

MAE 315: Lab 4 Report

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Abstract

This lab culminates in the effects of many fluid dynamic phenomena on various bluff bodies including cylinders and airfoils. The concepts explored in this lab were wake, vortex shedding frequency, boundary layer, airfoil theory, and 2D vs 3D cylinders. It was found that vortex shedding is closely related to the Reynolds number and the strouhal number of an object. The frequency at which the highest amplitude occurs can be predicted by knowing the natural frequency of the object at hand.

Within a boundary layer, there is a section of laminar flow followed by a turbulent flow section where the boundary layer becomes thicker and the velocity increases. No matter where along the surface interacting with the fluid, the velocity will always be zero due to the no slip condition.

Airfoils are a large part of how airplanes fly. They use precise designs to create specific ratios of drag and lift forces as well as pitching moments to allow them to maneuver. It has been found that a dirty configuration of an airfoil works far better than the clean configuration when it comes to avoiding stall - which would be catastrophic in a real life situation.

Further, it was emphasized that increasing the length of a bluff body in a fluid flow will decrease the amount of drag acting on the body. The longer the cylinder, the more it approaches the 2D cylinder which has less drag by eliminating the vortices on either side of the cylinder.

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Background

A deep understanding of fluid dynamics is vital for understanding flight, airplane wing design, airplane failure, and flight successes. Any aerospace engineer should be well versed in concepts such as turbulent flow, the fluid boundary layer, and lift and drag forces.

Anyone who has ever flown in a plane is familiar with the term turbulence. They may be associated with the feeling of a shaking aircraft or a bumpy ride. In more technical terms, turbulence occurs when moving fluids (air) behave in a certain way. This can have a major impact on planes - especially the wings.

The Reynolds number is a non-dimensional parameter given by equation 1. The Reynolds number can provide information regarding behavior of the flow, specifically whether the flow is turbulent. A low Reynolds number means that a specific flow is laminar (streamlines are parallel to one another) and generally, a Reynolds number greater than 3×10^5 means that the flow is turbulent and moving in ways that are very hard to predict.

$$R_e = \frac{\rho V d}{\mu}$$

Equation 1: Reynolds Number

Another incredibly important equation to fluid mechanics is Bernoulli's equation. This powerful formula works for the assumption that points one and two are on the same streamline, the fluid is incompressible, the flow is frictionless and irrotational. This equation has been used to determine velocity in this lab.

$$\begin{aligned} P + \frac{1}{2} \rho U^2 + \rho g h &= \text{const.} \\ P_1 + \frac{1}{2} \rho U_1^2 + \rho g h_1 &= P_2 + \frac{1}{2} \rho U_2^2 + \rho g h_2 \end{aligned}$$

Equation 2: Bernoulli's Equation

Another important concept is wake. A bluff body such as a cylinder can generate a wake when the fluid flows past it in such a way that the boundary layer separates causing a large amount of drag on the body. This also creates areas of very low pressure behind the body and areas of high pressure in front of the body (stagnation pressure). The low pressure behind the body causes turbulent condition and frequency shedding.

Frequency shedding of a bluff body is caused by an imbalance of forces acting on either side of the body as the fluid flows past. The forces tend to alternate back and forth causing a vibrating motion. The body will vibrate the most at its natural frequency. The actual frequency that the body will oscillate at is related to the Strouhal number and the Reynolds number.

$$S = 0.198 \left(1 - \frac{19.7}{R_e}\right)$$

Equation 3: Strouhal Number

The portion of this lab with the most obvious relation to aerospace engineering is airfoil theory. Airfoils come in many different shapes and they are all built strategically to influence the flow of fluids over the body and therefore affect the forces acting upon it. The lift force is always normal to the velocity of the fluid (this is how an aircraft ascends into the sky) and the drag force is always parallel to the fluid velocity. The angle of attack refers to the angle that the velocity interacts with the airfoil. Pitching moment of an airfoil determines how the airfoil can react when it encounters different angles of attack.

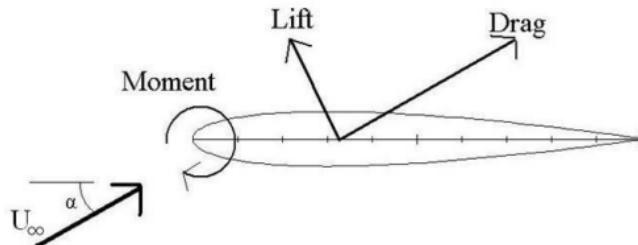


Figure 1: Forces acting on an airfoil. Source - class material

One additional aspect affecting airfoils is the wing tip vortices. Drag is increased at the tips of the wings by the fluid having to bend around the body. This phenomenon can be viewed by the difference in drag force between a 2D and a 3D object. The object that only has fluid flows affecting two of its dimensions experiences far less drag than an object that encounters fluid on all three.

Another assumption that is made in fluid dynamics is that there is a no slip condition. This means that the velocity of the fluid at the surface that it interacts with is zero. While this is true, the velocity in all other places does not just immediately jump to the value of the free stream velocity, so there exists a boundary layer. The boundary layer begins very thin and increases in thickness moving further upstream. Once a certain point is reached, the flow transitions from laminar to turbulent. The maximum boundary layer velocity also increases moving upstream. The characteristics and actions of flows when they are turbulent is very hard to predict, but turbulence also allows for a boundary layer that is less likely to detach from a blunt object and therefore reduce drag on the object.

Procedure

In order to conduct our experiment, we had two days of experimentation each with three sub-experiments. The first portion of the experiment conducted was the wind tunnel calibration. The purpose of this was to correlate air velocity with the frequency of the fan in the tunnel. Equipment needed to complete the calibration included the wind tunnel, a pitot-static tube, a pressure transducer, and a thermometer measuring air temperature inside the tunnel. The pitot-static tube was placed in the center of the test section of the tunnel (figure 2). Beginning with the fan turned off, three pressure and temperature readings were recorded. Next, the fan was turned on to a frequency of 5 Hz - three recordings of pressure and temperature were recorded again. This process was then repeated by increasing the fan speed in increments of 5 Hz until 40 Hz was reached. The pitot-static tube was then removed from the tunnel in preparation for the next phase of the experiment.

The second portion of the experiment utilized a PVC cylinder, an accelerometer, and the LabVIEW computer program “Accelerometer-Cylinder.vi.” The dimensions (diameter and length) of the cylinder were measured using a measuring tape. Then, the cylinder with the accelerometer attached was secured into the wind tunnel. Again the wind tunnel was turned on to 30 Hz for the collection of 16384 data points via LabVIEW with a sampling frequency of 6000 Hz. The procedure was then repeated for a fan frequency of 45 Hz. The data collected in this sub-experiment provides the voltage of the movement of the cylinder due to the air velocity.

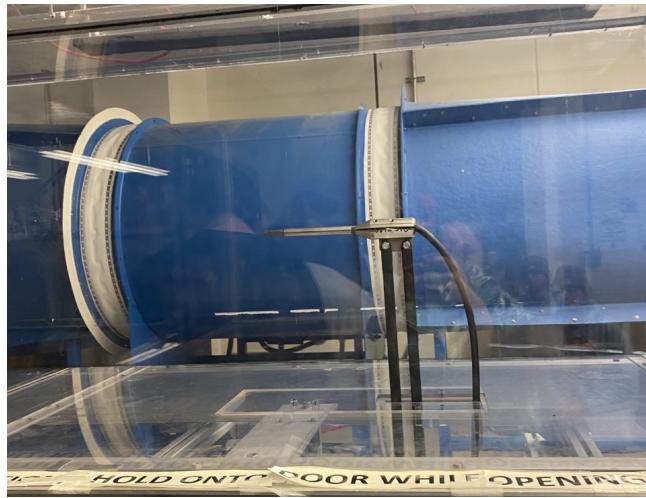


Figure 2: Wind tunnel configuration with a pitot-static tube for wind tunnel calibration

Part three of the first day of experimentation utilizes a pressure rake to measure the pressure of the wake caused by the cylinder. The distance between each prong on the pressure rake was measured with a pair of calipers. First, the pressure rake was set up inside of the wind tunnel without the cylinder and given a fan frequency of 25 Hz and 45 Hz. The pressure was recorded in LabView 20 times by each of the prongs on the pressure rake at both fan frequencies. Sequentially, the cylinder was added into the wind tunnel downstream of the pressure rake. The cylinder was placed about 8 diameter measures ahead of the rake (figure 3). Similarly, 20 pressure measurements were recorded by the rake for fan frequencies of 25 Hz and 45 Hz.

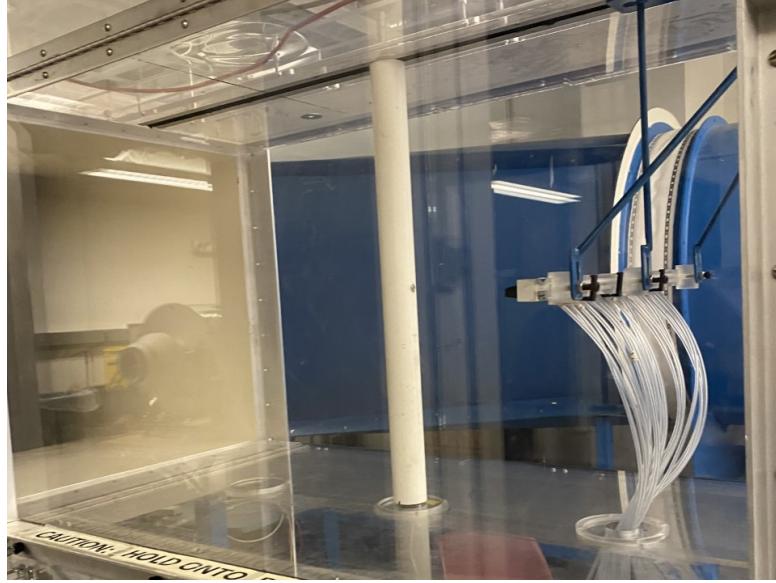


Figure 3: Wind tunnel configuration for wake experiment with cylinder

The first portion of the lab conducted on the second day of experimentation was the boundary layer experiment. This experiment used a second pressure rake which consequently had to be measured with calipers the same way as the first (figure 4). The rake was placed in the first hole of the test section. After the fan was turned on to 30 Hz, pressure was recorded using LabVIEW, and temperature was recorded by viewing the thermometer.

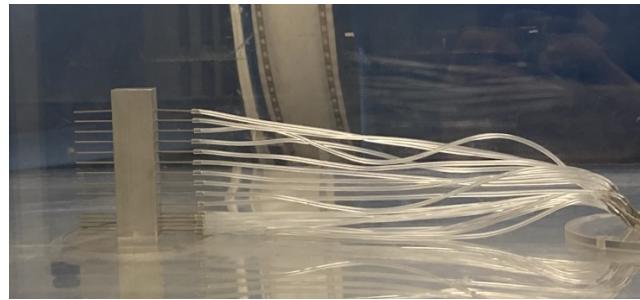


Figure 4: Boundary layer rake



Figure 5: Airfoil clean configured in wind tunnel with a negative angle of attack



Figure 6: Airfoil dirty configured in wind tunnel with positive angle of attack

The fifth portion of the experiment analyzed the effects of airflow on an airfoil. Initially, the dimensions of the airfoil - including chord, span, and depth - were measured with a tape measure and calipers. The resulting measurements were recorded in table 1. The airfoil used in this experiment has a profile shape that is named Clark y (figure 7). Using set screws, the airfoil was secured within the wind tunnel to prepare for experimentation. Four trials were run for this experiment; fan speeds of 30 and 45 Hz each both with and without flaps and slats. The experiment without flaps and slats is referred to from now on as the “clean” configuration, and the flaps and slats configuration is known as “dirty.” Once the airfoil was secured into the tunnel, the model positioning system was used to adjust the angle of the airfoil to -10 degrees. After force data collection at the initial angle using LabVIEW ‘Force_Balance_Airfoil.vi,’ the angle was increased by two degrees. The data collection and angle increase process was repeated until

the attack angle reached 18 degrees. The airfoil was considered to be stalling when small strings attached to the trailing edge of the airfoil demonstrated turbulent motion. The entire procedure was then reproduced for each variation of the trial.

Dimension	Length (m)
chord	0.089
span	0.251
depth	0.012

Table 1: Dimensions of airfoil

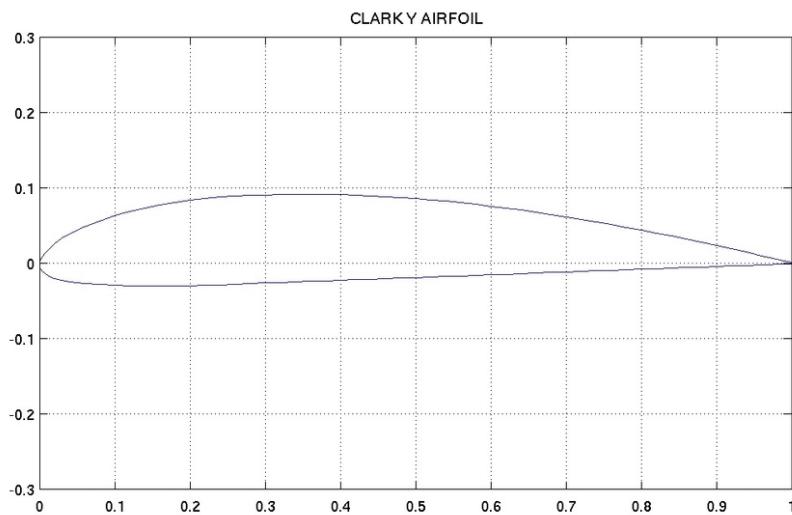


Figure 7: Clark Y airfoil profile. Source: [1: Scaled schematic of Clark Y airfoil. | Download Scientific Diagram \(researchgate.net\)](#)

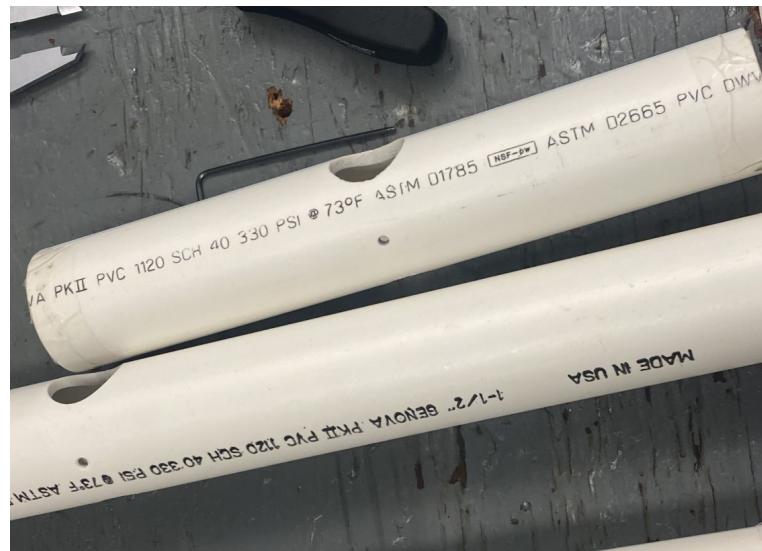


Figure 8: Two cylinders used in the cylinder drag experiment

Finally, the last portion of the experiment involved measuring the drag on cylinders in wind tunnels at a variety of lengths. First, the diameter and length of each cylinder was measured using a measuring tape. The dimensions of each cylinder are shown in table 2. Before any cylinders are placed in the wind tunnel, the wind tunnel is turned on to 30 Hz while LabVIEW “Force_Balance_Cylinders.vi” measures the drag force. Using the same procedure (and mounting the cylinders perpendicular to the flow) measure the drag force on each cylinder one at a time. A base value was obtained by running the experiment without a cylinder as denoted by ‘cylinder number 0.’

Cylinder Number	Diameter (m)	Length (m)
0	NA	NA
1	0.0476	0.60
2	0.0476	0.56
3	0.0476	0.51
4	0.0476	0.46
5	0.0476	0.27

Table 2: Dimensions of cylinders used in the cylinder drag experiment

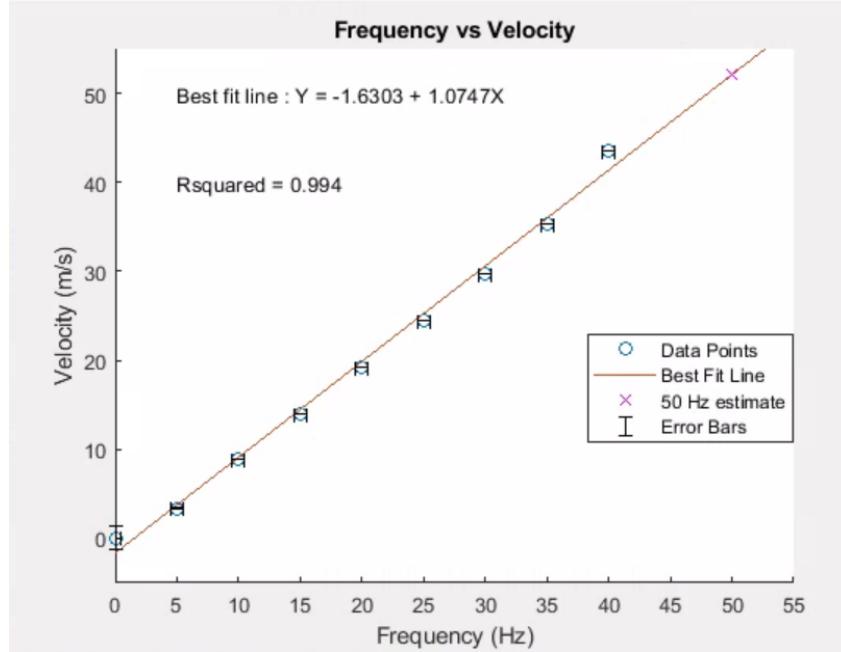
Following all experimentation and data collection, the data collected through LabVIEW was downloaded and all data was converted into SI units in preparation for analysis and code writing.

Results

This lab provided very useful insight and visualization into how moving fluids behave in the presence of bluff bodies. The various experiments of this lab allowed for a deep understanding of air flow in addition to many fundamental fluid dynamic principles.

