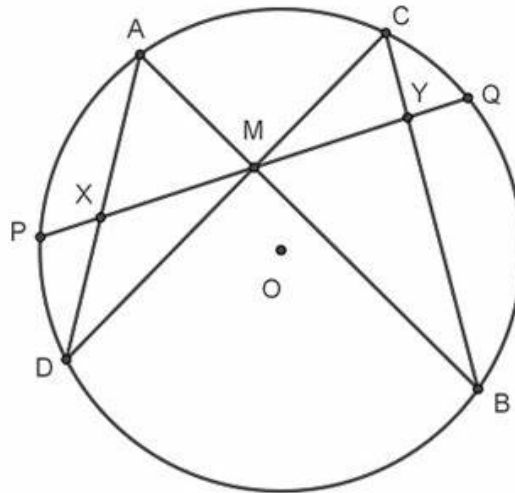


An Introduction To Butterfly Theorem

Let M be the midpoint of a chord PQ of a circle, through which two other chords AB and CD are drawn; AD cuts PQ at X and BC cuts PQ at Y .



Prove that M is also the midpoint of XY .

Author

nhoc_dddADMIN

Topic

Mathematics

Description

Let M be the midpoint of a chord PQ of a circle, through which two other chords AB and CD are drawn; AD cuts PQ at X and BC cuts PQ at Y . Prove that M is also the midpoint of XY .

Content

Let O be the center of the given circle. Since $OM \perp XY$, in order to show that $XM = MY$, we have to prove that $\angle XOM = \angle YOM$.

Drop perpendiculars OK and ON from O onto AD and BC , respectively. Obviously, K is the midpoint of AD and N is the midpoint of BC . Further,

$$\angle DAB = \angle DCB \text{ and } \angle ADC = \angle ABC,$$

as angles subtending equal arcs. Triangles ADM and CBM are therefore similar, and $AD/AM = BC/CM$, or $AK/AM = CN/CM$. In other words, in triangles AKM and CNM two pairs of sides are proportional. Also, the angles between the corresponding sides are equal. We infer that the triangles AKM and CNM are similar. Hence, $\angle AKM = \angle CNM$.