Honours Degree of Bachelor of Science in Artificial Intelligence

Batch 22 - Level 2 (Semester 2)

CM 2320: Mathematical Methods

Chapter 1: Special Functions

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Special Functions

Learning Outcomes

By the end of this chapter, students will be able to;

1. identify the delta function.
2. identify the properties of delta function.ons.

Contents

1 Dirac Delta Function

The **delta function** $\delta(t)$ is often also referred to as the **Dirac delta function**, and defined as the derivative of $H(t)$ with respect to t. The function $H(t)$ is constant for $t > 0$ and $t < 0$, the delta function vanishes almost everywhere. But the function $H(t)$ jumps discontinuously at $t = 0$, and this implies that its derivative is infinite at this point. We therefore have

$$\delta(t) = \frac{d}{dt}H(t) = \begin{cases} 0, & t \neq 0, \\ \infty, & t = 0. \end{cases} \tag{1}$$

Delta function is sketched in the Figure 1.

The main property of the delta function is in the fact that it reaches infinity at a single point and is zero at any other point. Its most important property is that its integral is always one:

$$\int_{-\infty}^{\infty} \delta(t)dt = 1. \tag{2}$$

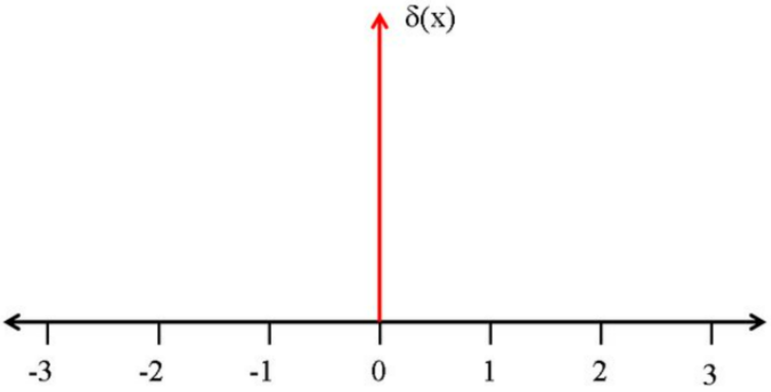

Figure 1: Graph of the Delta function.

1.1 Properties of Delta Function

The delta function $\delta(x)$ has the following properties:

(1) $\displaystyle\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$

(2) $\delta(t-a) = \dfrac{d}{dt}H(t-a) = \begin{cases} 0, & t \neq a, \\ \infty, & t = a. \end{cases}$

(3) $\displaystyle\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$

(4) $\displaystyle\int_{t_1}^{t_2} f(t)\delta(t-a)dt = \begin{cases} f(a), & \text{when } (t_1,t_2) \text{ includes } t=a, \\ 0, & \text{when } (t_1,t_2) \text{ excludes } t=a. \end{cases}$

(5) $\delta(t-a) = \delta(a-t)$

(6) $\delta(ct) = \dfrac{1}{|c|}\delta(t)$

(7) $\displaystyle\int_{-\infty}^{\infty} f(t)\delta'(t-a)dt = -f'(a)$

(8) $t\delta(t) = 0$

(9) $\delta(a^2-t^2) = (\delta(t-a)+\delta(t+a))/2|a|$