The first part of the lab - wind tunnel calibration - proved to be very important for the entire duration of the lab as it gave a way to estimate air velocity from fan frequency. By applying a linear regression to fan frequency and air temperature data, equation 4 was determined where X is the fan frequency and Y is the estimated air velocity. The model fit the data very well as proven by a high R² value of 0.994. Plot 1 gives a better visual of how closely the data actually fits the regression line. This proves that for most purposes, the linear regression can work well enough as a relationship between fan speed and air speed.

$$Y = -1.6303 + 1.0747X$$

Equation 4: Regression equation predicting air velocity from fan frequency**Plot 1:** Frequency vs Velocity plot from wind tunnel calibration

Initially, equation 5 had to be utilized to calculate velocity from the temperature and pressure data. Since temperature had been recorded for each fan frequency, the respective air density was interpolated from a table listing common properties of dry air [3]. The data provided by the pitot tube was the dynamic pressure - allowing for the use of only one pressure value as $P_D = P_T - P_S$. Figure 9 depicts the way the pitot-static tube functions with two pressure reading ports. The pressure difference between the two ports yields the pressure in the free stream and therefore the pressure in the wind tunnel for that fan speed.

$$U = \sqrt{\frac{2(P_T - P_S)}{\rho}}$$

Equation 5: Formula relating pressure, air density, and air velocity

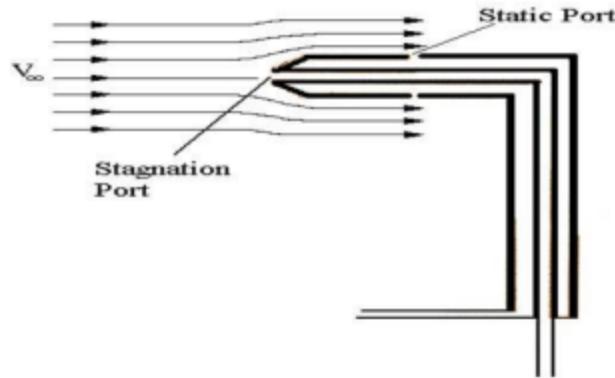


Figure 9: Pitot-static tube reading pressure

The results of the vortex shedding experiment have been tabulated (tables 3 and 4) to compare the results of applying different air velocities to the same bluff body (in this case a cylinder). In both cases, the empirical shedding frequency (equation 6) and the shedding frequency from the accelerometer were pretty similar to one another proving integrity of the experiment as the two frequencies were calculated in different ways but, they are both measuring the speed of the oscillating motion of the cylinder due to the airflow. Although in this case it seems that a higher air speed will cause a higher shedding frequency, the shedding frequency will actually increase as the velocity of the air gets closer to the natural frequency of the body. When the body is exposed to its natural frequency, the highest shedding frequency will occur. The reason the cylinder obtains a frequency is due to the high pressure at the leading edge of the body and the low pressure at the trailing edge of the body. When the air passes over the cylinder, it rushes in to fill the area of low pressure behind the body causing a complicated flow pattern where the forces on the sides of the body act periodically. The larger the shedding frequency, the faster the body vibrates. This is verified in our experiment as the cylinder with a fan speed of 30 Hz had a lower shedding frequency than the cylinder with a fan speed of 45 Hz.

$$\frac{f \cdot d}{U_\infty} = 0.198 \left(1 - \frac{19.7}{Re} \right)$$

Equation 6: formula for empirical shedding frequency (f) as well as strouhal number (fd/U)

Another unsurprising result was the fact that the reynold's number for the 30 Hz experiment was less than that of the 45 Hz experiment. This makes sense due to the fact that 45 Hz would cause a larger air velocity which is more likely to have turbulence than a lower velocity air flow. The tables 3 and 4 depict the same Strouhal number for both experiments; however, given enough significant figures, there would be a slight difference in the values. These strouhal numbers are of moderate value which verifies that vortex sheddings should be taking place [4].

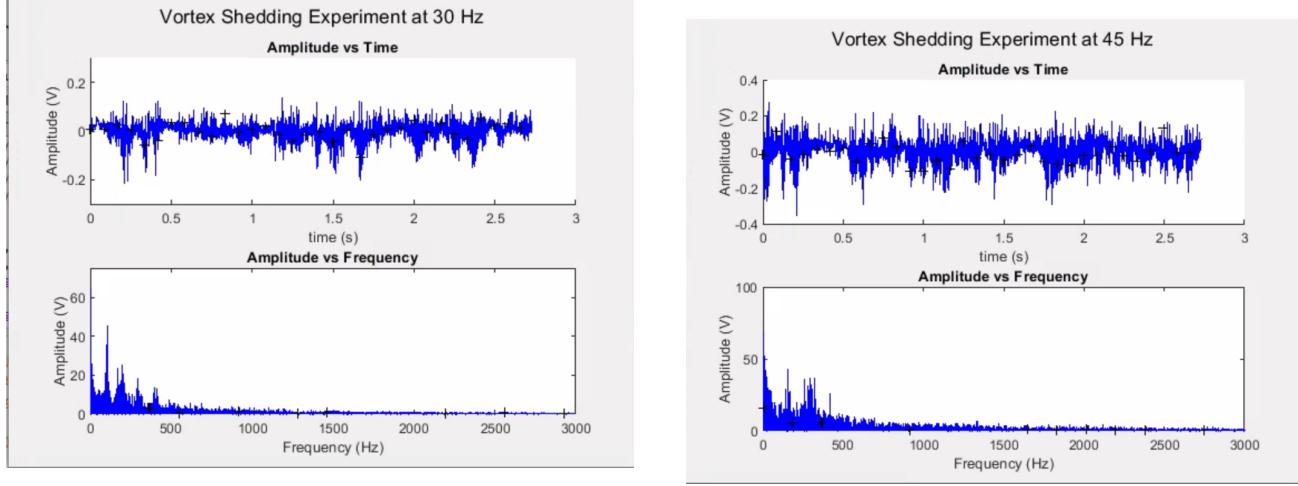
Table 3: Vortex shedding experiment at 30 Hz

	Calculated Value	Uncertainty
Frequency of Tunnel	30 Hz	
Strouhal Number	0.198	+/- 5.4261e-08
Reynold's Number	1.25e+06	+/- 2.2013e+04
Empirical Shedding Frequency	10.0429 Hz	+/- 0.1764
Shedding Frequency from Accelerometer	9.9423 Hz	+/- .5 Hz

Table 4: Vortex shedding experiment at 45 Hz

	Calculated Value	Uncertainty
Frequency of Tunnel	45 Hz	
Strouhal Number	0.198	+/- 2.3846e-08
Reynold's Number	1.90e+06	+/- 2.2033e+04
Empirical Shedding Frequency	15.3318 Hz	+/- 0.1766
Shedding Frequency from Accelerometer	15.1783 Hz	+/- .5 Hz

The plots made from the data collected in the vortex shedding portion of the experiment verify the results using data collected from the accelerometer. Plot 2 clearly shows that when the fan speed is set to 30 Hz, the amplitude of cylinder vibration is less than the amplitude of vibration for 45 Hz. This suggests that at 45 Hz the cylinder is moving more than at 30 Hz and 45 Hz must be closer to the natural frequency of the cylinder than 30 Hz. It was possible to use equipment such as the accelerometer which was used in a previous lab since the accelerometer measures the amount at which the object it is placed on moves from its initial position. In this case, the accelerometer measures how far to each side the cylinder moves due to the airflow as seen in the amplitude vs time plots.



Plot 2: Amplitude vs Time and Amplitude vs Frequency plots from the vortex shedding experiment conducted at 30 Hz (left) and 45 Hz (right)

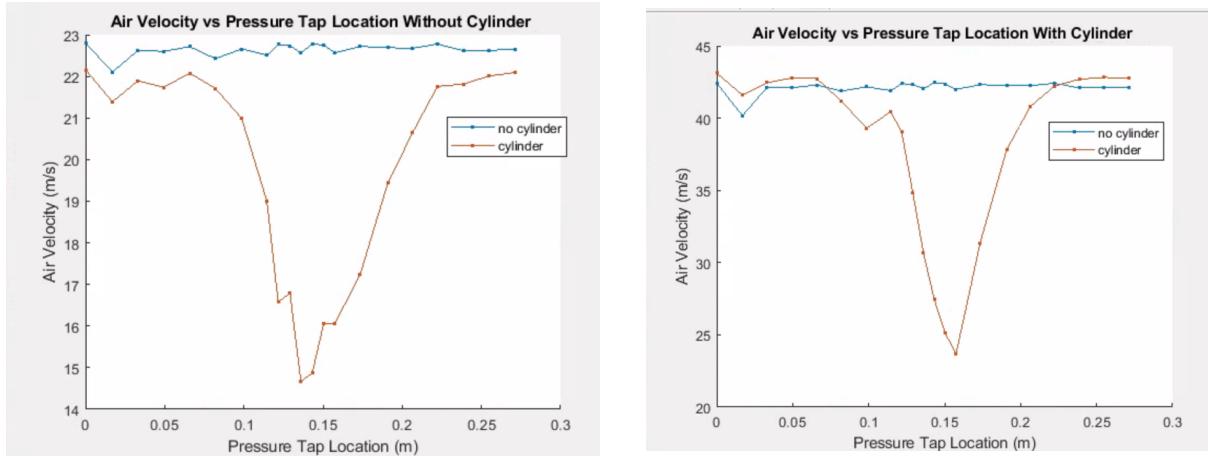
For the third part of the experiment, data that was collected by group M006 rather than group M002 was used for analysis. The reason for this decision was that the graphs seemed to better fit the shapes of the graphs given in the examples. Since M002 is the first group of the semester to run the experiment, it is likely that there could have been an error in data collection or experiment configuration.

The cylinder wake experiment was an attempt to show the speed of the air as it flowed past the cylinder at different starting velocities. As stated previously, there is a low pressure area behind the cylinder where air rushes in to fill causing a different flow pattern upstream of the cylinder. This difference in flow causes a wake that grows in width as one moves upstream. The difference in flow affects a larger area than just the width of the cylinder - the area spreads out as it gets further from the body.

As seen in plot 3, if the cylinder was placed about in line with the 0.15 m mark on the pressure rake, we have the lowest velocity due to the lowest pressure directly behind the cylinder. In the plot, the data points collected with no cylinder in the wind tunnel are meant to be used as a comparison to show that when there is nothing to disrupt the airflow, there is generally no variation in velocity across the width of the wind tunnel.

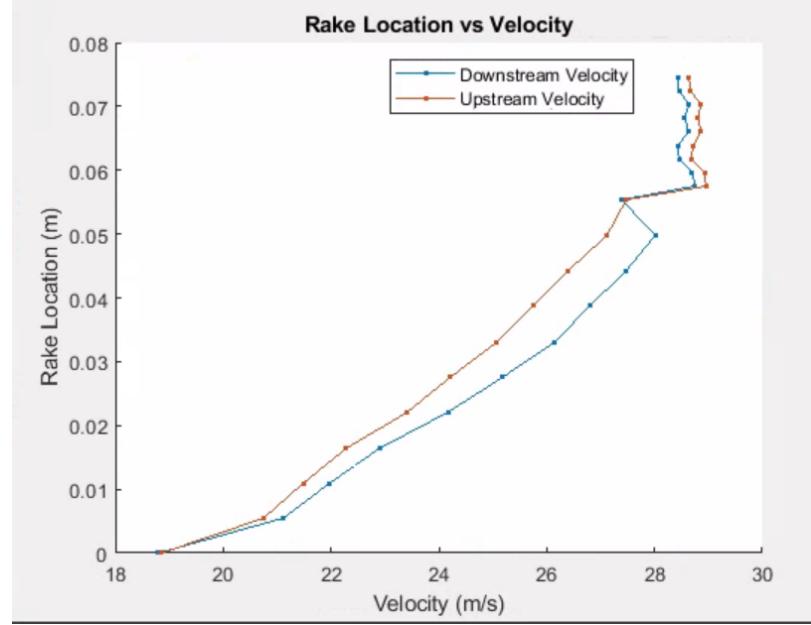
Based on collected data from this experiment, the drag force for the 25 Hz trial was calculated to be 109.9127 N while the drag force for the 45 Hz trial was calculated to be 408.9945 N. The larger air velocity causes a larger drag force on the cylinder. Anyone who has felt themselves being pulled back by a fast gust of wind can understand these results from their own experience. The coefficients of drag for each trial were very similar - only varying by a few decimals. This result makes sense because the coefficient of drag is largely based on the geometry of the object - the same object was used for both trials - and the Reynolds number. Published values of coefficient of drag range from 0.82 for long cylinders and 1.15 for short cylinders; the value calculated in this experiment was around 240 leading to the conclusion that something went terribly wrong in the calculations [5]. The error in this situation is incredibly

large and therefore cannot be attested to human error or equipment limitations. The only explanation would be that the coefficient of drag was simply calculated wrong



Plot 3: *Air velocity vs pressure tap location for pressure rake experiment conducted at 25 Hz (left) and 45 Hz (right)*

The fourth part of this experiment explores the boundary layer at different locations within the wind tunnel. The rake was in the center of the wind tunnel meaning that zero on the x-axis of plot 4 is the starting point of the rake, not the width of the tunnel. However, due to the no-slip condition, the velocity at each side of the wall of the wind tunnel must be zero. Directly adjacent to the wall begins the boundary layer which starts with a thickness of relatively zero downstream and increases moving upstream. As the boundary layer thickness increases, so does the maximum fluid velocity within it. Based on plot 4, both velocities begin at the same low value, as the distance from the wall increases, the velocity also increases until the boundary layer is seemingly surpassed (at about .06 m) and the velocity is relatively constant as a function of rake location. The overall result of this experiment is that there is a thin area adjacent to a flat, stationary plate where velocity increases from zero as a function of distance from the plate. Once a certain distance has been reached, the boundary layer has been surpassed and the velocity is now equal to the free stream velocity which is constant across the wind tunnel.



Plot 4: Lift vs angle of attack for airfoil experiment conducted at 30 Hz

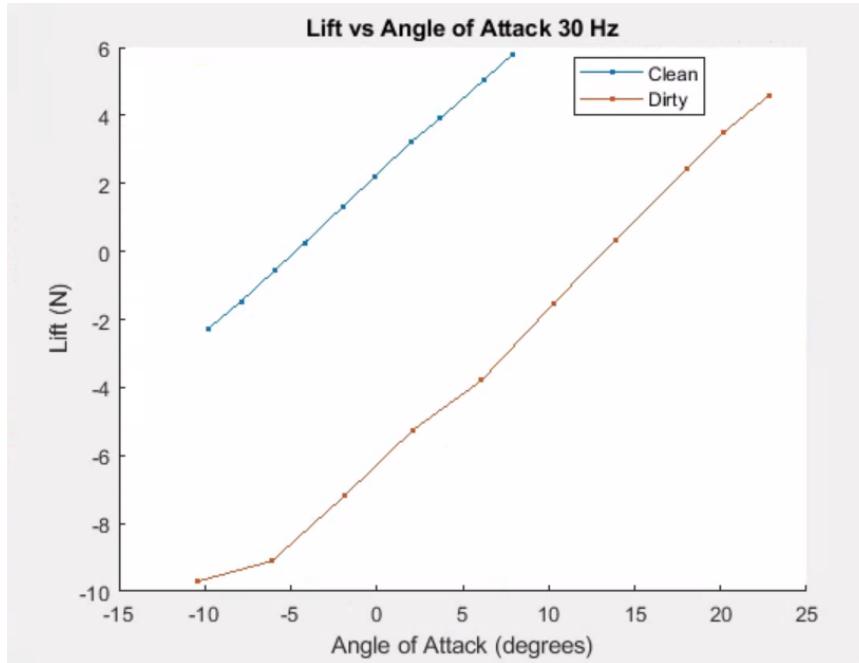
The numeric results of the fourth experiment (the airfoil experiment) have been recorded in table 5. The data that was collected by group M006 was also utilized for this portion of the experiment as there were slight problems getting the airfoil to change angles during the M002 section. As seen in previous experimentation and analysis, the Reynolds number is affected by the air velocity and therefore has a different value for each fan speed. Reynolds number is also affected by a geometric component of the airfoil which was changed when the flaps and slats were engaged.

Fan Frequency	Configuration	Reynolds Number	Uncertainty	Peak Efficiency
25 Hz	clean	5.1551e+05	+/- 1815.1638	4.20
	dirty	7.7528e+05	+/- 1.2259e+03	2.86
45 Hz	clean	5.1381e+05	+/- 812.4735	5.1
	dirty	7.7657e+05	+/- 1.2279e+03	3.47

Table 5: Airfoil experiment

Plots were created for lift and drag vs angle of attack for each fan speed and each foil configuration. Plot 5 shows a plot of how the lift force is affected by the angle of attack. When comparing the lines for the clean and dirty configurations, one may believe it is evident that

since the clean configuration can generate higher lift values, the favorable configuration for aircrafts is actually the dirty configuration since it can maintain a lift force at higher angles. An aircraft with dirty airfoils rather than clean is safer as it is able to continue flight at higher angles from the normal without stalling and ultimately crashing. By inspection in the lab, the airfoil with the clean configuration began to stall at an attack angle of 14 degrees and the dirty configuration didn't stall until an attack angle of about 18 degrees. The stall angle was determined by observing when the strings on the trailing edge of the airfoil began to move in faster, more unpredictable motion. A similar trend is seen in the drag vs angle of attack plot where the clean configuration generates a greater force but the dirty configuration is able to sustain that force for a larger attack angle (plot 6). Notably, the maximum value of the lift force is greater than the maximum value of the drag force which is a favorable situation for aircrafts.



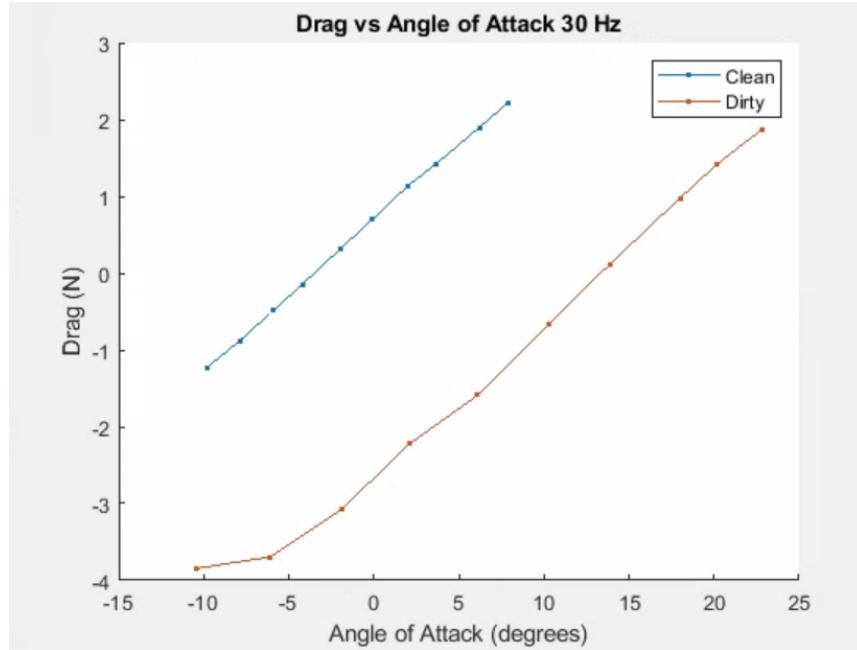
Plot 5: Lift vs angle of attack for airfoil experiment conducted at 30 Hz

Figure 10 plots an accepted trend for the Clark Y airfoil for comparison with the experimental graphs. It seems that the overall trend of force increasing with angle of attack holds true for both experimental and accepted plots, however it seems that the lift graph never had a chance to reach the value for stall where the line reaches a peak and begins to decrease.

