Identification of the Dynamic Parameters of a Closed Loop Robot

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Abstract

This paper presents the experimental results of the identification of the dynamic parameters of the 6 degree of freedom SR400 robot. This industrial robot is characterized by having a parallelogram closed loop and a mechanical coupling between the joints of the hand. The different steps starting from the modelling till the validation of the results will be given. Practical issues will be adressed to carry out with success such experimentation.

1. Introduction

Dynamic models of robots are required to control or simulate their motions. These models are functions of the geometric parameters of robots (length of links, angle between joint axis,...) and the dynamic parameters (inertia, first moments, masses, friction). This paper is focused on estimating the dynamic parameters using least squares identification techniques, while the geometric parameters are supposed known. The paper is organized following the main steps of the identification process. Section 2 defines the geometric description of the robot, section 3 presents the dynamic model of the robot and the base inertial parameters, which are the only identifiable parameters, section 4 contains the identification models and the experimental protocol to get the values of the dynamic parameters and to validate the obtained results. Finally, section 5 contains the conclusion.

2. Geometric description of the robot

The SR400 robot has 6 degrees of freedom and 10 Kg as nominal load. The number of moving links (denoted n) is equal to 8 and the number of joints (denoted N_j) is equal to 9. It contains a closed loop of parallelogram type. It is actuated with N=6 brushless motors. The description of the geometry of the robot is carried out using the modified Denavit and Hartenberg notation [1]. The coordinate frame j is fixed on link j, the $\mathbf{z}_j$ axis is along the axis of joint j, the $\mathbf{x}_j$ axis is along the common perpendicular of z_j and one of the succeeding axis on the same link (see Fig. 1). In the case of closed loop robot, the geometric parameters are determined for an equivalent tree structure by opening each closed loop. Then two frames are added on the opened joints [1], thus the number of frames is equal to : $N_f = n + 2B = 10$, where B is the number of closed loops ($B = N_j - n$).

The geometric parameters of the 10 frames of the SR400 Robot are given in [2]. Owing to the parallelogram loop, the following geometric constraint relations between the joint positions are verified :

$$\theta_3=\pi/2-\theta_2+\theta_7,\ \ \theta_8=\theta_2-\theta_7,\ \ \theta_9=\pi/2+\theta_3=\pi-\theta_2+\theta_7 \quad (1)$$

Since motors 5 and 6 are fixed on link 4, a mechanical coupling exists between the joints 5 and 6 such that :

$$\dot{\theta}_6 = \dot{\theta}_{m6} - \dot{\theta}_5 \quad (2)$$

$\dot{\theta}_{m6}$ is the velocity of motor 6 referred to the joint side, $\dot{\theta}_j$ is the velocity of joint j.

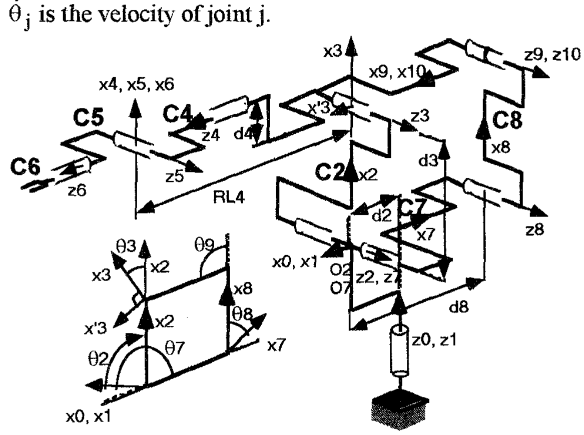

Figure 1. SR400 robot.

3. The dynamic model

Let the joint positions q_{tr} and the joint torques Γ_{tr} of the equivalent tree structure of the robot be written as :

IEEE International Conference
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$$q_{tr} = \begin{bmatrix} q_{tra} \\ q_{trp} \end{bmatrix}, \quad \Gamma_{tr} = \begin{bmatrix} \Gamma_{tra} \\ \Gamma_{trp} \end{bmatrix} \tag{3}$$

where :

q_{tra}, q_{trp} represent the active and passive joint positions respectively,

Γ_{tra}, Γ_{trp} are the active and passive joint torques respectively.

