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Lab 1.1 & 1.2
MEE 416
Due 9/24

Beam Bending and Stress Transformation and Column Buckling Lab Write Up

Introduction

1.1 This lab was assigned in order to teach us how a beam will react to forces acting perpendicular to its span. Engineers experience these types of loading situations all the time so it is important to understand how the material will react and it is especially important to know if certain loading conditions will pose any risk to the design. Our ultimate goal of this lab was to understand the internal forces and the moment acting in the beam due to the external forces that we applied using the load cell. In order to do so, we had to understand how to determine the strain within the beam by using a strain gauge and the relationship between resistance measured within the wheatstone bridge and the strain felt by the material.

1.2 As mechanical engineering students, it was important for us to perform this lab in order to understand basic solid mechanics concepts related to buckling of a thin, slender column. Buckling is a very common way that structural designs can fail if the buckling load of the column is not taken into account. Although buckling does not mean that the material fails, it can have detrimental results on the design the columns are incorporated in. This lab aimed to help us to understand the process of testing structural materials, understanding how materials in specific shapes will react to loads and how to calculate the collapse load of a specific column. Specifically, this lab dealt with a discrete system under pinned-pinned conditions.

Experimental Facility and Instrumentation

2.1 The experimental set up for this lab can be seen in **figure 1.1**. The materials needed to complete this lab are as follows: LLOYD LS5 Advanced Materials Testing System, Bridge completion circuit, Strain gauges - with different gauge factors, LLOYD NEXYGEN-PLUS 3 Data Acquisition Software, rulers, calipers, electronic measuring equipment and a 25.16 x 3.23 x 197 mm stainless steel bar.

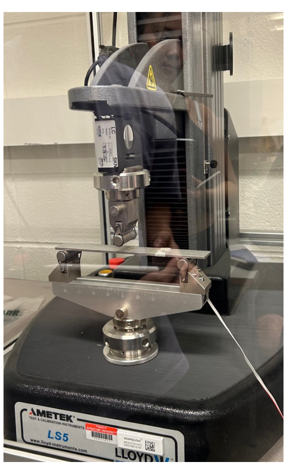
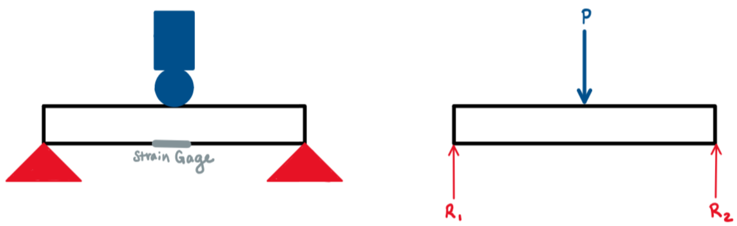


Fig. 1.1: *Bending experimental set up*

2.2 **Figure 2.1** represents how the equipment was set up for the buckling experiment. The instruments used in this experiment were the LLOYD LS5 Advanced Materials Testing System, LLOYD NEXYGEN-PLUS 3 Data Acquisition Software, a deflection extensometer, a ruler, a pair of calipers and electronic measuring equipment. The materials tested in this experiment were a steel bar with dimensions 300 mm x 25.37 mm x 1.62 mm. a n aluminum bar with dimensions 299 mm x 24.51 mm x 1.50 mm, and a carbon fiber bar with dimensions 299 mm x 25.32 mm x 1.64 mm.

Method

3.1 In order to assess the solid mechanics theories we set out to test, we needed to measure force applied to the beam, strain within the beam, and deflection of the beam by performing a three point bending test. The load cell was used to set the load value that would be applied to the beam. The load cell's initial position was also recorded so the deflection could be calculated. The data collected by the load cell was compiled into a spreadsheet that was downloaded from the data acquisition software. The strain gauge was used to determine the strain in the beam. Figure 1.2 shows the experimental set up and how the forces act on the beam. The figure also shows the location of the strain gage on the specimen. The wheatstone bridge was used in conjunction with the strain gauge due to its ability to read very small voltage differences. Figure 1.3 can be used to calculate the value of strain from the data given by the wheatstone bridge. The formula relates output voltage to strain by using the resistance measured by the wheatstone bridge. All resistance values were 121.2 Ω while the strain gage factor (k) is 1.77.



$$\frac{V_{measured}}{V_{supply}} = \frac{R_2 \cdot R_3}{(R_2+R_3)^2}\left[\frac{\Delta R}{R}\right]$$

$$\frac{\Delta R}{R} = k\varepsilon$$

Fig. 1.2: *Three point bending setup used for this experiment.* ***Fig. 1.3.:*** *Voltage to strain conversion*

The ruler used to measure the length of the beam had an accuracy of 0.5 mm and the calipers used to measure the width and thickness of the beam had an accuracy of 0.0005 mm. Due to the uncertainty in the measurements, our calculated values will also have a level of uncertainty associated with them.

3.2 The measurements sought to collect in this experiment were the axial and transverse displacement of the beam, the axial load applied to the beam, and various significant data points such as the buckling load and the yield load of the specimen. The load cell was used to determine the axial deflection of the beam. The initial position was recorded by the data acquisition software in conjunction with the load cell, as the beam buckled, the maximum deflection from the initial position was recorded. The load cell was also used to measure the loading value. The maximum load value was entered into the load cell through its interface, then the test was run and as the load increased, the force applied to the beam was collected at the same time that the displacement value was taken.

The deflection extensometer was placed directly next to the specimen at the set up of the experiment, as the column deflected, it pressed against the extensometer causing it to deflect from its initial position. The deflection was recorded by the data acquisition software.

All measurements taken in this lab had levels of uncertainty (0.0005 mm for caliper measurements and 0.5 mm for ruler measurements). Due to these uncertainties, the uncertainties of the calculated buckling loads can be seen in figure 2.2.

Material	Uncertainty in Buckling Load
Steel	0.42
Aluminum	0.035
Carbon Fiber	0.0000032

Fig. 2.2: *Uncertainties of the calculated buckling load for each material.*

Experimental Design

4.1 This lab consisted of two trials - one where the beam has a loading span of 140 mm and the second with a loading span of 160 mm. Prior to loading, the wheatstone bridge had to be zeroed to ensure an accurate reading. If the value of strain changed by more than 10μm between trials, the trial had to be repeated. The beam is then loaded, midspan, in 50 N increments up to 500 N. Each load is applied to the beam for a total of 30 seconds. All of the different trials with different loadings were intended to show us how strain changes as force applied to the beam increases. Figure 1.4 shows the tests that were performed during the experiment.

Span	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10
140 mm	50 N	100 N	150 N	200 N	250 N	300 N	350 N	400 N	450 N	500 N
160 mm	50 N	100 N	150 N	200 N	250 N	300 N	350 N	400 N	450 N	500 N

Fig. 1.4: *Trials to be completed during the Beam Bending experiment.*

The purpose of this lab was to learn how strain gauges work and how one can use a resistance measurement to calculate strain. After completing this lab, we should have a much better understanding of various instrumentation that can be used in stress analysis. Additionally, this lab is meant to supplement the theoretical knowledge we gained from taking Mechanics of Solids. This theoretical knowledge consists of the internal forces and shear and normal stresses occurring within a beam when it is acted upon by a load perpendicular to the span.

4.2 The purpose of this lab was to understand the theories associated with a column that is significantly longer than it is wide. The experimental conditions in this lab were three columns of different materials loaded for two minutes at a time within a load cell. The vertical and horizontal deflections were recorded as well. A load limit of 250 N was set into the load cell. The maximum speed given was 2mm/min. Table 2.3 shows the experimental conditions of the three trials conducted as a part of this experiment.

Trial #	**Material**	**Load Time**
1	Steel	2 min
2	Carbon Fiber	2 min
3	Aluminum	2 min

Table 2.3: *Trials to be completed during the Column Buckling Lab*

Experimental Procedure

5.1

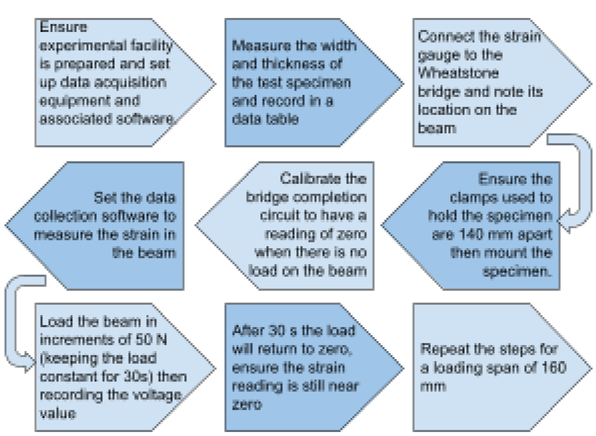


Fig. 1.5: *Beam Bending and Stress Transformation Lab Procedure*

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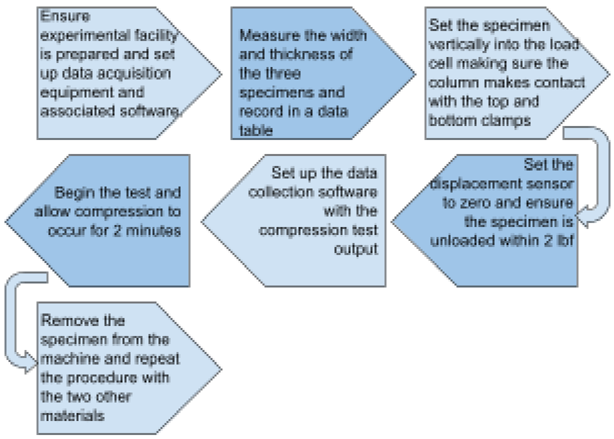
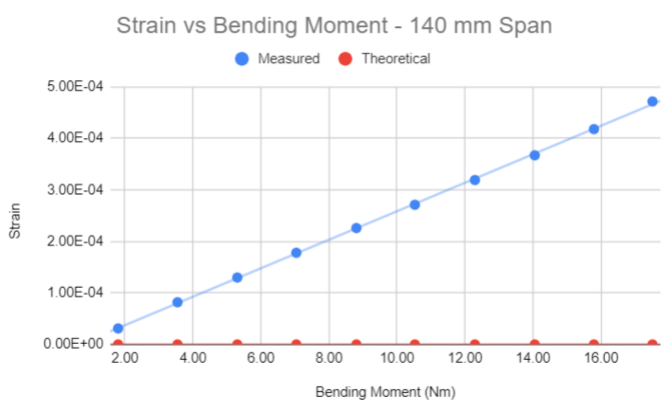


Fig. 2.4: *Column Buckling Lab Procedure*



<u>Results and Discussion</u>

6.1 Both the theoretical and measured values of strain have a positive relationship with bending moment. This is expected since as the beam bends more, the deflection within the beam will increase. The theoretical values are however, much smaller than the measured values. This could be due to an experimental error such as an error in the data acquisition software.