As seen in plot 7, the pitching moment has a negative relationship with the angle of attack. Interestingly, the dirty configuration has both a higher pitching moment for each angle. Perhaps this means that the dirty configuration is more equipped to correct itself when it takes on an angle of attack that is unfavorable.

Notably, the figures utilizing the data from the 45 Hz fan speed seem to be random and incorrect. Both data from M002 and M006 sections were compared, however neither seemed to

provide better results. The conclusion is that the equations used to plot these figures are incorrect. Somehow, this discrepancy only affected the 45 Hz plots and not the 30 Hz plots.



Plot 6: Drag vs angle of attack for airfoil experiment conducted at 30 Hz

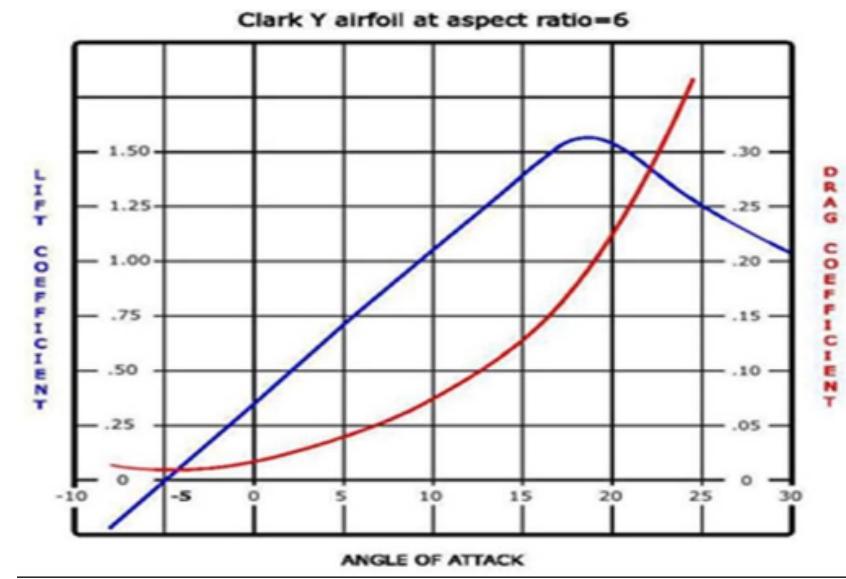
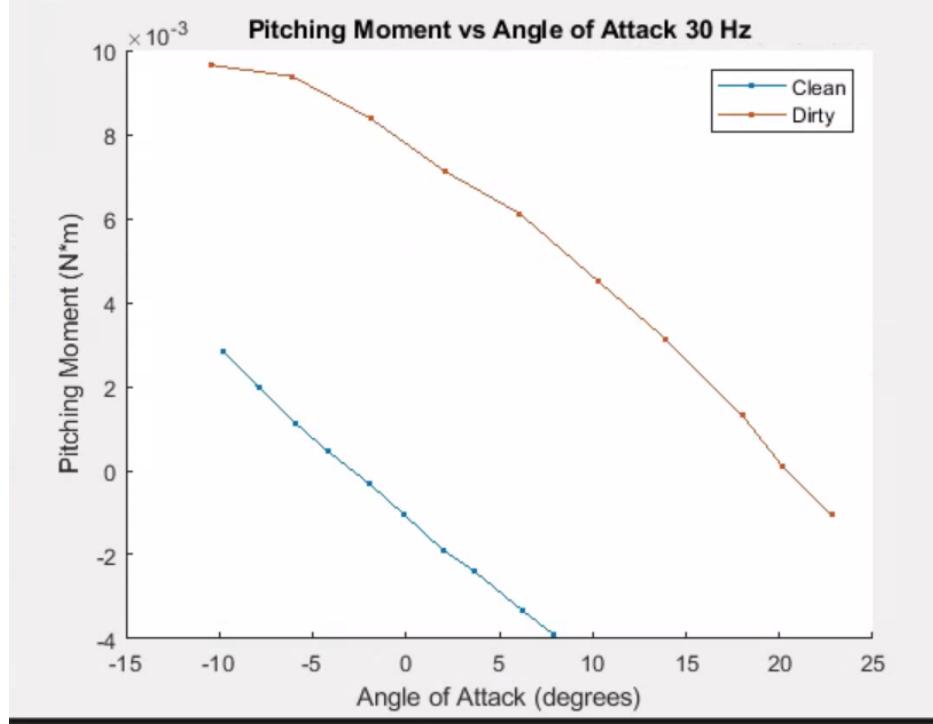


Figure 10: Accepted lift vs angle of attack and drag vs angle of attack plots. Source - [2: Lift and drag curves for the typical airfoil. | Download Scientific Diagram \(researchgate.net\)](#)



Plot 7: Pitching moment vs angle of attack for airfoil experiment conducted at 30 Hz

The Reynolds number, drag, and coefficient of drag for each cylinder with each fan speed is tabulated in table 6 as found in the fifth part of the experiment. This experiment luckily found coefficients of drag far closer to the published data compared to the third part of the experiment however, there is still a discrepancy. The values which are accepted for the coefficient of drag for cylinders range from .82 to 1.15 where the data shows a range from .02 to .08. Again, this must be due to an error in the calculation of the coefficient of drag.

As to be expected, as the length of the cylinder increases, the drag decreases because the drag caused by vortices on each edge of the cylinder begins to vanish. Conclusively, a cylinder that is the entire width of the wind tunnel would have the lowest value of drag force associated with it. The cylinder that is as wide as the wind tunnel would be considered to be in ‘2D flow’ since the air is only acting on the leading edge of the cylinder. ‘3D’ flow would be when a shorter cylinder experiences more drag since the air creates vortices from the flat edges of the cylinder.

	Cylinder Number	Length (m)	Reynolds Number	Drag	Uncertainty in Drag	Coefficient of Drag	Uncertainty in Coefficient of Drag
25 Hz	0						
	1	0.6	8.0656e+04	0.8081	0.0162	0.0729	+/- 0.0408i
	2	0.56	9.7848e+04	0.2660	0.0053	0.0257	+/- 0.0243i

	3	0.51	$8.0703e+04$	0.6912	0.0138	0.0740	+/- 0.0411i
	4	0.46	$8.0719e+04$	0.5543	0.0111	0.0661	+/- 0.0389i
	5	0.27	$8.0719e+04$	0.4147	0.0083	0.0841	+/- 0.0438i
45 Hz	0						
	1	0.6	$1.4935e+05$	0.8081	0.0162	0.0213	+/- 0.0162i
	2	0.56	$1.4938e+05$	0.2660	0.0053	0.0075	+/- 0.0096i
	3	0.51	$1.4944e+05$	0.6912	0.0138	0.0216	+/- 0.0163i
	4	0.46	$1.4947e+05$	0.5543	0.0111	0.0193	+/- 0.0154i
	5	0.27	$1.4947e+05$	0.4147	0.0083	0.0245	+/- 0.0174i

Table 6: 2D-3D cylinder experiment

Conclusion

The results of this lab verified many important fluid dynamic concepts. It was found that vortex shedding frequency is related to the natural frequency which we learned about in a previous lab. It became apparent that fluid flows are incredibly complex and many small changes can leave a large effect. Similarly, the difference between clean and dirty airfoils can mean the difference between a plane crash and making it to your destination.

There were many aspects of these results that seem inaccurate, such as some of my plots from the airfoil experiments. I also believe that I was not able to accurately calculate the drag coefficients for any of the cases. I am to assume that these are problems with my code which could be fixed with a better understanding of the material.

Another finding was that every Reynolds number calculated was above the range for turbulent flow. I am not sure if this is correct because I believe that at least some of the situations occurred in laminar flow. This again may be a source of error from calculations.

Some key takeaways from this lab were how to eliminate drag, how to avoid stalling, and the effect of attack angle on airfoils.

Overall, I enjoyed this class and found it to be more manageable than I had heard it was in prior years. I appreciate Professor Anderson being accommodating to all students and I also appreciate the TA's for making it a priority to help students. The thing I found challenging in this lab however was that each day there were many different experiments which made it difficult to remember each procedure and what data was collected when. I think it would be more beneficial to students if there were labs with less different experiments so they could fully grasp each one.

Appendix

Sources

[1] [Shedding Frequency - an overview | ScienceDirect Topics](#)

[2] [Vortex Flowmeters - Shedding Frequency vs. Volume Flow Rate \(engineeringtoolbox.com\)](#)

[3] [Dry Air - Thermodynamic and Physical Properties \(engineeringtoolbox.com\)](#)

[4][Strouhal Number - an overview | ScienceDirect Topics](#)

[5][Drag Coefficient: Definition, Formula, Equation, Applications \(testbook.com\)](#)

[6][*40565421 \(blackboardcdn.com\)](#)

[7][*39605788 \(blackboardcdn.com\)](#)

[8] Class materials provided by Professor Jackie Anderson for the use in MAE 315 Fall 2022

Drag Derivation

$$D = \rho L u_1^2 \int_0^w \left(1 - \frac{u_2^2}{u_1^2}\right) dy$$

$$D = \int_0^w \rho u_2 (u_1 - u_2) dy$$

- Incompressible therefore density is constant

$$D = \rho \int_0^w u_2 u_1 - u_2^2 dy$$

$$D = \left[\frac{-w^3}{3} \right] \rho \int_0^w u_2 u_1 dy$$

Equations

$$f_{es} = \frac{1}{5} \left(\frac{v}{d} \right)$$

Equation A.1: Empirical Shedding Frequency. Source - [Vortex Shedding in Water | Harvard Natural Sciences Lecture Demonstrations](#)

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 S}$$

Equation A.2: *Coefficient of Drag*

$$C_L = 2\pi\alpha$$

$$C_L = \frac{L}{\frac{1}{2}rhoV^2S}$$

Equation A.3: *Coefficient of Lift*

$$C_M = \frac{M}{\frac{1}{2}rhoV^2Sx}$$

Equation A.4: *Coefficient of Pitching Moment*

$$R_e = \frac{\rho V d}{\mu}$$

Equation A.5: *Reynolds Number*

Tables
Table A.1: *Wind tunnel calibration*

	Estimate	Uncertainty
Frequency	50 Hz	+/- 0.5 Hz
Velocity	52.1047 m/s	+/- 0.5373 m/s
R squared	0.994	

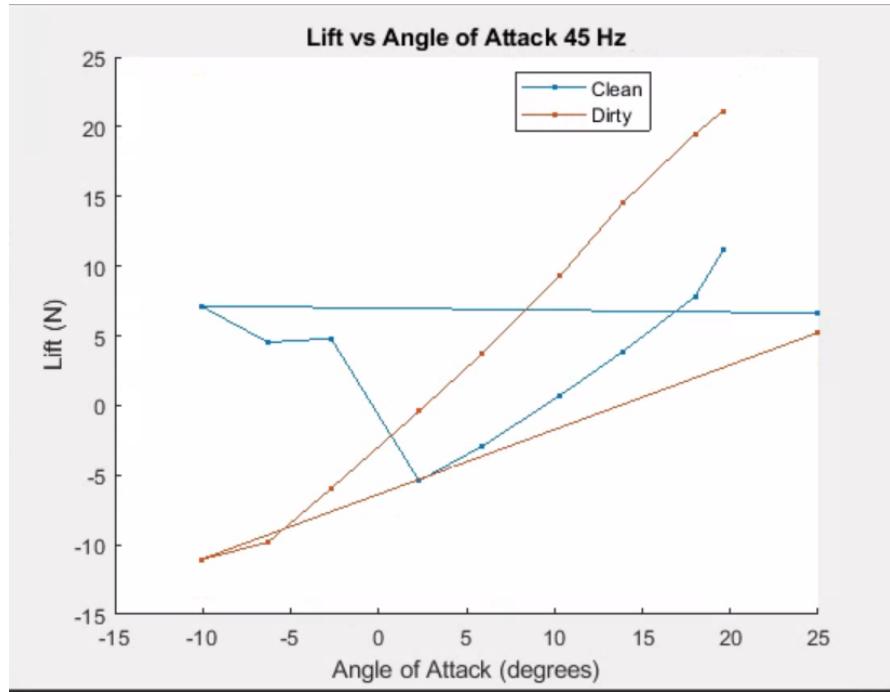
Table A.2: *Cylinder wake experiment at 25 Hz*

25 Hz	Calculated Value	Uncertainty
Frequency of Tunnel	25 Hz	
Drag	109.9127 N	+/- 2.1983 N
Coefficient of Drag	240.4058	+/- 59.8505

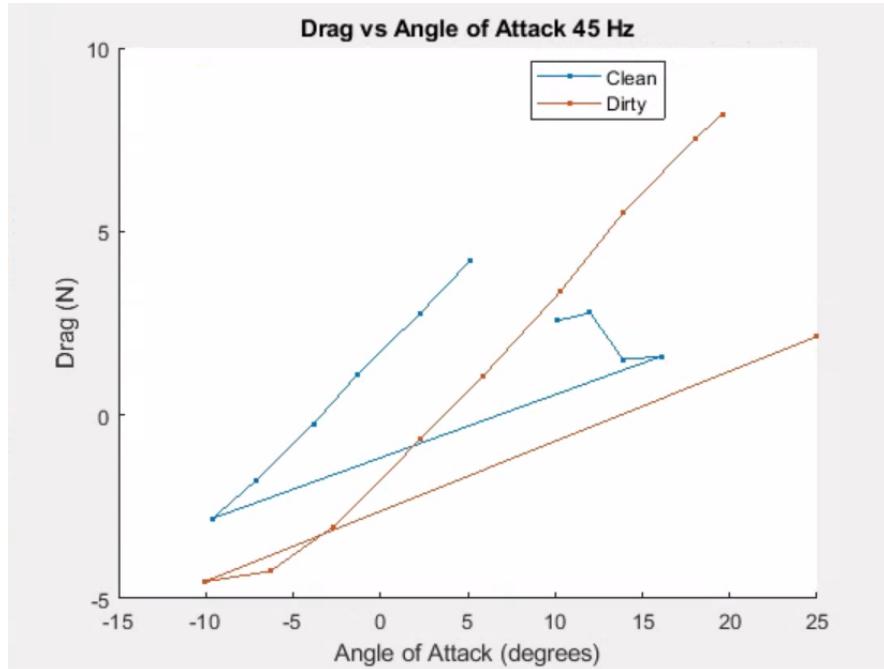
Table A.3: *Cylinder wake experiment at 45 Hz*

	Calculated Value	Uncertainty
Frequency of Tunnel	45 Hz	
Drag	408.9945 N	+/- 8.1799 N
Coefficient of Drag	239.0743	+/- 30.6782

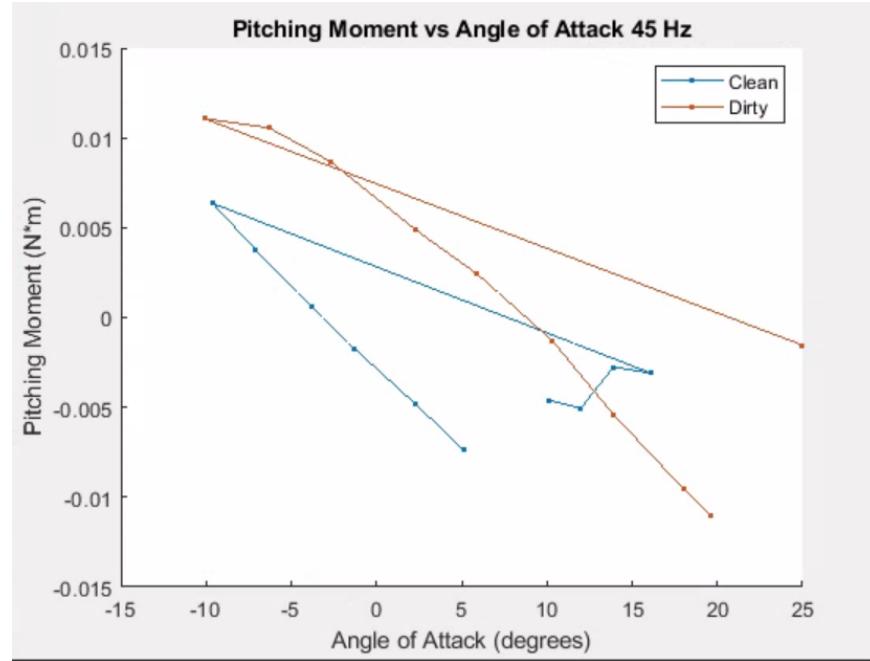
Plots



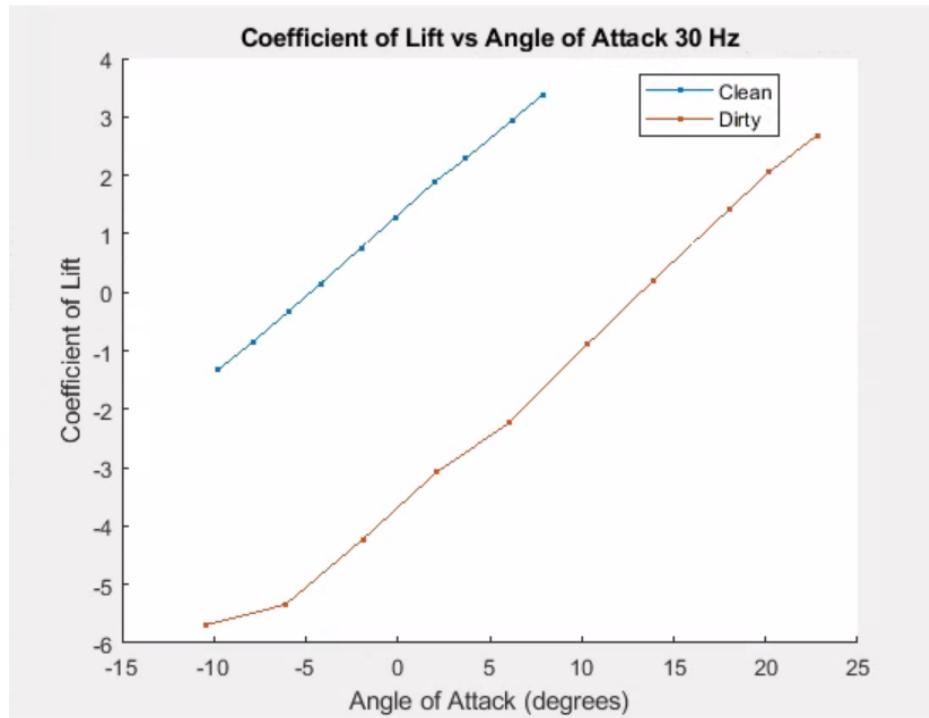
Plot A.1: Lift vs angle of attack for airfoil experiment conducted at 45 Hz