The inverse dynamic model calculates the motor torques as a function of the joint positions, velocities and accelerations. It can be calculated efficiently using customized Newton-Euler method for the equivalent tree structure [3] and by taking into account the constraint equations between the joint variables. It can be obtained using the following relation [4] :

$$\Gamma = G^T \Gamma_{tr} = \Gamma_{tra} + G_p{}^T \Gamma_{trp} \tag{4}$$

where :

Γ is the (Nx1) vector of the actuated joint torques of the robot.

$G = \dfrac{\partial q_{tr}}{\partial q_{tra}} = \begin{bmatrix} I_N \\ G_p \end{bmatrix}$, is the (nxN) Jacobian matrix of q_{tr} with respect to q_{tra}.

$G_p = \dfrac{\partial q_{trp}}{\partial q_{tra}}$, is the ((n-N)xN) Jacobian matrix of q_{trp} with respect to q_{tra}.

I_N is the (NxN) identity matrix.

In the SR400 case, we have :

$$q_{tra} = [\theta_1 \ \theta_2 \ \theta_4 \ \theta_5 \ \theta_6 \ \theta_7]^T, \qquad q_{trp} = [\theta_3 \ \theta_8]^T$$
$$\Gamma_{tra} = [\Gamma_{tr1} \ \ \Gamma_{tr2} \ \ \Gamma_{tr4} \ \ \Gamma_{tr5} \ \ \Gamma_{tr6} \ \ \Gamma_{tr7}]^T$$
$$\Gamma_{trp} = [\Gamma_{tr3} \ \Gamma_{tr8}]^T \tag{5}$$

Using relations (1) and (4), taking into account the coupling between 5 and 6, and adding the effect of motor inertia and frictions, we get :

$$\Gamma_{m1} = \Gamma_{tr1} + I_{a1}\ddot{q}_1 + \Gamma_{f1}$$
$$\Gamma_{m2} = \Gamma_{tr2} - \Gamma_{tr3} + \Gamma_{tr8} + I_{a2}\ddot{q}_2 + \Gamma_{f2}$$
$$\Gamma_{m4} = \Gamma_{tr4} + I_{a4}\ddot{q}_4 + \Gamma_{f4}$$
$$\Gamma_{m5} = \Gamma_{tr5} - \Gamma_{tr6} + I_{a5}\ddot{q}_5 + \Gamma_{f5}$$
$$\Gamma_{m6} = \Gamma_{tr6} + I_{a6}\ddot{q}_{m6} + \Gamma_{fm6}$$
$$\Gamma_{m7} = \Gamma_{tr7} + \Gamma_{tr3} - \Gamma_{tr8} + I_{a7}\ddot{q}_7 + \Gamma_{f7} \tag{6}$$

Γ_m is the vector of motor torques referred to the joint side.

$\ddot{q}_j$ is the joint acceleration.

I_{aj} is the inertia of motor j referred to the joint side.

Γ_{fj} is the friction torque referred to the joint side.

A classical mean friction torque model at non zero speed has been chosen for j=1,2,7,4 :

$$\Gamma_{fj} = F_{sj}\,\mathrm{Sign}(\dot{q}_j) + F_{vj}\,\dot{q}_j \tag{7}$$

Fvj, Fsj, are the viscous and Coulomb friction coefficients of joint j.

$\dot{q}_j$ is the joint velocity, with $\dot{q}_{m6} = \dot{q}_6 + \dot{q}_5$.

