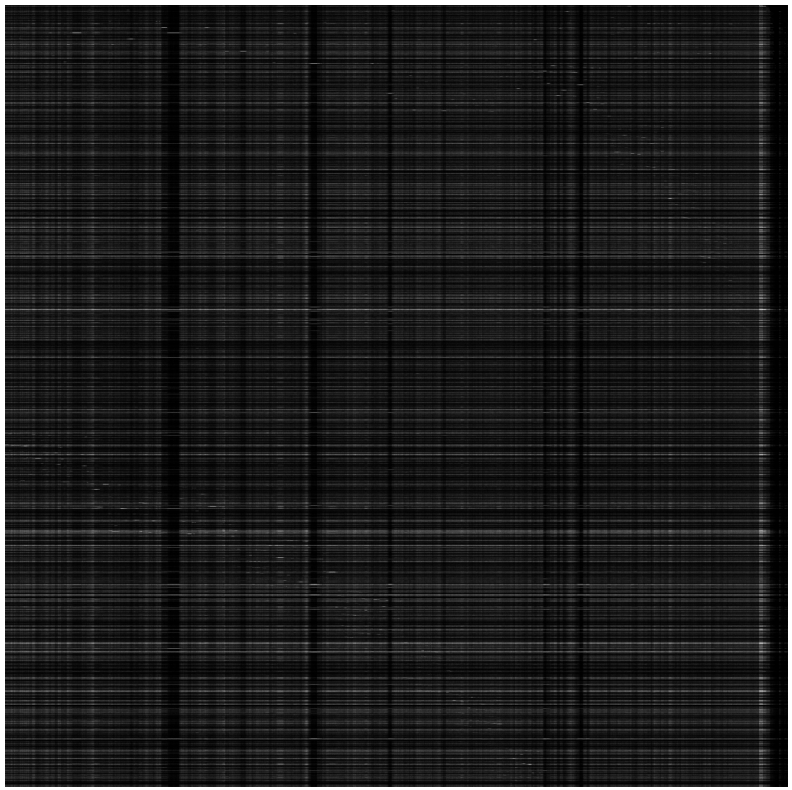
- In this setting authority factors ϕ_i and τ_j are essentialy stationary probabilities for random walk process defined by stohastic matrices $\mathbf{Q}_1\mathbf{Q}_2^T$ and $\mathbf{Q}_1^T\mathbf{Q}_2$ respectively and can be described by following equations:

$$\phi = \mathbf{Q}_1\tau, \text{ and } \tau = \mathbf{Q}_2^T\phi.$$

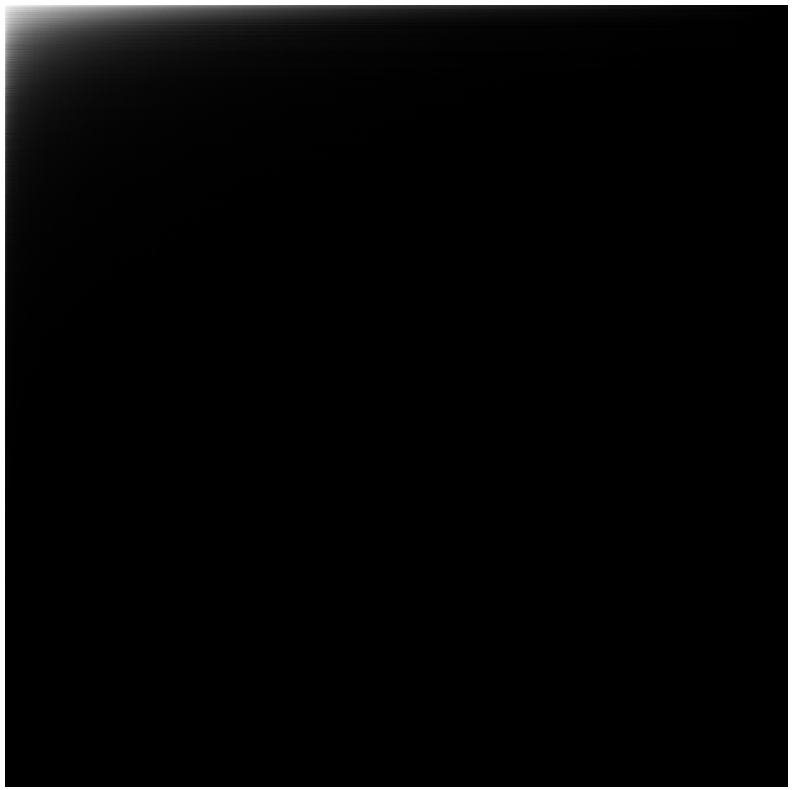
- The algoritm is very similar to one used by Google which assigns PageRank authority factor to each web-page in the Internet as logarithm of stationary probability for random walk through the Internet graph defined by hyperlinks.

