Shooter Design and Calculations

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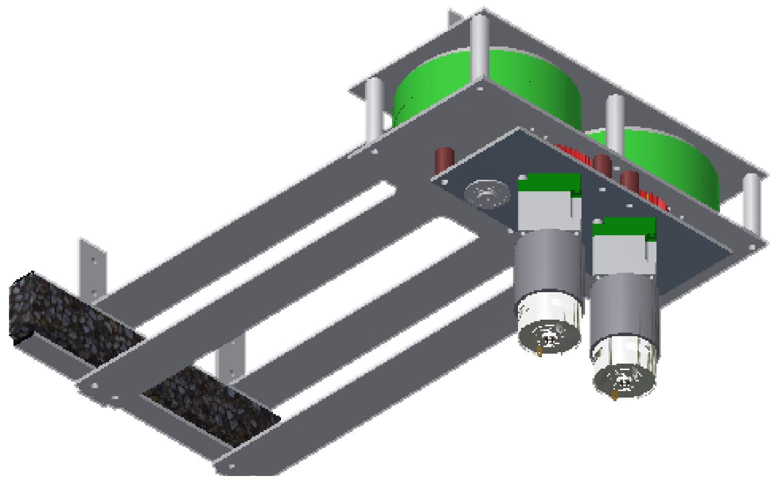


In the process of designing our shooter for this year's robot, we sought a wheel with enough friction on the edge of the Frisbee so it would not slip as it accelerates and launches. The speed on the outer edge of the Frisbee travels at twice the speed of the center. (See Figure S.1 and S.2) If the Frisbee does not slip, through conservation of momentum, we calculated the speed of wheel that shoots at our ideal speed.

Velocity of a Rolling Disk:

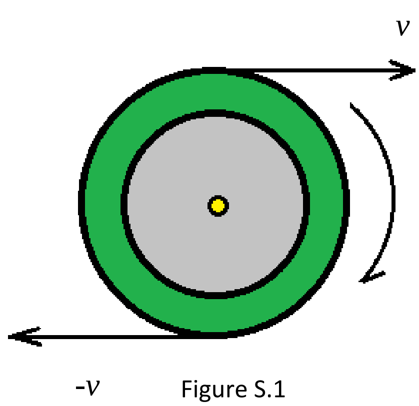


Figure S.1

Figure S.1
Assume a wheel is spinning in place about its center at velocity *v*. If one is standing at the yellow diamond, the top of wheel would appear to be moving at velocity *v.* In contrast, the bottom of the wheel would move at velocity –*v* for it is moving back-wards.

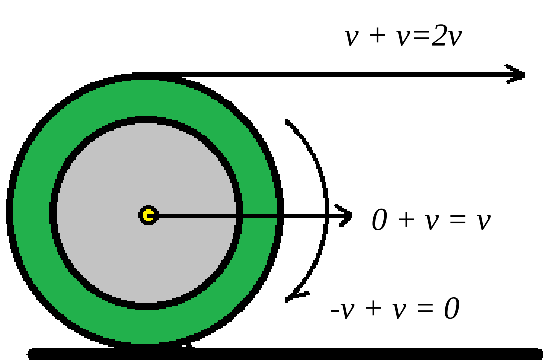


Figure S.2

Figure S.2
Assume the wheel is now rolling across a surface without slipping with velocity *v*. A vector of *v* is added to the speed components of the wheel, making the top of the wheel move at twice the speed relative to the center of the wheel.

Frisbee Efficiency and Recovery (Single-Wheel Shooter):

$$Efficiency = \frac{output}{input} = \frac{E_d}{E_{wi} - E_{wf}}$$

$$E_{wi} - E_{wf} = \frac{1}{2} k_w m_w (V_{wi}^2 - V_{wf}^2)$$

$$E_d = \frac{1}{2} m_d V_d^2 + \frac{1}{2} I_d \omega_d^2$$

$$E_d = \frac{1}{2} m_d V_d^2 + \frac{1}{2} k_d m_d V_d^2 = \frac{1}{2} V_d^2 (1 + k_d) m_d$$