Due to the coupling between joints 5 and 6, the classical model at non zero speed is taken as :

$$\Gamma_{f5} = F_{s5}\,\mathrm{Sign}(\dot{q}_5) + F_{v5}\,\dot{q}_5 + F_{s6}\,\mathrm{Sign}(\dot{q}_6) + F_{v6}\,\dot{q}_6$$
$$\Gamma_{fm6} = F_{sm6}\,\mathrm{Sign}(\dot{q}_{m6}) + F_{vm6}\,\dot{q}_{m6} - F_{s6}\,\mathrm{Sign}(\dot{q}_6) - F_{v6}\,\dot{q}_6 \tag{8}$$

F_{v6}, F_{s6} are the viscous and Coulomb friction coefficients due to the coupling between joints 5 and 6. This model has been validated using velocity step tests for positive and negative values of the product $(\dot{q}_6 * \dot{q}_5)$ while joints 1, 2, 3, 4 are locked in a configuration which cancels the gravity effect on joints 5 and 6. The six friction parameters are estimated using least squares and are given in table 4. Predictions of Γ_{f5} and Γ_{f6} are compared with the experimental values on Fig. 2 for constant velocities $\dot{q}_{m6}$=2.4rad/s and $\dot{q}_5$=-3rd/s respectively. Note the discontinuities when $\dot{q}_6$=0 $(\dot{q}_{m6} = \dot{q}_5)$, which validate the proposed model.

To use the dynamic model in the control or simulation of the robot, the links inertial parameters and friction coefficients appearing in the previous equations must be identified.

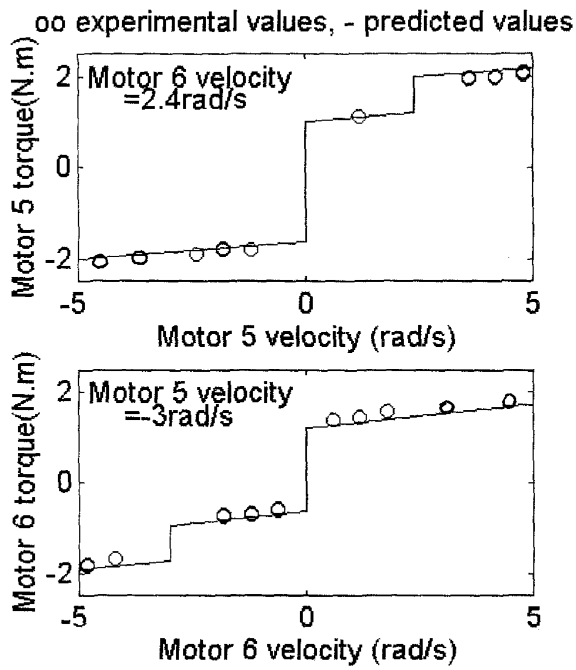

Figure 2. Coupled friction model validation.

3.1 Standard Inertial parameters of the SR400 robot

For each link the following 10 standard inertial parameters are defined :

•(XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j) are the 6 coefficients of the inertia matrix of link j with respect to frame j,
•(MX_j, MY_j, MZ_j) are the 3 first moments of link j, around the origin of frame j,
•M_j is the mass of link j.
These 10 inertial parameters, the inertia of motor Ia_j and the friction parameters F_{sj} and F_{vj} constitute the 13 standard dynamic parameters of the link j :

$$X_s^j = [XX_j\ XY_j\ XZ_j\ YY_j\ YZ_j\ ZZ_j\ MX_j\ MY_j\ MZ_j\ M_j\ Ia_j\ Fv_j\ Fs_j]^T$$

The dynamic model is seen to be linear in relation to these parameters [5, 6, 7].

3.2. The base inertial parameters

The base inertial parameters are defined as the minimum parameters which can be used to get the dynamic model. They represent the set of parameters which can be identified using the dynamic or energy model, thus its determination is essential for the identification of the inertial parameters of robots [5, 6, 7]. The use of the base parameters in Newton-Euler algorithm leads to reduce the computation complexity of the model [7, 8].
These parameters can be obtained from the standard inertial parameters by eliminating those which have no effect on the dynamic model and by regrouping some others. In [9, 10] we have presented a symbolic method to calculate these parameters for serial robots, tree structure robots and robots with parllelogram closed loop robots respectively, in [11] numerical method to obtain these parameters is also developed. The final results of the minimum inertial parameters of the SR400 robot can be summarized as follows :
The following 11 parameters have no effect on the dynamic model :
XX_1, XY_1, XZ_1, YY_1, YZ_1, MX_1, MY_1, MZ_1, M_1, MZ_2, and MZ_7.
The number of base inertial parameters of the SR400 robot is 42. They are given in table 1, where the letter "R" indicates the parameter on which other parameters have been regrouped. Twenty parameters are the standard ones and 22 parameters are regrouped using linear relations given in [2]. Thus the dynamic model is given as a function of a vector X of 56 identifiable parameters, composed of 42 base inertial parameters X_b, of the 7 coefficients of viscous friction F_v and of the 7 coefficients of Coulomb friction F_s :

