Final Project Part 1

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Takeoff:

$P_{01} = 100 kPa$

$T_{01} = 15°C = 288 K$

$\gamma = 1.4$

$C_p = 1004\ J/kgK$

$R = 287\ J/kgK$

compression ratio: $\frac{P_{0e}}{P_{0i}} = 6$

$\eta_{cp} = .92 \rightarrow$ const

$\dot{m} = 20 kg/s$

Design Choices:

$C_{\theta 1} = 0$

const axial velocity $(C_{x1} = C_{x2})$

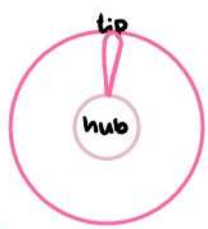
$\left(\frac{hub}{tip}\right)_1 = K_1 = .6$

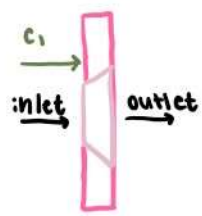
$M_i = .34$

$U_t = 259.2\ m/s$

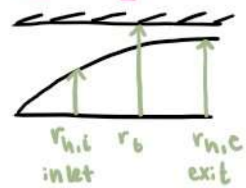
Size & Design 1st Stage of Compressor

1. size the compressor (D_t is const, D_h increases)





$r_m = \frac{r_t + r_h}{2}$



Inlet:

$\frac{T_{0i}}{T_i} = \left(1 + \frac{\gamma - 1}{2} M_i^2\right)$ $\quad T_i = T_{0i}\left(1 + \frac{\gamma-1}{2} M_i^2\right)^{-1}$

$T_i = 288K\left(1 + \frac{1.4-1}{2}(.34)^2\right)^{-1}$

$\boxed{T_i = 281.49K}$

$a_i = \sqrt{\gamma R T_i} = \sqrt{1.4(287 J/kgK)(281.49K)}$ $\quad \boxed{a_i = 336.31 m/s}$

$M_i = \frac{C_i}{a_i} \quad C_i = M_i a_i = 336.31 m/s \cdot .34$ $\quad \boxed{C_i = 114.3 m/s}$

$\frac{P_{0i}}{P_i} = \left(1 + \frac{\gamma-1}{2} M_i^2\right)^{\frac{\gamma}{\gamma-1}}$ $\quad P_i = P_{0i}\left(1 + \frac{\gamma-1}{2} M_i^2\right)^{-\gamma/\gamma-1} \quad -1.4/1.4-1$

$P_i = 100kPa\left(1 + \frac{1.4-1}{2}(.34)^2\right)$

$\boxed{P_i = 92.31 kPa}$

$P_i = \rho_i R T_i \quad \rho_i = \frac{P_i}{RT_i}$ $\quad$ * $C_i = C_{xi}$ b/c $C_{\theta i} = 0$

$\rho_i = \frac{92310 Pa}{(287 J/kgK)(281.49K)}$ $\quad \boxed{\rho_i = 1.14 kg/m^3}$

Inlet Annulus Area: $\dot{m} = \rho_i C_{ix} A_i$ $\quad A_i = \pi(r_t^2 - r_h^2)_i$

$\dot{m} = \rho_i C_{ix} \pi r_{ti}^2 (1 - K_i^2)$ $\quad = \pi r_{ti}^2\left(1 - \frac{r_h^2}{r_t^2}\right)_i$

$r_{ti} = \sqrt{\frac{\dot{m}}{\rho_i C_{ix} \pi (1 - K_i^2)}} = \sqrt{\frac{20 kg/s}{(1.14\frac{kg}{m^3})(114.3 m/s)\pi(1 - .6^2)}}$

$\boxed{r_{ti} = 0.28m}$

$K = \frac{r_{hi}}{r_{ti}} \quad r_{hi} = K r_{ti} = (.6)(.28m)$ $\quad \boxed{r_{hi} = 0.17m}$

Exit:

$\frac{P_{0e}}{P_{0i}} = 6$ $\quad P_{0e} = 6 \cdot 100 kPa = 600 kPa$

$\eta_p = \frac{\gamma - 1}{\gamma} \frac{\ln\left(\frac{P_{0e}}{P_{0i}}\right)}{\ln\left(\frac{T_{0e}}{T_{0i}}\right)}$ $\quad e^{\ln\left(\frac{T_{0e}}{T_{0i}}\right)} = e^{\frac{\gamma-1}{\gamma \eta_p} \ln\left(\frac{P_{0e}}{P_{0i}}\right)}$

