**Problem 3.** Say whether the following is true or false and support your answer by a proof: For any integer $n$, the number $n^2 + n + 1$ is odd.

*Proof.* By cases.

For $n = -2$,

$$n^2 + n + 1 = (-2)^2 + (-2) + 1 = 3$$

For $n = -1$,

$$n^2 + n + 1 = (-1)^2 + (-1) + 1 = 1$$

For $n = 0$,

$$n^2 + n + 1 = (0)^2 + (0) + 1 = 1$$

For $n = 1$,

$$n^2 + n + 1 = (1)^2 + (1) + 1 = 3$$

1 and 3 are odd numbers, so the statement seems true.

Using the equation we have,

$$n^2 + n + 1 = n(n + 1) + 1 \quad \text{[Factoring out the common factor n]}$$

Let $n$ be any arbitrary integer. So $n + 1$ is the consecutive number.

There can be two cases then

- **Case 1 ($n$ is odd)**
  If $n$ is odd, $n + 1$ has to be even.
  odd $\times$ even = even.
  even + 1 is an odd number.
  Hence, if $n$ is odd, $n^2 + n + 1$ is odd and our initial statement is **TRUE**.
- **Case 2 ($n$ is even)**
  If $n$ is even, $n + 1$ has to be odd.
  even $\times$ odd = even.
  even + 1 is an odd number.
  Hence, if $n$ is even, $n^2 + n + 1$ is also odd and our initial statement is **TRUE** yet again.

$\therefore$ It has been proved that for any integer $n$, $n^2 + n + 1$ is odd.

$\blacksquare$