$$X = [X_b^T\ F_v^T\ F_s^T]^T \qquad (9)$$

4. The identification model of the dynamic parameters

The identification of the dynamic parameters of robots can be carried out using the energy or the dynamic model. Since the parameters are linear in these models, linear least squares techniques are used [5, 6, 12].

Table 2: Base inertial parameters of the SR400 robot.

j	XX_j	XY_j	XZ_j	YY_j	YZ_j	ZZ_j
1	0	0	0	0	0	ZZR_1
2	XXR_2	XYR_2	XZR_2	0	YZR_2	ZZR_2
3	XXR_3	XYR_3	XZR_3	0	YZR_3	ZZR_3
4	XXR_4	XY_4	XZ_4	0	YZ_4	ZZR_4
5	XXR_5	XY_5	XZ_5	0	YZ_5	ZZR_5
6	XXR_6	XY_6	XZ_6	0	YZ_6	ZZ_6
7	0	0	0	0	0	0
8	0	0	0	0	0	0

j	MX_j	MY_j	MZ_j	M_j	Ia_j
1	0	0	0	0	0
2	MXR_2	MY_2	0	0	0
3	MXR_3	MYR_3	0	0	0
4	MX_4	MYR_4	0	0	Ia_4
5	MX_5	MYR_5	0	0	Ia_5
6	MX_6	MY_6	0	0	Ia_6
7	MXR_7	MY_7	0	0	0
8	0	MY_8	0	0	0

4.1 The energy identification model

The energy identification model is given as [12] :

$$y(i) = \int_{t_a(i)}^{t_b(i)} \Gamma_m^T \dot{q}_m \, dt = H(t_b(i)) - H(t_a(i))) +$$

$$\sum_{j=1,2,4,7} F_{sj} \int_{t_a(i)}^{t_b(i)} |\dot{q}_j| dt + F_{vj} \int_{t_a(i)}^{t_b(i)} \dot{q}_j^2 dt + \int_{t_a(i)}^{t_b(i)} (\Gamma_{f5}\dot{q}_5 + \Gamma_{fm6}\dot{q}_{m6}) dt \qquad (10)$$

where $H(t_i)=h(q(t_i),\dot{q}(t_i))\ X_b$, is the total energy (kinetic and potential) at time t_i and $\dot{q}_{mj}$ and $\dot{q}_j$ are the velocities of motor j and joint j, respectively. The coefficients of **h** are very easy to calculate either numerically or symbolically, they are function of joint positions and velocities and geometric parameters.
Since H is linear in the inertial parameters then (10) can be written as a linear equation in the dynamic parameters of the robot :

$$y = w(q, \dot{q})\ X \qquad (11)$$

where $w(q, \dot{q})$ denotes the (1xNp) regressor row matrix of the energy model. The main advantage of this model is that it is not function of the joint accelerations.
To identify the dynamic parameters a sufficient number of equations can be obtained by calculating relation (11) between different intervals of time $t_{a(i)}$, $t_{b(i)}$, i = 1, ..., r, r ≥ Np, in order to get the following overdetermined linear system :

$$Y = W\,X + \rho \qquad (12)$$

where W is the (rxNp) observation matrix, r is the number of samples, N_p is the number of the identifiable parameters, and ρ is the vector of errors.
The least squares solution is generally used to identify X. In order to improve the identification process, an exciting trajectory must be used [13, 14]. This method has been applied in [2] to identify the parameters of links 5 and 6 of the SR400 robot. Although that the model is simple, the selection of the identification trajectory must be done by an optimization procedure which minimizes a function depending on the condition number of the observation matrix W. This optimization procedure is somewhat difficult when the number of parameters is important, which is the case if we want to consider more than two links together. Therefore in this paper we will use the dynamic identification model, which is a vectoriel relation that gives more information than the energy model which is just a scalar quantity.

