
Honours Degree of Bachelor of Science in Artificial Intelligence

Batch 21 - Level 2 (Semester 2)

CM 2320 - Mathematical Methods

Tutorial 2

1. Show that

a) $\displaystyle\int_{-a}^{a} e^{-t^2} dt = \sqrt{\pi}\ \ erf(a)$

b) $\displaystyle\int_{a}^{b} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}\left[erf(b) - erf(a)\right] = \frac{\sqrt{\pi}}{2}\left[erfc(a) - erfc(b)\right]$

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2. Show that $\dfrac{d}{dz} erf(z) = \dfrac{2}{\sqrt{\pi}} e^{-z^2}$.

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3. If X is a normal random variable, its probability density function is

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-m)^2/2\sigma^2},$$

where m is the mean value of X and σ^2 the variance. The probability that $X \leq y$ is defined by

$$P(X \leq y) = \int_{-\infty}^{y} p(x) dx.$$

a) Show that

$$P(X \leq y) = \frac{1}{2}\left[1 + \ erf\left(\frac{y-m}{\sqrt{2}\sigma}\right)\right].$$

b) What is the probability $P(X \leq y)$ in the limit $y \to \infty$?

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