Data availability

Densities before and after sorting by authority factors



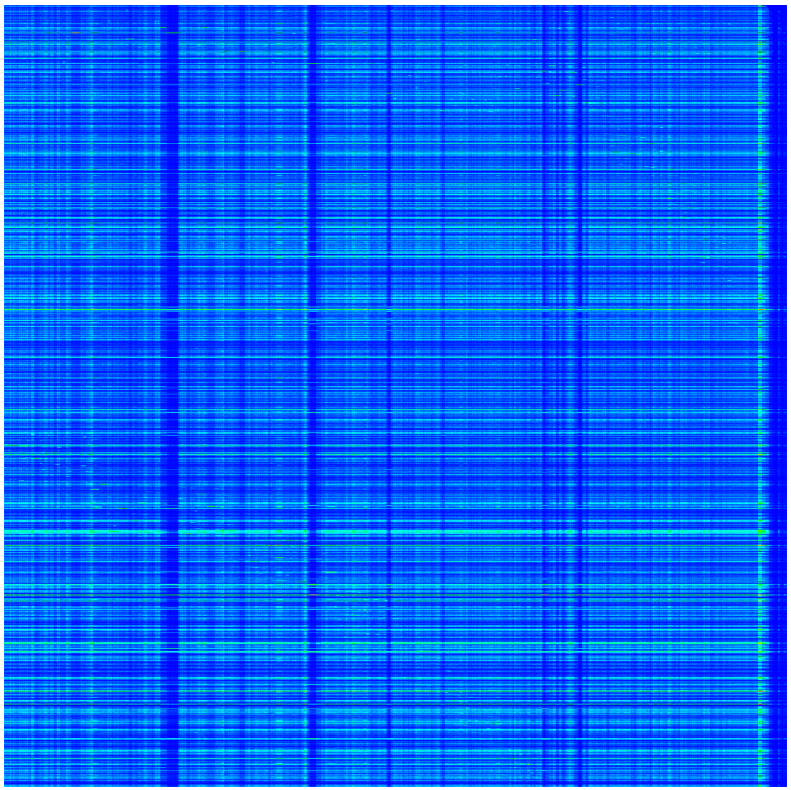
initial



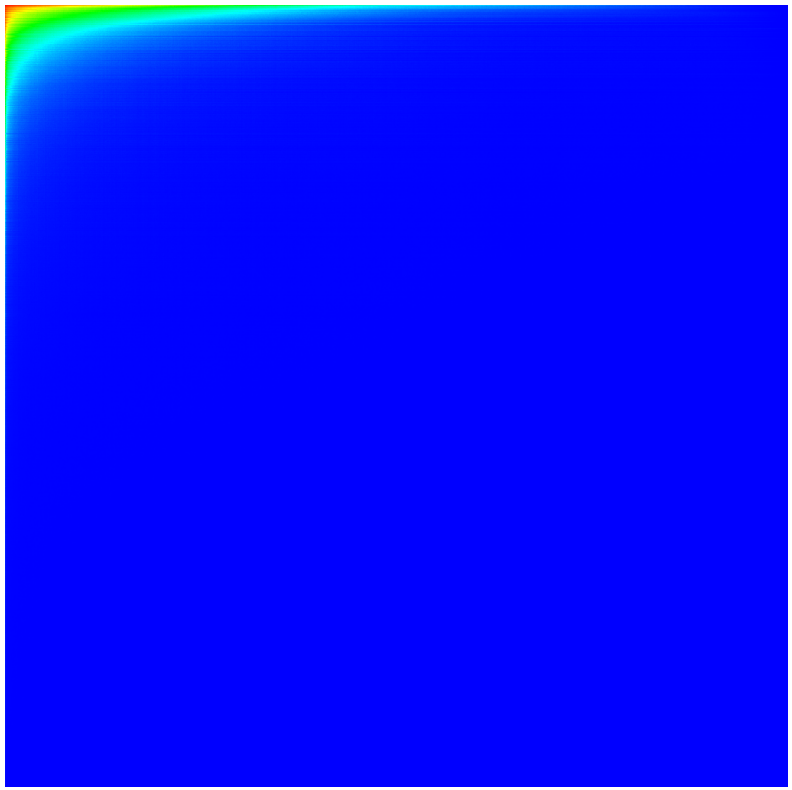
after 5 iterations

Data availability

Densities before and after sorting by authority factors