$$E_d = \frac{1}{2} \frac{V_d^2}{4} (1 + k_d) m_d$$

$$\frac{E_d}{E_{wi} - E_{wf}} = \left(\frac{V_{wf}^2}{V_{wi}^2 - V_{wf}^2} \right) \frac{(1 + k_d) m_d}{4 k_w m_w}$$

substitute

$$\frac{V_{wi}}{V_{wf}} = 1 + \frac{(1 + k_d) m_d}{2 k_w m_w} = 1 + C \quad where\ C \equiv \frac{(1 + k_d)}{2 k_w m_w}$$

$$\frac{E_d}{E_{wi} - E_{wf}} = \frac{1}{\left(\frac{V_{wi}}{V_{wf}} \right)^2 - 1} = \frac{1}{\left(\frac{V_{wi}}{V_{wf}} \right) - 1} \cdot \frac{1}{\left(\frac{V_{wi}}{V_{wf}} \right) + 1}$$

$$\frac{E_d}{E_{wi} - E_{wf}} = \frac{4 k_w m_w}{4 k_w m_w + (1 + k_d) m_d} \cdot \frac{(1 + k_d) m_d}{4 k_w m_w} = \frac{(1 + k_d) m_d}{4 k_w m_w + (1 + k_d) m_d} = \frac{C}{2 + C}$$

An important factor in a shooter is efficiency and recovery after shooting. During our tests, we discovered that the wheel's small moment of inertia made the wheel's speed drop significantly, losing about 60 to 70 percent of its energy. This is a serious problem if we want to shoot Frisbees quickly. So, by adding a flywheel and a second wheel to the original shooting wheel, we increased the moment of inertia. With these adjustments, our efficiency improved by more than a factor of two. In combination with our closed-loop control, we can accurately shoot 4 Frisbees in less than 4 seconds.

Setting Speeds for Tandem Shooter:

Ideally, we want each of our two identical wheels to pass off exactly $1/2$ of the disc's total energy. As the Frisbee pass through each shooter wheel, the traction force is limited to $\mu F_{squeeze}$, where μ is the coefficient of friction and $F_{squeeze}$ is the force of compression on the Frisbee. The maximum energy we can transfer over a contact angle ϑ is $\Delta E_{max} = r_d \theta \mu F_{squeeze}$. So for each wheel, we can transfer ΔE_{max}.

$$E = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$

$$E = \frac{1}{2} m V^2 + \frac{1}{2} k m V^2$$

Where $0.5 \leq k \leq 1$

For our Frisbee, $k \approx 0.71$

So, the energy for the first wheel, is $$E_{d1} = \frac{1}{2} m (1 + k) V_1^2 \qquad (1)$$

We can apply max force on the second wheel, so that

$$E_{d2} = E_{d1} + \mu F_{squeeze} = 2 E_{d1} \qquad (2)$$

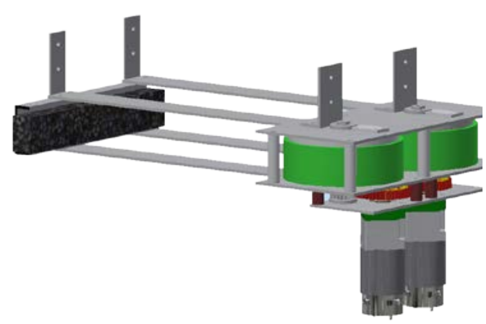
$$E_{d2} = \frac{1}{2} m (1 + k) V_2^2 \qquad (3)$$

So, from (2), (1), and (3), we have

$$(2) \frac{1}{2} m (1 + k_d) V_1^2 = \frac{1}{2} m (1 + k_d) V_2^2$$

$$2 V_1^2 = V_2^2$$

$$\frac{V_2}{V_1} = \sqrt{2} \approx 1.414$$



Determining Initial Speeds

Having determined the final speeds, we need to calculate the initial speed when the Frisbee enters each wheel. We have already deduced the speed drop of a single wheel (see Frisbee Efficiency), however, since the Frisbee enters the second shooter wheel with an initial speed, a considerable amount of energy can be reduced to elicit the same result.