Fig. 1.6: *Theoretical and Measured Strain vs Bending Moment for 140 mm span.*

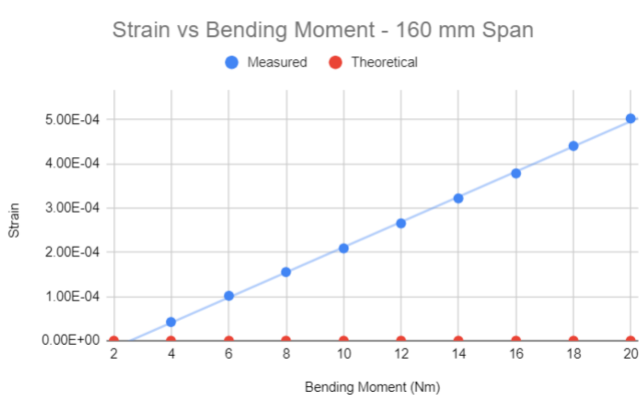


Fig. 1.7: *Theoretical and measured strain vs bending moment for 160 mm span.*

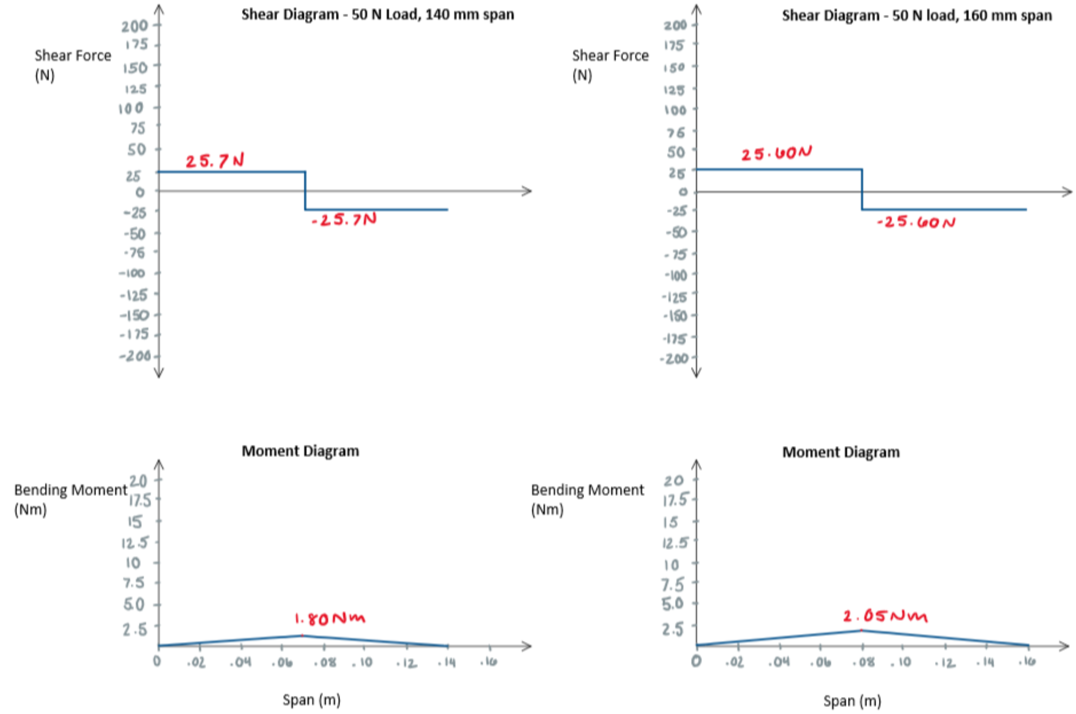


Fig. 1.8: *Shear force and bending moment diagram for both trials when the loading condition is 50 N.*

Figure 1.8 shows the resulting span- moment relationship and span- shear force diagram for the beam when it was loaded with 50 N. The results seen in these plots is expected as you can see a strong correlation between the moment and shear diagrams. This relationship is that the shear force is the derivative of the moment diagram.

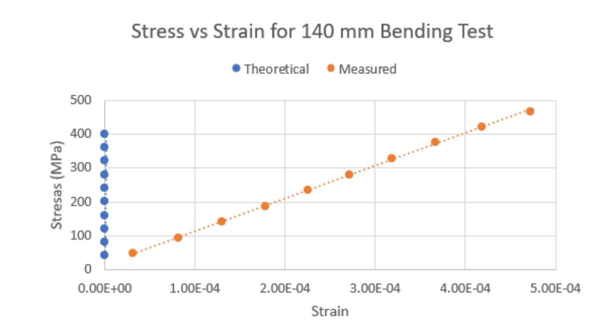


Fig 1.9: *Plot comparing both the theoretical and measured values of normal stress with the theoretical and measured values of strain, respectively, for the trial with a 140 mm span.*

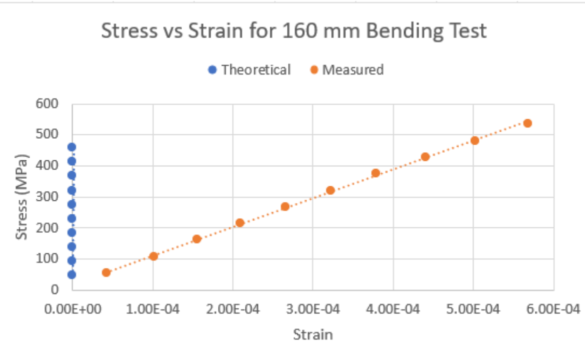


Fig 1.10: *Plot comparing both the theoretical and measured values of normal stress with the theoretical and measured values of strain, respectively, for the trial with a 160 mm span.*

As stated previously, the results with the theoretical and measured values look to be accurate when examined separately except, when the two relationships are looked at together, we can assume that there was an error in the measured data.

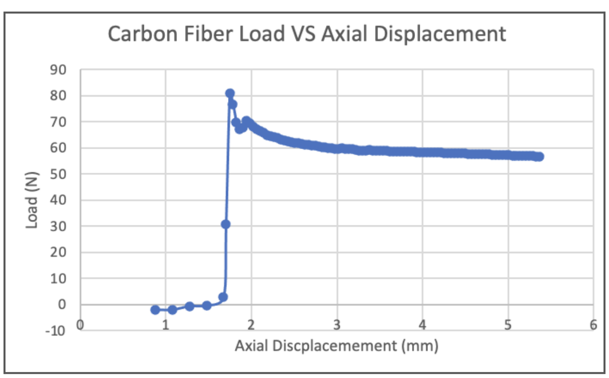
6.2 Due to the data from my lab section having a significant flaw, I utilized the timed data from section M006 subgroup 1. Using this data, I was able to attain expected results as seen in figure A2.10. In most cases, as the load increases, the deflection angle increases. This makes sense because the member will begin buckling once it reaches the buckling load (the point seen in figure 2.6 right before the column deflects) and it will continue to deflect therefore increasing its angle of deflection until its yield point. Due to the load limit placed on the load cell, none of the specimens were brought to yield during this lab.

One interesting outcome that can be seen is that Steel was able to reach the highest load out of any of the other materials, however it was the material that had the smallest value of deflection. This is presumably due to the fact that steel has a far higher modulus of elasticity than the other two materials allowing it to be put under higher forces while feeling less strain. The lower value of strain hints that steel will not deflect as much as some other materials.

Additionally, carbon fiber happened to deflect the most and be able to carry the smallest load. Looking at figure 2.5 it is obvious that carbon fiber has an elastic modulus that is much smaller than the two metals. This would cause the material to yield under lower stresser therefore the load cell could not apply as much force as it could to the other materials without the carbon fiber yielding.

Material	Max Deflection	Max Load	Elastic Modulus
Carbon Fiber	5.4038 mm	85.1 N	0.5 GPa
Steel	4.4393 mm	160 N	190 GPa
Aluminum	5.0068 mm	127 N	70 GPa

Fig. 2.5: *Maximum loading conditions and deflections that each material was able to reach as collected by the load cell.*



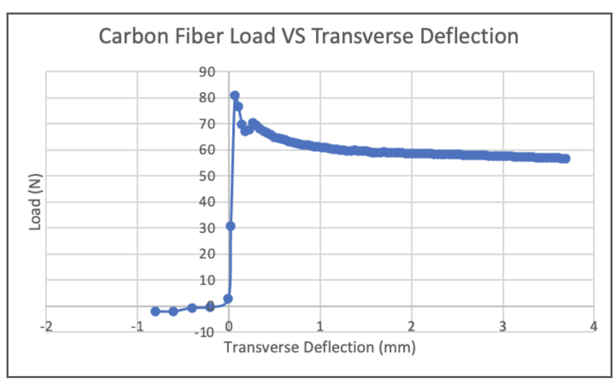


Fig 2.6: *Load vs deflection in both directions for carbon fiber.*

<u>Conclusion and Recommendations</u>

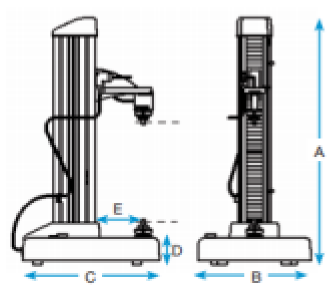
7.1 This lab taught me a lot about bending and how materials react under these conditions. I now understand the importance of considering shear forces when designing as an engineer. At times, I think the lab manual was hard to understand and directions were unclear.

7.2 I enjoyed this experiment however, I think the data given to use from the data acquisition software was very difficult to understand. There were many headings of data that seemed to be saying the same thing. The biggest takeaway from this lab was that it was hard to ensure that the column would deflect in the direction of the extensometer - this caused my group and I to have to repeat the experiment.

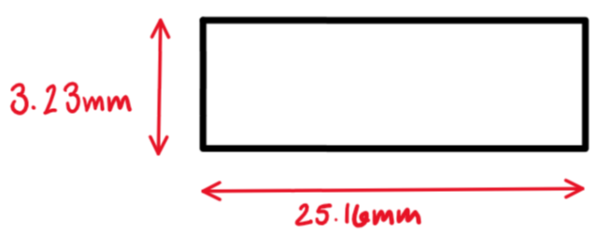
References

- Lab manual T1.1
- Lab manual T1.2
- [Aluminium - Strength - Hardness - Elasticity - Crystal Structure (material-properties.org)]
- [Carbon Fiber | Density, Strength,, Melting Point (material-properties.org)]

9.1 Appendix: Additional Information for Beam Bending and Stress Transformation Lab



A1.1: *LLOYD LS5 Advanced Materials Testing System*



A1.1: *Cross section of specimen used in the bending experiment*

1) Check the experimental facility for its apparent integrity and readiness to operate (TA to assure with assistance from a lab technician as needed); 2) Set up the data acquisition system (TA); 3) Connect a strain gage to the bridge completion circuit as described by TA. Also, check Appendix 3 for schematics of bridge completion circuit 4) Take down the span (length between loading points) and align beam with the loading points 5) Measure the thickness and width of the specimen cross-section and take note of the strain gage factor 6) Zero out the bridge circuit using the nulling adjust on the bridge completion circuit 7) Set up compression test, and data output (TA will assist) 8) Load beam in increments of 50 N, do not load beam to failure (do not exceed 500 N)

9) Once load is released, read output of bridge to make sure zero strain reference has not changed 10) If it has changed by more than 10 µm strain, measurement must be repeated 11) Repeat above steps for a new span between the loading points 12) Draw a sketch of the location of the strain gage on the beam

Fig A1. : *Detailed procedure for the bending experiment. Source: T1.1 lab manual.*

Load (N)	Voltage Out - 140 mm Span	Voltage Out - 160 mm Span
50 N	**0.11 mV**	**0.15 mV**
100 N	**0.29 mV**	**0.36 mV**
150 N	**0.46 mV**	**0.55 mV**
200 N	**0.63 mV**	**0.74 mV**
250 N	**0.80 mV**	**0.94 mV**
300 N	**0.96 mV**	**1.14 mV**
350 N	**1.13 mV**	**1.34 mV**
400 N	**1.30 mV**	**1.56 mV**
450 N	**1.48 mV**	**1.78 mV**
500 N	**1.67 mV**	**2.01 mV**

A1.3: *Raw data collected from the strain gage reading.*

3 Point Bending Test - 140 mm Span							
Load (P)	**Measured Load (P_m)**	**Shear Force (V)**	**Theoretical Max Normal Stress (σ_m)**	**Measured Max Normal Stress (σ_m)**	**Bending Moment (M)**	**Theoretical Strain (ε_m)**	**Measured Strain (ε_m)**
50 N	51.487 N	25.7435 N	41.19 MPa	48.055 MPa	1.80 Nm	2.53E-10	3.11E-05
100 N	101.39 N	50.695 N	81.11 MPa	94.63 MPa	3.55 Nm	4.98E-10	8.19E-05
150 N	151.621	75.8105 N	121.30	141.51	5.31 Nm	7.45E-10	1.30E-04

