Complete Reference Guide: 4-Wheel Skid-Steer Rover Kinematics, Dynamics, Control & Turning Strategy

1 Geometry & 4WD Skid-Steer Kinematics

1.1 Notation

- L = lateral distance between left & right wheel centers (track width)
- T = longitudinal distance between front & rear axles (wheelbase length)
- Wheels indexed: LF, LR, RF, RR (left-front, left-rear, right-front, right-rear)
- (v_L, v_R) = linear velocities of left-side and right-side virtual wheels (m/s) — usually desired commands
- (x, y, θ) = robot pose in world frame

1.2 Basic Body Kinematics

$$v = \frac{v_R + v_L}{2}, \qquad \omega = \frac{v_R - v_L}{L} \tag{1}$$

Pose rates:

$$\dot{x} = v\cos\theta, \quad \dot{y} = v\sin\theta, \quad \dot{\theta} = \omega \tag{2}$$

1.3 Wheel Radii About ICC

Each physical wheel travels a different radius about the ICC (instantaneous center of curvature). If ICC is at distance R from robot center, wheel radii are:

$$R_{RF} = R - \frac{L}{2} - \frac{T}{2} \tag{3}$$

$$R_{RR} = R - \frac{L}{2} + \frac{T}{2} \tag{4}$$

$$R_{LF} = R + \frac{L}{2} - \frac{T}{2} \tag{5}$$

$$R_{LR} = R + \frac{L}{2} + \frac{T}{2} \tag{6}$$

Wheel linear speeds (geometric, no slip):

$$v_i = \omega R_i \quad \text{for wheel } i \tag{7}$$

In practice, lateral slip happens; real v_i from encoders will differ.

2 Skid (Lateral Slip) Modeling

When wheels are fixed (no steering), lateral motion creates tangential scrub forces. Two useful metrics:

2.1 Slip Ratio (Longitudinal)

For traction wheels on deformable ground — skip if hard floor.

2.2 Side-Slip/Skid Angle (α)

For each wheel, the lateral force F_y grows with slip angle; approximate linear at small angles:

$$F_{y,i} \approx -C_{\alpha,i}\alpha_i \tag{8}$$

where C_α is cornering stiffness (depends on tire and terrain).

Practical implication: Turning torque $\gg$ ideal formula because you must overcome lateral friction at 4 contact patches.

3 Dynamics — Force/Torque Balance for 4WD

Let wheel radii $= r$. Motor torques $T_{RF}, T_{RR}, T_{LF}, T_{LR}$. Wheel forces at rim:

$$F_i = \frac{T_i}{r} \tag{9}$$

Total forward force:

$$F_x = \sum_i F_i - F_{\text{resist}} \tag{10}$$

Yaw moment:

$$M_z = \frac{L}{2}\left[(F_{RF} + F_{RR}) - (F_{LF} + F_{LR})\right] + \Delta_z \tag{11}$$

where Δ_z accounts for asymmetries (CG offset etc.).

Equations of motion:

$$m\dot{v} = F_x, \qquad I_z\dot{\omega} = M_z \tag{12}$$

4 Torque Required — Practical Formulas

4.1 Rolling Resistance

For baseline estimates on **flat ground**, to overcome rolling resistance and accelerate:

Rolling resistance force:

$$F_{\text{roll}} = \mu_r mg \tag{13}$$

Torque per wheel for steady speed & no turning:

$$T_{\text{wheel, steady}} = \frac{F_{\text{roll}}}{n_w} \cdot r \tag{14}$$

where $n_w = 4$ wheels (assuming equal sharing).

4.2 Turning In Place

Turning in place ($v_L = -v_R$) is worst-case scrub torque. Approximate extra torque per wheel from scrub:

Approximate *scrub friction torque* to rotate at low speed:

$$T_{\text{scrub,total}} \approx \mu_s mg \cdot \rho \tag{15}$$

where ρ is effective lever arm $\sim$ half the diagonal footprint (rough approx).

A more conservative estimate: treat all four wheels dragging laterally so:

$$T_{\text{scrub,total}} \approx 4 \cdot (\mu_s N_i) \cdot r_{\text{eff}} \tag{16}$$

with $N_i \approx \frac{mg}{4}$, $r_{\text{eff}} \sim$ half the wheelbase or some small radius depending how scrub transmits to motor.

4.3 Practical Calculation Recipe

1. Measure/know: m, r, L, T, g, μ_r (rolling), μ_s (static lateral), motor stall torque T_{stall}, gearbox ratio G, wheel radius r.
2. Required torque per wheel to move straight at steady speed (neglect accel):
$$T_{\text{req,straight}} \approx \frac{\mu_r m g}{n_w} \cdot r \tag{17}$$
3. For in-place rotation estimate (conservative):
$$T_{\text{req,turn}} \approx \frac{\mu_s m g}{2} \cdot \frac{L/2}{n_{\text{drive}}/2} \tag{18}$$

If T_{stall} (after gearing) $< T_{\text{req}}$, motors will stall or barely move.

5 Motor & Electrical Model

5.1 DC Motor Basics

$$T = K_t I \tag{19}$$
$$V = IR + K_e \omega_m \tag{20}$$

- Stall current: $I_{\text{stall}} = \frac{V}{R}$ at zero speed
- Stall torque: $T_{\text{stall}} = K_t I_{\text{stall}}$

6 Traction Control & Anti-Skid

- Monitor each wheel encoder. If one wheel's velocity $\gg$ other wheel on same side and command equal, that wheel is slipping — reduce torque to that wheel (open-loop traction control).
- Use current sensing: sudden low current + high speed = slip.
- Use IMU yaw rate vs commanded ω to detect slip and close the loop.

