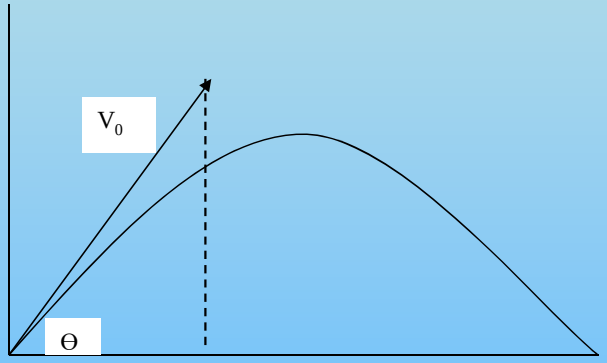
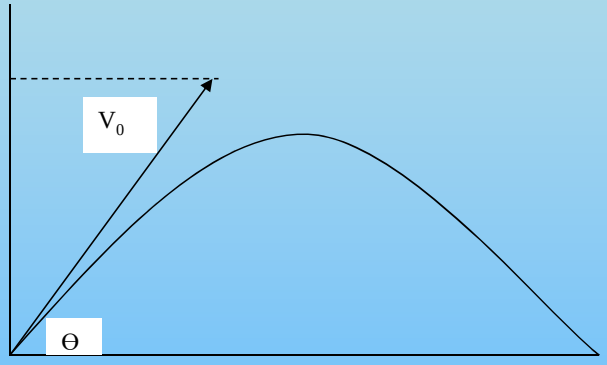
Projectile Motion

Derivation of the Projectile Equations

$V_x = \cos\Theta$



$$V_y = V_0 \sin \Theta - gt$$



$$h=\frac{v_0^2\sin^2\Theta}{2g}$$

$V^2 = V_0^2 + 2ax$

$0 = V_0^2 + 2ax$

$0 = V_0^2 + 2ah$

$\frac{-V_0^2}{2a} = h$

$\frac{V_0^2}{2g} = h$

$h=\frac{v_0^2\sin^2\Theta}{2g}$

$$T = \frac{2V_0 \sin\Theta}{g}$$

$v = v_0 + at$

$t = v/a$

$t = v/g$

$t = 2v/g$

$T = \frac{2V_0 \sin\Theta}{g}$

$$R=\frac{V_0^2 \sin 2\Theta}{g}$$

$X=VT$

$T=2V_0 \sin\Theta$

$V=V_0 \cos\Theta$

$X=2v_0 \sin\Theta \; V_0 \cos\Theta/g$

$$R=\frac{V_0^2 \sin 2\Theta}{g}$$

$x = (V_0 \cos\Theta)t$

$x = vt$

$V = V_0 \cos\Theta$

$X = (V_0 \cos\Theta)t$

$$Y=(V_0 \sin\Theta)t - \tfrac{1}{2} gt^2$$

$Y = vt + \tfrac{1}{2} at^2$

$Y = (V_0 \sin\Theta)t - \tfrac{1}{2} gt^2$

The big trajectory Equation

$y = (\tan\Theta)x - gX^2 / 2V_0^2\cos^2\Theta$

$X = vt + \frac{1}{2} at^2$

$Y = vt + \frac{1}{2} at^2$

$T = X/V_0\cos\Theta$

$X = V(X/V_0\cos\Theta) + \frac{1}{2} a[X/V_0\cos\Theta]^2$

$X = V(X/V_0\cos\Theta) - \frac{1}{2} g[X/V_0\cos\Theta]^2$

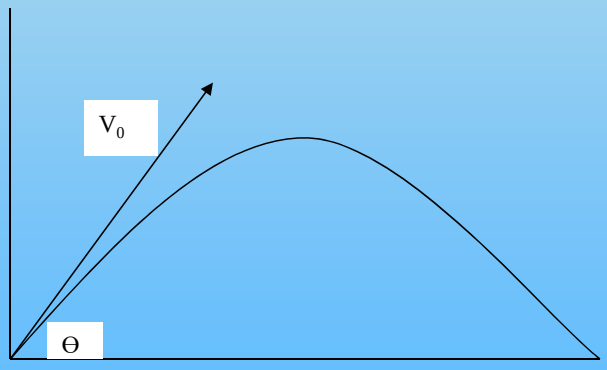
$V = V_0\sin\Theta$

$X = V_0\sin\Theta(X/V_0\cos\Theta) - \frac{1}{2} g[X/V_0\cos\Theta]^2$

$$X = (\tan\Theta)x - \frac{gx^2}{2(V_0^2\cos^2\Theta)}$$

Find the initial vertical and horizontal velocity.

The initial velocity is 30 m/s. The angle is 40º. What is the initial horizontal and vertical velocity?



Answers

Use the $V_x=\cos\Theta$ and $V_y=V_0\sin\Theta - gt$ to get

V_x = 23.0 m/s

V_y= 19.3 m/s

Height, Time and Range

The initial velocity is 30 m/s. The angle is 40º. What is the height the projectile attains, how long is it in the air and how far does it go?

Answers

Use $h=\frac{v_0^2 \sin^2 \Theta}{2g}$ to get 19.0 m for the height.

Use $T=\frac{2V_0 \sin \Theta}{g}$ to get 3.9 sec for the time

Use $R=\frac{V_0^2 \sin 2\Theta}{g}$ to get 90.4 m for the range.

Jumping Angles

Greta Greyhound is jumping hurdles. She is running a velocity of 18.3 m/s and the hurdles are .75 m high. At what angle must she leave the ground in order to clear the hurdles?

Answer

Use the height equation and solve for θ.

$$h = \frac{V_o^2 \sin^2 \theta}{2g}$$

$$\frac{\sqrt{2gh}}{V_o} = \sin\theta$$

$$\sin^{-1}\left(\frac{\sqrt{2gh}}{V_o}\right) = \theta$$

θ = 12.1 degrees

Go, Greta!!!

References

- Schaum's Outlines Applied Physics Fourth Edition Arthur Beiser, Ph.D. McGraw-Hill 2004 p 42.