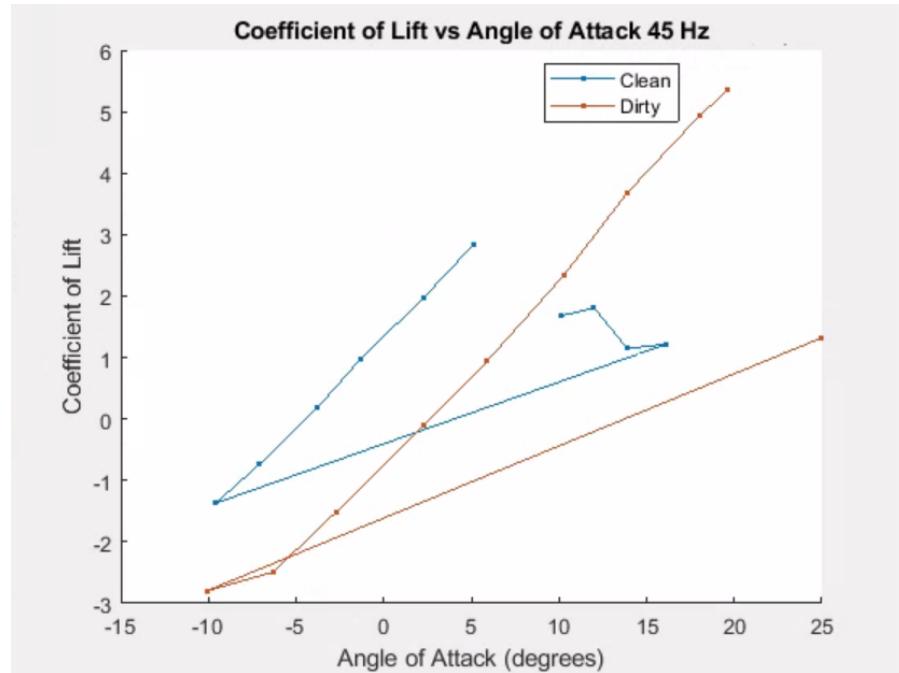
Plot A.2: Drag vs angle of attack for airfoil experiment conducted at 45 Hz



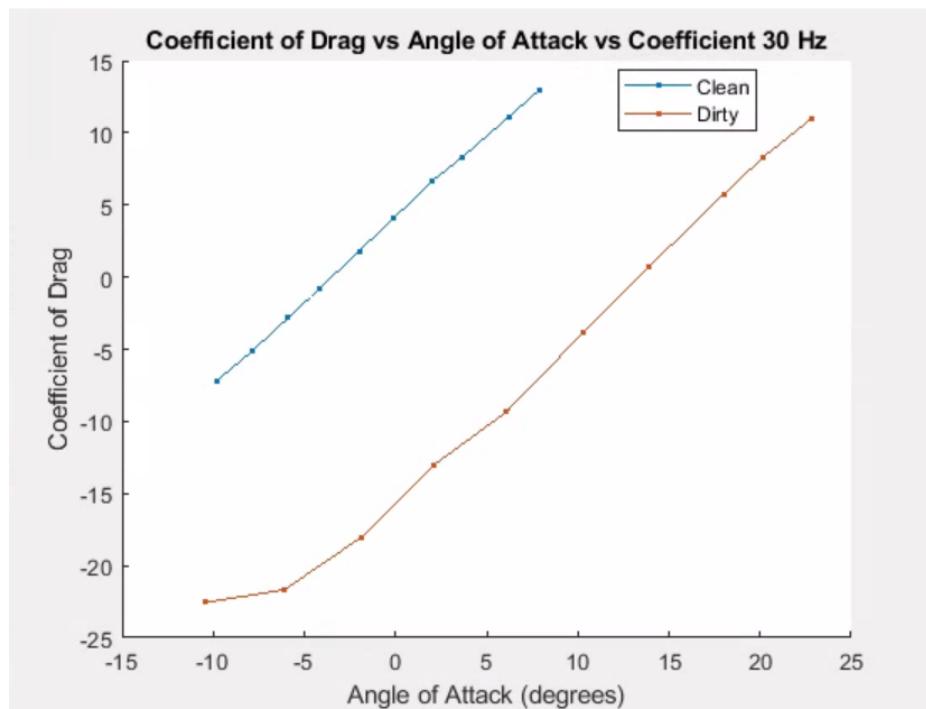
Plot A.3: Pitching moment vs angle of attack for airfoil experiment conducted at 45 Hz



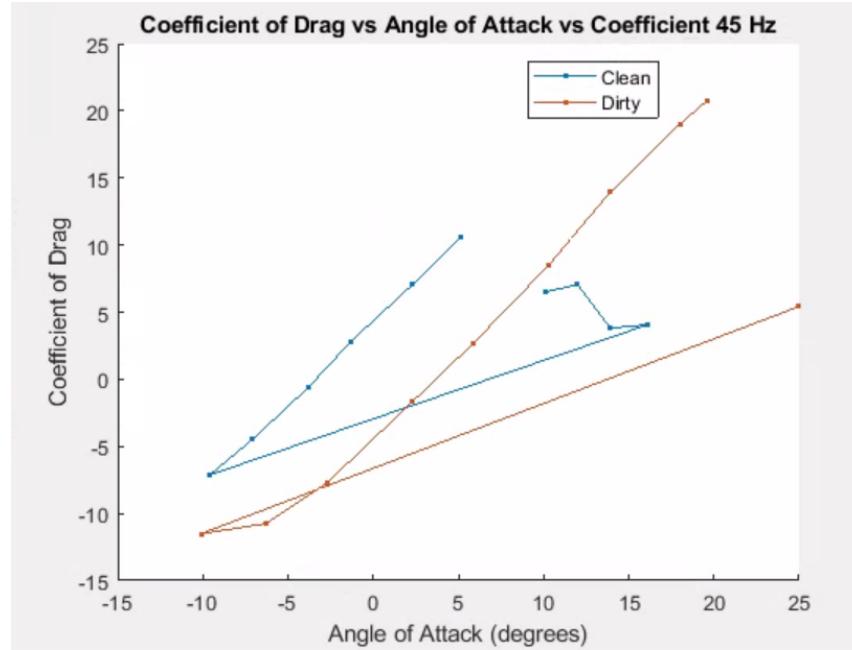
Plot A.4: Coefficient of lift vs angle of attack for airfoil experiment conducted at 30 Hz



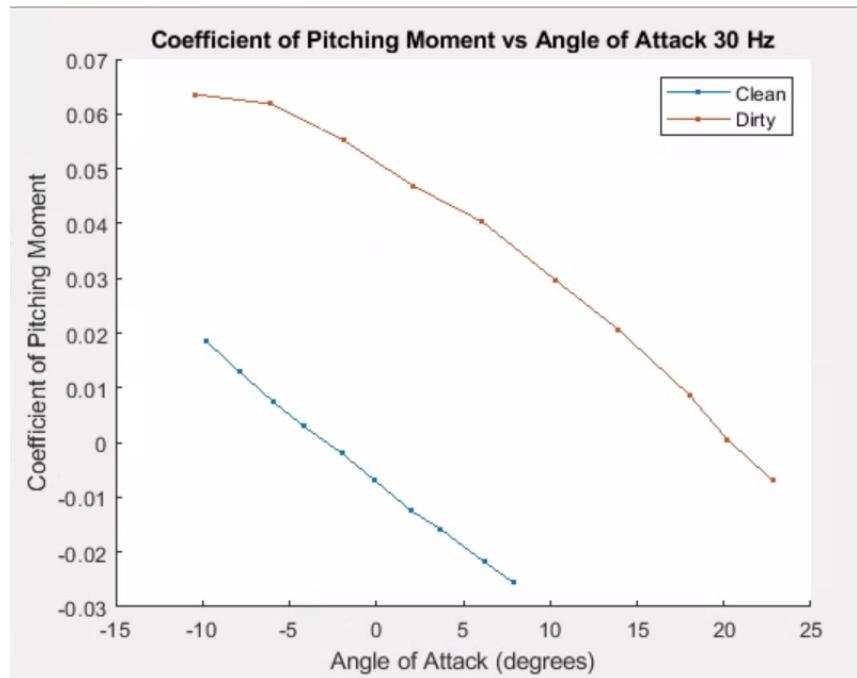
Plot A.5: Coefficient of lift vs angle of attack for airfoil experiment conducted at 45 Hz



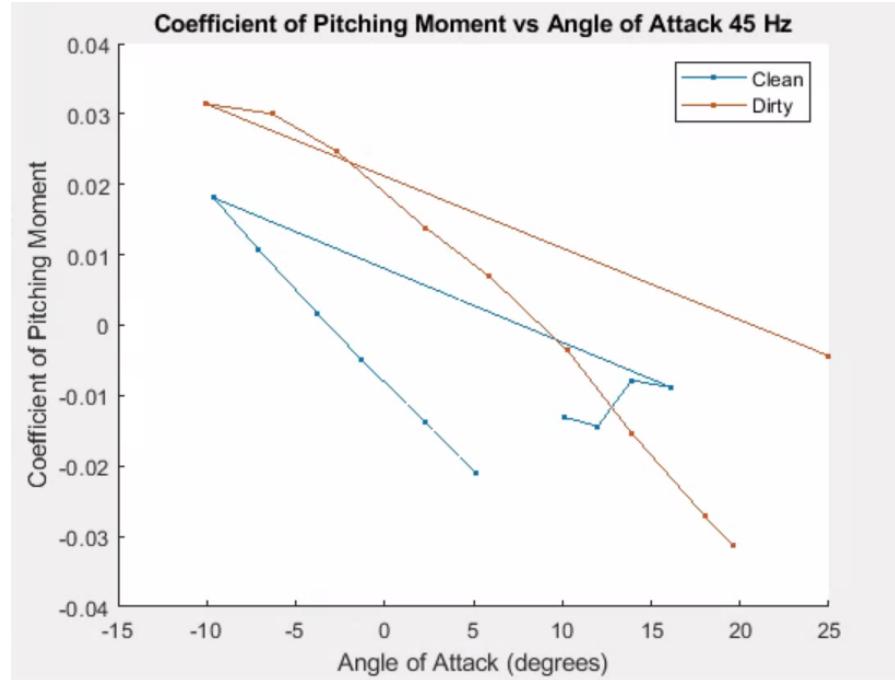
Plot A.6: Coefficient of drag vs angle of attack for airfoil experiment conducted at 30 Hz



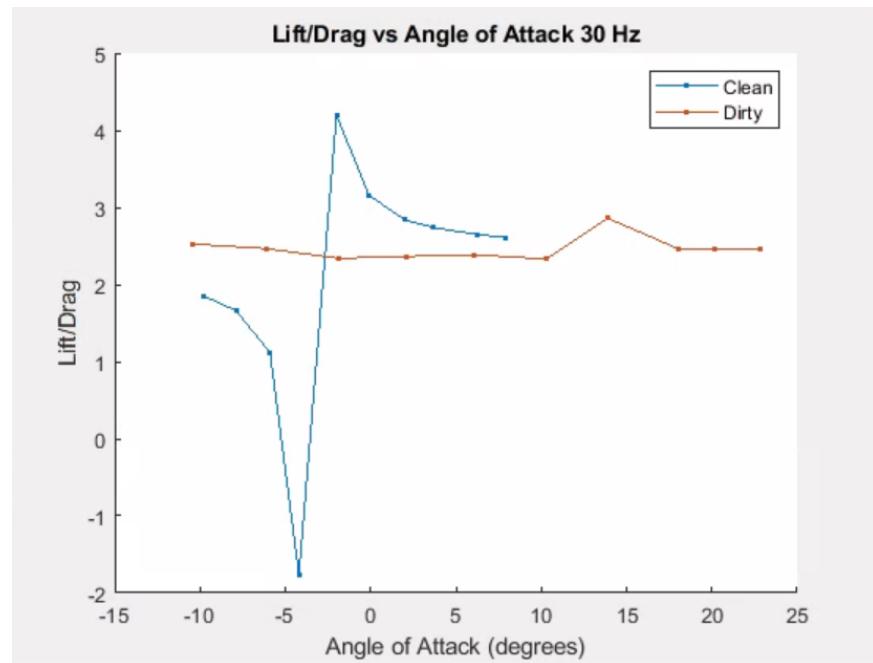
Plot A.7: Coefficient of drag vs angle of attack for airfoil experiment conducted at 45 Hz



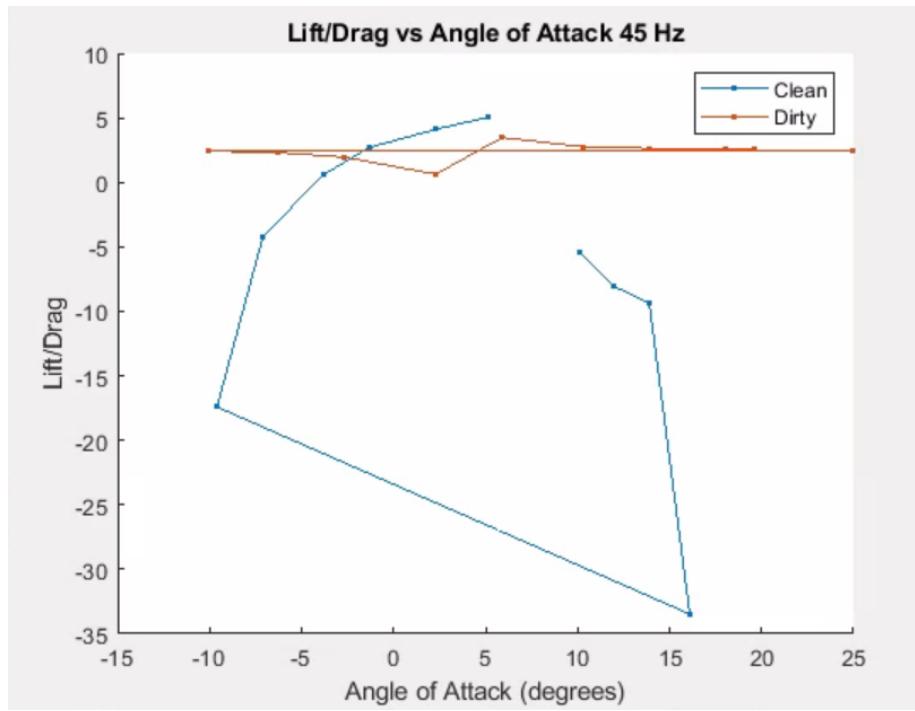
Plot A.8: Coefficient of pitching moment vs angle of attack for airfoil experiment conducted at 30 Hz



Plot A.9: Coefficient of pitching moment vs angle of attack for airfoil experiment conducted at 45 Hz



Plot A.10: Lift/drag vs angle of attack for airfoil experiment conducted at 30 Hz



Plot A.11: *Lift/drag vs angle of attack for airfoil experiment conducted at 45 Hz*

Contents

- IMPORT DATA
 - rake experiment
 - vortex shedding experiment
 - PART B DATA
 - 2D and 3D cylinder experiment
 - airfoil experiments
 - Uncertainties
 - Governing Equations
 - Wind Tunnel Calibration
 - Vortex Shedding
 - Rake Experiment
 - Boundary Layer Experiment
 - Airfoil
 - 2D and 3D Cylinders
 - fan speed is 25 Hz
 - fan speed is 45 Hz
-

```
% Teagan Kilian
% MAE 315
% Lab 4
```

IMPORT DATA

```
% PART A DATA

% tunnel calibration
norake25 = importdata('m002norake25');
norake45 = importdata('m002norake45');
pitot = readtable('MAE 315 lab 4.xlsx','sheet','Pitot Tube', 'Range', 'I6:L14');
pitot = pitot{1:9,[ "Var1", "Var2", "Var3", "Var4"]};
freqpitot = pitot(:,1); % frequency in Hz
prespitot = pitot(:,2); % pressure in Pa
temppitot = pitot(:,3); % temperature in K
rhopot = pitot(:,4); % density in kg/m^3
```

rake experiment

```
norake25raw = importdata('M006nocyl25');
norake25 = norake25raw(:,2:22);
norake45raw = importdata('M006nocyl45');
norake45 = norake45raw(:,2:22);
rake25raw = importdata('M006cy125');
rake25 = rake25raw(:,2:22);
rake45raw = importdata('M006cy145');
rake45 = rake45raw(:,2:22);
prake1 = readtable('MAE 315 lab 4.xlsx','sheet', 'Pressure Rake', 'Range', 'I4:K23');
prake1 = prake1{1:20, [ "Var1", "Var3"]};
gap_n1 = prake1(:,1);
gap_w1 = prake1(:,2); % gap width in m
T_30_nocyl = 20.2778 + 273; % temperature in K
T_45_nocyl = 23.5 + 273; % temperature in K
T_30_cyl = 21.222 + 273; % temperature in K
T_45_cyl = 23.3889 + 273; % temperature in K
position1 = readtable('MAE 315 lab 4.xlsx', 'sheet', 'Pressure Rake', 'Range', 'L3:L23');
position1 = position1{1:21, "Var1"};
```

vortex shedding experiment

```
Vcyl30 = importdata('M002cylinder30.txt');
Vcyl45 = importdata('M002cylinder45.txt');
Lcyl = 0.6096; % m
```

```

dia      = 0.050673;                      % m
n_samp   = 16384;
freq_samp = 6000;                         % Hz
time     = (0:1/freq_samp:2.7305)';        % s
T_30     = 21.222;                         % temperature in deg C
T_45     = 23.444;                         % temperature in deg C
fd       = (0:n_samp - 1) .* freq_samp ./ n_samp; % frequency domain