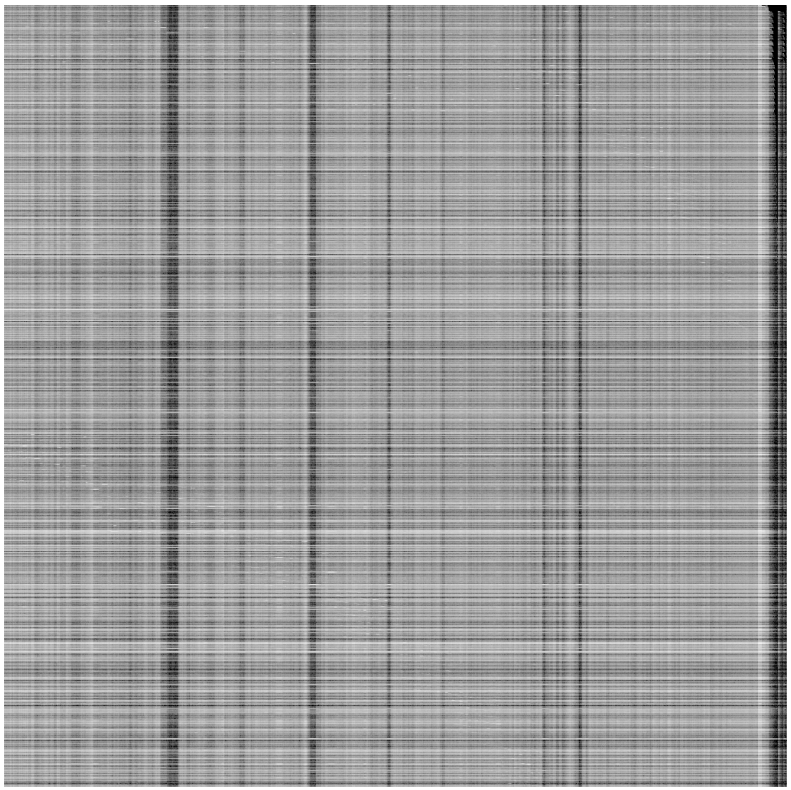
initial



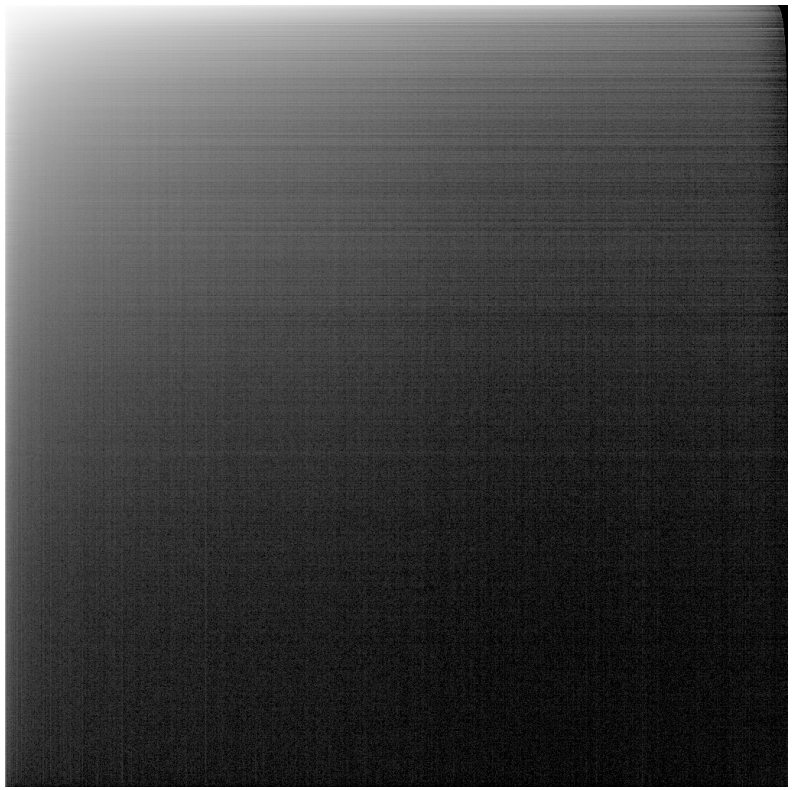
after 5 iterations

Data availability

Densities before and after sorting by authority factors: log-scale



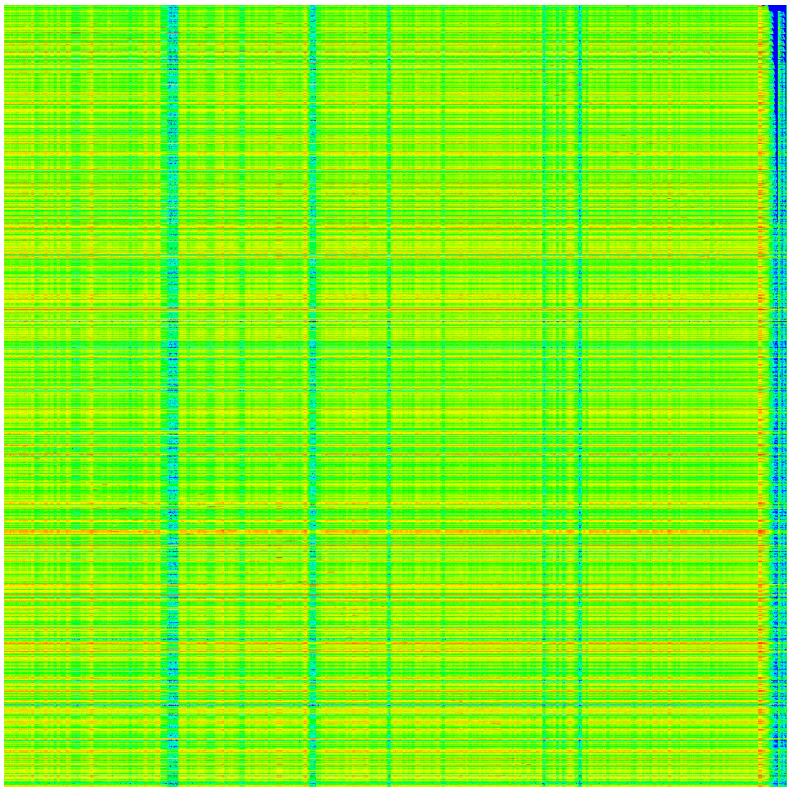
initial



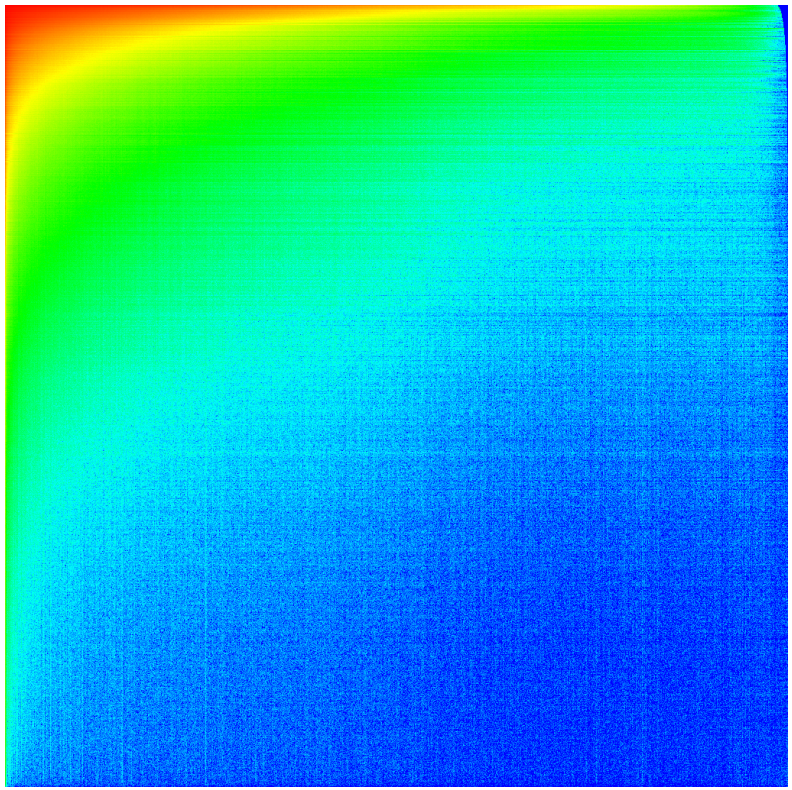
