

Problem 2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

Proof. Let n be any arbitrary integer.

Taking the first 5 integers 1, 2, 3, 4, 5, we have the summation to be

$$1 + 2 + 3 + 4 + 5 = 15$$

15 is divisible by 5, so the statement seems true.

Let n be the first integer of the 5 consecutive integers. The 5 integers will then be n , $n + 1$, $n + 2$, $n + 3$, and $n + 4$.

Hence

$$\begin{aligned} n + (n + 1) + (n + 2) + (n + 3) + (n + 4) &= 5n + 10 \\ &= 5(n + 2) \text{ [Factoring out the common factor 5]} \end{aligned}$$

$5(n + 2)$ is divisible by 5.

Hence the sum of any five consecutive integers is divisible by 5 which proves that the original statement is **TRUE**.