$T_{0e} = {}_{e}\left(\frac{\gamma-1}{\gamma\eta_p} \ln\left(\frac{P_{0e}}{P_{0i}}\right)\right)$

$$\eta_p = \frac{\gamma-1}{\gamma} \frac{\ln\left(\frac{P_{oe}}{P_{oi}}\right)}{\ln\left(\frac{T_{oe}}{T_{oi}}\right)} \qquad e^{\ln\left(\frac{T_{oe}}{T_{oi}}\right)} = e^{\frac{\gamma-1}{\gamma\eta_p}\ln\left(\frac{P_{oe}}{P_{oi}}\right)}$$

$$\frac{T_{oe}}{T_{oi}} = e^{\left(\frac{\gamma-1}{\gamma\eta_p}\ln\left(\frac{P_{oe}}{P_{oi}}\right)\right)}$$

$$\frac{T_{oe}}{T_{oi}} = e^{\frac{1.4-1}{1.4(.92)}\ln(6)} = 1.74$$

$T_{oe} = 1.74(288K)$

$\boxed{T_{oe} = 520K}$

* C_x is const $\therefore$ $C_{xi} = C_{xe} = 114.3\,\frac{m}{s}$

$C_{xe} = C_e$

$$T_{oe} = T_e + \frac{C_e^2}{2c_p}$$

$$T_e = T_{oe} - \frac{C_e^2}{2c_p} = 520K - \frac{(114.3\,m/s)^2}{2(1004\,J/kgK)} \qquad \boxed{T_e = 495.9\,K}$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{1.4(287\,J/kgK)(495.9K)} \qquad \boxed{a_e = 446.37\,m/s}$$

$$M_e = \frac{C_e}{a_e} = \frac{114.3\,m/s}{446.37\,m/s} \qquad \boxed{M_e = 0.26}$$

$$\frac{P_{oe}}{P_e} = \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{\gamma/\gamma-1} \qquad P_e = P_{oe}\left(1 + \frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1}$$

$$P_e = 600kPa\left(1 + \frac{1.4-1}{2}(.26)^2\right)^{\frac{-1.4}{1.4-1}}$$

$\boxed{P_e = 573.3\,kPa}$

$$\rho_e = \frac{P_e}{RT_e} = \frac{573300}{(287\,J/kgK)(495.9K)}$$

$\boxed{\rho_e = 4.03\,kg/m^3}$

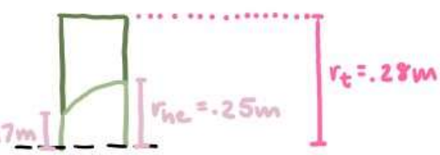
$r_{ti} = r_{te} = 0.28m$

$$\dot{m} = \rho C_{ex} A_e = \rho_e C_{ex} \pi (r_{te}^2 - r_{he}^2)$$

$$r_{he} = \sqrt{\frac{-\dot{m}}{\rho C_{ex}\pi} + r_{te}^2}$$

$$r_{he} = \sqrt{\frac{-20kg/s}{(4.03\,kg/m^3)(114.3\,m/s)\pi} + (0.28m)^2}$$

$\boxed{r_{he} = 0.25\,m}$ $\boxed{r_{te} = .28m}$



2. Mean-line station

$$\frac{P_{o3}}{P_{o1}} = 1.1$$

$\dot{m} = 20kg/s$	$U_t = 259.2m/s$
$T_{oi} = 288K$	$r_{hi} = .17m$
$T_i = 281.49K$	$r_{he} = .25m$
$a_i = 336.31m/s$	$r_t = .28m$
$M_i = .34$	$\gamma = 1.4$
$C_i = 114.3m/s$	$R = 287\,J/kgK$
$P_{oi} = 100kPa$	$c_p = 1004\,J/kgK$
$P_i = 92.31kPa$	$K = .6$
$\rho_i = 1.14kg/m^3$	$\eta_p = .92$
$T_{oe} = 520K$	
$T_e = 495.9K$	
$a_e = 446.37m/s$	
$M_e = .26$	
$C_e = 114.3m/s$	
$P_{oe} = 600kPa$	
$P_e = 573.3kPa$	
$\rho_e = 4.03kg/m^3$	

$\underline{\gamma-1}$

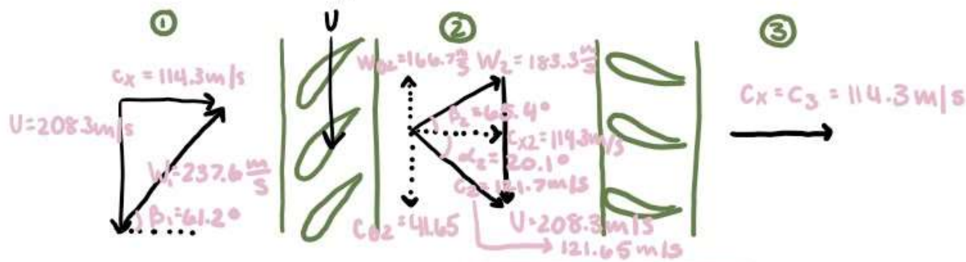
$P_e = 573.3\,kPa$
$\rho_e = 4.03\,kg/m^3$

