SCS 2112
Automata Theory

Regular Grammars

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What is a Grammar?

He Plays Basketball.

Language that accepts 0 and 1

- T -> 0 | 1
- N ->
- S -> S
- P ->

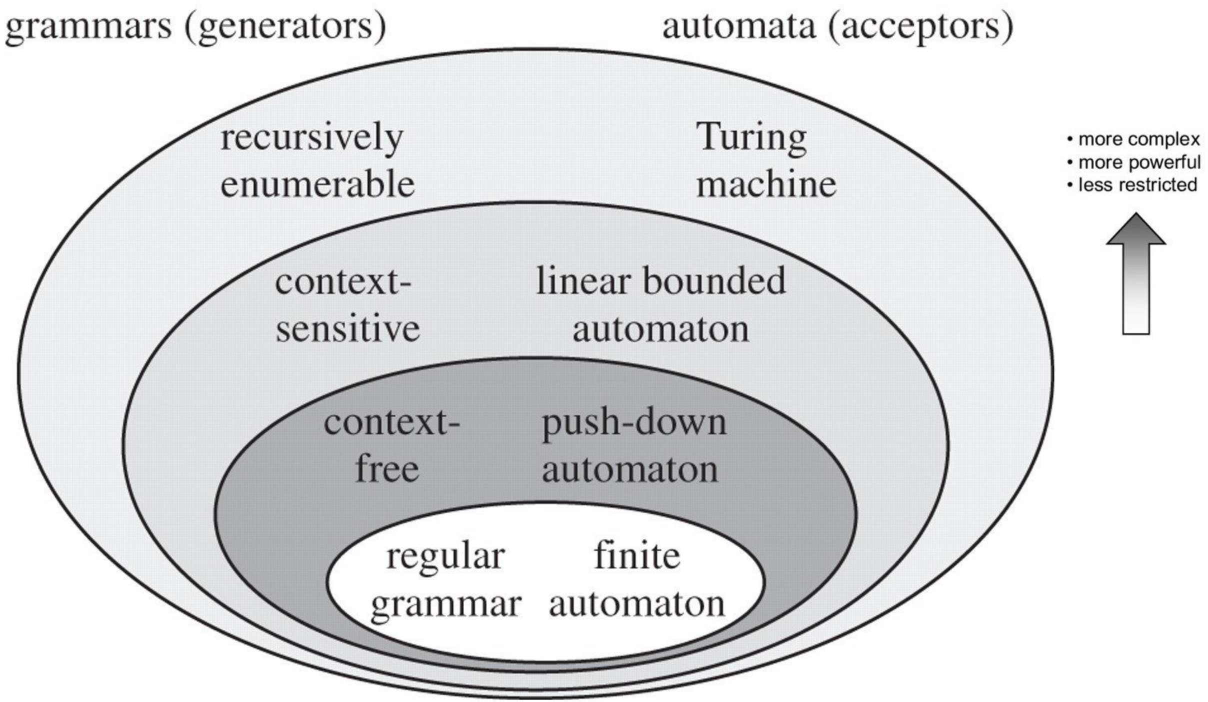
Grammars

- **Grammars are used to express languages.**
- **Definition of Grammar: G = (N,T,S,P)**
  - N : Set of variables (non terminals)
  - T : Set of terminal symbols
  - S : Start variable
  - P : Set of Production rules

A Simple Grammar

- Grammar
  - $N : \{S\}$
  - $T : \{a\}$
  - $S : S$
  - $P : S \longrightarrow aS \mid \varepsilon$
- Derivation of sentences
  - $S \longrightarrow aS \longrightarrow a$
  - $S \longrightarrow aS \longrightarrow aaS \longrightarrow aa$

Chomsky Hierarchy of Grammars



Linear Grammars

- Grammars with at most one variable at the right side of a production.
- Productions:
    - $S \rightarrow a$
    - $S \rightarrow \varepsilon$
    - $S \rightarrow uAv$

Exercise 01

- **G1**
  - $S \rightarrow aAb$
  - $A \rightarrow a \mid b \mid \varepsilon$
- G3
  - $S \rightarrow aBAb$
  - $A \rightarrow a \mid \varepsilon$
  - $B \rightarrow b \mid \varepsilon$
- **G2**
  - $S \rightarrow A$
  - $A \rightarrow aB \mid \varepsilon$
  - $B \rightarrow Ab$
- G4
  - $S \rightarrow A \mid bB$
  - $A \rightarrow bB \mid a \mid \varepsilon$
  - $B \rightarrow Ab$

https://forms.gle/fuhdJvAKt7JPzAGT8

Left and Right Linear Grammars

- **A grammar G = (N,T,S,P) is said to be right linear if all productions are of the form**
  - $A \rightarrow xB \mid x$

    where $A, B \in N$, $x \in T^*$
- **A grammar G = (N,T,S,P) is said to be left linear if all productions are of the form**
  - $A \rightarrow Bx \mid x$

    where $A, B \in N$, $x \in T^*$

Left Linear Grammars

- **Consider this rule from a left linear grammar**
  - A → Babc
- **Can that rule be used to recognize this string?**
  - abbabc
- **We need to check rule for B**
  - B → Cb | D
- **Now we need to check rules for C and D.**

Right Linear Grammars

- **Consider this rule from a right linear grammar**
  - A → abcB
- **Can that rule be used to recognize this string?**
  - abcabb
- **We immediately see that the first part of the sring “abc” matches the first part of the rule.**
- **Thus, the problem simplifies to this:**
  Can this rule B be used to recognize “abb” string?

Convert left linear grammar to right linear grammar

1. If the left linear grammar has a rule with the start symbol S on the right hand side, simply add this rule: $S_0 \rightarrow S$
2. If the left linear grammar has a rule $S \rightarrow p$, then make that a rule in the right linear grammar
3. If the left linear grammar has a rule $A \rightarrow p$, then add the following rule to the right linear grammar: $S \rightarrow pA$
4. If the left linear grammar has a rule $B \rightarrow Ap$, add the following rule to the right linear grammar: $A \rightarrow pB$
5. If the left linear grammar has a rule $S \rightarrow Ap$, then add the following rule to the right linear grammar: $A \rightarrow p$

Exercise 02

- **G1**
  - $S \rightarrow Aa$
  - $A \rightarrow ab$
- **G2**
  - $S \rightarrow Ab$
  - $S \rightarrow Sb$
  - $A \rightarrow Aa$
  - $A \rightarrow a$
- G3
  - $S \rightarrow Ab \mid Ba$
  - $A \rightarrow a \mid \varepsilon$
  - $B \rightarrow b \mid \varepsilon$
- G4
  - $S \rightarrow A \mid Bb$
  - $A \rightarrow Bb \mid a \mid \varepsilon$
  - $B \rightarrow Ab$

Regular Grammars

- Regular grammars generate regular languages.
- A regular grammar should be either right linear or left linear.

Finite Automata and Regular Grammar

- For every regular language there is a NFA.
- For every regular grammar there is regular grammar.
- We can convert regular grammar into NFA.

Exercise 03

- **G1**
  - $S \longrightarrow aA$
  - $A \longrightarrow ab$
- **G2**
  - $S \longrightarrow aA$
  - $A \longrightarrow bC$
  - $A \longrightarrow aA$
  - $C \longrightarrow bC$
  - $C \longrightarrow \varepsilon$