	N		MPa	MPa			
200 N	201.23 N	100.615 N	160.99 MPa	187.81 MPa	7.04 Nm	9.88E-10	1.78E-04
250 N	251.697 N	125.8485 N	201.36 MPa	234.92 MPa	8.81 Nm	1.24E-09	2.26E-04
300 N	300.839 N	150.4195 N	240.68 MPa	280.78 MPa	10.53 Nm	1.48E-09	2.71E0-4
350 N	351.27 N	175.635 N	281.02 MPa	327.85 MPa	12.29 Nm	1.73E-09	3.19E-04
400 N	401.545 N	200.7725 N	321.25 MPa	374.78 MPa	14.05 Nm	1.97E-09	3.67E-04
450 N	451.316 N	225.658 N	361.06 Mpa	421.23 MPa	15.80 Nm	2.22E-09	4.18E-04
500 N	500.466 N	250.233 N	400.39 MPa	467.1 MPa	17.52 Nm	2.46E-09	4.72E-04

Fig. A1.4: *Measured and Calculated values from the 3 point bending test with a span of 140 mm*

3 Point Bending Test - 160 mm Span								
Load (P)	**Measured Load (P_m)**	**Shear Force (V)**	**Theoretical Max Normal Stress (σ_m)**	**Measured Max Normal Stress (σ_m)**	**Bending Moment (M)**	**Theoretical Strain (ε_m)**	**Measured Strain (ε_m)**	**Theoretical Strain (ε_m)**
50 N	51.198 N	25.599	46.8 MPa	54.612 MPa	2.04792 Nm	2.87E-10	4.24E-05	2.87E-10
100 N	100.885 N	50.4425	92.24 MPa	107.61 MPa	4.0354 Nm	5.66E-10	1.02E-04	5.66E-10
150 N	151.336 N	75.668	138.37 MPa	161.42 MPa	6.05344 Nm	8.50E-10	1.55E-04	8.50E-10
200 N	200.965 N	100.4825	183.75 MPa	214.36 MPa	8.0386 Nm	1.13E-09	2.09E-04	1.13E-09
250 N	251.21 N	125.605	229.68 MPa	267.96 MPa	10.0484 Nm	1.41E-09	2.66E-04	1.41E-09

300 N	301.088 N	150.544	275.29 MPa	321.16 MPa	12.04352 Nm	1.69E-09	3.22E-04	1.69E-09
350 N	351.005 N	175.5025	320.93 MPa	374.41 MPa	14.0402 Nm	1.97E-09	3.79E-04	1.97E-09
400 N	401.178 N	200.589	366.80 MPa	427.92 MPa	16.04712 Nm	2.25E-09	4.41E-04	2.25E-09
450 N	451.192 N	225.596	412.53 MPa	481.27 MPa	18.04768 Nm	2.53E-09	5.03E-04	2.53E-09
500 N	500.916 N	250.458	457.99 MPa	534.31 MPa	20.03664 Nm	2.81E-09	5.68E-04	2.81E-09

Fig. A1.5: *Measured and Calculated values from the 3 point bending test with a span of 160 mm*

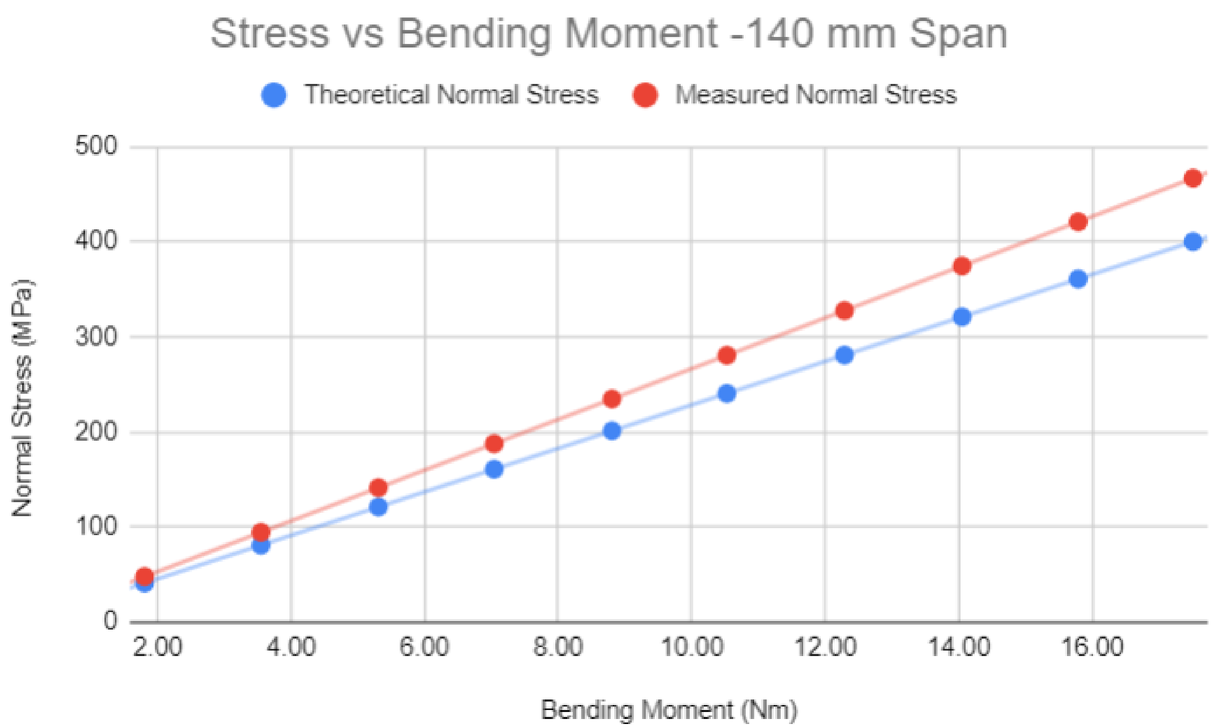


Fig A1.6: *Plot comparing both the theoretical and measured values of normal stress with the bending moment for the trial with a 140 mm span.*

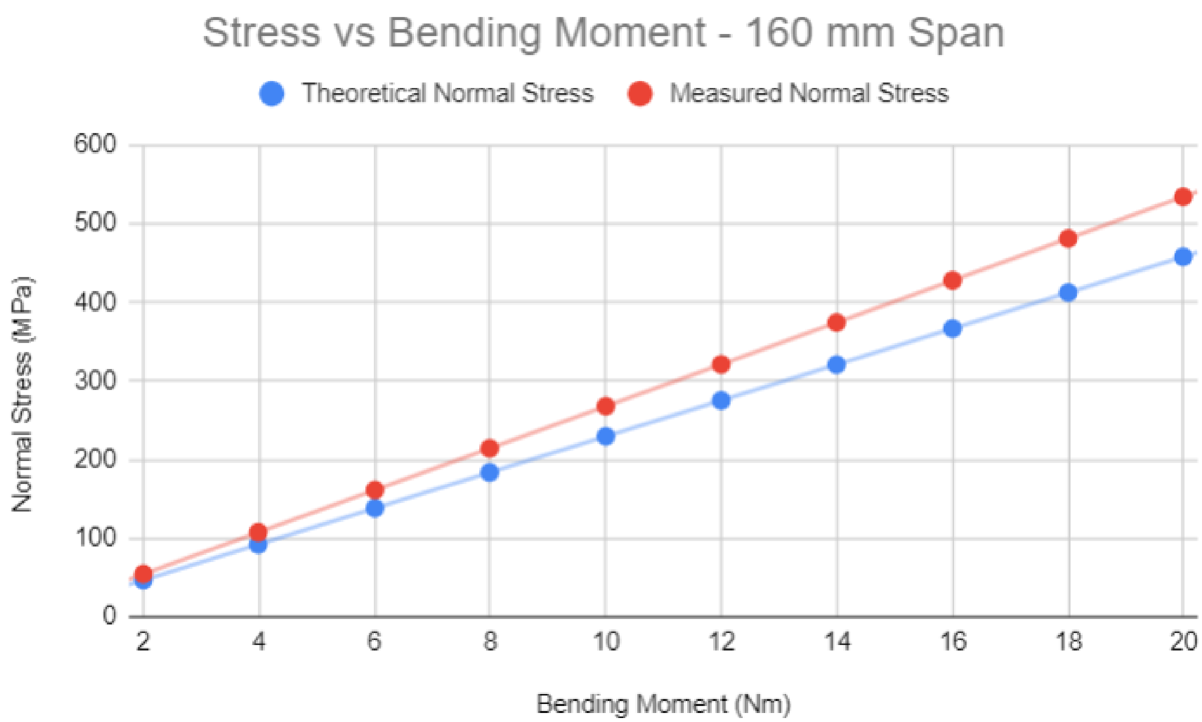


Fig 1.9: *Plot comparing both the theoretical and measured values of normal stress with the bending moment for the trial with a 160 mm span.*

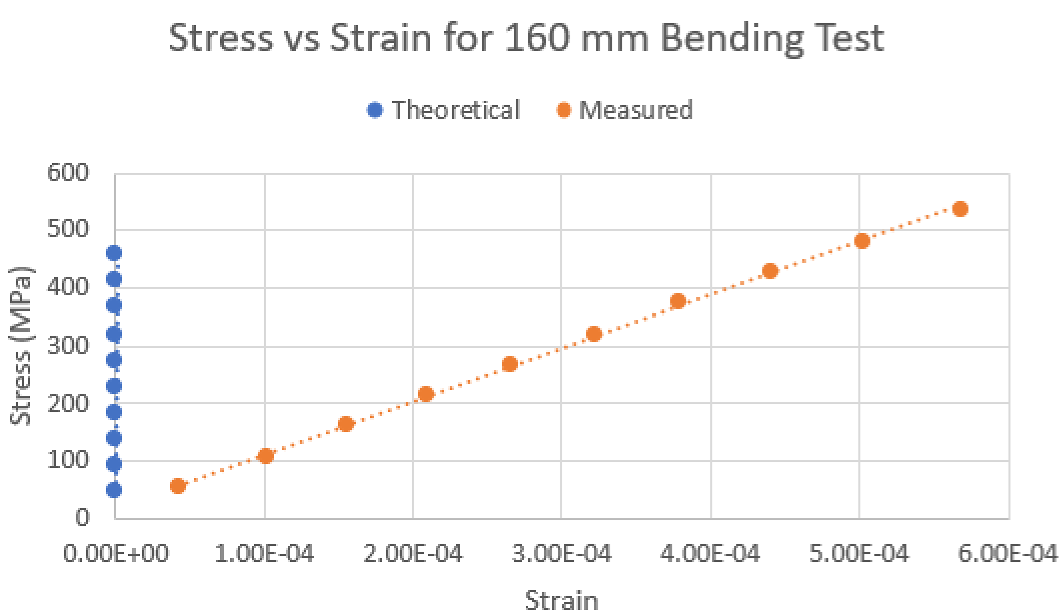


Fig A1.9: *Plot comparing both the theoretical and measured values of normal stress with the theoretical and measured values of strain, respectively, for the trial with a 160 mm span.*