- $T_{03} = ?$ $\quad \eta_c = \dfrac{\left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03}}{T_{01}} - 1}$ $\qquad T_{03} = \left(\dfrac{\left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} + 1\right) T_{01}$

$$T_{03} = \left(\frac{(1.1)^{\frac{1.4-1}{1.4}} - 1}{.92} + 1\right)(288K)$$

$\boxed{T_{03} = 296.64}$

- Velocity Triangles:



$\omega = \dfrac{U_t}{r_t} = \dfrac{259.2\,m/s}{.28\,m}$ $\qquad \boxed{\omega = 925.7\,rad/s}$

$r_m = \dfrac{(r_t + r_h)_i}{2} = \dfrac{(.28m + .17m)}{2}$ $\qquad \boxed{r_m = 0.225\,m}$

$U_m = \omega r_m = (925.7\,rad/s)(.225\,m)$ $\qquad \boxed{U_m = 208.3\,m/s}$

① $W_1 = \sqrt{U^2 + C_{x1}^2} = \sqrt{(208.3\,m/s)^2 + (114.3\,m/s)^2}$

$\boxed{W_1 = 237.6\,m/s}$ $\qquad \alpha_1 = 0°$ $\qquad$ * $U_1 = U_2$ for axial flow machines

$\beta_1 = \cos^{-1}\left(\dfrac{114.3\,m/s}{237.6\,m/s}\right)$ $\qquad \boxed{\beta_1 = 61.2°}$

② Euler Turbine: $\dot{W}_s = \dot{m}U(C_{\theta 2} - \cancel{C_{\theta 1}})$

Energy Eq: $\dot{W}_s = \dot{m}(h_{02} - h_{01}) = \dot{m}c_p(T_{02} - T_{01})$ $\qquad T_{0i} = T_{01} = 288K$

$\cancel{\dot{m}}U C_{\theta 2} = \cancel{\dot{m}} c_p (T_{03} - T_{01})$ $\qquad C_{\theta 2} = \dfrac{c_p(T_{03} - T_{01})}{U}$

$C_{\theta 2} = \dfrac{1004\,J/kgK\,(296.6K - 288K)}{208.3\,m/s}$ $\qquad \boxed{C_{\theta 2} = 41.65\,m/s}$

$C_2 = \sqrt{(41.65\,m/s)^2 + (114.3\,m/s)^2}$ $\qquad \boxed{C_2 = 121.7\,m/s}$

$\alpha_2 = \cos^{-1}\left(\dfrac{114.3\,m/s}{121.7\,m/s}\right)$ $\quad \alpha_2 = 20.1°$

$\dot{W}_s = \dot{m}c_p(T_{03} - T_{01}) = (20\,kg/s)(1004\,J/kg\,K)(296.6K - 288K)$

$\boxed{\dot{W}_s = 172688\,J}$

$\tan\beta_2 = \dfrac{\dot{W}_s}{\dot{m}UC_x} + \tan\beta_1$ $\qquad \beta_1 = \tan^{-1}\left(\dfrac{172688\,J}{(20\,\frac{kg}{s})(208.3\,\frac{m}{s})(114.3\,m/s)} + \tan(61.2°)\right)$

$\boxed{\beta_2 = 65.4°}$

$W_{\theta 2} = U - C_{\theta 2} = 208.3\,\frac{m}{s} - 41.65\,\frac{m}{s}$ $\qquad \boxed{W_{\theta 2} = 166.7\,m/s}$

$W_2 = \dfrac{W_{\theta 2}}{\sin\beta_2} = \dfrac{166.7\,m/s}{\sin(65.4°)}$ $\qquad \boxed{W_2 = 183.3\,m/s}$

- De-Haller Number:

$W_2 \quad 183.3\,m/s$ $\qquad \boxed{D_r = 0.77 > .72}$

· De-Haller Number:

Rotor → $D_r = \frac{W_2}{W_1} = \frac{183.3\,m/s}{237.6\,m/s}$

$D_r = 0.77 > .72$ ∴ the rotor design is good

Stator → $D_s = \frac{C_3}{C_2} = \frac{114.3\,m/s}{121.7\,m/s}$

$D_s = 0.94 > .72$ ∴ the stator design is good

· Rotor Diffusion Factor:

$h/c = 3$

$N_b = 30$

$D_r \equiv 1 - \frac{W_2}{W_1} + \frac{|W_{\theta 2} - W_{\theta 1}|}{2\sigma_r W_1}$ $\quad \sigma = \frac{c}{s} \quad s = \frac{2\pi r}{N}$

$h = r_b - r_{h\ell} = .28m - .17m$ $\quad h = 0.11m$

$c = \frac{h}{3} = \frac{.11m}{3}$ $\quad c = 0.037m$

$\sigma = \frac{.037m}{.047m} = .785$

$s = \frac{2\pi(.225m)}{30}$ $\quad s = 0.047m$

$D_r = 1 - .77 + \frac{|166.7m/s - 208.3m/s|}{2(.785)(237.6m/s)}$

$D_r = .34 < .5$ ∴ AR & blade # is good

· Stator Diffusion Factor:

$h/c = 3$

$N = 20$

$s = .07m \quad \sigma = \frac{.037m}{.07m} \quad \sigma = .52$

$s = \frac{2\pi(.225m)}{20}$

$D_s = 1 - \frac{C_3}{C_2} + \frac{|C_{\theta 3} - C_{\theta 1}|}{2\sigma C_2}$

$D_s = 1 - .94 + \frac{\cancel{|0|}}{2(.52)(121.7\,m/s)}$ $\quad D_s = .06 < .5$ ∴ AR & vane # is good

· Stage Efficiency? Is $\eta_c = \eta_p$ a good assumption?

$\bar{\omega}_r = \bar{\omega}_s = .03$

$\bar{\omega}_r = \frac{(P_{02} - P_{01})_{rel}}{\frac{1}{2}\rho W_1^2}$ $\quad \frac{T_{03}}{T_{01}} = \frac{296.64K}{288K} = \frac{T_{03}}{T_{01}} = 1.03$

$C_p T_{02} = C_p T_2 + \frac{C_2^2}{2}$

$\bar{\omega}_s = \frac{P_{03} - P_{01}}{\frac{1}{2}\rho C_2^2}$ $\quad T_2 = T_{02} - \frac{C_2^2}{2c_p} = 296.64K - \frac{(121.7m/s)^2}{2(1004J/kgK)}$

$T_2 = 289.26K$

$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4(287J/kgK)(289.26K)}$

$a_2 = 340.9\,m/s$ $\quad M_2 = \frac{C_2}{a_2} = \frac{121.7m/s}{340.9m/s}$ $\quad M_2 = .36$

$T_1 = T_{01} - \frac{C_1^2}{2C_p}$ $\quad M_{2,r} = \frac{W_2}{a_2} = \frac{183.3m/s}{340.9m/s}$ $\quad M_{2,r} = .57$

$T_1 = 288K - \frac{(114.3\,m/s)^2}{2(1004J/kgK)}$ $\quad T_1 = 281.5\,m/s$

$M_1 = \frac{C_1}{a_1} = \frac{114.3m/s}{336.3m/s}$ $\quad M_1 = .34$

$a_1 = \sqrt{\gamma R T_1}$

$a_1 = \sqrt{1.4(287J/kgK)(281.5m/s)}$

$M_{1,r} = \frac{W_1}{a} = \frac{237.6m/s}{336.3m/s}$

$a_1 = 336.3m/s$

$$M_{1,r} = \frac{W_1}{a_1} = \frac{287.6\,m/s}{336.3\,m/s}$$

$$a_1 = \sqrt{\gamma R T_1}$$
$$a_1 = \sqrt{1.4(287\,J/kgK)(281.5\,m/s)}$$
$$a_1 = 336.3\,m/s$$

$$M_{1,r} = 0.71$$

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_1 = P_{01}\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\gamma/\gamma-1} = 100kPa\left(1 + \frac{1.4-1}{2}(.34)^2\right)^{\frac{-1.4}{1.4-1}}$$

$$P_1 = 92.3\,kPa \qquad \rho_1 = \frac{P_1}{RT_1} = \frac{92300Pa}{(287\frac{J}{kgK})(281.5\frac{m}{s})} \qquad \rho_1 = 1.14\frac{kg}{m^3}$$

$$P_{01,r} = P_1\left(1 + \frac{\gamma-1}{2} M_{1r}^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{01,r} = 92.3kPa\left(1 + \frac{1.4-1}{2}(.71)^2\right)^{\frac{1.4}{1.4-1}} \qquad P_{01,r} = 129.2\,kPa$$

$$\bar{\omega}_r = \frac{P_{01,r} - P_{02,r}}{\frac{1}{2}(\rho_1 W_1^2)} \qquad -\bar{\omega}_r .5\rho_1 W_1^2 + P_{01,r} = P_{02,r}$$