```

PART B DATA

```

% boundary layer experiment
bl_down  = importdata('m002bled');
bl_down  = bl_down(:,2:21);
bl_up    = importdata('m002bleu');
bl_up    = bl_up(:,2:21);
T_down   = 20.0556 + 273; % temperature in K
T_up     = 21.111 + 273; % temperature in K
prake2   = readable('MAE 315 lab 4.xlsx','sheet', 'Pressure Rake','Range', 'D4:F23');
prake2   = prake2{1:20, ["Var1", "Var3"]};
gap_n2   = prake2(:,1);
gap_w2   = prake2(:,2);
position2 = readable('MAE 315 lab 4.xlsx', 'sheet', 'Pressure Rake', 'Range', 'G3:G23');
position2 = position2{1:20, "Var1"};

```

2D and 3D cylinder experiment

```

cyls_data = importdata('M002_cylinders');
cyls     = cyls_data.data;
cyl0    = cyls(1,:);
cyl1    = cyls(2,:);
cyl2    = cyls(3,:);
cyl3    = cyls(4,:);
cyl4    = cyls(5,:);
cyl5    = cyls(6,:);
cyl_data = readable('MAE 315 lab 4.xlsx','sheet', 'Cylinders','Range', 'A5:G10');
cyl_data = cyl_data{1:6, ["Var1", "Var3", "Var5", "Var7"]};
n_cyl   = cyl_data(:,1);
L_cyl   = cyl_data(:,2); % m
D_cyl   = cyl_data(:,3); % m
T_cyl   = cyl_data(:,4); % deg C

```

airfoil experiments

```

af_dirty = importdata('M006_afdirty');
af_clean = importdata('M006_afclean');
v_aird  = af_dirty.data(:,1);
v_aird30 = v_aird(1:10,:);
v_aird45 = v_aird(11:20,:);
angled   = af_dirty.data(:,2);
angled30 = angled(1:10,:);
angled45 = angled(11:20,:);
dragd   = af_dirty.data(:,3);
drag_d30 = -dragd(1:10,:).* 4.448;
drag_d45 = -dragd(11:20, :).* 4.448;
Dstdd   = af_dirty.data(:,4);
Dstdd30 = Dstdd(1:10,:);
Dstdd45 = Dstdd(11:20,:);
liftd   = af_dirty.data(:,5);
lift_d30 = -liftd(1:10, :).* 4.448;
lift_d45 = -liftd(11:20, :).* 4.448;
Lstdd   = af_dirty.data(:,6);
Lstdd30 = Lstdd(1:10,:);
Lstdd45 = Lstdd(11:20,:);
Md      = af_dirty.data(:,7);
M_d30   = -Md(1:10,:).*113;
M_d45   = -Md(11:20,:).*113;
Mstdd   = af_dirty.data(:,8);
Mstdd30 = Mstdd(1:10,:);
Mstdd45 = Mstdd(11:20,:);
v_airc  = af_clean.data(:,1);
v_airc30 = v_airc(1:10,:);
v_airc45 = v_airc(11:20,:);

```

```

anglec   = af_clean.data(:,2);
anglec30 = anglec(1:10,:);
anglec45 = anglec(11:20,:);
dragc   = af_clean.data(:,3);
drag_c30 = -dragc(1:10,:).* 4.448;
drag_c45 = -dragc(11:20,:).* 4.448;
Dstdc   = af_clean.data(:,4);
Dstdc30 = Dstdc(1:10,:);
Dstdc45 = Dstdc(11:20,:);
liftc   = af_clean.data(:,5);
lift_c30 = -liftc(1:10,:).* 4.448;
lift_c45 = -liftc(11:20,:).* 4.448;
Lstdc   = af_clean.data(:,6);
Lstdc30 = Lstdc(1:10,:);
Lstdc45 = Lstdc(11:20,:);
Mc      = af_clean.data(:,7);
M_c30   = -Mc(1:10,:).*113;
M_c45   = -Mc(11:20,:).*113;
Mstdc   = af_clean.data(:,8);
Mstdc30 = Mstdc(1:10,:);
Mstdc45 = Mstdc(11:20,:);
T_c30   = 18.64957 + 273; % temperature in K
T_d30   = 19.67901 + 273; % temperature in K
T_c45   = 21.76667 + 273; % temperature in K
T_d45   = 21.89815 + 273; % temperature in K

chord = 0.089; % m
span   = 0.251; % m
depth  = 0.012; % m

```

Uncertainties

```

res_tape = 1/32 * .0254; % measuring tape resolution (m)
res_calip = .001 * .0254; % caliper resolution (m)
res_f    = 1; % frequency measurement resolution (Hz)
res_pres = .001 * 133.332; % pressure resolution (Pa)
u_calip = .5 * res_calip; % uncertainty in calipers (m)
u_tape  = .5 * res_tape; % uncertainty in measuring tape (m)
u_freq   = .5 * res_f; % uncertainty in frequency (Hz)
u_F     = .02; % uncertainty in force (N)
u_pres   = .5 * res_pres; % uncertainty in pressure (Pa)
u_t     = .5.*(.1/.000).*ones(16384,1); % uncertainty in time (s)
u_V     = .5.*(.5/(2.^24-1)).*ones(16384,1); % uncertainty in voltage (V)

```

Governing Equations

```

syms P rho CD CL CM d mu c span

v   = sqrt((2 * P) / rho); % velocity (m/s)
Re  = (rho .* v .* d) ./ mu; % reynolds number
S   = .198 .* (1 - (19.7 ./ Re)); % strouhal number
f_s = (S .* v) ./ d; % vortex shedding frequency (Hz)
SA  = span .* c; % wing area (m^2)
L   = .5 .* rho .* (v.^2) .* CL .* SA; % lift force (N)
D   = .5 .* rho .* (v.^2) .* CD .* SA; % drag force (N)
ac  = .25 .* c; % position of aerodynamic center (m)
M   = .5 .* rho .* (v.^2) .* CM .* ac .* SA; % pitching moment (N*m)

```

Wind Tunnel Calibration

```

% calculate air velocity
v_air = sqrt((2 .* prespitot)./rhopitot); % m/s

% calculate uncertainty in velocity
dvdP = diff(v,P); % partial derivative with respect to pressure

% exclude uncertainties for density
u_v  = dvdP.* u_pres;
u_v  = subs(u_v, P, prespitot);
u_v  = double(subs(u_v, rho, rhopitot));
u_v  = u_v(1:9:81);
u_ff = u_freq .* ones(size(freqpitot));

```

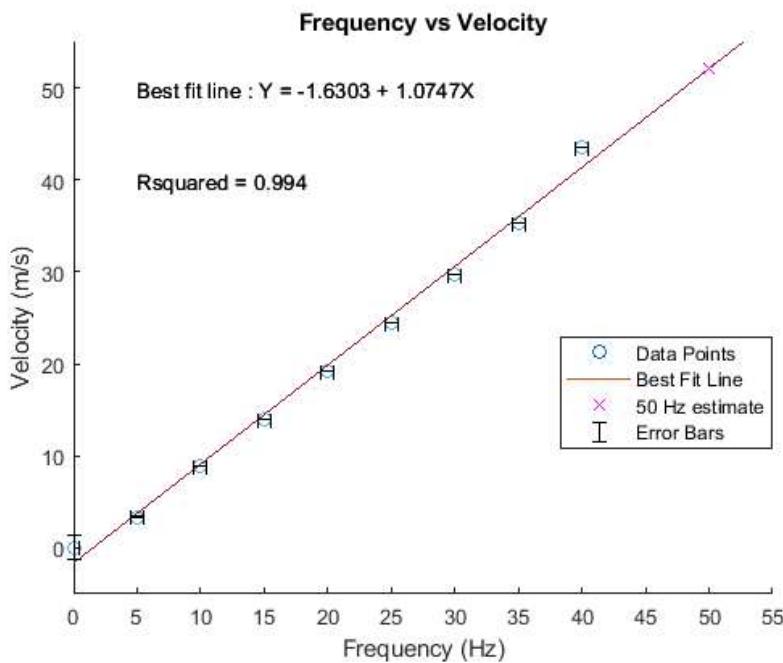
```

figure (1)
hold on
fit = polyfit(freqpitot, v_air, 1);
fit1 = fitlm(freqpitot, v_air);
fitline = polyval(fit,0:55);
scatter (freqpitot, v_air)
plot (0:5:55, fitline, '-')
v_50 = -1.6303 + 1.0747*50; % point estimate for frequency of 50 Hz
plot (50, v_50, 'xm') % plot the point estimate

% plot error bars
e1 = errorbar(freqpitot, v_air, u_v, 'Vertical', 'k', 'LineStyle','none');
e2 = errorbar(freqpitot, v_air, u_ff, 'Horizontal', 'k', 'LineStyle','none' );
xlim ([0,55])
ylim ([-5,55])
legend ('Data Points', 'Best Fit Line', '50 Hz estimate','Error Bars')
title 'Frequency vs Velocity';
xlabel 'Frequency (Hz)';
ylabel 'Velocity (m/s)';
text (5,50,'Best fit line : Y = -1.6303 + 1.0747X ')
text (5,40, 'Rsquared = 0.994')

% uncertainty in estimated velocity
syms f
v_fit = -1.6303 + 1.0747*f;
u_v_fit = diff(v_fit, f).* u_freq;
u_v_50 = double(subs(u_v_fit, f, 50));

```



Vortex Shedding

```

% 30 Hz Test

% amplitude vs time
figure (2)
subplot (2,1,1)
hold on
plot (time, Vcyl30, 'b')
freq30 = fft(Vcyl30);
e3 = errorbar(time(1:500:end), Vcyl30(1:500:end), u_t(1:500:end), 'Vertical', 'k', 'LineStyle', 'none');
e4 = errorbar(time(1:500:end), Vcyl30(1:500:end), u_V(1:500:end), 'Horizontal', 'k', 'LineStyle', 'none');
xlim ([0,3])
ylim ([-.3, .3])
xlabel 'time (s)'
ylabel 'Amplitude (V)'
title 'Amplitude vs Time'

```

```

% amplitude vs frequency
subplot (2,1,2)
plot (fd , freq30, 'b' )
hold on
u_freq1 = u_freq .* ones(size(freq30));
e5 = errorbar(fd(1:500:end), freq30(1:500:end), u_t(1:500:end), 'Vertical', 'k', 'LineStyle', 'none');
e6 = errorbar(fd(1:500:end), freq30(1:500:end), u_freq1(1:500:end), 'Horizontal', 'k', 'LineStyle', 'none');
xlim ([0,3000])
ylim ([0,75])
xlabel 'Frequency (Hz)'
ylabel 'Amplitude (V)'
title 'Amplitude vs Frequency'
sgtitle 'Vortex Shedding Experiment at 30 Hz'
hold off

% 45 Hz Test

% amplitude vs time
figure (3)
subplot (2,1,1)
hold on
plot (time, Vcyl45, 'b')
e7 = errorbar(time(1:500:end), Vcyl45(1:500:end), u_t(1:500:end), 'Vertical', 'k', 'LineStyle', 'none');
e8 = errorbar(time(1:500:end), Vcyl45(1:500:end), u_V(1:500:end), 'Horizontal', 'k', 'LineStyle', 'none');
xlim ([0,3])
ylim ([-.4, .4])
xlabel 'time (s)'
ylabel 'Amplitude (V)'
title 'Amplitude vs Time'

% amplitude vs frequency
freq45 = fft(Vcyl45);
subplot (2,1,2)
plot (fd , freq45, 'b')
hold on
e9 = errorbar(fd(1:500:end), freq45(1:500:end), u_t(1:500:end), 'Vertical', 'k', 'LineStyle', 'none');
e10 = errorbar(fd(1:500:end), freq45(1:500:end), u_freq1(1:500:end), 'Horizontal', 'k', 'LineStyle', 'none');
xlim ([0,3000])
ylim ([0,100])
xlabel 'Frequency (Hz)'
ylabel 'Amplitude (V)'
title 'Amplitude vs Frequency'
sgtitle 'Vortex Shedding Experiment at 45 Hz'
hold off

d_vs = .6096;      % cylinder diameter (m)
rho_vs30 = 1.2014; % air density (kg/m^3)
rho_vs45 = 1.19222;
v_vs30 = -1.6303 + 1.0747*30; % air velocity (m/s)
v_vs45 = -1.6303 + 1.0747*45;
mu_vs = .00001789; % viscosity (kg/m*s)

% calculate reynolds number
Re30 = (rho_vs30 .* v_vs30.* d_vs)./ mu_vs;
Re45 = (rho_vs45 .* v_vs45.* d_vs)./ mu_vs;

% uncertainty in reynolds number

% fan speed is 30 Hz
syms Re v d u_v u_tp u_Re

Re = (rho .* v .* d ) ./ mu;
u_Re30 = sqrt((diff(Re,v).* u_v).^2 + (diff(Re,d).* u_tp).^2);
v = v_vs30;
u_v_fit = diff(v_fit, f).* u_freq;
u_v_30 = double(subs(u_v_fit, f, 30));
u_v = u_v_30;
u_tp = u_tape;
rho = rho_vs30;
mu = mu_vs;
d = d_vs;
u_Re30 = eval(u_Re30);

% fan speed is 45 Hz
syms Re v d u_v u_tp

```

```

Re      = (rho .* v .* d) ./ mu;
u_Re45 = sqrt((diff(Re,v).* u_v).^2 + (diff(Re,d).* u_tp).^2);
v      = v_vs45;
u_v_fit = diff(v_fit, f).* u_freq;
u_v_45  = double(subs(u_v_fit, f, 45));
u_v_25  = double(subs(u_v_fit, f, 25));
u_v     = u_v_45;
u_tp    = u_tape;
rho     = rho_vs45;
mu     = mu_vs;
d      = d_vs;
u_Re45 = eval(u_Re45);

% calculate strouhal number
S30   = .198 .* (1 - (19.7 ./ Re30));
S45   = .198 .* (1 - (19.7 ./ Re45));

% uncertainty in strouhal number
syms Re

S     = .198 .* (1 - (19.7 ./ Re));
u_S  = diff(S,Re).* u_Re;

% fan speed is 30 Hz
u_S30 = subs(u_S, Re, Re30);
u_S30 = double(subs(u_S30, u_Re30));

% Fan speed is 45 Hz
u_S45 = subs(u_S, Re, Re45);
u_S45 = double(subs(u_S45, u_Re45));

% calculate empirical shedding frequency

esf30 = (1./5) .* (v_vs30 ./d_vs); % Hz
esf45 = (1./5) .* (v_vs45 ./d_vs); % Hz

% uncertainty in empirical shedding frequency

% fan speed is 30 Hz
syms v d u_v u_d

esf   = (1/5) .* (v./d);
u_esf = sqrt((diff(esf,v).*u_v).^2 + (diff(esf,d).*u_d).^2);

v      = v_vs30;
d      = d_vs;
u_v    = u_v_30;
u_d    = u_tape;
u_esf30 = eval(u_esf);

% fan speed is 45 Hz
syms v d u_v u_d

esf   = (1/5) .* (v./d);
u_esf = sqrt((diff(esf,v).*u_v).^2 + (diff(esf,d).*u_d).^2);

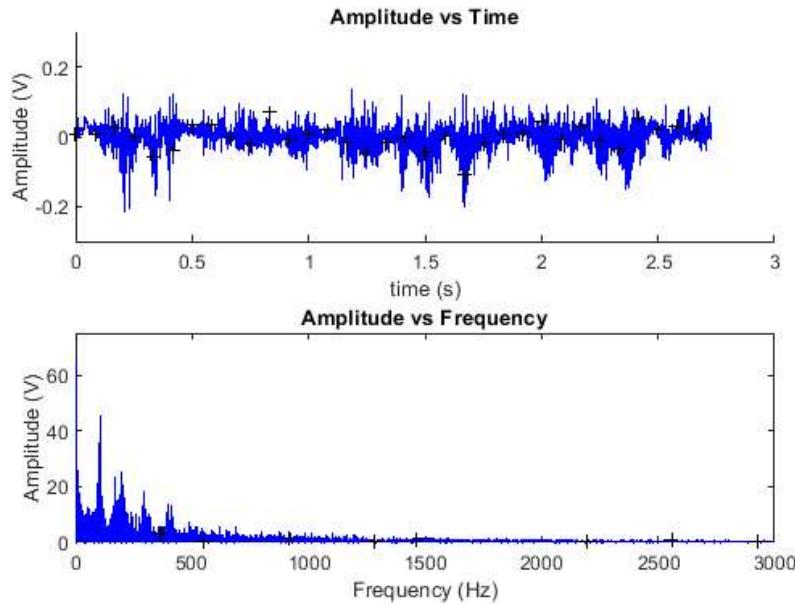
v      = v_vs45;
d      = d_vs;
u_v    = u_v_45;
u_d    = u_tape;
u_esf45 = eval(u_esf);

% calculate shedding frequency from accelerometer data
sh_f30 = (S30.*v_vs30)./d_vs;
sh_f45 = (S45.*v_vs45)./d_vs;

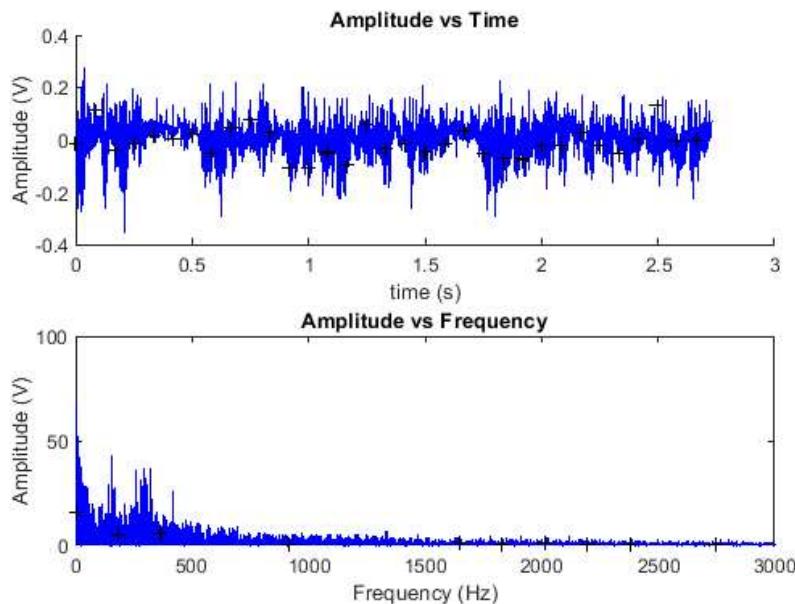
```

Warning: Imaginary parts of complex X and/or Y arguments ignored.
 Warning: Using only the real component of complex data.
 Warning: Using only the real component of complex data.
 Warning: Imaginary parts of complex X and/or Y arguments ignored.
 Warning: Using only the real component of complex data.
 Warning: Using only the real component of complex data.