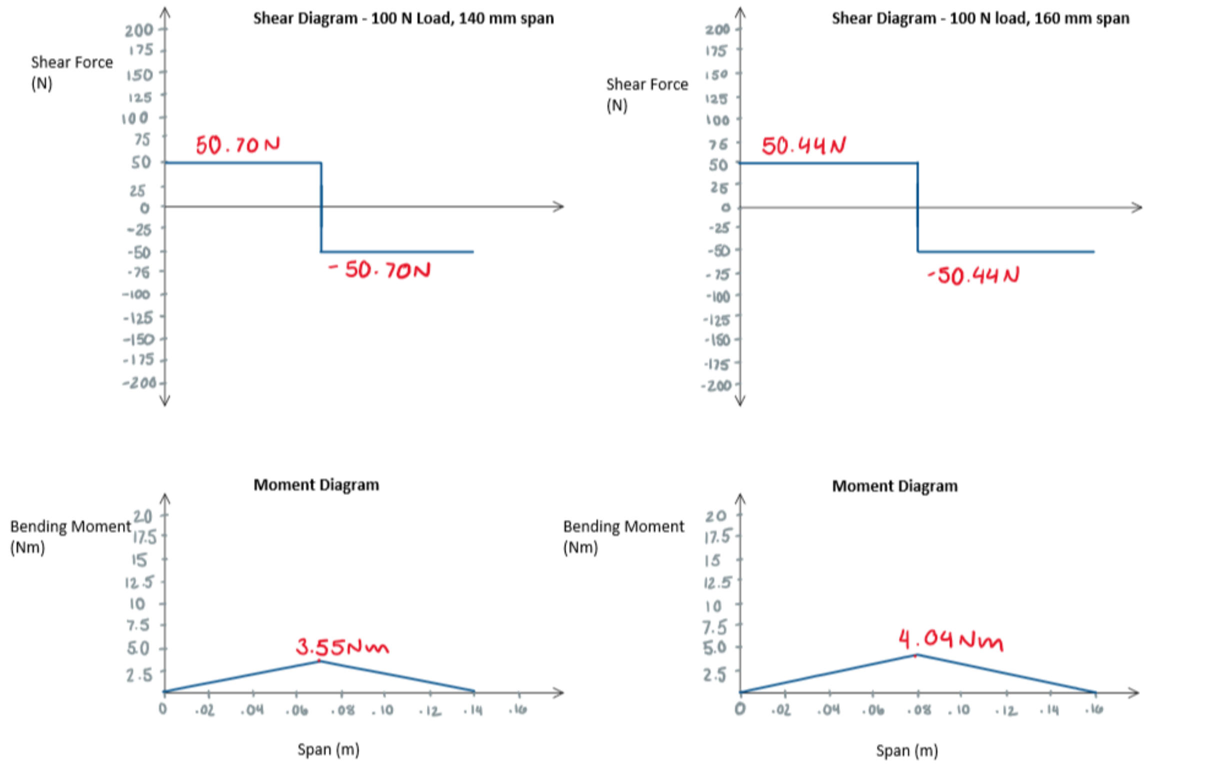


Fig A1.10: *Plots containing the shear force and bending moments for both trials conducted with a 100 N load.*

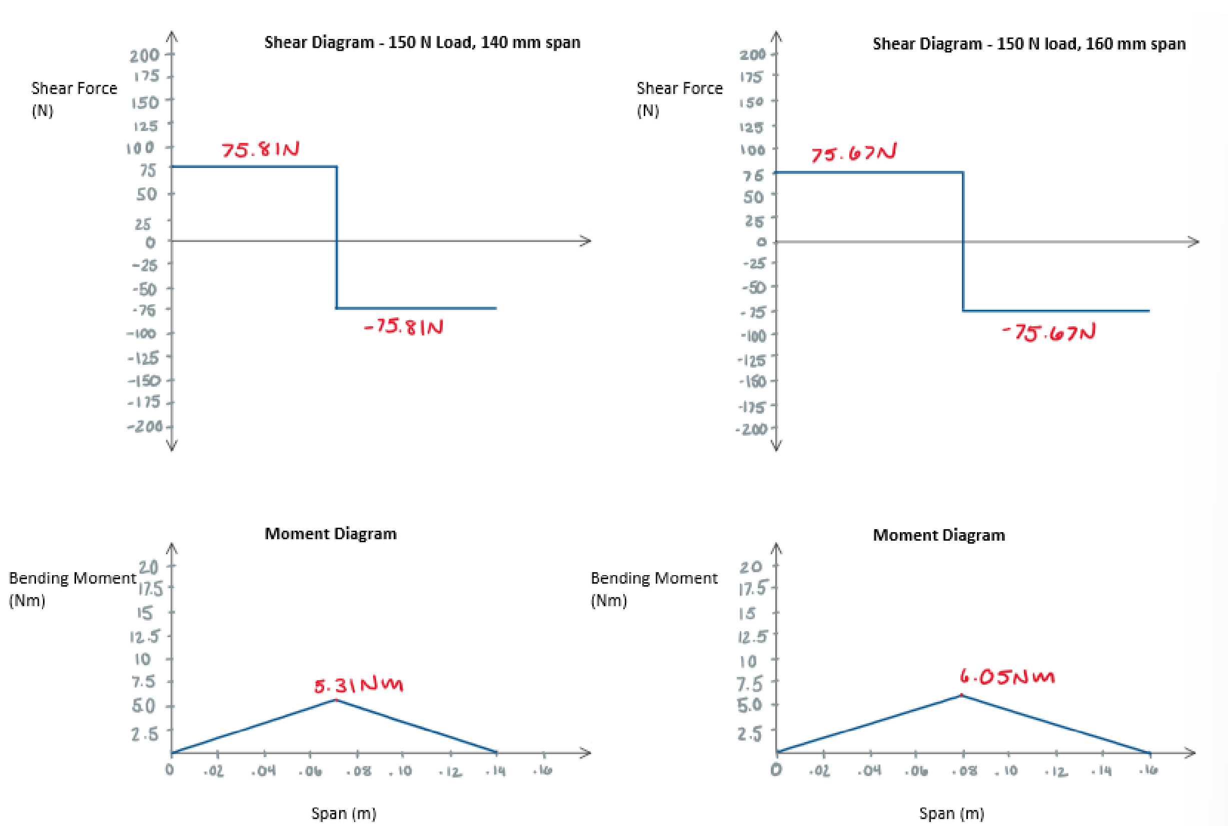


Fig A1.11: *Plots containing the shear force and bending moments for both trials conducted with a 150 N load.*

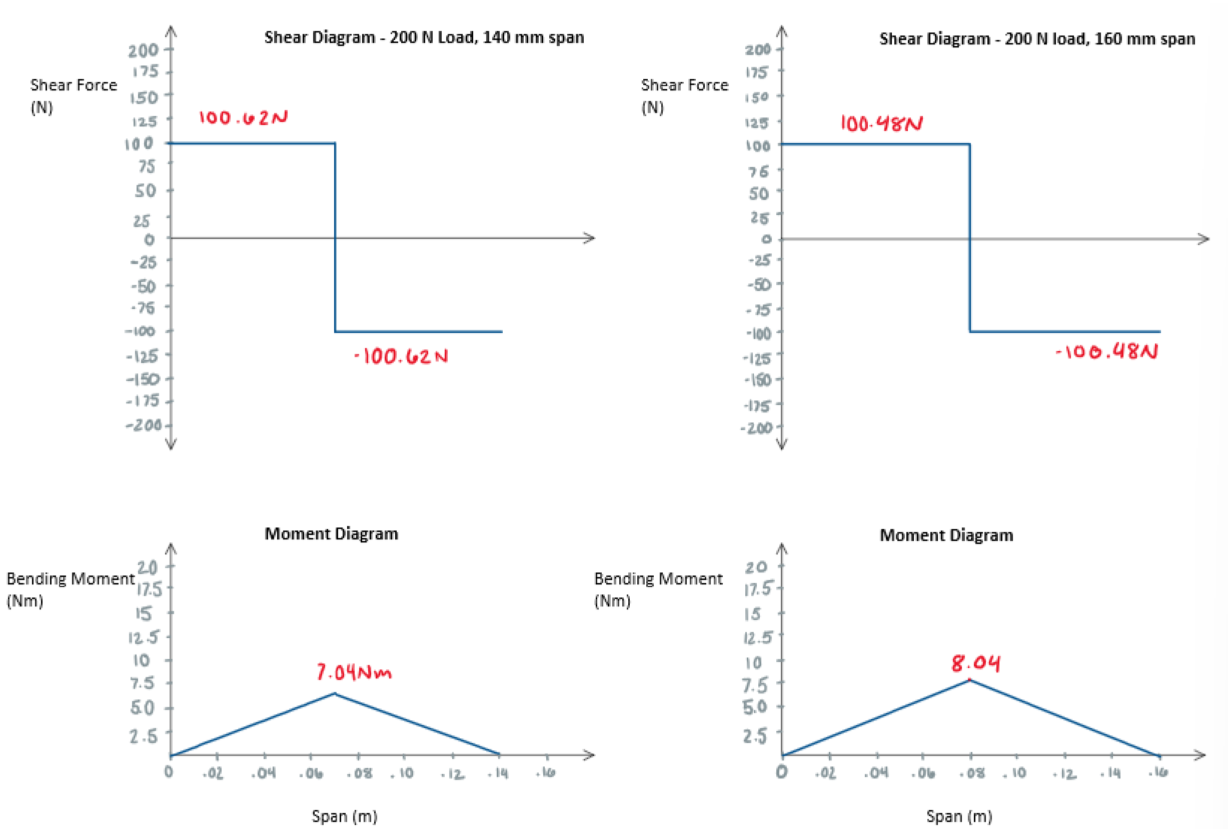


Fig A1.12: *Plots containing the shear force and bending moments for both trials conducted with a 200 N load.*

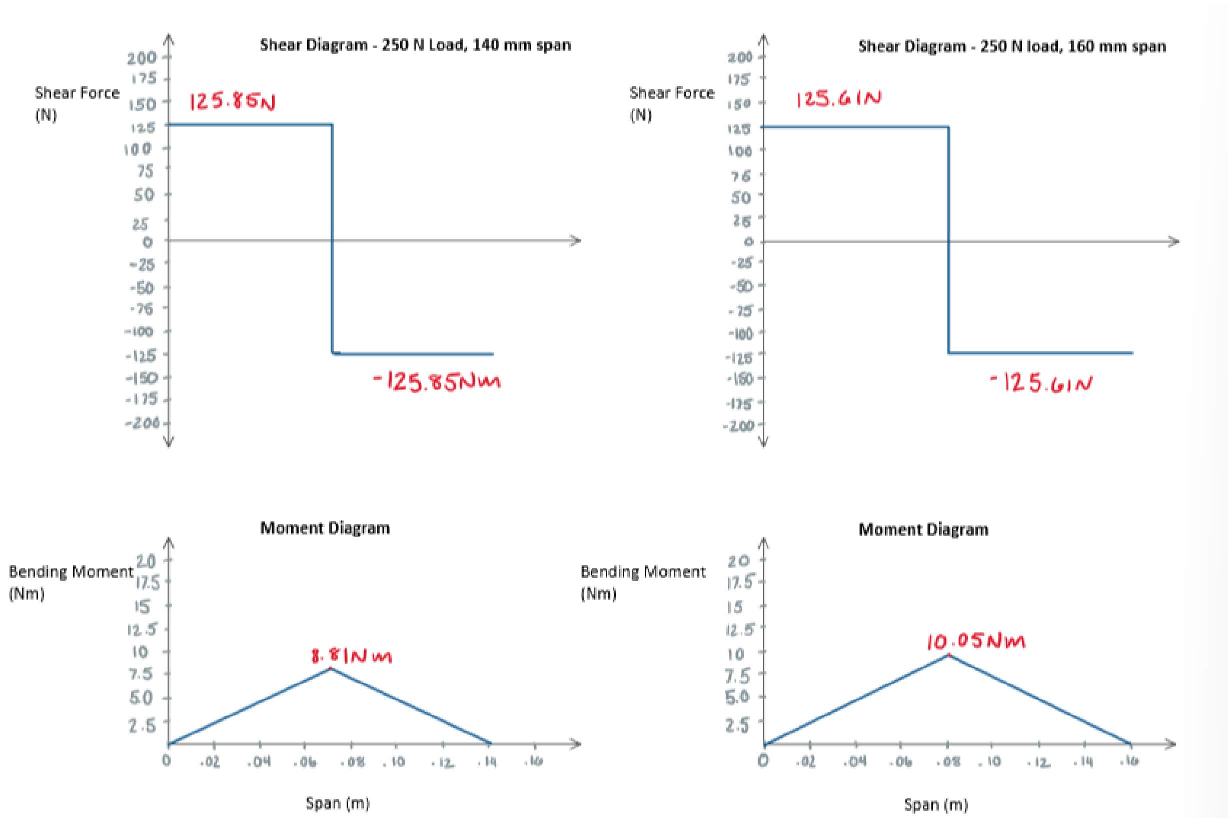


Fig A1.13: *Plots containing the shear force and bending moments for both trials conducted with a 250 N load.*