$$P_{02,r} = 129200Pa - .03(.5)(1.14\tfrac{kg}{m^3})(287.6\,m/s)$$

$$P_{02,r} = 129196\,Pa$$

$$P_2 = P_{02,r}\left(1 + \frac{\gamma-1}{2} M_{2,r}^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$P_2 = 129196\,Pa\left(1 + \frac{1.4-1}{2}(.57)^2\right)^{\frac{-1.4}{1.4-1}}$$

$$P_2 = 103646\,Pa \qquad \frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{02} = P_2\left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{02} = 103646Pa\left(1 + \frac{1.4-1}{2}(.36)^2\right) \wedge \left(\frac{1.4}{1.4-1}\right)$$

$$P_{02} = 113358\,Pa \qquad \frac{P_{02}}{P_{01}} = \frac{113358\,Pa}{100{,}000\,Pa} \qquad \frac{P_{02}}{P_{01}} = 1.13$$

$$\bar{\omega}_s = \frac{P_{02} - P_{03}}{.5\rho_2 C_2^2} \qquad \rho_2 = \frac{P_2}{RT_2} = \frac{103646\,Pa}{(287\,J/kgK)(289.26\,K)}$$

$$\rho_2 = 1.25\,kg/m^3$$

$$P_{03} = P_{02} - \bar{\omega}_s .5\rho_2 C_2^2$$

$$P_{03} = 113358Pa - .03(.5)(1.25\tfrac{kg}{m^3})(121.7\,m/s)^2$$

$$P_{03} = 113080\,Pa \qquad \frac{P_{03}}{P_{02}} = \frac{113080Pa}{113358Pa}$$

$$\frac{P_{03}}{P_{02}} = 0.998 \qquad \frac{P_{03}}{P_{01}} = \left(\frac{P_{02}}{P_{01}}\right)\left(\frac{P_{03}}{P_{02}}\right) = (1.13)(.998)$$

$$\frac{P_{03}}{P_{01}} = 1.1 \qquad \frac{T_{03}}{T_{02}} = 1.03$$

$$\eta_c = \frac{\left[\left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\left(\frac{T_{03}}{T_{01}} - 1\right)} = \frac{(1.1)^{\frac{1.4-1}{1.4}} - 1}{1.03 - 1}$$

$\eta_c = 0.92 = \eta_p$ ∴ using polytropic efficiency was a good assumption

· find δ^* & δ^*_s

efficiency was a good assumption

· find δ^*_r & δ^*_s

$\delta^* = m\theta\left(\frac{s}{c}\right)^n$ $n = .5$ $\frac{a}{c} = .5$ $\beta_2 = 65.4°$ $\beta_1 = 61.2°$ $\alpha_3 = 0°$

$m_r = .23\left(\frac{2a}{c}\right)^2 + \frac{\beta_2}{500}$ $m_r = .23(1)^2 + (65.4°/500)$ $m_r = .36$

$m_s = .23\left(\frac{2a}{c}\right)^2 + \frac{\alpha_3}{500}$ $m_s = .23(1)^2 + 0 = .23$

$i = 0 \longrightarrow K_1 = \beta_1$ $\delta^* = \beta_2 - K_2$

$\theta = K_1 - K_2 = \beta_1 - K_2$ $\beta_1 - \beta_2 = \theta - \delta^*$ $\frac{s}{c_r} = \frac{1}{.785} = 1.27$

$\beta_1 - \beta_2 = \theta - m\theta\left(\frac{s}{c}\right)^n$ $\frac{s}{c_s} = \frac{1}{.125} = 8$

$\beta_1 - \beta_2 = \theta\left(1 - m\left(\frac{s}{c}\right)^n\right)$

$\theta = \dfrac{\beta_1 - \beta_2}{1 - m\left(\frac{s}{c}\right)^n}$ $\alpha_2 = 20.1°$ $\alpha_3 = 0°$

$\theta_r = \dfrac{61.2° - 65.4°}{1 - .36(1.27)^{.5}}$ $\theta_r = -7.07°$

$\theta_s = \dfrac{\alpha_3 - \alpha_2}{1 - m_s\left(\frac{s}{c}\right)^n} = \dfrac{-20.1°}{1 - .23(8)^{.5}}$ $\theta_s = -57.5°$

$\delta^*_r = -\beta_1 + \beta_2 + \theta_r = 61.2° + 65.4° - 7.07°$ $\delta^*_r = -2.87°$

$\delta^*_s = -\alpha_2 + \alpha_3 + \theta_s = -20.1° - 57.5°$ $\delta^*_s = -77.6°$