after 5 iterations

Data availability

Densities before and after sorting by authority factors: log-scale



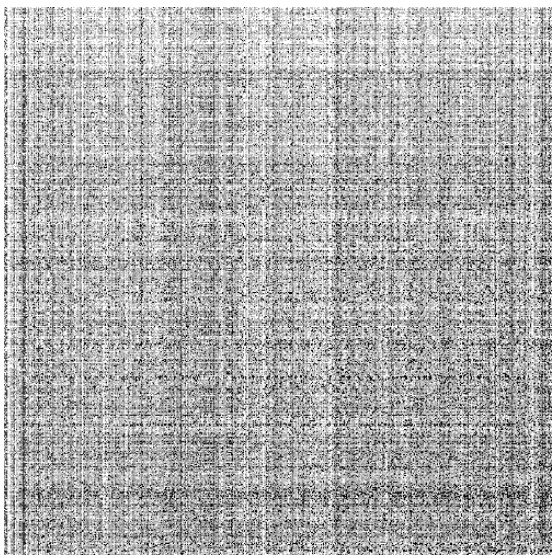
initial



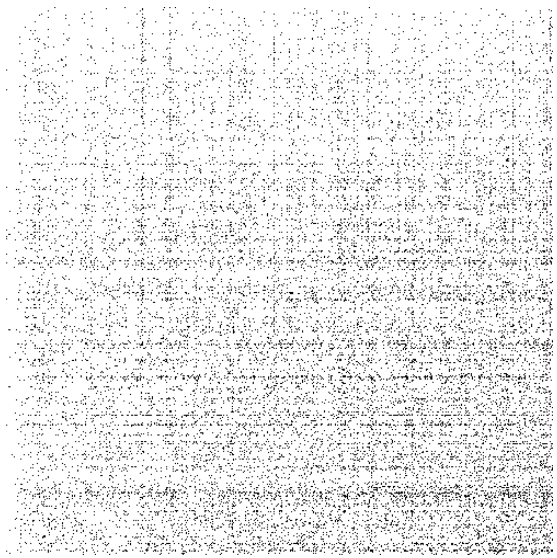
after 5 iterations

Submatrix selection

- 500×500 – upper left corner of data sorted according to authority factors;
- missing ratio – 7% (score fill ratio increased from 1.18% to 93%).



scores



missing data (black pixels)

[1] J. Bennett, S. Lanning, and N. Netflix, “The netflix prize,” in *In KDD Cup and Workshop in conjunction with KDD*, 2007.