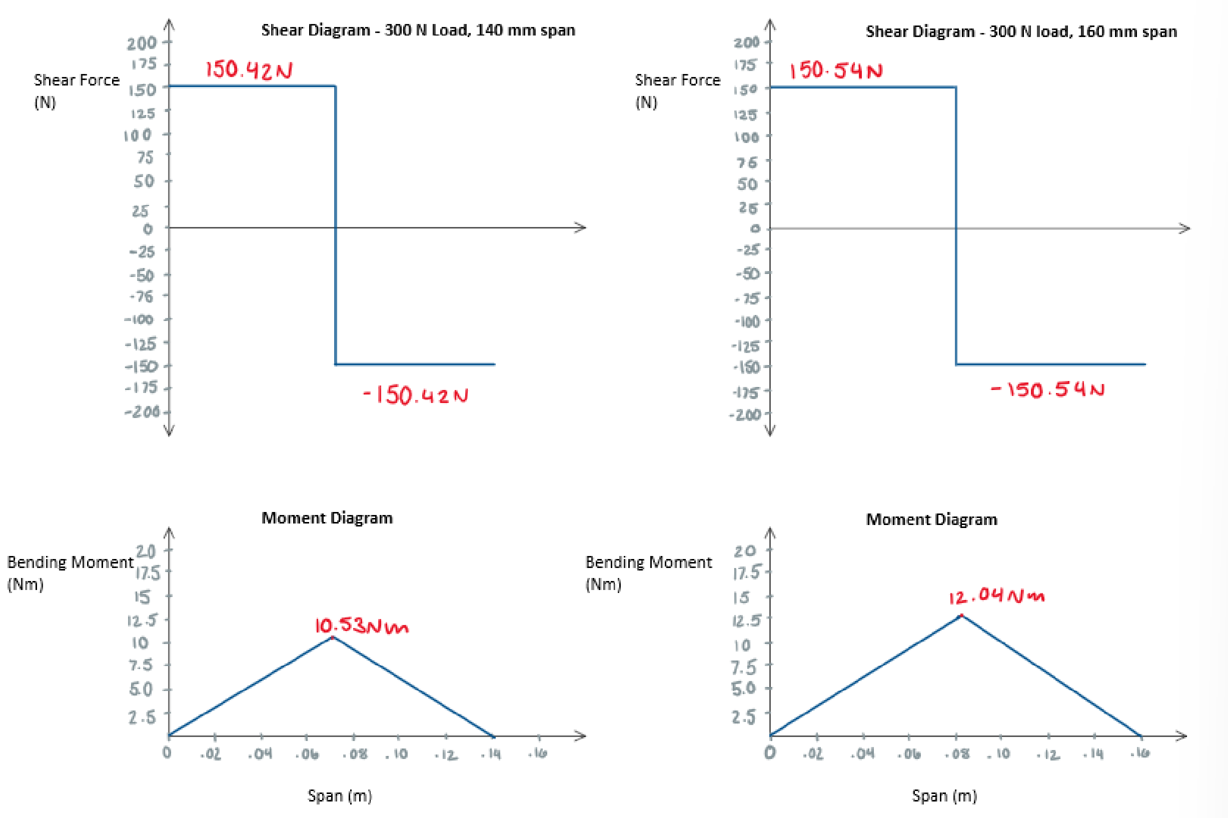


Fig A1.14: *Plots containing the shear force and bending moments for both trials conducted with a 300 N load.*

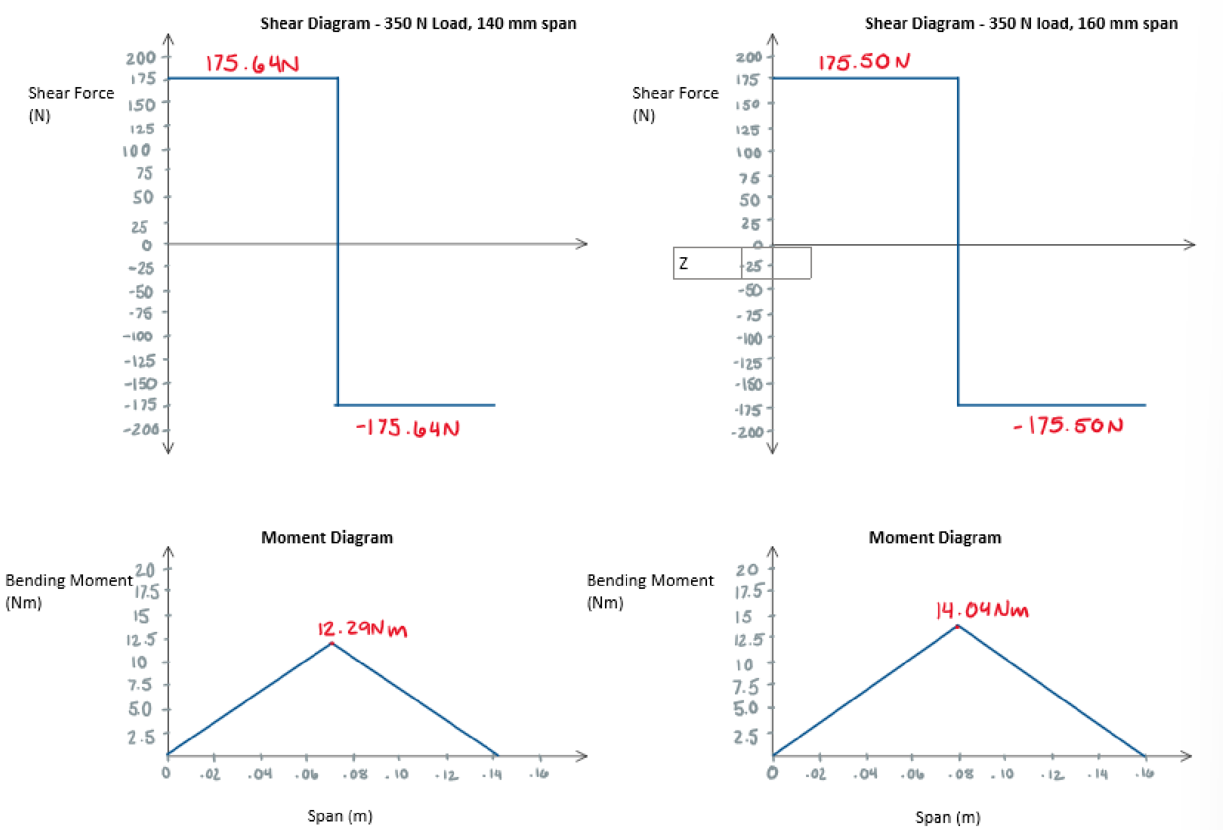


Fig A1.15: *Plots containing the shear force and bending moments for both trials conducted with a 350 N load.*

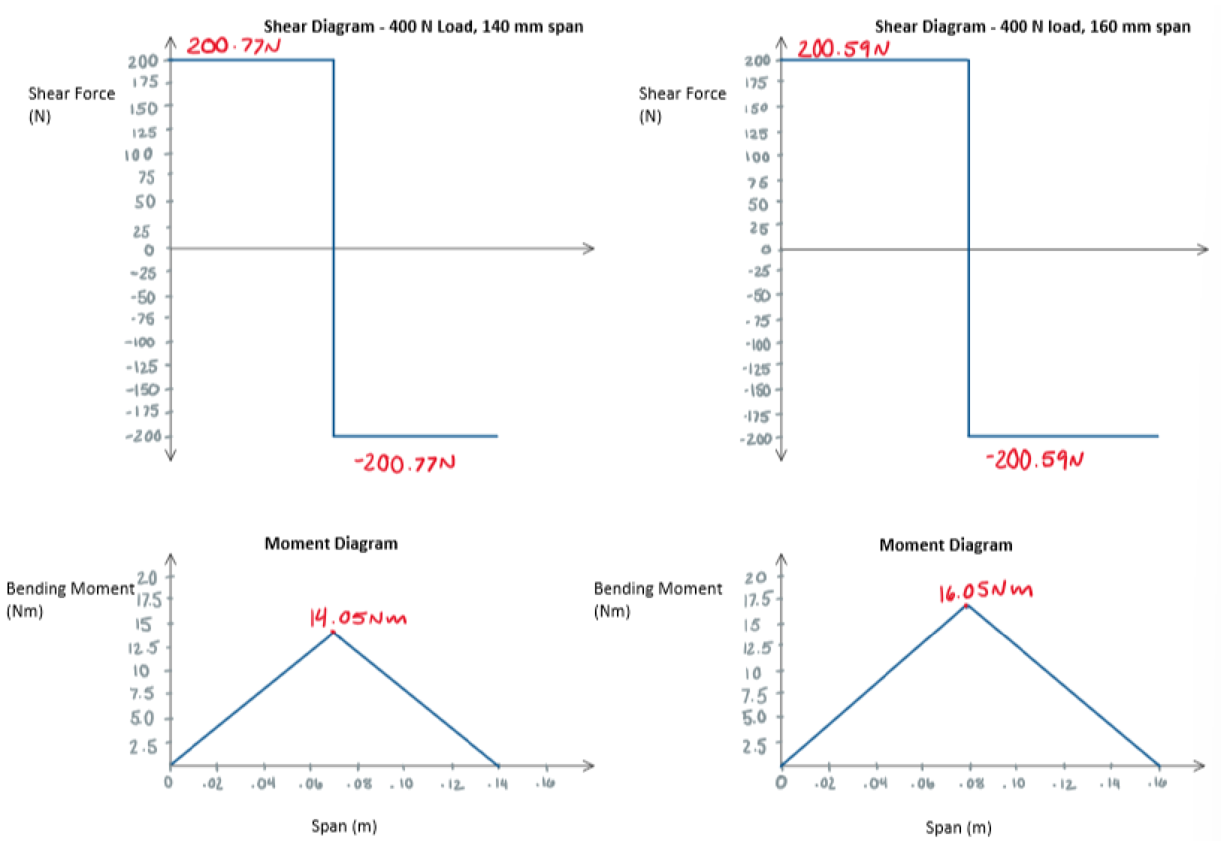


Fig A1.16: *Plots containing the shear force and bending moments for both trials conducted with a 400 N load.*

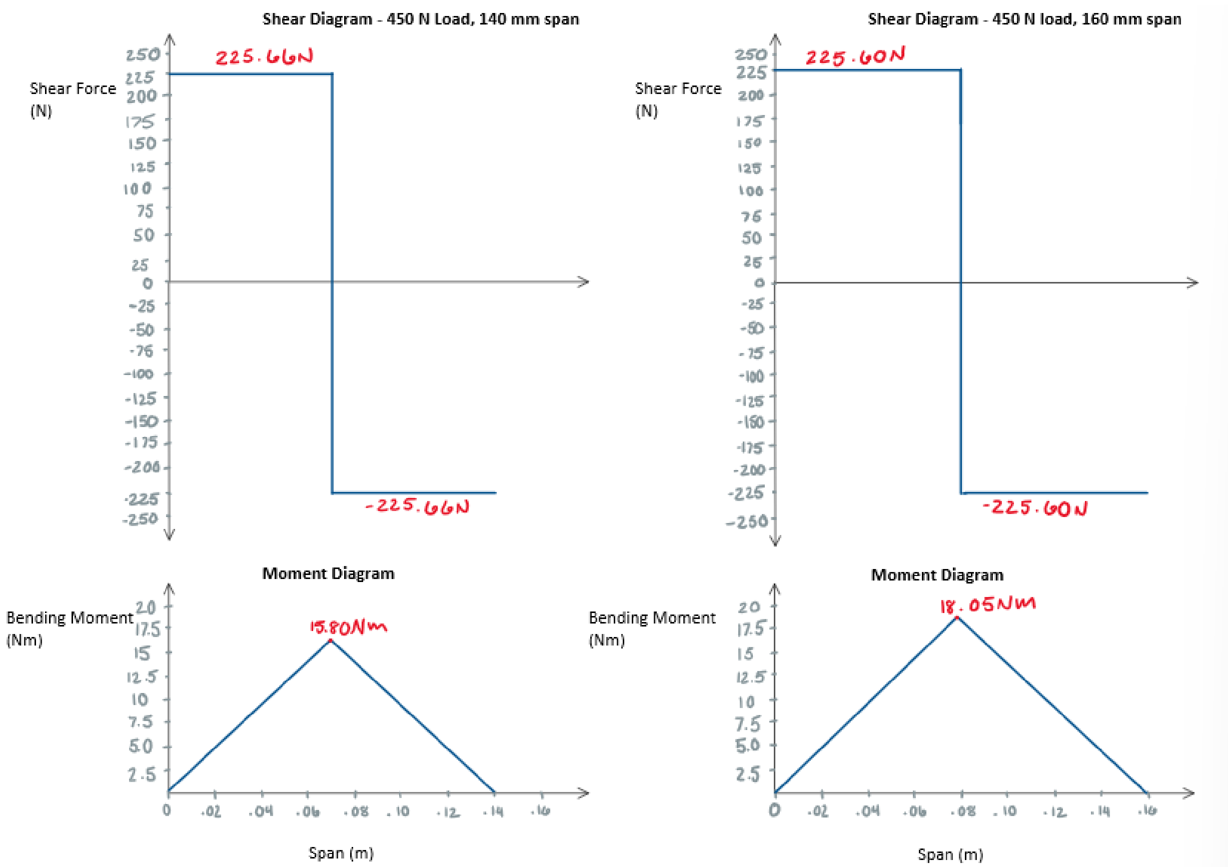


Fig A1.17: *Plots containing the shear force and bending moments for both trials conducted with a 450 N load.*