· Number of Stages: $\Delta T_{stage} = \text{const}$

$T_e - T_i = 495.9K - 281.49K = 214.41K$

$T_3 - T_1 = 290K - 281.49K = 8.6K$

$T_3 = T_{03} - \frac{C_3^2}{2C_p}$

$T_3 = 296.6K - \frac{114.3 m/s^2}{2(1004 J/kgK)}$

$T_3 = 290 K$

$\# \text{Stages} = \dfrac{T_e - T_i}{T_3 - T_1} = \dfrac{214.41K}{8.6K}$

$\# \text{Stages} = 24.9 \approx 25$ Stages

3. Create Velocity Triangles for tip & hub:

$C_x = \text{const} = 114.3 m/s$ $\omega = 925.7 \, rad/s$

Mean:
$r_m C_\theta = .225m \cdot 41.65 m/s$
$rC_\theta = 9.37$
$U_m = 208.3 m/s$

$rC_\theta = \text{const}$

Tip: $r = .28m$
$C_\theta = 9.37/.28$
$C_{\theta t} = 33.5 m/s$
$U_t = \omega \cdot r_t = 925.7 \frac{rad}{s}(.28m)$
$U_t = 259.2 m/s$

Hub: $r_h = .17m$
$C_\theta = 9.37/.17m$
$C_{\theta h} = 55 m/s$
$U_h = \omega r_h = 925.7 \frac{rad}{s}(.17m)$
$U_h = 159.4 m/s$