7 Example Numeric Walk-Through

Say:

- mass $m = 15$ kg, wheel radius $r = 0.05$ m, $\mu_r = 0.03$, 4 wheels

Compute rolling torque per wheel (steady):

$$F_{\text{roll}} = \mu_r m g = 0.03 \times 15 \times 9.81 \approx 4.41 \text{ N} \tag{21}$$

Torque per wheel:

$$T = \frac{F_{\text{roll}}}{4} \times r \approx \frac{4.41}{4} \times 0.05 \approx 0.055 \text{ N·m} \tag{22}$$

(This is tiny — acceleration and turn will need far more.)

If you want to rotate in place and $\mu_s \approx 0.6$ (rubber on concrete), very roughly:

$$\text{Yaw resisting moment} \approx \mu_s m g \cdot (L/2) \tag{23}$$

If $L = 0.3$ m:

$$M_{\text{resist}} \approx 0.6 \times 15 \times 9.81 \times 0.15 \approx 13.2 \text{ N·m} \tag{24}$$

Dividing by two driven sides then by two wheels per side gives torque per wheel $\sim 13.2/4 \approx 3.3$ N·m. That's orders of magnitude larger than steady rolling torque above — shows why turning-in-place needs strong motors/gearbox.

Part I
Practical Turning Strategy for 50 kg Rover

8 Understanding the Situation

8.1 Key Constraints

- **In-place turn (yaw in place) is impossible:** Required torque per wheel ≈ 22.3 N·m, motor rated torque $= 3.92$ N·m.
- **Minimum turning radius along a curve** (forward + yaw) ≈ 1.48 m.
- Track width $L = 0.52$ m, wheel radius $r = 0.065$–0.07 m.
- Motors are rated for 210 RPM output (≈ 1.54 m/s linear at $r = 0.07$ m).

Strategy: Turn while moving forward, not rotate in place.

9 Skid-Steer Kinematics for Turning

For a skid-steer vehicle:

$$v_L = v_c \left(1 - \frac{L}{2R}\right), \quad v_R = v_c \left(1 + \frac{L}{2R}\right) \tag{25}$$

Where:

- v_L, v_R = linear speeds of left & right wheels
- v_c = robot center forward speed
- L = track width
- R = turning radius

9.1 Example Calculation

For minimum radius $R = 1.48$ m and track $L = 0.52$ m:
Compute wheel speed ratio:

$$\frac{v_R}{v_L} = \frac{1 + L/(2R)}{1 - L/(2R)} = \frac{1 + 0.26/1.48}{1 - 0.26/1.48} \approx \frac{1 + 0.1757}{1 - 0.1757} = \frac{1.1757}{0.8243} \approx 1.426 \tag{26}$$

Result: The right wheels move $\sim 1.43\times$ faster than left wheels to achieve that turn radius.

10 Speed Limitation Analysis

10.1 Torque Requirement

Max wheel torque must not exceed rated torque. Torque required decreases with forward motion because wheels are not fighting each other:

$$T_{\text{req,curve}} = T_{\text{req,in-place}} \cdot \frac{v_R - v_L}{v_R + v_L} \tag{27}$$

Substitute the ratio:

$$\frac{v_R - v_L}{v_R + v_L} = \frac{1.426 - 1}{1.426 + 1} = \frac{0.426}{2.426} \approx 0.175 \tag{28}$$

Torque required per wheel:

$$T_{\text{req,curve}} \approx 22.31 \cdot 0.175 \approx 3.90 \text{ N·m} \tag{29}$$

Result: Matches rated torque perfectly. The rover can turn safely on this radius without overloading motors.

11 Implementation in Practice

11.1 Step 1: Set Motor Speeds Using Ratio

- Pick a forward center speed $v_c \leq$ maximum allowable linear speed.
- Compute left & right wheel speeds:

$$v_L = \frac{2v_c}{1 + v_R/v_L}, \quad v_R = (v_R/v_L) \cdot v_L \tag{30}$$

- Use PWM motor control to match these wheel speeds.

11.2 Step 2: Smooth Acceleration

- Gradually ramp speeds to avoid spikes in torque.
- Start turn slowly → avoid skidding or wheel slip.

11.3 Step 3: Verify with Low-Speed Test

- Use small trial v_c (e.g., 0.2 m/s)
- Observe turn radius
- Gradually increase until motors reach rated torque.

11.4 Step 4: Optional Improvements

- Slightly reduce radius by adding caster wheels (reduce lateral friction)
- Use trajectory planning: curve turns instead of abrupt point turns.

12 Summary

12.1 Key Takeaways

- **Cannot rotate in place** (torque too high)
- **Can turn on ∼1.48 m radius** by moving forward with wheel speed ratio ∼1.426
- **Forward motion reduces torque demand** from 22.31 N·m → 3.9 N·m per wheel
- **Safe with current motors** if you control speeds carefully

Parameter	**Value/Conclusion**
In-place rotation	Cannot rotate in place (torque too high)
Minimum turn radius	~ 1.48 m
Wheel speed ratio	~ 1.426 (right/left)
Torque reduction	From 22.31 N·m $\rightarrow$ 3.9 N·m per wheel
Safety	Safe with current motors if speeds controlled carefully

Table 1: Turning strategy summary for 50 kg rover