Problem 6. A classic unsolved problem in number theory asks if there are infinitely many pairs of 'twin primes', pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

Proof. By using the theorem in Problem 5 and definition of primes.

The definition of prime number states that
"A prime number is a positive integer n, greater than 1, whose only exact divisors are 1 and n"

The theorem in Problem 5 states that
"For any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3"

If n is the first of the 3 integers that are separated by 2, then we have the integers n, $n+2$, and $n+4$.

However using the theorem in Problem 5, we know that one of these three integers must be divisible by 3.

The integers 3, 5 and 7 form a prime triple because the integer that is divisible by 3, in this case, is 3 itself. 3 is a prime number.

Every integer in the set of integers $\{n, n+2, n+4\}$ greater than this, that is divisible by 3, will have to be a multiple of 3 and cannot therefore be a prime number.

For example when $n = 5$, we have 5, 7, and 9. Here, 9 is divisible by 3 and hence not a prime number. Taking other examples greater than this, will also give the same result.

Hence it is proved that 3, 5, and 7 is the only prime triple.

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