$C_{\theta t}: 33.5\ m/s$

$U_t = \omega \cdot r_t = 925.7 \frac{rad}{s}(.28m)$

$U_t: 259.2\ m/s$

$W_{\theta 2} = U_t - C_\theta = 259.2 \frac{m}{s} - 33.5 \frac{m}{s}$

$W_{\theta 2} = 225.7\ m/s$

$W_2 = \sqrt{225.7 m/s^2 + 114.3 m/s^2}$

$W_2: 253\ m/s$

$\beta_2 = \sin^{-1}(225.7/253)$

$\beta_2 = 63.1°$

$C_2 = \sqrt{33.5 \frac{m}{s}^2 + 114.3 \frac{m}{s}^2}$

$C_2 = 119.1\ m/s$

$\alpha_2 = \tan^{-1}(33.5/114.3)$

$\alpha_2 = 16.3°$

$W_1 = \sqrt{259.2^2 + 114.3^2}$

$W_1 = 283.3\ m/s$

$\beta_1 = \cos^{-1}(114.3/283.3)$

$\beta_1 = 66.2°$

$C_{\theta h} = 55\ m/s$

$U_h = \omega r_h = 925.7 \frac{rad}{s}(.17m)$

$U_h = 159.4\ m/s$

$W_{\theta 2} = U_h - C_\theta = 159.4 \frac{m}{s} - 55 \frac{m}{s}$

$W_{\theta 2} = 104.4\ m/s$

$W_2 = \sqrt{104.4 m/s^2 + 55 m/s^2}$

$W_2 = 118\ m/s$

$\beta_2 = \sin^{-1}(104.4/118)$

$\beta_2 = 62.2°$

$C_2 = \sqrt{55 m/s^2 + 114.3 m/s^2}$

$C_2: 126.8\ m/s$

$\alpha_2 = \tan^{-1}(55/114.3)$

$\alpha_2 = 25.7°$

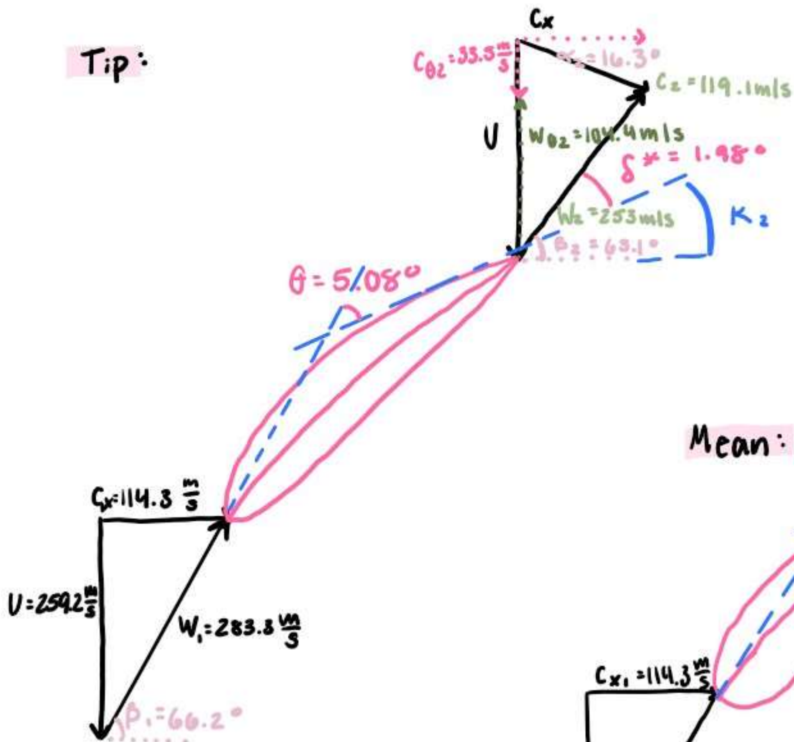
$W_1 = \sqrt{159.4^2 + 114.3^2}$

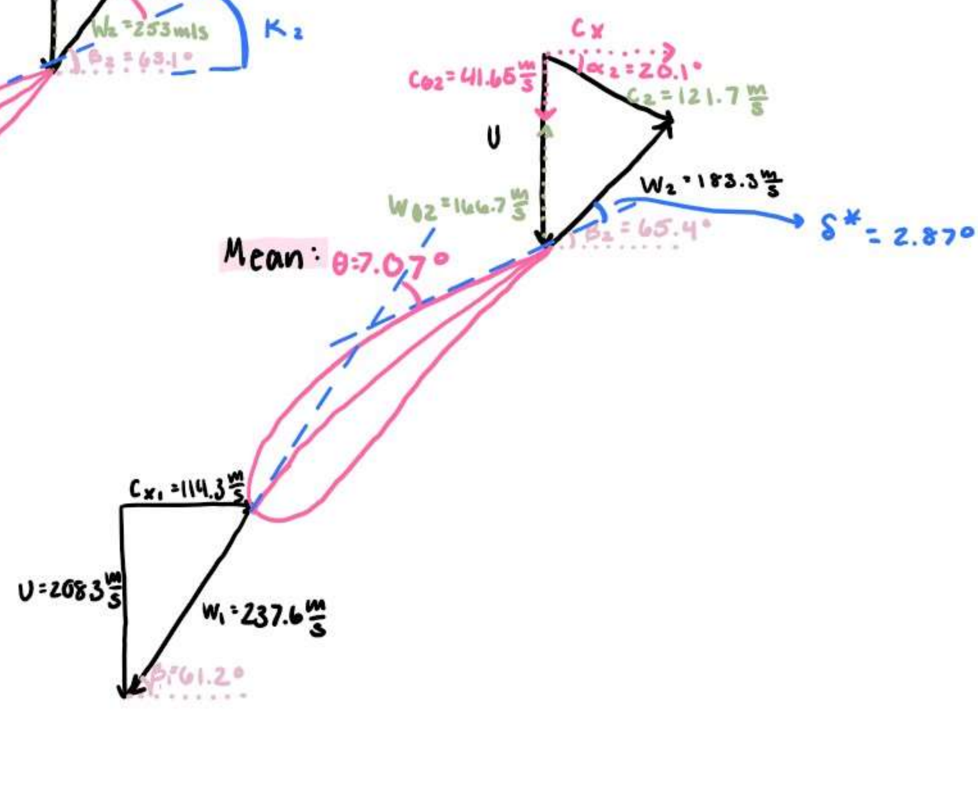
$W_1 = 196.1\ m/s$

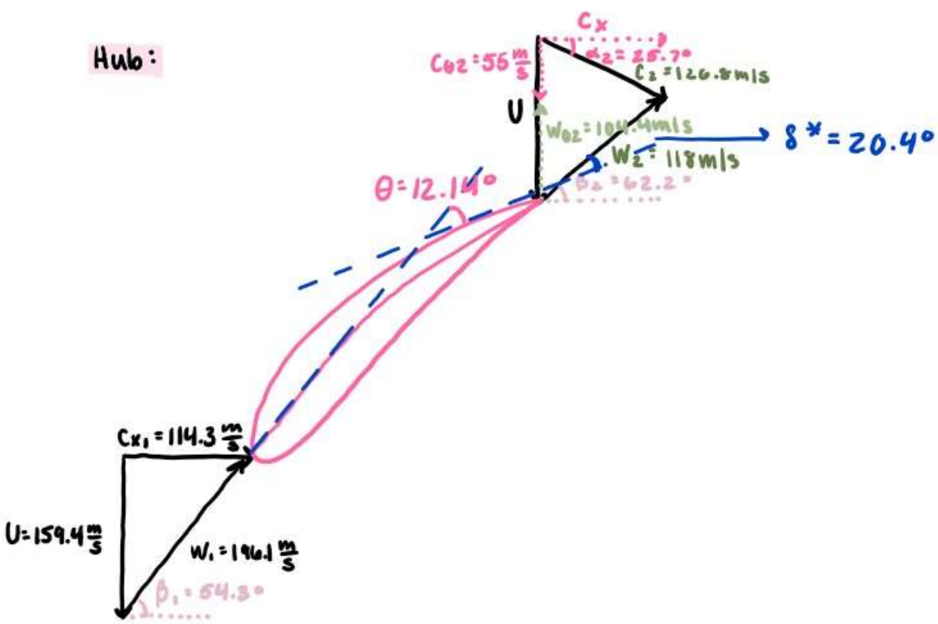
$\beta_1 = \cos^{-1}(114.3/196.1)$

$\beta_1 = 54.8°$

Tip:







4. Decide if rotor & stator are good designs

Tip:

$D_{Rt} = \frac{W_{2t}}{W_{1t}} = \frac{253 m/s}{283.3 m/s}$

$D_{Rt} = 0.89 > .72$

$D_{St} = \frac{C_{3t}}{C_{2t}} = \frac{114.3 m/s}{119.1 m/s}$

$D_{St} = 0.96 > .72$

Hub:

$D_{Rh} = \frac{W_{2h}}{W_{1h}} = \frac{118 \frac{m}{s}}{196.1 \frac{m}{s}}$

$D_{Rh} = .6 < .72$

$D_{Sh} = \frac{C_{3h}}{C_{2h}} = \frac{114.3 m/s}{126.8 m/s}$

$D_{Sh} = .90 > .72$

The tip rotor, tip stator & hub stator are good designs but the hub rotor is not

5. Deviation angle?

Tip:

$\alpha_3 = 0 \quad \alpha_2 = 16.3°$

$n = .5$

$a/c = .5$