4.2 The dynamic identification model

The dynamic model can be written as linear in the dynamic parameters [5, 6, 7] as follows :

$$\Gamma_m = D\,(q, \dot{q}, \ddot{q})\,X \qquad (13)$$

D is the regressor (NxNp) matrix of the linear dynamic model.
The calculation of the different columns of D can be performed using Newton Euler algorithm and a customized symbolic method. A program has been developed in our software package SYMORO+ [15] to get these coefficients automatically.
To identify the dynamic parameters a sufficient number of equations can be obtained by calculating relation (13) at different time, in order to get an overdetermined linear system similar to that given in (12). Then the least squares solution is used to identify X.
It is to be noted that the dynamic model is more rich in information than the energy model, but its use needs to handle well the filtering of the position signals q to get $\dot{q}$ and $\ddot{q}$, and that of the model.

4.3 Experimental issues

The robot has been driven to different point to point trajectories using a classical 5th order trajectory generator. The current references and joint positions have been recorded using a sampling frequency of 1 KHz. The following practical issues must be observed :
1- The motor torques are calculated from the current reference using the relation :

$\Gamma_{mj} = G_{Tj} \cdot V_{Tj}$,

V_{Tj} is the current reference of the amplifier current loop which can be measured easily,
G_{Tj} is the gain of the joint j drive chain, which is taken as constant in the operating range of the robot.
The good determination of G_{Tj} is essential for the success of the identification.
Table 2 gives the G_{Tj} values for the six joints of the SR400 robot.

Table 2 : Drive chain gains.

G_{T1}	G_{T2}	G_{T3}	G_{T4}	G_{T5}	G_{T6}
94±1	-160±1	160±1	-14.9±0.2	-16±0.2	16.1±0.2

2- The robot is provided by synchro-resolver as joint position sensors, and tachometers to give the joint velocities. It has been shown that the tachometer signals are contaminated with bias and noise and it is better to get the joint velocities by differentiating the position signals. The joint positions have been recorded using 1 KHz sampling rate, then they have been low pass filtered using a decimate filter of order 5 from MATLAB, the joint velocities have been calculated from the filtered position signals using central difference to avoid phase shift.
3- The joints accelerations are calculated from the calculated joint velocities using central difference algorithm.
4- After calculating matrices Γ_m and D to get system (12), Y and colums of W are both filtered in a process called parallel filtering, using a decimate of order 10 from MATLAB, in order to eliminate high frequency torque noises and ripples from Y and to keep available the linear system (12) in the filter bandwith which must be chosen to include the robot dynamic range.
5- Standard deviation are estimated considering the matrix W to be a deterministic one, and ρ to be a zero mean additive independent noise, with standard deviation σ_ρ such that $C_\rho = E[\rho\rho^T] = \sigma_\rho^2 I_r$, where E is the expectation operator. The variance-covariance matrix of the estimation error and standard deviations can be calculated by :

$$C_{\hat{X}} = E\left[(X-\hat{X})(X-\hat{X})^T\right] = \sigma_\rho^2 (W^T W)^{-1}$$

$$\sigma_{\hat{X}i}{}^2 = C_{\hat{X}i} \qquad (14)$$

An unbiased estimation of σ_ρ is used to get the relative standard deviation $\sigma_{\hat{X}ri}$ by the expression :

$$\hat{\sigma}_\rho^2 = \frac{\|Y - W\hat{X}\|^2}{r - Np} \qquad \sigma_{\hat{X}ri} = \frac{\sigma_{\hat{X}i}}{\hat{X}i}$$

A parameter with $100{*}\sigma_{\hat{X}ri} \geq 15$ is not significant, it means that this parameter has a little effect on the model and cannot be identified with acceptable accuracy on the actual trajectory, it will be eliminated from the model to get the essential parameters of the dynamic model.