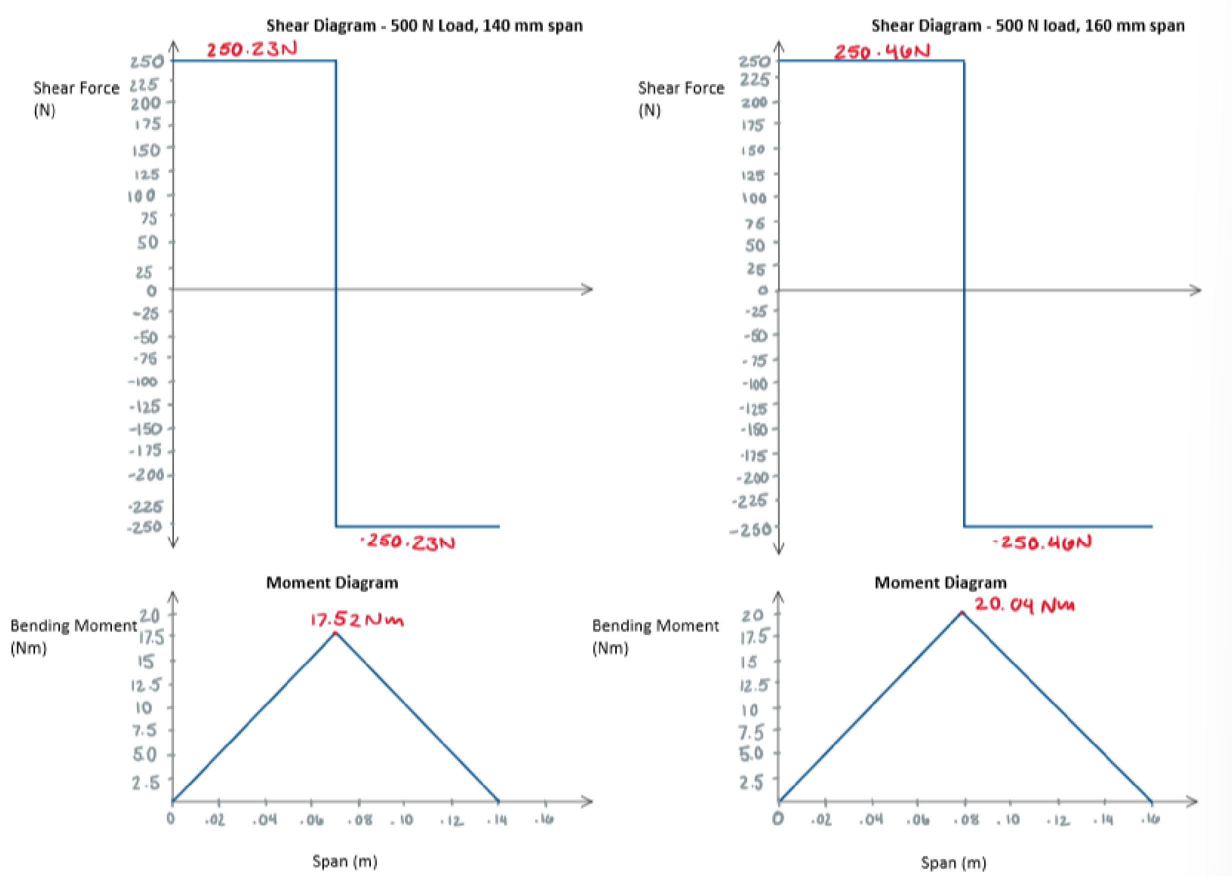


Fig A1.18: *Plots containing the shear force and bending moments for both trials conducted with a 500 N load.*

9.2 Appendix: Additional Information for Column Buckling Lab

1) Check the experimental facility for its apparent integrity and readiness to operate (TA to assure with assistance from a lab technician as needed);
2) Set up the data acquisition system (TA will assist);
3) Using the calipers and a tape measure, measure the length, width and thickness of each specimen, and record the values
4) Install the specimen in the testing machine and zero the load cell output
5) Make sure the specimen contacts the top and bottom of the clamps and the specimen is unloaded within ± 2 lbf

6) Set up the compression test and data output (TA will assist)
7) Start the test, and allow compression for 2min
8) Once test completed remove specimen
9) Repeat steps for remaining specimen

Fig A2.1: *Detailed procedure for the buckling experiment. Source: T1.2 lab manual.*

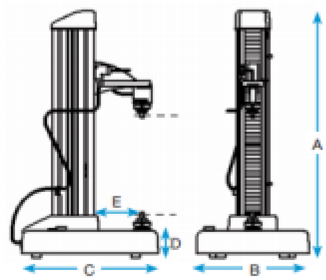
$$I = \frac{ba^3}{12}$$

Fig. A2.2: *Formula used to calculate moment of inertia of the specimen.*

Material	**Width (mm)** **± 0.0005 mm**	**Thickness (mm)** **± 0.0005 mm**	**Length (mm)** **± 0.5 mm**
Steel	45.37	1.62	300.0
Aluminum	24.51	1.50	299.0
Carbon Fiber	25.32	1.64	299.0

Fig. A2.3: *Data collected from the test materials using calipers and a ruler.*

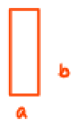
Fig. A2.4: *The three specimens tested in the buckling lab. From left to right: Carbon Fiber, Aluminum, Steel.*



A2.5: *LLOYD LS5 Advanced Materials Testing System*

* Uncertainty of Buckling Load:

$$I = \frac{ba^3}{12}$$



$$P_{cr} = \frac{\pi^2 E I}{Le^2} = \frac{\pi^2 E b a^3}{12\, Le^2}$$

$$\frac{dP_{cr}}{da} = \frac{\pi^2 E b a^2}{4 Le^2}$$

$$\frac{dP_{cr}}{db} = \frac{\pi^2 E a^3}{12 Le^2}$$

$$\frac{dP_{cr}}{dLe} = \frac{-\pi^2 E b a^3}{6 Le^3}$$

u_C = .0005mm

u_R = .5mm

$$u\_P_{cr} = \left(\frac{dP_{cr}}{da}\cdot u\_C\right)^2 + \left(\frac{dP_{cr}}{db}\cdot u\_C\right)^2 + \left(\frac{dP_{cr}}{dLe}\cdot u\_R\right)^2$$

Steel: $E = 1.9\times10^{11}$ Pa, $Le = 300$ mm, $a = 1.62$ mm, $b = 25.37$ mm

$$u\_P_{cr}\_S = \left(\frac{\pi^2(1.9\times10^{11}Pa)(.02537m)(.00162m)^2}{4(.3m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(\frac{\pi^2(1.9\times10^{11}Pa)(.00162m)^3}{12(.3m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(-\frac{\pi^2(1.9\times10^{11}Pa)(.02537m)(.00162m)^3}{6(.3m)^3}\cdot 5\times10^{-4}\right)^2 = 0.42$$

Aluminum: $E = 70\times10^9$ Pa, $Le = 299$ mm, $a = 1.5$ mm, $b = 24.51$ mm

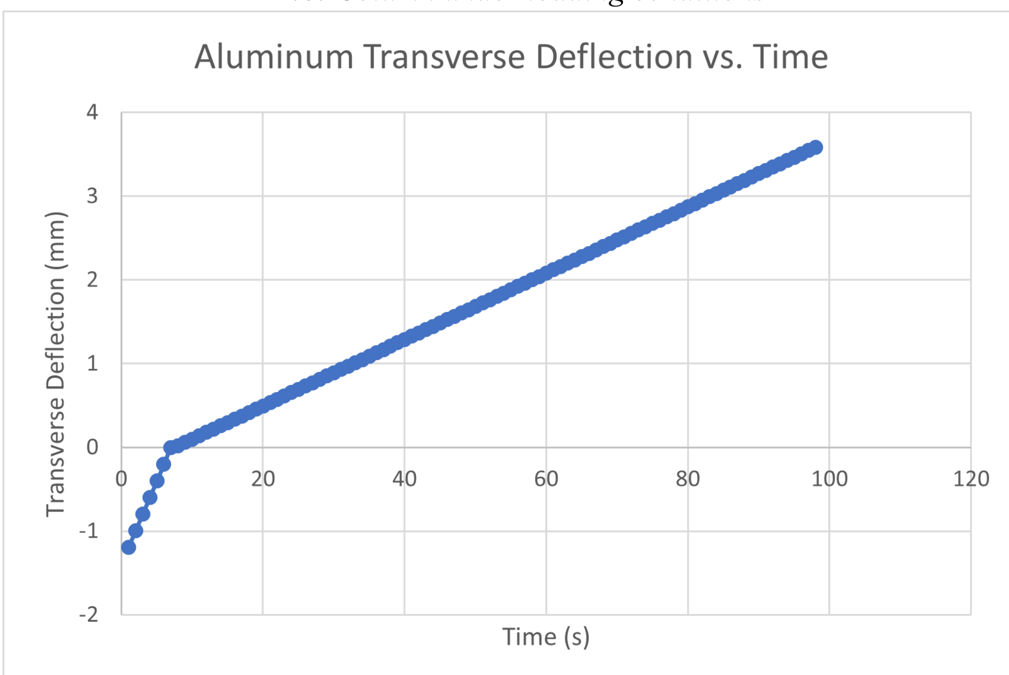
$$u\_P_{cr}\_A = \left(\frac{\pi^2(70\times10^9Pa)(.02451\,m)(.0015\,m)^2}{4(.299m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(\frac{\pi^2(70\times10^9Pa)(.0015\,m)^3}{12(.299m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(-\frac{\pi^2(70\times10^9Pa)(.02451\,m)(.0015\,m)^3}{6(.299m)^3}\cdot 5\times10^{-4}\right)^2 = 0.035$$

