Choosing Gear Ratios

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Outline

Where are we going today?

- Goals
- The general DE and Behavior
- Solving the Calculus Way
- Simulating
- Making practical use of solutions/simulations

Here Be Dragons

This presentation contains a lot of math. It's mostly for your interest, and doesn't really reinforce the points abotu design too much, but it might be interesting to you.

Goals of a Mechanism

- Move/lift a load to a position
- Move/lift a load to a velocity
- Minimize electrical consumption
- Maximize precision/controllability

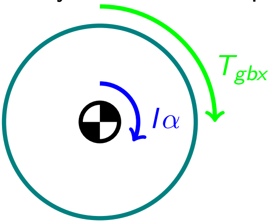
What is motor behavior like? (Flywheel Example)

Building on Motor Behavior

$$T_{motor} = T_{max} \frac{(\omega_{max} - \omega)}{\omega_{max}} \tag{1}$$

$$T_{gearbox} = G T_{motor} \tag{2}$$

A flywheel with torques acting on it can be modeled as:



$$\sum M = I\alpha \tag{3}$$

$$T_{gbx} = I\alpha_{wheel} \tag{4}$$

What is motor behavior like? (Continued)

$$\alpha_{wheel} = \frac{d\omega_{wheel}}{dt} \tag{5}$$

$$\omega_{motor} = G\omega_{wheel} \tag{6}$$

$$GT_{max}\frac{\omega_{max} - \omega}{\omega_{max}} = \frac{I}{G}\frac{d\omega}{dt} \tag{7}$$

This is a "differential equation" ($\frac{d\omega}{dt}$ and ω are in the same equation).

Apology

This math isn't too imperative to the overall point. If you prefer to just use the calculator and look at pretty plots and trends, that's alright. There be dragons (calculus) ahead.

The Mathy Approach

We can solve differential equations with math!

$$\text{let } B = \frac{G^2 T_{max}}{I} \tag{8}$$

$$\text{Substitute: } B \frac{\omega_{max} - \omega}{\omega_{max}} = \frac{d\omega}{dt} \tag{9}$$

$$\text{Separate and integrate: } \int B \ dt = \int \frac{\omega_{max}}{\omega_{max} - \omega} d\omega \tag{10}$$

$$\text{Solve integral: } Bt + C = -\omega_{max} \ ln[\omega_{max} - \omega] \tag{11}$$

$$\text{Solve for } \omega\text{: } \omega = \omega_{max} - C \ e^{-\frac{Bt}{\omega max}} \tag{12}$$

$$\text{Solve for C with initial condition } \omega(0) = 0 \rightarrow C = \omega_{max} \tag{13}$$

$$\omega = \omega_{max} \ [1 - e^{-\frac{G^2}{I}\frac{T_{max}}{\omega max}\frac{t}{}}] \tag{14}$$

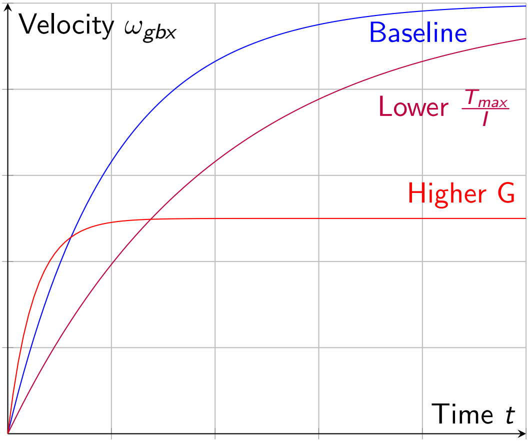
$$\omega_{gbx} = \frac{\omega_{max}}{G} \ [1 - e^{-\frac{G^2}{I}\frac{T_{max}}{\omega max}\frac{t}{}}] \tag{15}$$

Plotting the Math

Acceleration is the slope of the velocity curve.

Increasing the mass of the system (or decreasing power!) will decrease acceleration.

Increasing the gear ratio of the system will increase acceleration, but decrease maximum speed.



Warning

This assumes that there is no constant load, or friction. This behavior is generally true, but not exactly true.

Simulating (Actually... is easier!)

Newton's Method of numerically solving differential equations:

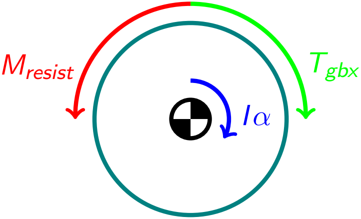
$$\frac{d}{dt}f(t) \approx \frac{\Delta f(t)}{\Delta t} \tag{16}$$

$$f(t_{i+1}) = f(t_i) + \frac{d}{dt}[f(t_i)]\ \Delta t \tag{17}$$

Another way of putting it... "the next value is the current value, plus the rate of change times the timestep of the simulation"
We just need to get an expression for the $\frac{d}{dt}f(t)$ we are interested in.
This process is sometimes called 'discretization' since we are taking a continuous field of time t and separating it into little Δt chunks.

Simulating a more complex example

Let's consider a flywheel with resistance M_{resist} on it.



$$\sum M = I\alpha \tag{18}$$

$$T_{gbx} - M_{resist} = I\alpha_{wheel} \tag{19}$$

$$\alpha_{wheel} = \frac{d}{dt}\omega_{wheel} \tag{20}$$

$$\frac{d}{dt}\omega_{wheel} = \frac{T_{gbx} - M_{resist}}{I} \tag{21}$$

$$\frac{d}{dt}\theta_{wheel} = \omega_{wheel} \tag{22}$$

This model in equations 20 and 21 actually contains all elements that we care about, if we consider M_{resist} could be a function of the gearbox velocity or other factors as well.

Using Simulators

Ari Meles-Braverman (AMB) has a nice little spreadsheet that builds hugely on the work of Jon Von Neun (JVN).
It actually takes into consideration even more than we've talked about here (like voltage drop).