6- The number r of equations in (12) must be large to get good accuracy. Classicaly, standard deviations decrease as the square root of r which is practically chosen greater than 50 times the number of parameters to identify.

7- Owing to the great difference between the values of the inertial parameters of the links 4, 5, 6 with respect to those of links 1, 2, 3, it has been seen that it is preferable to carry out the identification in two experiments :

–The first one, all the joints are moved, but the dynamic equations corresponding to motors 4, 5, and 6 are used in order to identify the dynamic parameters of links 4, 5, and 6.

–In the second experiment the joints 4, 5 and 6 are locked, while the other joints are moving. This permits to identify the rest of the dynamic parameters.

Estimated values $\hat{X}$ of the essential dynamic parameters of the unloaded robot are given in table 5.

4.4 Validation of the identified values

The validation of the estimated values is carried out using the following methods :

- carrying out the identification process using the energy model and then by using the dynamic model and comparing the obtained values. This validation is carried out for the parameters of links 5 and 6, the two results are comparable.
- reidentify the parameters of the robot while it is loaded by a known load. In this case the values of some base parameters will be changed as a function of the load parameters. We have to check that the values calculated from the base parameters identified without load plus the load effect are the same as those of the identified values with the load. Table 3 gives the parameters of the load used for this validation. One can see that 7 parameters which are not essential for the unloaded robot (0 values), become essential for the loaded robot.

Table 3 : Parameters of the load, SI units.

XX_L	XY_L	XZ_L	YY_L	YZ_L
0.454	-0.02	-0.09	0.445	-0.115
ZZ_L	MX_L	MY_L	MZ_L	M_L
0.065	0.48	0.6	2	10.9

The following 19 parameters will be modified owing to the load :

$XXR_{6L} = XXR_6 + XX_L - YY_L$, $XY_{6L} = XY_6 + XY_L$
$XZ_{6L} = XZ_6 + XZ_L$, $YZ_{6L} = YZ_6 + YZ_L$
$ZZ_{6L} = ZZ_6 + ZZ_L$, $MX_{6L} = MX_6 + MX_L$
$MY_{6L} = MY_6 + MY_L$, $XXR_{5L} = XXR_5 + YY_L$
$ZZR_{5L} = ZZR_5 + YY_L$, $MYR_{5L} = MYR_5 + MZ_L$
$XXR_{3L} = XXR_3 + (RL_4^2 - d_4^2)\, M_L$
$XYR_{3L} = XYR_3 - d_4\, RL_4\, M_L$
$ZZR_{3L} = ZZR_3 + (d_4^2 + RL_4^2)\, M_L$
$MXR_{3L} = MXR_3 + d_4\, M_L$
$MYR_{3L} = MYR_3 + RL_4\, M_L$
$XXR_{2L} = XXR_2 - d_3^2\, M_L$
$ZZR_{2L} = ZZR_2 + d_3^2\, M_L$
$MXR_{2L} = MXR_2 + d_3\, M_L$
$ZZR_{1L} = ZZR_1 + (d_4^2 + d_3^2 + d_2^2)\, M_L$

where : XR_j is the values without load, XR_{jL} is the equivalent value with load.

Table 4 gives the identified values $\hat{X}_L$ of the inertial parameters when the robot is loaded, and its predicted values Z_L using the previous relations.

Table 4 : Essential dynamic parameters. SI units.

