

# An Introduction To Stolz-Cesaro Theorem

**Stolz-Cesaro Theorem**

Let  $a_n$  and  $b_n$  be two sequences of real numbers.

Assume that:

- I.  $b_n \rightarrow \infty$  as  $n \rightarrow \infty$ ,
- II.  $b_n$  is increasing for sufficiently large  $n$ ,
- III.  $\lim_{n \rightarrow \infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = L$ .

Then  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ .

**Question 1.**  $a_n = \ln n$ ,  $b_n = n$ ,  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = ?$

**Solution.** The conditions I and II of the Stolz-Cesaro theorem are satisfied.

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## Topic

Mathematics

## Description

In mathematics, the Stolz–Cesaro theorem is a criterion for proving the convergence of a sequence. The theorem is named after mathematicians Otto Stolz and Ernesto Cesaro, who stated and proved it for the first time.

The Stolz–Cesaro theorem can be viewed as a generalization of the Cesaro mean, but also as a L'Hopital rule for sequences.

## Content

Let  $\{a_n\}_{n \geq 1}$  and  $\{b_n\}_{n \geq 1}$  be two sequences of real numbers. Assume that  $\{b_n\}_{n \geq 1}$  is a strictly monotone and divergent sequence (i.e. strictly increasing and approaching  $+\infty$ , or strictly decreasing and approaching  $-\infty$ ) and the following limit exists:

$$\lim_{n \rightarrow \infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = l.$$

Then, the limit

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = l.$$

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