Vortex Shedding Experiment at 30 Hz



Vortex Shedding Experiment at 45 Hz



Rake Experiment

```

Pavg25 = ((sum(norake25))./21)';
Pavg45 = ((sum(norake45))./21)';
Pavg25c = ((sum(rake25))./21)';
Pavg45c = ((sum(rake45))./21)';

% covert pressure from psi to Pa
Pavg25 = abs(Pavg25) .* 6894.757;
Pavg45 = abs(Pavg45) .* 6894.757;
Pavg25c = abs(Pavg25c) .* 6894.757;
Pavg45c = abs(Pavg45c) .* 6894.757;

% densities in kg/m^3
rho_ncyl25 = 1.20172889;
rho_ncyl45 = 1.192455556;
rho_cyl25 = 1.205771111;
rho_cyl45 = 1.19198;

v_no25 = sqrt((2 .* Pavg25)./rho_ncyl25);

```

```

v_no45 = sqrt((2 .* Pavg45)./rho_ncyl45);
v_cyl125 = sqrt((2 .* Pavg25c)./rho_cyl125);
v_cyl45 = sqrt((2 .* Pavg45c)./rho_cyl45);

figure (4)
hold on
plot (position1, v_no25,'.-')
plot (position1, v_cyl125,'.-')
legend ('no cylinder', 'cylinder')
title 'Air Velocity vs Pressure Tap Location Without Cylinder '
xlabel 'Pressure Tap Location (m)'
ylabel 'Air Velocity (m/s)'
hold off

figure (5)
hold on
plot (position1, v_no45,'.-')
plot (position1, v_cyl45,'.-')
legend ('no cylinder', 'cylinder')
title 'Air Velocity vs Pressure Tap Location With Cylinder '
xlabel 'Pressure Tap Location (m)'
ylabel 'Air Velocity (m/s)'

% calculate drag

% cylinder dimensions
Lcyl = 0.6096; % length of cylinder (m)
Dcyl = 0.050673; % diameter of cylinder (m)

% when fan speed is 25 Hz
Drag25 = 1.20577 .* 22.1584 .* trapz(position1, v_cyl125)-1.20577.*trapz(position1,v_cyl125).^2; % drag force (N)
Dcoef_25 = Drag25./(.05*1.20577.*22.1584.^2.* (Dcyl/2).*Lcyl);

% when fan speed is 45 Hz
Drag45 = 1.19198 .* 43.1099 .* trapz(position1, v_cyl45)-1.19198.*trapz(position1,v_cyl45).^2; % drag force (N)
Dcoef_45 = Drag45./(.05*1.19198.*43.1099.^2.* (Dcyl/2).*Lcyl);

% calculate uncertainties in drag and coefficient of drag
u_Drag25 = Drag25 .* .02;
u_Drag45 = Drag45 .* .02;

syms u_Dr Dr rho v u_v D u_D S u_S

Dcoef = Dr./(.05*rho.*v.^2.*D.*S);
u_Dcoef = sqrt((diff(Dcoef, D).*u_D).^2 + (diff(Dcoef,v).*u_v).^2 + (diff(Dcoef,S).*u_S).^2 + (diff(Dcoef,D).*u_D).^2);
u_Dr = u_Drag25;
Dr = Drag25; % N
rho = 1.20577; % kg/m^3
v = 22.1584; % m/s
u_v = u_v_25;
D = 0.012; % m
S = 0.251; % m
u_D = u_calip;
u_S = u_tape;

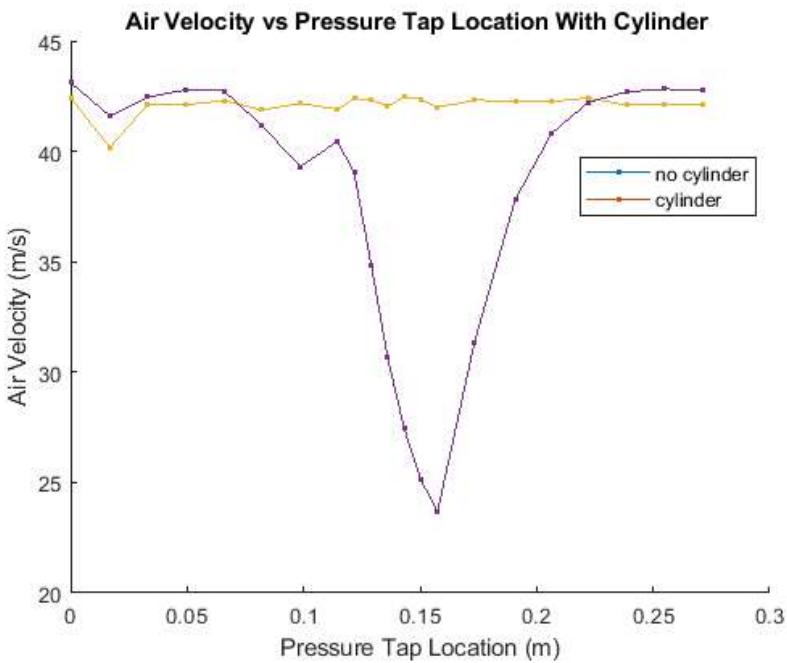
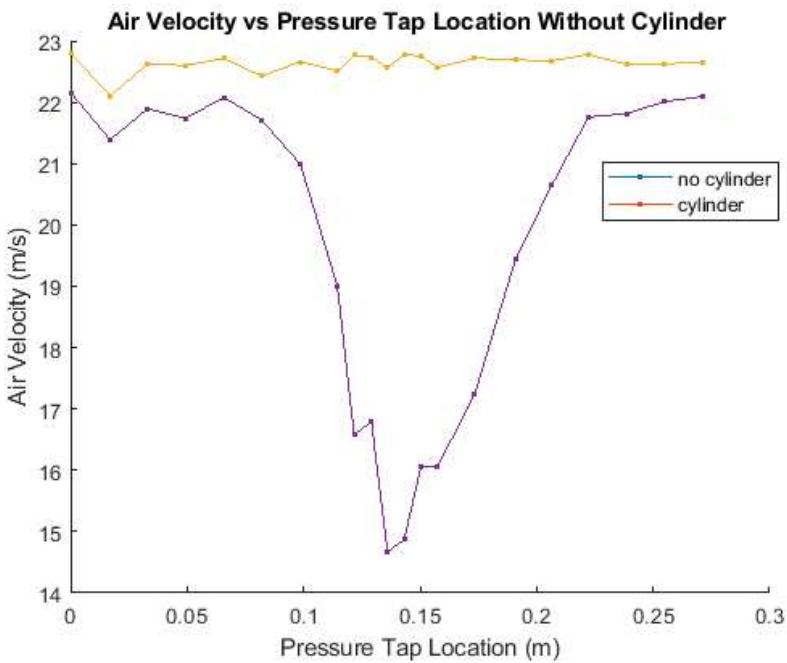
u_Dcoef25 = eval(u_Dcoef);

syms u_Dr Dr rho v u_v D u_D S u_S

Dcoef = Dr./(.05*rho.*v.^2.*D.*S);
u_Dcoef = sqrt((diff(Dcoef, D).*u_D).^2 + (diff(Dcoef,v).*u_v).^2 + (diff(Dcoef,S).*u_S).^2 + (diff(Dcoef,D).*u_D).^2);
u_Dr = u_Drag45;
Dr = Drag45; % N
rho = 1.19198; % kg/m^3
v = 43.1099; % m/s
u_v = u_v_45;
D = 0.012; % m
S = 0.251; % m
u_D = u_calip;
u_S = u_tape;

u_Dcoef45 = eval(u_Dcoef);

```



Boundary Layer Experiment

```

P_ds_avg = ((sum(bl_down))./21)';
P_us_avg = ((sum(bl_up))./21);

% Convert pressure from psi to Pa
P_ds = P_ds_avg .* 6894.757;
P_us = P_us_avg .* 6894.757;

rhod = 1.202205;
rho = 1.206722;

v_u = sqrt((2 .* P_ds)./rhod);
v_d = sqrt((2 .* P_us)./rho);

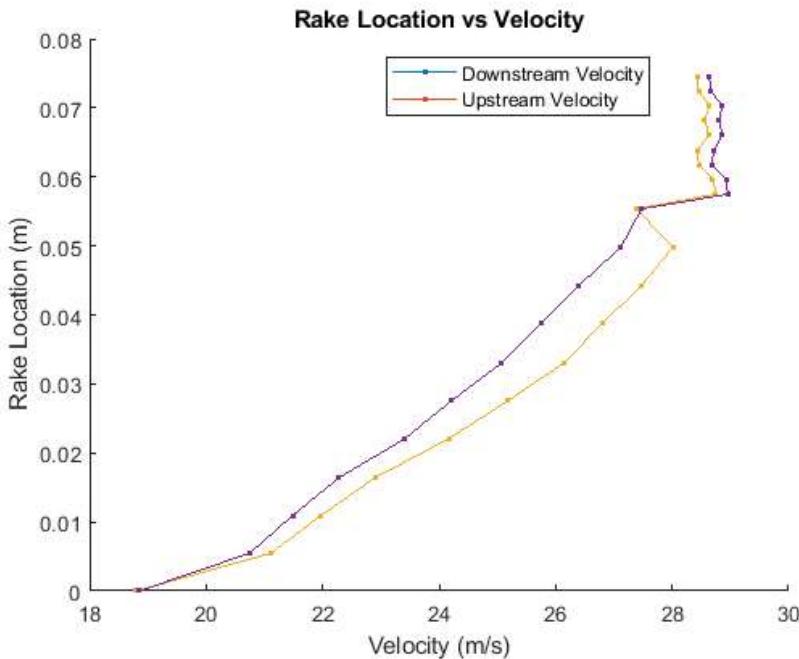
figure (6)
hold on
plot (v_d, position2, '.-')
plot (v_u, position2, '.-')
legend ('Downstream Velocity', 'Upstream Velocity')

```

```

title 'Rake Location vs Velocity'
xlabel 'Velocity (m/s)'
ylabel 'Rake Location (m)'

```



Airfoil

```

% solve equations symbolically
syms dragc30 dragc45 liftc30 liftc45 Mc30 Mc45 rhoc30 rhoc45 dragd30...
    dragd45 liftd30 liftd45 Md30 Md45 rhod30 rhod45 D S C vel30 vel45...
    u_vel30 u_vel45 u_tape u_calip mu30 mu45 fr u_fr

vel = -1.6303 + 1.0747.*fr;

% calculate reynolds number

% clean
Re_c30 = (rhoc30.*vel30.*S./mu30);
Re_c45 = (rhoc45.*vel45.*S./mu45);

% dirty
Re_d30 = (rhod30.*vel30.*S./mu30);
Re_d45 = (rhod45.*vel45.*S./mu45);

% calculate drag coefficients

% clean
Dcoef_c30 = dragc30/.(0.05*rhoc30.*vel30.^2.*D.*S);
Dcoef_c45 = dragc45/.(0.05*rhoc45.*vel45.^2.*D.*S);

% dirty
Dcoef_d30 = dragd30/.(0.05*rhod30.*vel30.^2.*D.*S);
Dcoef_d45 = dragd45/.(0.05*rhod45.*vel45.^2.*D.*S);

% calculate lift coefficients

% clean
Lcoef_c30 = liftc30/.(0.5.*rhoc30.*vel30.^2.*D.*S);
Lcoef_c45 = liftc45/.(0.5.*rhoc45.*vel45.^2.*D.*S);

% dirty
Lcoef_d30 = liftd30/.(0.5.*rhod30.*vel30.^2.*D.*S);
Lcoef_d45 = liftd45/.(0.5.*rhod45.*vel45.^2.*D.*S);

% calculate moment coefficients

% clean
Mcoef_c30 = Mc30/.(0.5.*rhoc30.*vel30.^2.*C.*D.*S);

```

```

Mcoef_c45 = Mc45./(.5.*rhoc45.*vel45.^2.*C.*D.*S);

% dirty
Mcoef_d30 = Md30./(.5.*rhod30.*vel30.^2.*C.*D.*S);
Mcoef_d45 = Md45./(.5.*rhod45.*vel45.^2.*C.*D.*S);

% calculate uncertainty in reynolds number
u_Re_c30 = sqrt((diff(Re_c30,vel30).*u_vel30).^2 + (diff(Re_c30, S).*u_tape).^2);
u_Re_c45 = sqrt((diff(Re_c45,vel45).*u_vel45).^2 + (diff(Re_c45, S).*u_tape).^2);
u_Re_d30 = sqrt((diff(Re_d30,vel30).*u_vel30).^2 + (diff(Re_d30, S).*u_tape).^2);
u_Re_d45 = sqrt((diff(Re_d45,vel45).*u_vel45).^2 + (diff(Re_d45, S).*u_tape).^2);
u_vel    = diff(vel, fr).* u_fr;

% assign values
rhoc30 = 1.212; % density of air (kg/m^3)
rhoc45 = 1.200; % density of air (kg/m^3)
rhod30 = 1.208; % density of air (kg/m^3)
rhod45 = 1.202; % density of air (kg/m^3)

vel30 = -1.6303 + 1.0747*30; % air velocity (m/s)
vel45 = -1.6303 + 1.0747*45; % air velocity (m/s)

mu30 = 1.80639.*10.^-5; % dynamic viscosity (kg/m*s)
mu45 = 1.81553.*10.^-5; % dynamic viscosity (kg/m*s)

C  = 0.089; % m
S  = 0.251; % m
D  = 0.012;

u_tape  = .5 * 1/32 * .0254;
u_vel30 = subs(u_vel, fr,30);
u_vel30 = subs(u_vel30, u_fr, .0005);
u_vel45 = subs(u_vel, fr,45);
u_vel45 = subs(u_vel45,u_fr,.0005);

dragc30 = drag_c30;
dragc45 = drag_c45;
liftc30 = lift_c30;
liftc45 = lift_c45;
Mc30    = M_c30;
Mc45    = M_c45;
dragd30 = drag_d30;
dragd45 = drag_d45;
liftd30 = lift_d30;
liftd45 = lift_d45;
Md30    = M_d30;
Md45    = M_d45;

Re_c30  = double(eval(Re_c30));
Re_c45  = double(eval(Re_c45));
Re_d30  = double(eval(Re_d30));
Re_d45  = double(eval(Re_d45));

u_Re_c30 = double(eval(u_Re_c30));
u_Re_c45 = double(eval(u_Re_c45));
u_Re_d30 = double(eval(u_Re_d30));
u_Re_d45 = double(eval(u_Re_d45));

Dcoef_c30 = eval(Dcoef_c30);
Dcoef_c45 = eval(Dcoef_c45);
Dcoef_d30 = eval(Dcoef_d30);
Dcoef_d45 = eval(Dcoef_d45);

Lcoef_c30 = eval(Lcoef_c30);
Lcoef_c45 = eval(Lcoef_c45);
Lcoef_d30 = eval(Lcoef_d30);
Lcoef_d45 = eval(Lcoef_d45);

Mcoef_c30 = eval(Mcoef_c30);
Mcoef_c45 = eval(Mcoef_c45);
Mcoef_d30 = eval(Mcoef_d30);
Mcoef_d45 = eval(Mcoef_d45);

figure (7)
hold on

```

```

plot (anglec30, liftc30,'.-')
plot (angled30, liftd30,'.-')
title 'Lift vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Lift (N)'
legend ('Clean', 'Dirty')

figure (8)
hold on
plot (angled45, liftc45,'.-')
plot (angled45, liftd45,'.-')
title 'Lift vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Lift (N)'
legend ('Clean', 'Dirty')

figure (9)
hold on
plot (anglec30, dragc30,'.-')
plot (angled30, dragd30,'.-')
title 'Drag vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Drag (N)'
legend ('Clean', 'Dirty')

figure (10)
hold on
plot (anglec45, dragc45,'.-')
plot (angled45, dragd45,'.-')
title 'Drag vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Drag (N)'
legend ('Clean', 'Dirty')

figure (11)
hold on
plot (anglec30, Mc30,'.-')
plot (angled30, Md30,'.-')
title 'Pitching Moment vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Pitching Moment (N*m)'
legend ('Clean', 'Dirty')

figure (12)
hold on
plot (anglec45, Mc45,'.-')
plot (angled45, Md45,'.-')
title 'Pitching Moment vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Pitching Moment (N*m)'
legend ('Clean', 'Dirty')

figure (13)
hold on
plot (anglec30, Lcoef_c30,'.-')
plot (angled30, Lcoef_d30,'.-')
title 'Coefficient of Lift vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Lift'
legend ('Clean', 'Dirty')

figure (14)
hold on
plot (anglec45, Lcoef_c45,'.-')
plot (angled45, Lcoef_d45,'.-')
title 'Coefficient of Lift vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Lift'
legend ('Clean', 'Dirty')

figure (15)
hold on
plot (anglec30, Dcoef_c30,'.-')

