Choosing Gear Ratios

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Outline

Where are we going today?

- Goals
- The general DE and Behavior

This may or may not be review for you. If it is, skim through, make sure you're comfy. If not, let this be a fun first foray!

Goals of a Mechanism

- Move/lift a load to a position
- Move/lift a load to a velocity
- Minimize electrical consumption
- Maximize precision/controllability

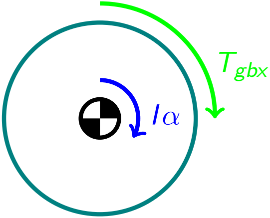
What is motor behavior like? (Flywheel Example)

Building on Motor Behavior

$$T_{motor} = T_{max}\frac{(\omega_{max} - \omega)}{\omega_{max}} \tag{1}$$

$$T_{gearbox} = GT_{motor} \tag{2}$$

A flywheel with torques acting on it can be modeled as:



$$\sum M = I\alpha \tag{3}$$

$$T_{gbx} = I\alpha_{wheel} \tag{4}$$

What is motor behavior like? (Continued)

$$\alpha_{wheel} = \frac{d\omega_{wheel}}{dt} \tag{5}$$

$$\omega_{motor} = G\omega_{wheel} \tag{6}$$

$$GT_{max}\frac{\omega_{max} - \omega}{\omega_{max}} = \frac{I}{G}\frac{d\omega}{dt} \tag{7}$$

This is a "differential equation" ($\frac{d\omega}{dt}$ and ω are in the same equation).

Apology

This math isn't too imperative to the overall point. If you prefer to just use the calculator and look at pretty plots and trends, that's alright. There be dragons (calculus) ahead.

The Mathy Approach

We can solve differential equations with math!

$$\text{let } B = \frac{G^2 T_{max}}{I} \tag{8}$$

$$\text{Substitute: } B\ \frac{\omega_{max} - \omega}{\omega_{max}} = \frac{d\omega}{dt} \tag{9}$$

$$\text{Separate and integrate: } \int B\ dt = \int \frac{\omega_{max}}{\omega_{max} - \omega} d\omega \tag{10}$$

$$\text{Solve integral: } Bt + C = -\omega_{max}\ ln[\omega_{max} - \omega] \tag{11}$$

$$\text{Solve for } \omega\text{: } \omega = \omega_{max} - C\ e^{-\frac{Bt}{\omega max}} \tag{12}$$

$$\text{Solve for C with initial condition } \omega(0) = 0 \rightarrow C = \omega_{max} \tag{13}$$

$$\omega = \omega_{max}\ [1 - e^{-\frac{G^2}{I}\frac{T_{max}}{\omega max}\frac{t}{}}] \tag{14}$$

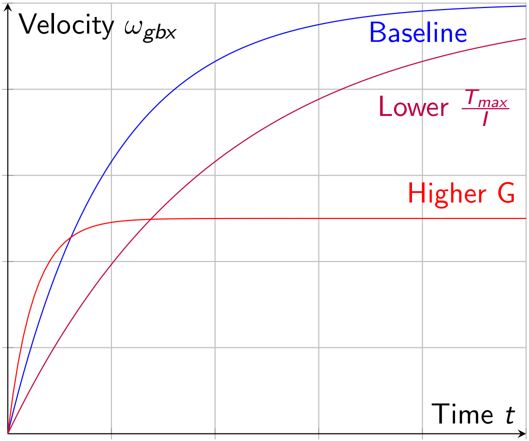
$$\omega_{gbx} = \frac{\omega_{max}}{G}\ [1 - e^{-\frac{G^2}{I}\frac{T_{max}}{\omega max}\frac{t}{}}] \tag{15}$$

Plotting the Math

Acceleration is the slope of the velocity curve.

Increasing the mass of the system (or decreasing power!) will decrease acceleration.

Increasing the gear ratio of the system will increase acceleration, but decrease maximum speed.



Warning

This assumes that there is no constant load, or friction. This behavior is generally true, but not exactly true.

Simulating (Actually... is easier!)