

# REVIEW TEST 3



**MSc. Nguyen Cam Nhiem**

Email: [nhiemnc@fe.edu.vn](mailto:nhiemnc@fe.edu.vn)

Find the point of intersection (if any) of the following pairs of lines.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 12 \end{bmatrix} + s \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

Find normal vector of the plane through  $P(1,2,1)$ ,  $Q(0,-3,2)$  and  $R(1,1,4)$

Find a vector equation of the line. Passing through  $P(3, -1, 4)$  and perpendicular to the plane  $3x - 2y - z = 0$

$$\text{Let } u = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, w = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$$

In each case, find  $x$  such that:  $3(2u + x) + w = 2x - v$ .

The area of the parallelogram determined by the given vector  $u$  and  $v$ .  $u=(-1,1,2)$ ,  $v=(0,2,1)$

Find the angle between the following pair of vectors.

$$u = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

Let  $A$  be a  $30 \times 70$  matrix and  $\text{rank}(A) = 40$ . Find

$\text{Dim}(\text{col}A)$

$\text{Dim}(\text{row}A)$

$\text{Dim}(\text{null}A)$



In each case, compute the projection of  $u$  on  $v$

$$u = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}, v = \begin{bmatrix} -6 \\ 4 \\ 2 \end{bmatrix}$$

Find the parametric equation of the line passing through  $P(1,0,-3)$  and parallel to the line with parametric equation

$$x = -1 + 2t, y = 2 - t, z = 3 + 3t$$

Find all real numbers  $x$  such that  $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ x \\ -1 \end{bmatrix}$  are orthogonal.

Let  $\|X\| = 4, \|Y\| = 3, XY = 2$ . Calculate  $\|2X - 3Y\|$ ?

A. 11

B. 1

C. -1

D.  $\sqrt{145}$

Find the three internal angles of the triangle with vertices:  
 $A(3, 1, -2)$ ,  $B(3, 0, -1)$ , and  $C(5, 2, -1)$ .

Let  $A$  be a  $4 \times 6$  matrix of rank 2. Find the dimension of the null space of  $A$ .

Find the dimension of the subspace

$$U = \{ [a \quad a-b \quad a+3b \quad a] \mid a, b \text{ are real numbers} \}$$

Find the area of the triangle with the following vertices.

$A(3, -1, 1)$ ,  $B(4, 1, 0)$ , and  $C(2, -3, 0)$

a)  $\sqrt{2}$

b) 2

c)  $\sqrt{5}$

d)  $\sqrt{3}$



Consider the triangle with vertices P (2, 0, -3), Q (5, -2, 1), and R (7, 5, 3). Find the lengths of the three sides

The dimension of the subspace

$U = \text{span}\{(-2, 0, 3), (1, 2, -1), (-2, 8, 5), (-1, 2, 2)\}$  is...

- a. 2
- b. 4
- c. 3
- d. 1

Let  $u$  and  $v$  be vectors in  $\mathbb{R}^3$  and  $w \in \text{span}\{u, v\}$ .

Then ...

- a.  $\{u, v, w\}$  is linearly dependent.
- b.  $\{u, v, w\}$  is linearly independent.
- c.  $\{u, v, w\}$  is a basis of  $\mathbb{R}^3$
- d. the subspace is spanned by  $\{u, v, w\}$  has the dimension 3.

Find a basis for the subspace of  $\mathbb{R}^3$  defined by

$$U = \{(a, b, c) : 2a - b + 3c = 0\}$$

- |                                |  |
|--------------------------------|--|
| a. $\{(1, 2, 0), (0, 3, 1)\}$  | b. $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ |
| c. $\{(1, 2, 0)\}$             | d. $\{(1, 0, 0), (1, 2, 0)\}$            |
| e. $\{(3, 0, -2), (1, 0, 0)\}$ | f. $\{(2, -1, 3)\}$                      |

Which of the following are subspaces of  $\mathbb{R}^3$ ?

- (i)  $U = \{(x, y, z + 2) \mid x, y, z \in \mathbb{R}\}$
- (ii)  $U = \{(x, y, z) \mid x + 3y - 2z = 0\}$
- (iii)  $U = \{(x, y, z) \mid x + y + z^2 = 3\}$
- (iv)  $U = \{(x, y, |z|) \mid x, y, z \in \mathbb{R}\}$

Let  $U = \text{span}\{(1, 1, 2, 1), (0, 1, 1, -2)\}$ .

Find all values of  $t$  such that  $(1, t, 3, 4)$  is in  $U$

- a. There is no such  $t$
- b.  $-2$
- c. All nonzero numbers
- d. None of the other choices is correct
- e. All number different from  $-1$

Which of the following subsets are independent in  $\mathbb{R}^4$  ?

$$(i) \left\{ [1 \ 2 \ 3 \ 4]^T, [2 \ 0 \ 1 \ -1]^T, [1 \ -1 \ 0 \ 3]^T \right\}$$

$$(ii) \left\{ [2 \ 0 \ 1 \ -1]^T, [1 \ 2 \ -1 \ 1]^T, [3 \ 2 \ 0 \ 0]^T \right\}$$

Let  $u, v$  be linearly independent vectors in  $\mathbb{R}^5$ . Let  
 $U = \text{span}\{u, v, 2u + 3v, u - v\}$ .  
 Find  $\dim U$ .

- a. There is not enough information
- b. 3
- c. None of the other choices is correct
- d. 5
- e. 2



Let  $X$  and  $Y$  be vectors in  $\mathbb{R}^n$ . Which of the following statement are true?

- (i) If  $\{X + Y, X - Y\}$  is an orthogonal set then  $X = Y$
- (ii) If  $\{X + Y, X - Y\}$  is an orthogonal set then  $X = -Y$
- (iii) If  $\{X + Y, X - Y\}$  is an orthogonal set then  $\|X\| = \|Y\|$

Find the dimension of the column space of

$$\begin{bmatrix} -1 & 7 & 0 & 3 & 1 \\ 1 & -1 & 0 & -1 & -1 \\ 0 & -3 & 0 & -1 & -1 \\ 0 & 5 & 3 & 4 & -3 \end{bmatrix}$$