	Unloaded robot		Loaded, M_L=10.9Kg		
Name	$\hat{X}$	$2\,\sigma_{\hat{X}}$	Z_L	$\hat{X}_L$	$2\sigma_{\hat{X}_L}$
ZZR_1	50	3	54	57	5
XXR_2	-22	3	-26	-26	5
ZZR_2	42	0.5	47	47	1
MXR_2	37	0.5	44.3	44.3	0.3
XXR_3	5	2	12	10	4
XYR_3	-2.6	0.5	-4.4	-4.4	0.8
ZZR_3	30	0.5	37	37	0.6
MXR_3	7.5	2	9	9	3
MYR_3	12	0.7	20.6	20	1
Ia_4	0.3	0.05		0.3	0.05
XXR_5	0		0.44	0.42	0.01
ZZR_5	0		0.44	0.42	0.01
MYR_5	0.04	0.01	2	2	0.04
Ia_5	0.27	0.01		0.27	0.01
XZ_6	0		-0.09	-0.07	0.02
YZ_6	0		-0.11	-0.15	0.05
ZZ_6	0		0.06	0.08	0.01
MX_6	0		0.47	0.47	0.01
MY_6	0		0.6	0.6	0.01
Ia_6	0.27	0.01		0.27	0.01
Fv_1	8	2		10	2
Fs_1	30	2		30	2
Fv_2	55	2		57	2
Fs_2	38	2		36	2
Fs_4	2	0.5		3	0.5
Fv_5	0.11	0.04			
Fs_5	1.3	0.1			
Fv_6	0.034	0.005			
Fs_6	-0.4	0.01			
Fv_{m6}	0.14	0.03			
Fs_{m6}	0.92	0.01			

One can see that they are perfectly compatible, which validates the identification process.

- comparing the calculated torques using the dynamic model for some selected trajectories with those obtained on the real robot. The trajectory used in this validation step must be different than those used in the identification process. As shown in Fig. 3, the predicted torques match the actual torques closely (both are filtered to eliminate high frequency noise and ripple).

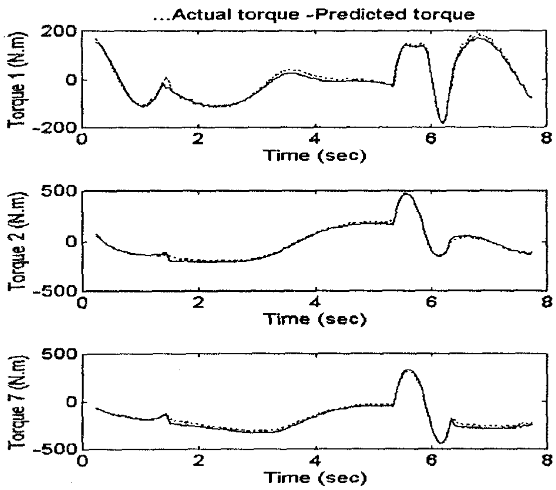

Figure. 3-1. Cross validation with load.

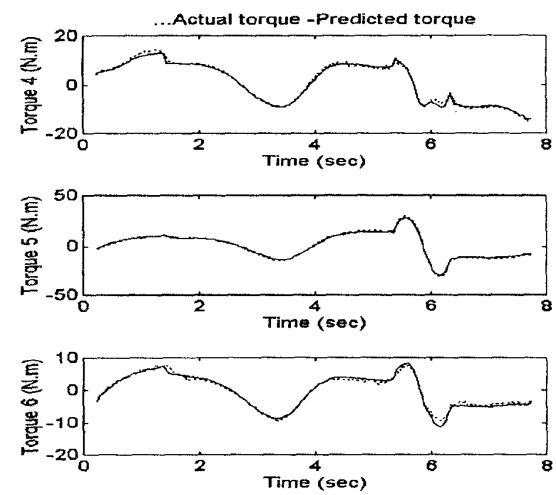

Figure. 3-2. Cross validation with load.

6. Conclusion

This paper presents the complete identification steps of the dynamic parameters of the SR400 robot. The base inertial parameters are given, and the identification model using the energy theorem and the dynamic model are presented. The use of the energy model, which can be seen as a low pass filter, is not sensitive to noise but must be associated with exciting trajectories, while the use of the dynamic model in the identification must be associated with good filtering process for the joint positions to get the joint velocities and accelerations and for the model itself (parallel filtering) to suppress the high frequency noise and ripple in the torques measurements. The paper treats practical aspects concerning the filtering, the validation of the identified values, the elimination of non significant parameters, in order to carry out with success the identification of the dynamic parameters.

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