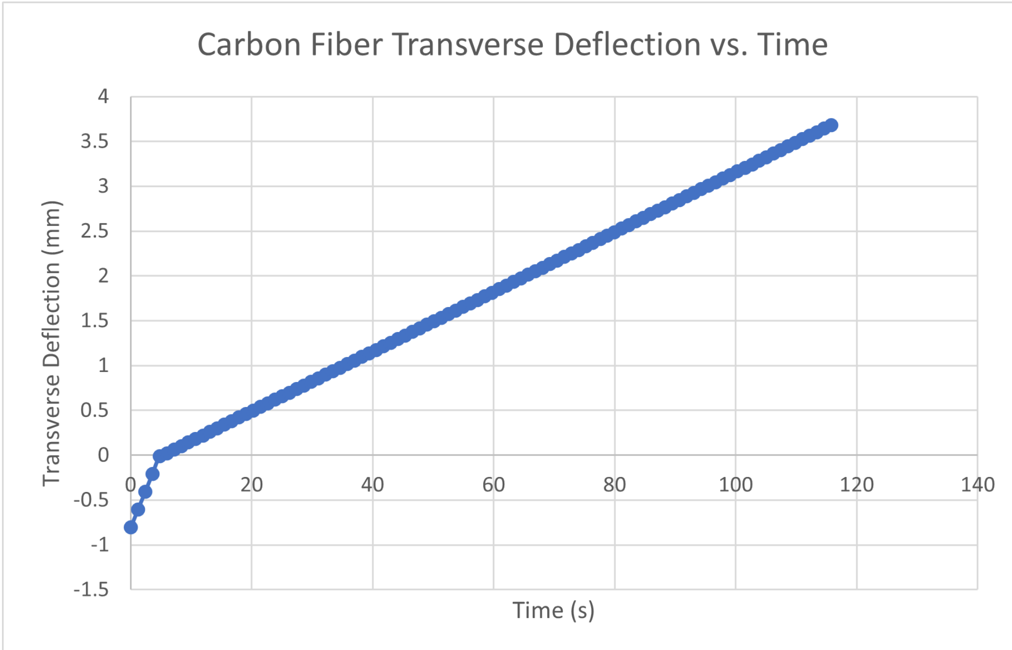
Carbon Fiber: $E = 500\times10^6$ Pa, $Le = 299$ mm, $a = 1.64$ mm, $b = 25.32$ mm

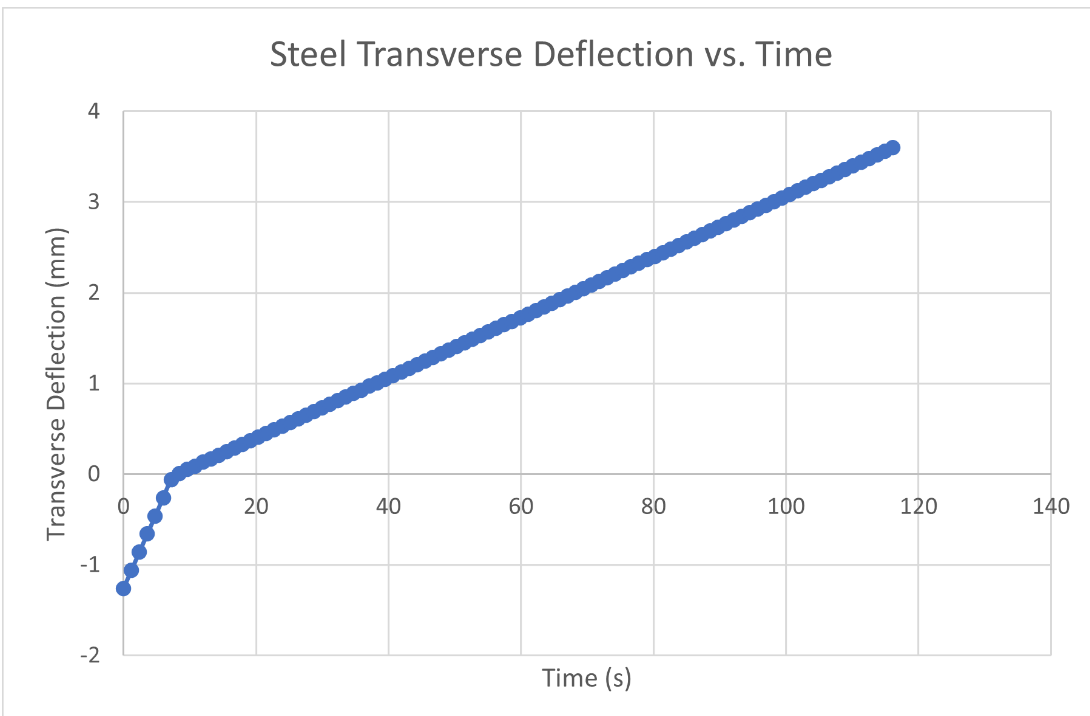
$$u\_P_{cr}\_C = \left(\frac{\pi^2(500\times10^6Pa)(.02532m)(.00164m)^2}{4(.299m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(\frac{\pi^2(500\times10^6Pa)(.00164m)^3}{12(.299m)^2}\cdot 5\times10^{-7}m\right)^2 + \left(-\frac{\pi^2(500\times10^6Pa)(.02532m)(.00164m)^3}{6(.299m)^3}\cdot 5\times10^{-4}\right)^2 = .0000032$$

A2.6: *Hand calculations used to calculate the uncertainties of the buckling load for each material.*

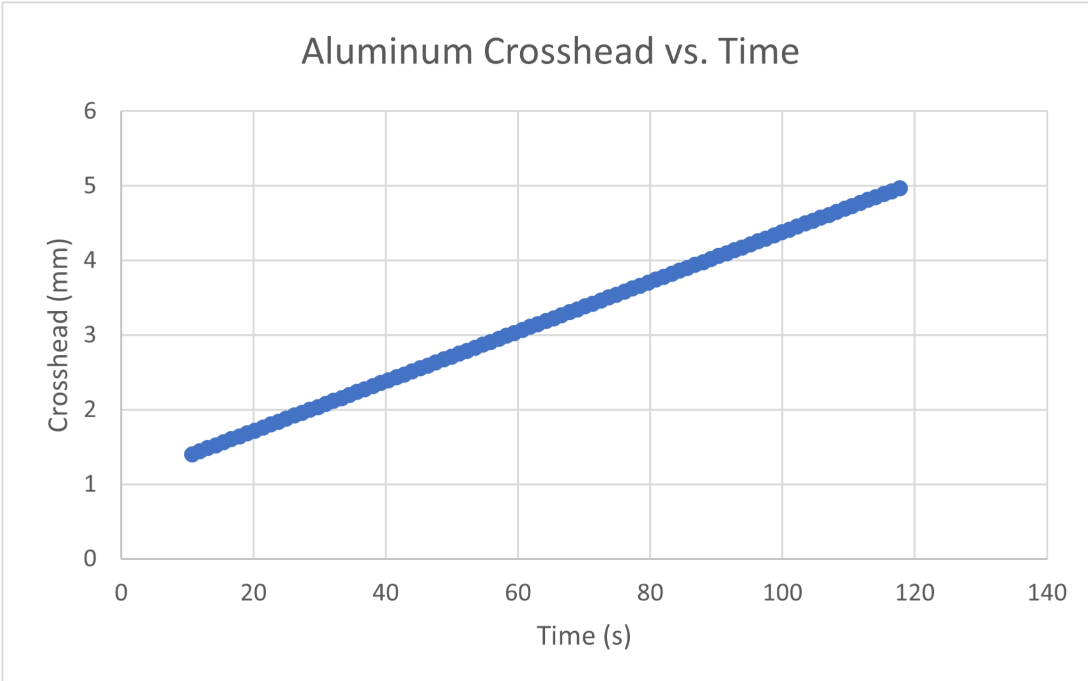
A2.6: *Column under loading conditions*

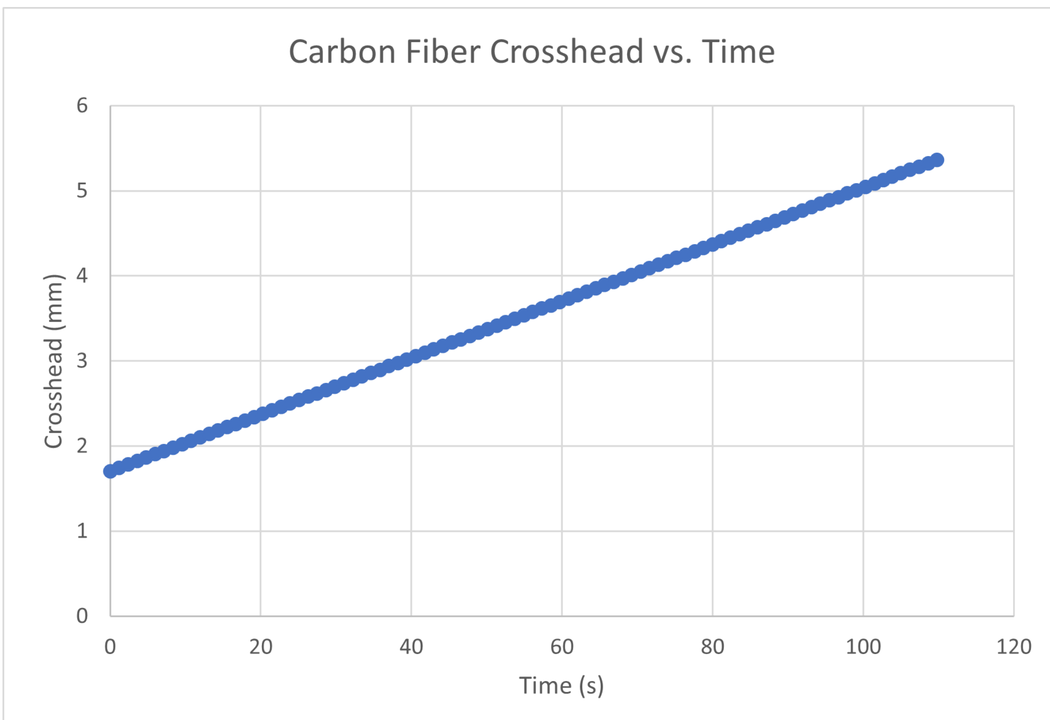


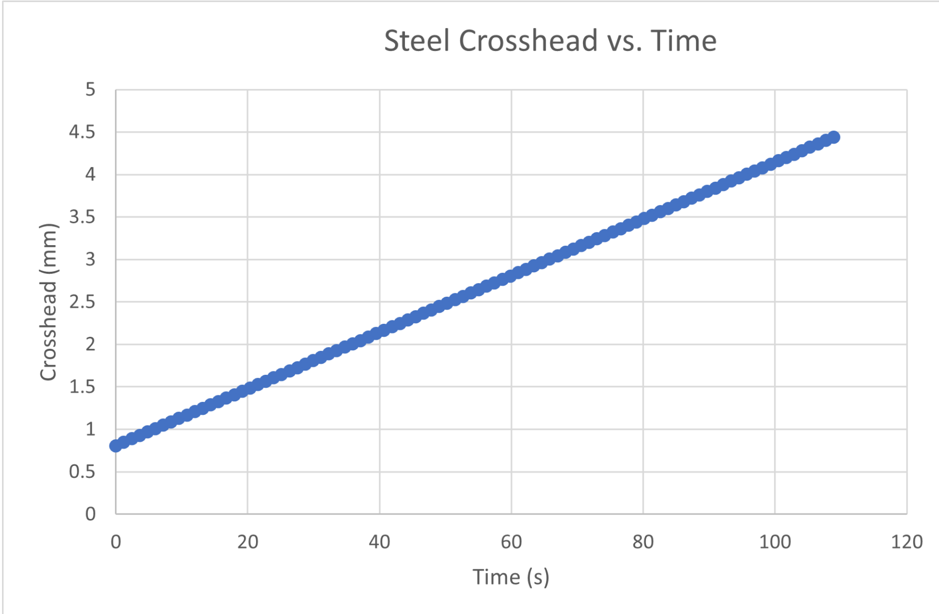




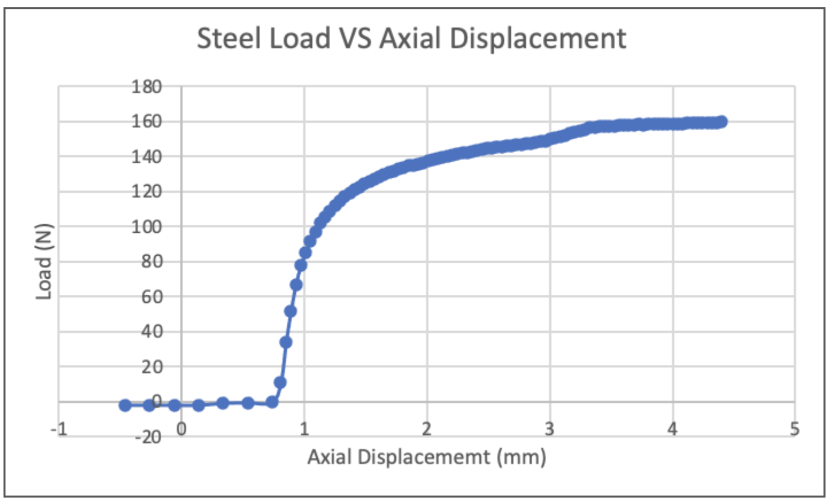
A2.7: *Deflection vs Time Plots for each material tested*