```

```

plot (angled30, Dcoef_d30,'.-')
title 'Coefficient of Drag vs Angle of Attack vs Coefficient 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Drag'
legend ('Clean', 'Dirty')

figure (16)
hold on
plot (anglec45, Dcoef_c45,'.-')
plot (angled45, Dcoef_d45,'.-')
title 'Coefficient of Drag vs Angle of Attack vs Coefficient 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Drag'
legend ('Clean', 'Dirty')

figure (17)
hold on
plot (anglec30, Mcoef_c30,'.-')
plot (angled30, Mcoef_d30,'.-')
title 'Coefficient of Pitching Moment vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Pitching Moment'
legend ('Clean', 'Dirty')

figure (18)
hold on
plot (anglec45, Mcoef_c45,'.-')
plot (angled45, Mcoef_d45,'.-')
title 'Coefficient of Pitching Moment vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Coefficient of Pitching Moment'
legend ('Clean', 'Dirty')

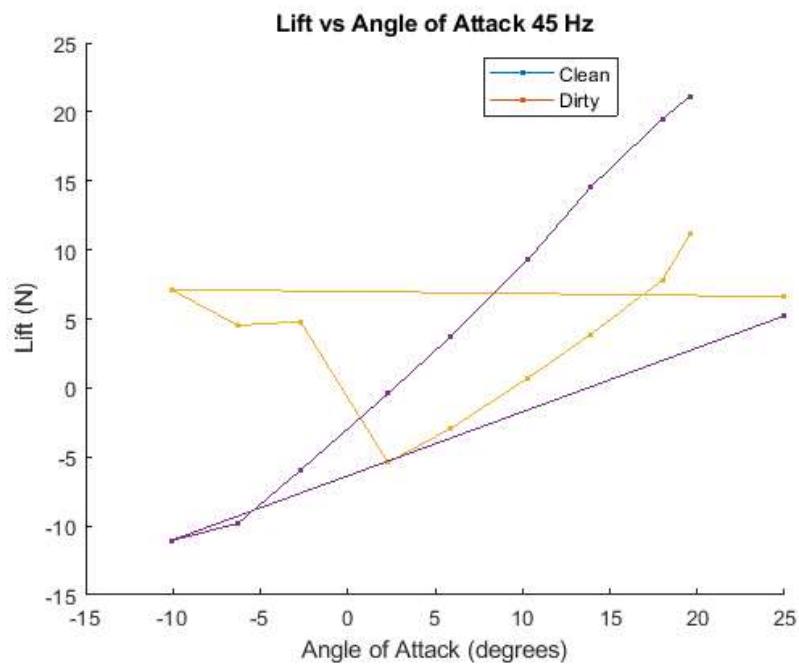
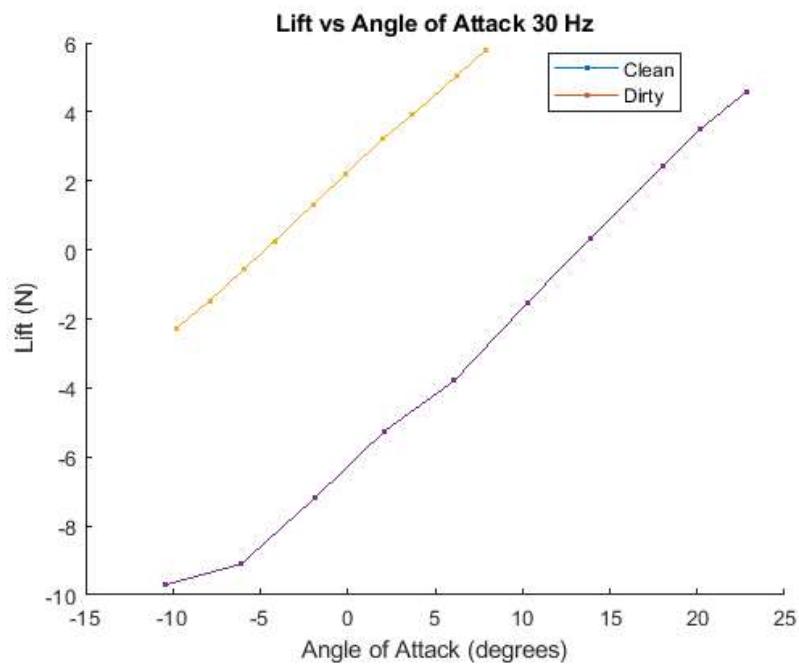
figure (19)
hold on

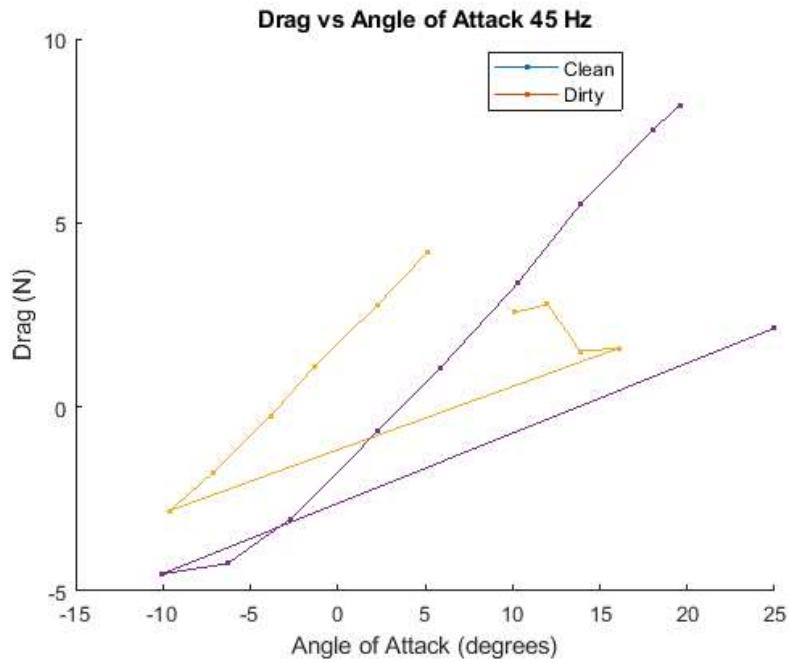
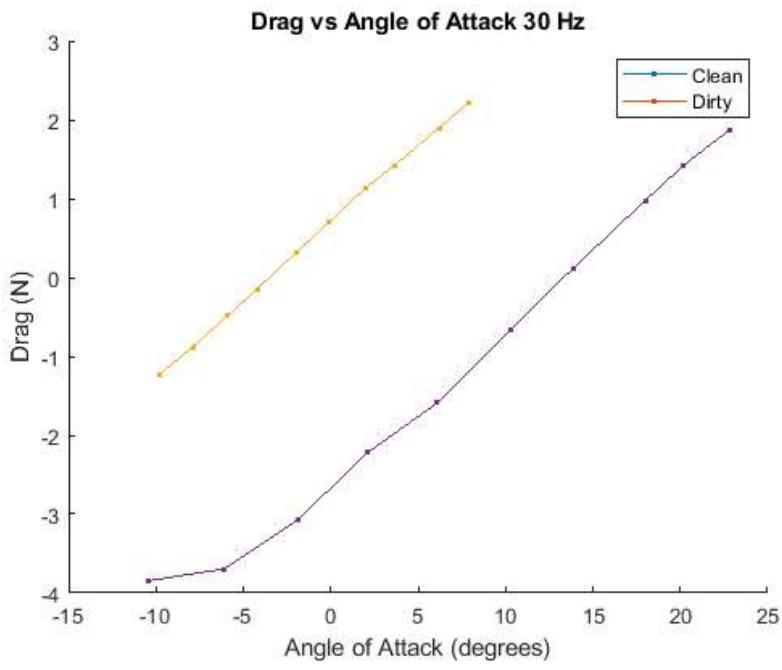
% create new variable for Lift/Drag
LD30c = liftc30./dragc30;
LD30d = liftd30./dragd30;
LD45c = liftc45./dragc30;
LD45d = liftd45./dragd45;

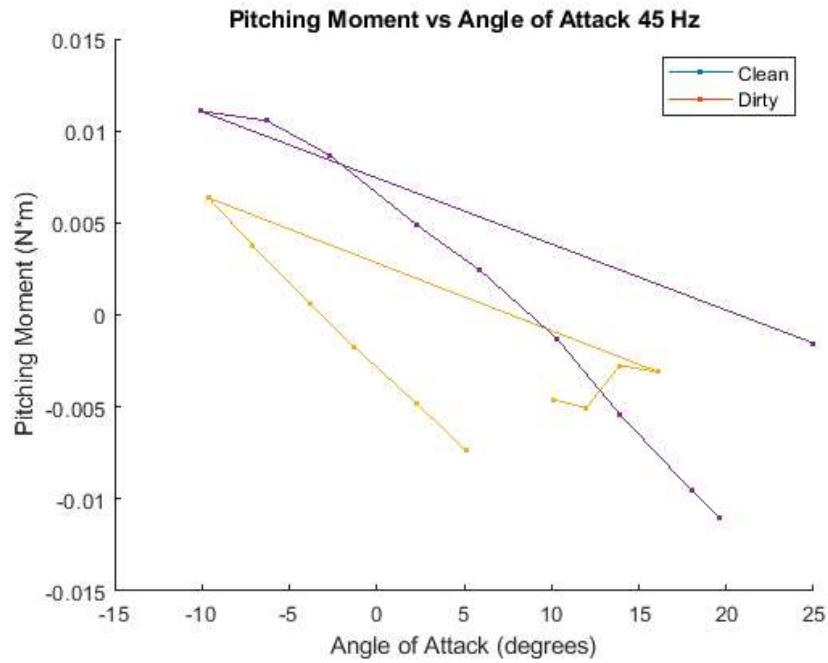
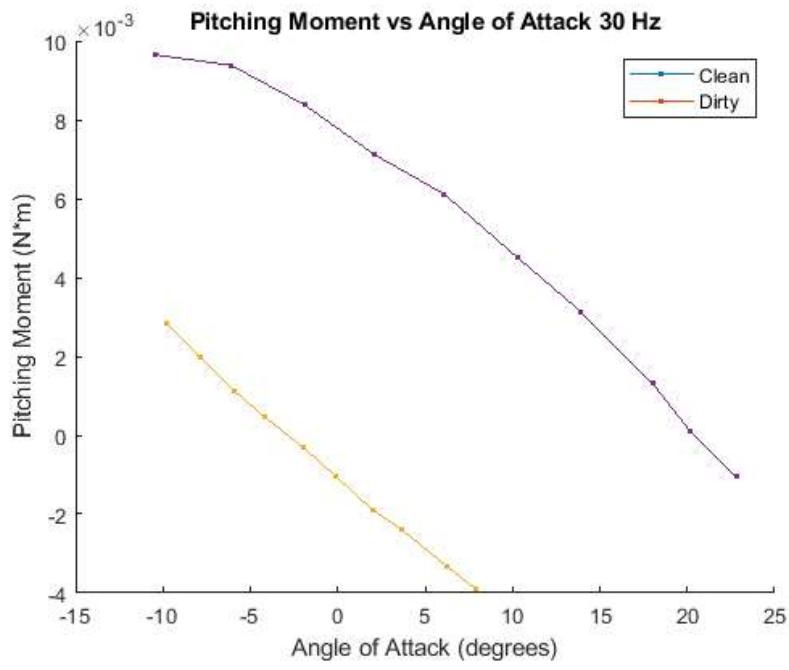
plot (anglec30, LD30c,'.-')
plot (angled30, LD30d,'.-')
title 'Lift/Drag vs Angle of Attack 30 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Lift/Drag'
legend ('Clean', 'Dirty')

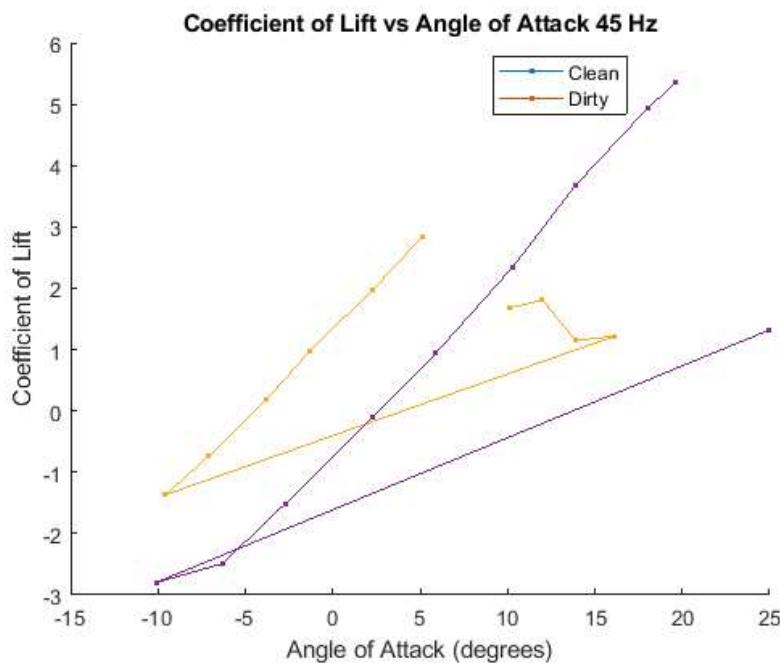
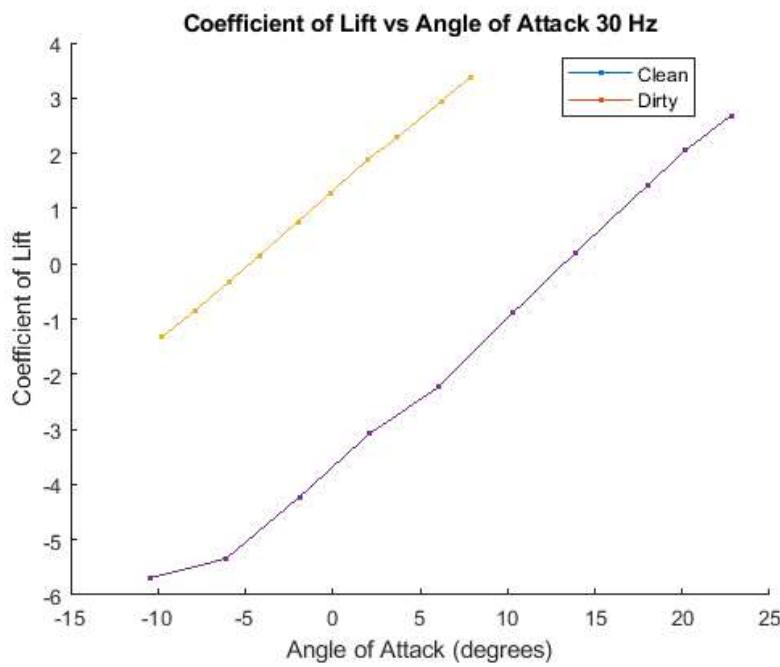
figure (20)
hold on
plot (anglec45, LD45c, '-.')
plot (angled45, LD45d, '-.')
title 'Lift/Drag vs Angle of Attack 45 Hz'
xlabel 'Angle of Attack (degrees)'
ylabel 'Lift/Drag'
legend ('Clean', 'Dirty')

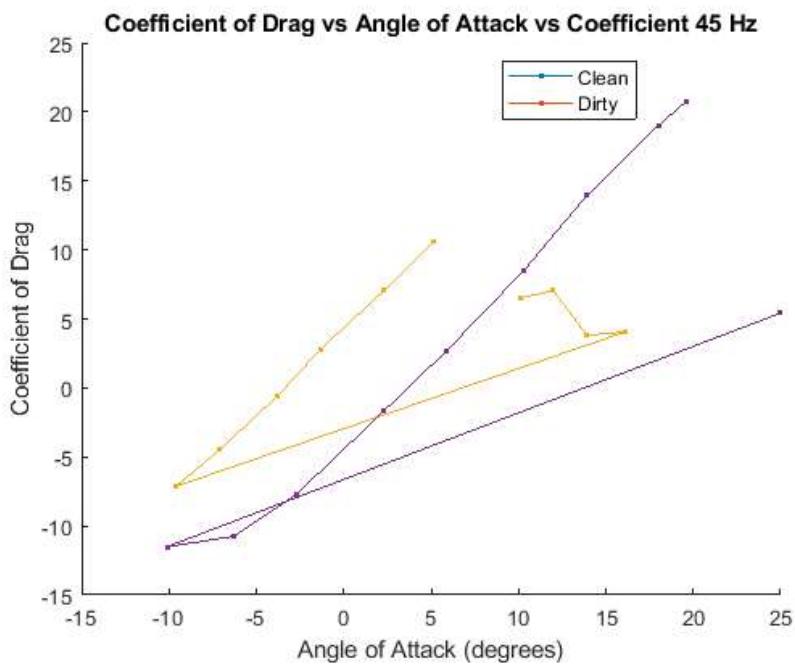
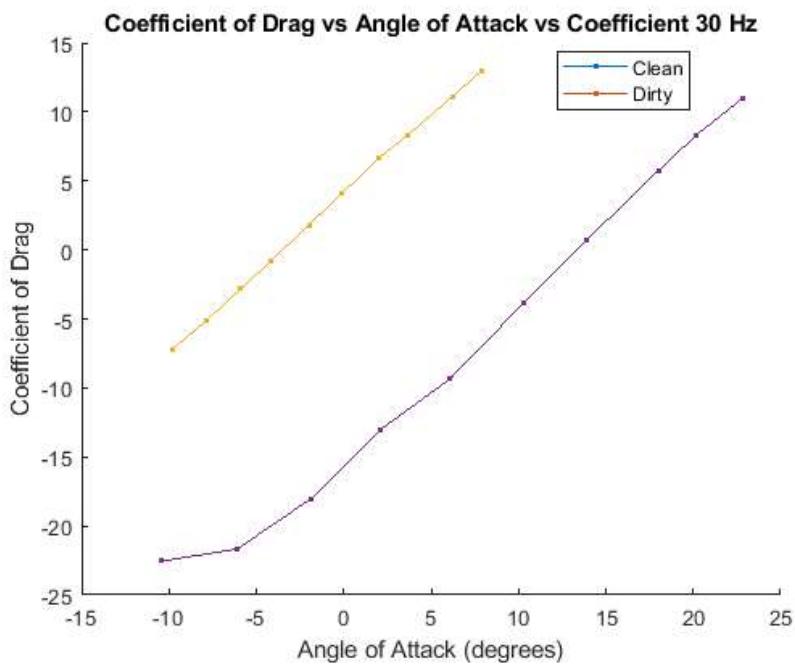
```