$\beta_2 = 63.1°$

$\beta_1 = 66.2°$

$i = 0$

Rotor: $\delta^* = m\theta\left(\frac{s}{c}\right)^n$

$\frac{\frac{2\pi(.28)}{30}}{.037} = 1.58$

$m = .23\left(\frac{2a}{c}\right)^2 + \frac{\beta_2}{500}$

$\beta_1 - \beta_2 = \theta - \delta^*$

$66.2° - 63.1° = \theta - \left(.23 + \frac{63.1°}{500}\right)(1.58)^{.5}\theta$

$\frac{3.1°}{1 - .39} = \theta = 5.08°$

$\delta^* = -\beta_1 + \beta_2 + \theta$
$= -66.2° + 63.1° + 5.08°$

$\delta^*_{Rt} = 1.98°$

Hub:

$\alpha_3 = 0 \quad \alpha_2 = 25.7°$

$\beta_2 = 62.2°$

$\beta_1 = 54.3°$

$\frac{s}{c} = \frac{\frac{2\pi(.17)}{30}}{.087} = .96$

$54.3° - 62.2° = \theta - \left(.23 + \frac{62.2}{500}\right)(.96)$

$\theta = \frac{-7.9°}{1 - .35} \qquad \theta = -12.14°$

$\delta^* = -\beta_1 + \beta_2 + \theta$
$= -62.2 + 54.3 - 12.14$

$\delta^*_{Rh} = -20.04°$

Stator: $m = .23\left(\frac{2a}{c}\right)^2 + \frac{\alpha_2}{500}$

$\frac{s}{c} = \frac{\frac{2\pi(.28)}{20}}{.37} = .24$

$\frac{s}{c} = \frac{\frac{2\pi(.17)}{20}}{.37} = .144$

Stator: $m = .23\left(\frac{2a}{c}\right)^2 + \frac{\alpha_3}{500}$ $\frac{s}{c} = \frac{\frac{2\pi(.28)}{20}}{.37} = .24$

$m = .23$

$\alpha_2 - \alpha_3 = \theta - .23\left(\frac{s}{c}\right)^n \theta$

$\frac{16.3°}{1 - .23(.24)^{.5}} = \theta$ $\theta = 18.38°$

$\delta^* = -\alpha_2 + \theta = -16.3° + 18.38°$

$\delta^*_{st} = 2.1°$

$\frac{s}{c} = \frac{\frac{2\pi(.17)}{20}}{.37} = .144$

$\frac{25.70}{1 - .23(.144)^{.5}} = \theta = 28.1°$

$\delta^* = -\alpha_2 + \theta = -25.7 + 28.1°$

$\delta^*_{sh} = 2.4°$

7. Comment on Velocity Triangles:

As you move from hub to mean to tip, the rotational velocity increases. Since the axial velocity (Cx) is constant, the relative velocity increases as the rotational velocity increases. Additionally, the camber decreases from hub to tip.