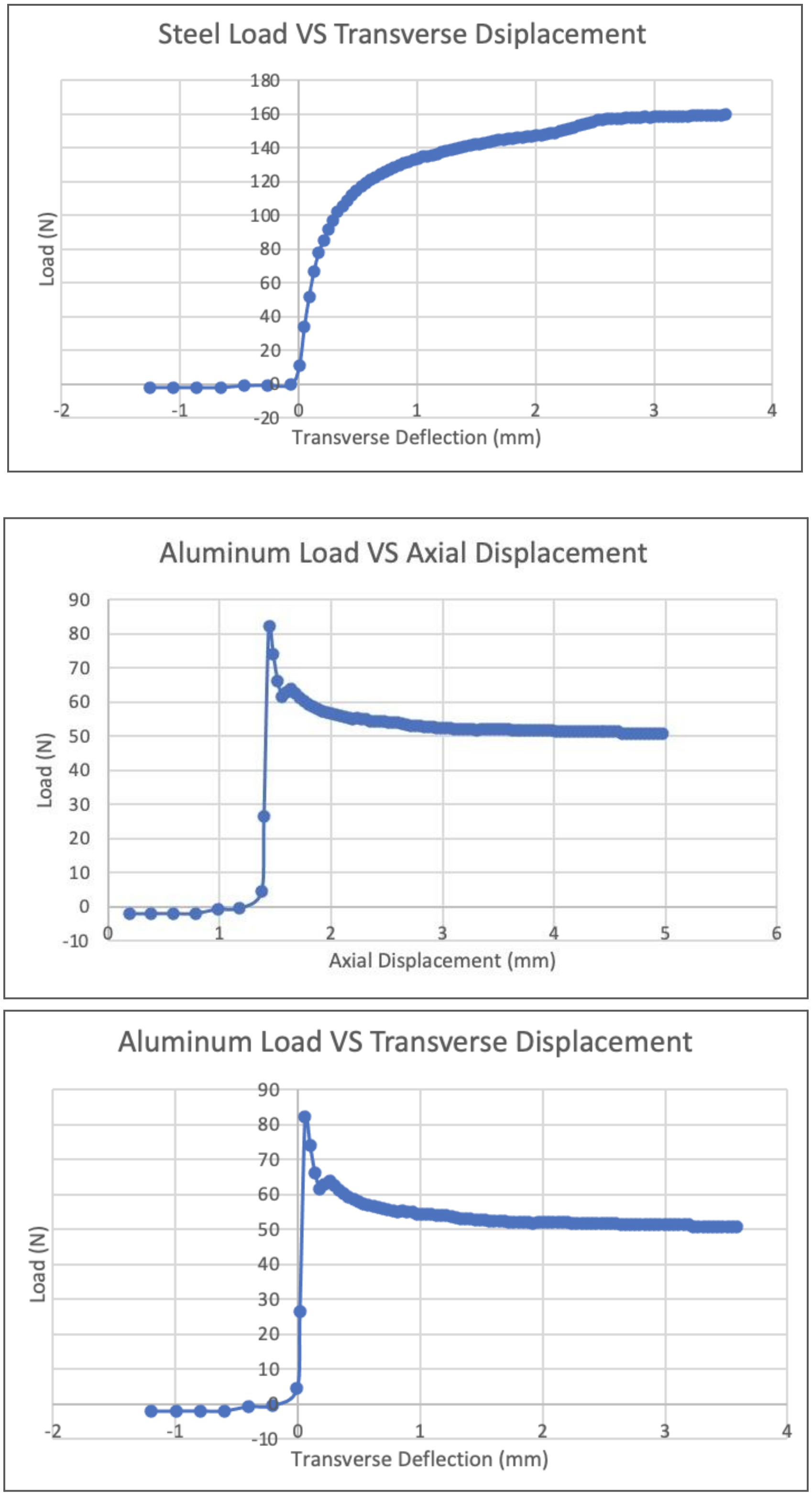

Aluminum Crosshead vs. Time
Crosshead (mm)
0
1
2
3
4
5
6
0
20
40
60
80
100
120
140
Time (s)


Carbon Fiber Crosshead vs. Time
Crosshead (mm)
0
1
2
3
4
5
6
0
20
40
60
80
100
120
Time (s)

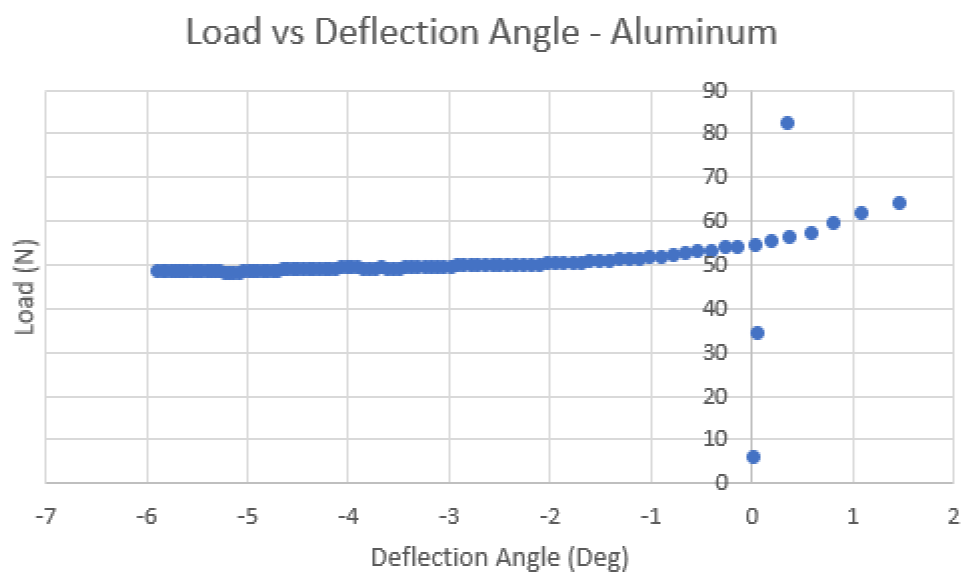


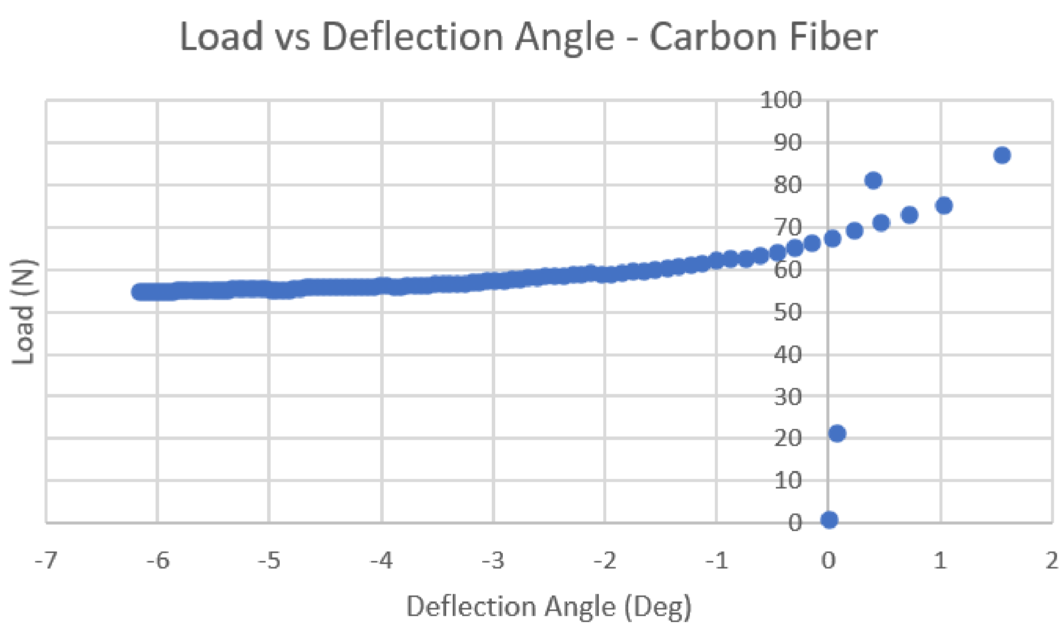
A2.8: *Crosshead vs Time Plots for each material tested*

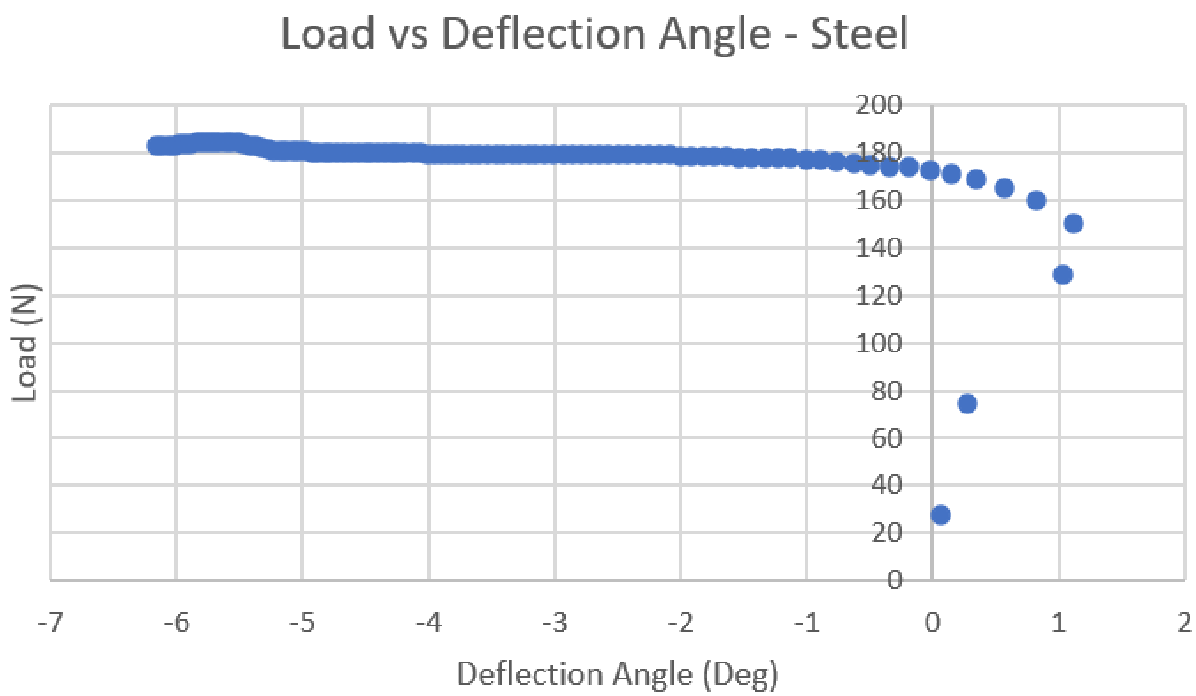




A2.9: *Plots for materials aluminum and steel showing load vs displacement (both transverse and axial)*


Load vs Deflection Angle - Aluminum
Load (N)
90
80
70
60
50
40
30
20
10
0
-7
-6
-5
-4
-3
-2
-1
0
1
2
Deflection Angle (Deg)


Load vs Deflection Angle - Carbon Fiber
Load (N)
100
90
80
70
60
50
40
30
20
10
0
-7
-6
-5
-4
-3
-2
-1
0
1
2
Deflection Angle (Deg)



A2.10: *Plots for each material depicting load vs deflection angle.*