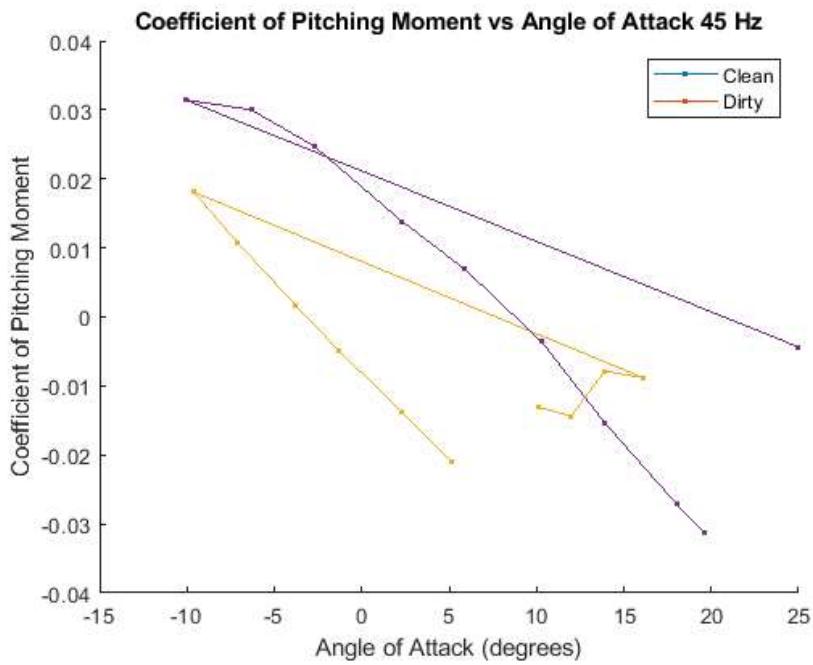
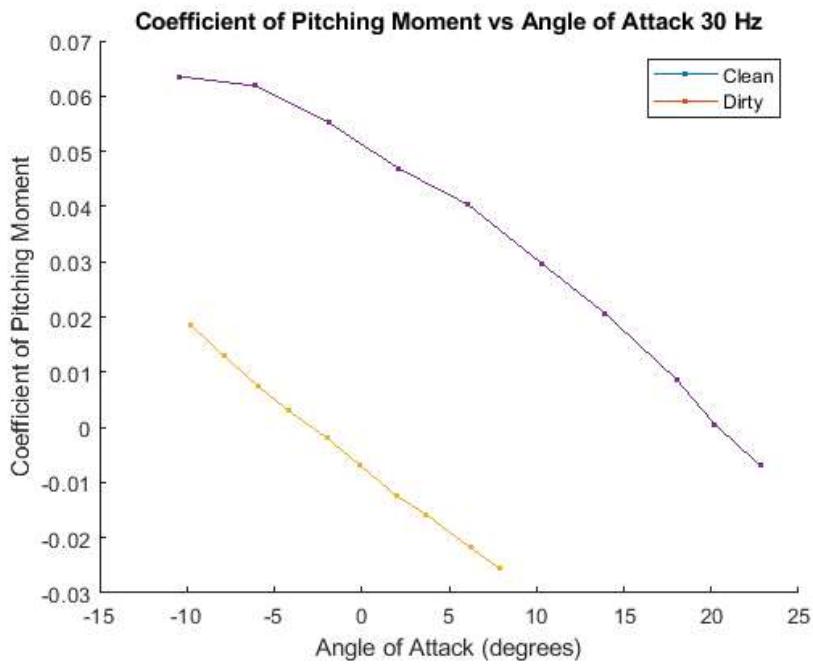


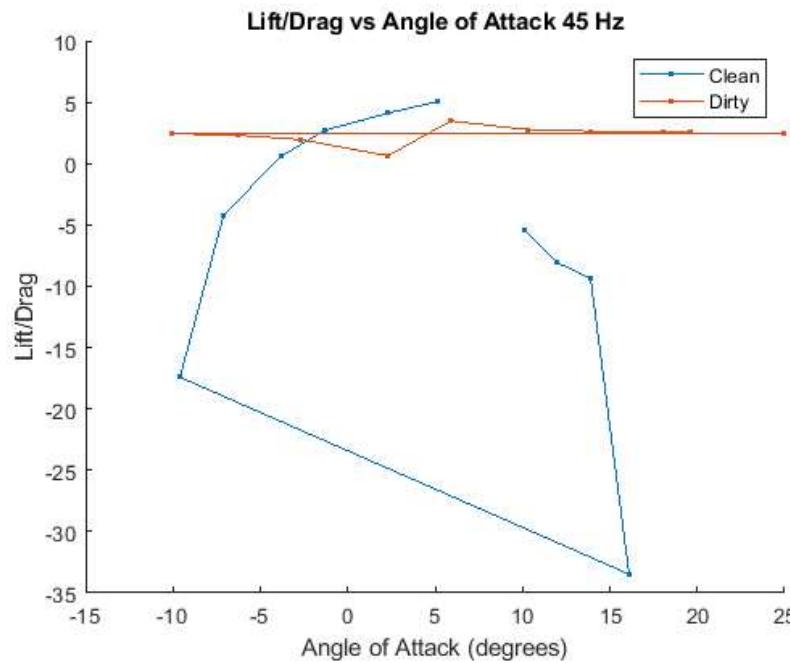
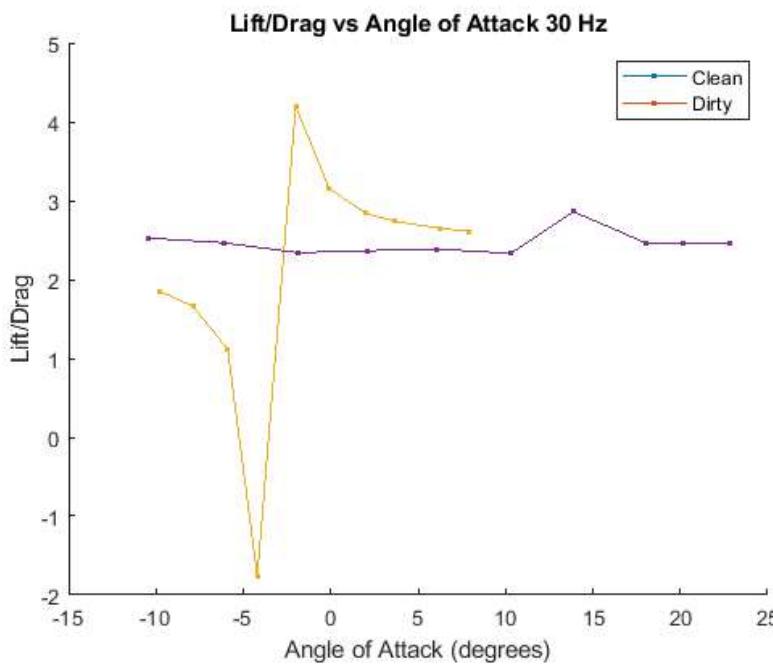












2D and 3D Cylinders

fan speed is 25 Hz

```
% solve equations symbolically

syms rho1 rho2 rho3 rho4 rho5 v_25 mu_25 d_cyl1 d_cyl11 d_cyl12 d_cyl13 ...
d_cyl14 d_cyl15 l0 l1 l2 l3 l4 l5 dr0 dr1 dr2 dr3 dr4 dr5 u_c...
u_v u_tp u_F

% reynolds number for each cylinder
Re1 = (rho1 .* v_25 .* d_cyl1) ./ mu_25;
Re2 = (rho2 .* v_25 .* d_cyl2) ./ mu_25;
Re3 = (rho3 .* v_25 .* d_cyl3) ./ mu_25;
Re4 = (rho4 .* v_25 .* d_cyl4) ./ mu_25;
Re5 = (rho4 .* v_25 .* d_cyl5) ./ mu_25;

% coefficient of drag for each cylinder
c_Dr1 = dr1 ./ (.5 .* rho1 .* v_25.^2 .* d_cyl1 .* l1);
c_Dr2 = dr2 ./ (.5 .* rho2 .* v_25.^2 .* d_cyl2 .* l2);
```

```

c_Dr3 = dr3 ./ (.5 .* rho3 .* v_25.^2 .* d_cyl3 .* 13);
c_Dr4 = dr4 ./ (.5 .* rho4 .* v_25.^2 .* d_cyl4 .* 14);
c_Dr5 = dr5 ./ (.5 .* rho5 .* v_25.^2 .* d_cyl5 .* 15);

% calculate uncertainties

% uncertainties of reynolds number
u_Re1 = sqrt((diff(Re1, v_25).*u_v).^2 + (diff(Re1,d_cyl1).*u_c).^2);
u_Re2 = sqrt((diff(Re2, v_25).*u_v).^2 + (diff(Re2,d_cyl2).*u_c).^2);
u_Re3 = sqrt((diff(Re3, v_25).*u_v).^2 + (diff(Re3,d_cyl3).*u_c).^2);
u_Re4 = sqrt((diff(Re4, v_25).*u_v).^2 + (diff(Re4,d_cyl4).*u_c).^2);
u_Re5 = sqrt((diff(Re5, v_25).*u_v).^2 + (diff(Re5,d_cyl5).*u_c).^2);

% uncertainties of drag
u_dr0 = dr0 .* u_F;
u_dr1 = dr1 .* u_F;
u_dr2 = dr2 .* u_F;
u_dr3 = dr3 .* u_F;
u_dr4 = dr4 .* u_F;
u_dr5 = dr5 .* u_F;

% uncertainties of drag coefficient
u_cDr1 = sqrt((diff(c_Dr1, v_25).*u_v.^2) + (diff(c_Dr1, 11).*u_tp).^2 + (diff(c_Dr1, d_cyl1).*u_c).^2 + (diff(c_Dr1, dr1).*u_dr1).^2);
u_cDr2 = sqrt((diff(c_Dr2, v_25).*u_v.^2) + (diff(c_Dr2, 12).*u_tp).^2 + (diff(c_Dr2, d_cyl2).*u_c).^2 + (diff(c_Dr2, dr2).*u_dr2).^2);
u_cDr3 = sqrt((diff(c_Dr3, v_25).*u_v.^2) + (diff(c_Dr3, 13).*u_tp).^2 + (diff(c_Dr3, d_cyl3).*u_c).^2 + (diff(c_Dr3, dr3).*u_dr3).^2);
u_cDr4 = sqrt((diff(c_Dr4, v_25).*u_v.^2) + (diff(c_Dr4, 14).*u_tp).^2 + (diff(c_Dr4, d_cyl4).*u_c).^2 + (diff(c_Dr4, dr4).*u_dr4).^2);
u_cDr5 = sqrt((diff(c_Dr5, v_25).*u_v.^2) + (diff(c_Dr5, 15).*u_tp).^2 + (diff(c_Dr5, d_cyl5).*u_c).^2 + (diff(c_Dr5, dr5).*u_dr5).^2);

% assign values

% air velocity at fan speed 30 Hz
v_25 = -1.6303 + 1.0747*25; % m/s

% air densities
rho0 = 1.210288889; % kg/m^3
rho1 = 1.212191111; % kg/m^3
rho2 = 1.212428889; % kg/m^3
rho3 = 1.212904444; % kg/m^3
rho4 = 1.213142222; % kg/m^3
rho5 = 1.215044444; % kg/m^3

% dynamic viscosity
mu_25 = 1.80639.*10.^-5; % kg/m*s

% uncertainty values
u_F = .02; % N
u_tp = u_tape; % m
u_c = 1.2700e-05; % m
u_v = u_v_25; % m/s

% cylinder lengths
l0 = l_cyl(1); % m
l1 = l_cyl(2); % m
l2 = l_cyl(3); % m
l3 = l_cyl(4); % m
l4 = l_cyl(5); % m
l5 = l_cyl(6); % m

% cylinder diameter
d_cyl0 = D_cyl(1); % m
d_cyl1 = D_cyl(2); % m
d_cyl2 = D_cyl(3); % m
d_cyl3 = D_cyl(4); % m
d_cyl4 = D_cyl(5); % m
d_cyl5 = D_cyl(6); % m

% drag force
dr0 = cyl0(:,3).* 4.448; % N
dr1 = cyl1(:,3).* 4.448; % N
dr2 = cyl2(:,3).* 4.448; % N
dr3 = cyl3(:,3).* 4.448; % N
dr4 = cyl4(:,3).* 4.448; % N
dr5 = cyl5(:,3).* 4.448; % N

% Evaluate symbolic expressions

```

```
% Reynolds number
Re1 = double(eval(Re1));
Re2 = double(eval(Re2));
Re3 = double(eval(Re3));
Re4 = double(eval(Re4));
Re5 = double(eval(Re5));

% coefficient of drag
c_Dr1 = double(eval(c_Dr1));
c_Dr2 = double(eval(c_Dr2));
c_Dr3 = double(eval(c_Dr3));
c_Dr4 = double(eval(c_Dr4));
c_Dr5 = double(eval(c_Dr5));

% uncertainty in coefficient of drag
u_cDr1 = double(eval(u_cDr1));
u_cDr2 = double(eval(u_cDr2));
u_cDr3 = double(eval(u_cDr3));
u_cDr4 = double(eval(u_cDr4));
u_cDr5 = double(eval(u_cDr5));
```

fan speed is 45 Hz

```
% solve equations symbolically

syms rho1 rho2 rho3 rho4 rho5 v_45 mu_45 d_cyl10 d_cyl11 d_cyl12 d_cyl13 ...
d_cyl14 d_cyl15 10 11 12 13 14 15 dr0 dr1 dr2 dr3 dr4 dr5 u_c...
u_v u_tp u_F

% reynolds number for each cylinder
Re1 = (rho1 .* v_45 .* d_cyl11) ./ mu_45;
Re2 = (rho2 .* v_45 .* d_cyl12) ./ mu_45;
Re3 = (rho3 .* v_45 .* d_cyl13) ./ mu_45;
Re4 = (rho4 .* v_45 .* d_cyl14) ./ mu_45;
Re5 = (rho4 .* v_45 .* d_cyl15) ./ mu_45;

% coefficient of drag for each cylinder
c_Dr1 = dr1 ./ (.5 .* rho1 .* v_45.^2 .* d_cyl11 .* 11);
c_Dr2 = dr2 ./ (.5 .* rho2 .* v_45.^2 .* d_cyl12 .* 12);
c_Dr3 = dr3 ./ (.5 .* rho3 .* v_45.^2 .* d_cyl13 .* 13);
c_Dr4 = dr4 ./ (.5 .* rho4 .* v_45.^2 .* d_cyl14 .* 14);
c_Dr5 = dr5 ./ (.5 .* rho5 .* v_45.^2 .* d_cyl15 .* 15);

% calculate uncertainties

% uncertainties of reynolds number
u_Re1 = sqrt((diff(Re1, v_45).*u_v).^2 + (diff(Re1,d_cyl11).*u_c).^2);
u_Re2 = sqrt((diff(Re2, v_45).*u_v).^2 + (diff(Re2,d_cyl12).*u_c).^2);
u_Re3 = sqrt((diff(Re3, v_45).*u_v).^2 + (diff(Re3,d_cyl13).*u_c).^2);
u_Re4 = sqrt((diff(Re4, v_45).*u_v).^2 + (diff(Re4,d_cyl14).*u_c).^2);
u_Re5 = sqrt((diff(Re5, v_45).*u_v).^2 + (diff(Re5,d_cyl15).*u_c).^2);

% uncertainties of drag
u_dr0 = dr0 .* u_F;
u_dr1 = dr1 .* u_F;
u_dr2 = dr2 .* u_F;
u_dr3 = dr3 .* u_F;
u_dr4 = dr4 .* u_F;
u_dr5 = dr5 .* u_F;

% uncertainties of drag coefficient
u_cDr1 = sqrt((diff(c_Dr1, v_45).*u_v.^2) + (diff(c_Dr1, 11).*u_tp).^2 + (diff(c_Dr1, d_cyl11).*u_c).^2 + (diff(c_Dr1, dr1).*u_dr1).^2);
u_cDr2 = sqrt((diff(c_Dr2, v_45).*u_v.^2) + (diff(c_Dr2, 12).*u_tp).^2 + (diff(c_Dr2, d_cyl12).*u_c).^2 + (diff(c_Dr2, dr2).*u_dr2).^2);
u_cDr3 = sqrt((diff(c_Dr3, v_45).*u_v.^2) + (diff(c_Dr3, 13).*u_tp).^2 + (diff(c_Dr3, d_cyl13).*u_c).^2 + (diff(c_Dr3, dr3).*u_dr3).^2);
u_cDr4 = sqrt((diff(c_Dr4, v_45).*u_v.^2) + (diff(c_Dr4, 14).*u_tp).^2 + (diff(c_Dr4, d_cyl14).*u_c).^2 + (diff(c_Dr4, dr4).*u_dr4).^2);
u_cDr5 = sqrt((diff(c_Dr5, v_45).*u_v.^2) + (diff(c_Dr5, 15).*u_tp).^2 + (diff(c_Dr5, d_cyl15).*u_c).^2 + (diff(c_Dr5, dr5).*u_dr5).^2);

% assign values

% air velocity at fan speed 30 Hz
v_45 = -1.6303 + 1.0747*45; % m/s
```

```

% air densities
rho0 = 1.210288889; % kg/m^3
rho1 = 1.212191111; % kg/m^3
rho2 = 1.212428889; % kg/m^3
rho3 = 1.212904444; % kg/m^3
rho4 = 1.213142222; % kg/m^3
rho5 = 1.215044444; % kg/m^3

% dynamic viscosity
mu_45 = 1.80639.*10.^-5; % kg/m*s

% uncertainty values
u_F = .02; % N
u_tp = u_tape; % m
u_c = 1.2700e-05; % m
u_v = u_v_45; % m/s

% cylinder lengths
l0 = l_cyl(1); % m
l1 = l_cyl(2); % m
l2 = l_cyl(3); % m
l3 = l_cyl(4); % m
l4 = l_cyl(5); % m
l5 = l_cyl(6); % m

% cylinder diameter
d_cyl0 = D_cyl(1); % m
d_cyl1 = D_cyl(2); % m
d_cyl2 = D_cyl(3); % m
d_cyl3 = D_cyl(4); % m
d_cyl4 = D_cyl(5); % m
d_cyl5 = D_cyl(6); % m

% drag force
dr1 = cyl1(:,3).* 4.448; % N
dr2 = cyl2(:,3).* 4.448; % N
dr3 = cyl3(:,3).* 4.448; % N
dr4 = cyl4(:,3).* 4.448; % N
dr5 = cyl5(:,3).* 4.448; % N

% Evaluate symbolic expressions

% Reynolds number
Re1 = double(eval(Re1));
Re2 = double(eval(Re2));
Re3 = double(eval(Re3));
Re4 = double(eval(Re4));
Re5 = double(eval(Re5));

% coefficient of drag
c_Dr1 = double(eval(c_Dr1));
c_Dr2 = double(eval(c_Dr2));
c_Dr3 = double(eval(c_Dr3));
c_Dr4 = double(eval(c_Dr4));
c_Dr5 = double(eval(c_Dr5));

% uncertainty in coefficient of drag
u_cDr1 = double(eval(u_cDr1));
u_cDr2 = double(eval(u_cDr2));
u_cDr3 = double(eval(u_cDr3));
u_cDr4 = double(eval(u_cDr4));
u_cDr5 = double(eval(u_cDr5));

```
