Problem 1. Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

Proof. By contradiction.

The negation of the proposition will be

$$(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(3m + 5n \neq 12)$$

We will need to show that the above is TRUE.

The natural numbers start from 1 and do not include 0.

$$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$$

Rearranging the equation in the original proposition to get the value of n on the left hand side

$$\begin{aligned} & 3m + 5n = 12 & \\ \implies & 5n = 12 - 3m & \text{[Subtracting both sides by } 3m\text{]} \\ \implies & 5n = 3(4 - m) & \text{[Factoring out the common factor 3]} \\ \implies & n = \frac{3}{5}(4 - m) & \text{[Dividing both sides by 5]} \end{aligned}$$

For n to be in the natural numbers, it must be the case that $5|(4 - m)$ i.e.

$$(x \in \mathbb{N})(4 - m = 5x)$$

Taking the smallest value of x i.e. $x = 1$. We have

$$\begin{aligned} & 4 - m = 5 & \\ \implies & 4 = 5 + m & \text{[Adding } m \text{ to both sides]} \\ \implies & m = -1 & \text{[Subtracting 5 from both sides]} \end{aligned}$$

But the above conclusion is FALSE by the definition of m, i.e. $m \in \mathbb{N}$ $(-1 \notin \mathbb{N})$. Larger values of x will give even smaller values of m.

Hence for all values of m and n, the negation $3m + 5n \neq 12$ is TRUE which means that the original proposition was **FALSE**.

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