Using the conservation of linear and angular momentum, we were able to figure out our ideal ratio of our inner wheel speed to outer wheel speed. For our design, the relative initial speed is 1.098, which is less than we excepted when we started this analysis.

$$\frac{I_w \omega_{w_2 i}}{R_w} + m_d V_{d_1} + \frac{I_d \omega_{d_1}}{R_d} = \frac{I_w \omega_{w_2 f}}{R_w} + m_d V_{d_2} + \frac{I_d \omega_{d_2}}{R_d}$$

$$\frac{I_w V_{w_2 i}}{R_w^2} + m_d V_{d_1} + \frac{I_d V_{d_1}}{R_d^2} = \frac{I_w R_{w_2 f}}{R_w^2} + m_d V_{d_2} + \frac{I_d V_{d_2}}{R_d^2}$$

$$k_w m_w V_{w_2 i} + m_d V_{d_1} + k_d m_d V_{d_1} = k_w m_w V_{w_2 f} + m_d V_{d_2} + k_d m_d V_{d_2}$$

$$k_w m_w V_{w_2 i} + m_d V_{d_1}(1 + k_d) = k_w m_w V_{w_2 f} + m_d V_{d_2}(1 + k_d)$$

$$k_w m_w V_{w_2 i} = k_w m_w V_{w_2 f} + m_d (1 + k_d)\left(V_{d_2} - V_{d_1}\right)$$

$$\textit{substitute} \quad \left(V_{d_2} - V_{d_1}\right) = (\sqrt{2} - 1)V_{d_1} = \left(1 - \frac{1}{\sqrt{2}}\right)V_{d_2}$$

$$k_w m_w V_{w_2 i} = 2 k_w m_w V_{d_2} + m_d (1 + k_d)\left(1 - \frac{1}{\sqrt{2}}\right)V_{d_2}$$

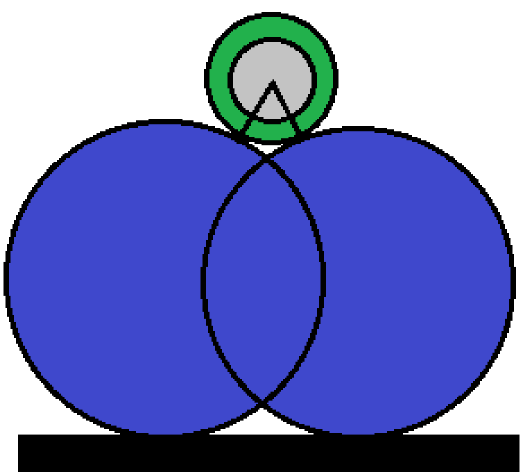
$$k_w m_w V_{w_2 i} = k_w m_w V_{w_2 f} + m_d (1 + k_d)\left(\frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}}\right)V_{w_2 f}$$

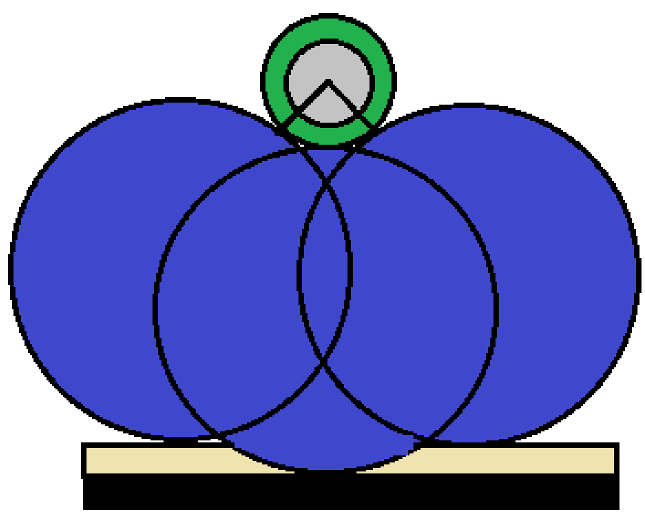
$$k_w m_w V_{w_2 i} = V_{w_2 f}\left(k_w m_w + m_d (1 + k_d)\left(\frac{2 - \sqrt{2}}{4}\right)\right)$$

$$\frac{V_{w_2 f}}{V_{w_2 i}} = \frac{k_w m_w}{k_w m_w + m_d (1 + k_d)\left(\frac{2 - \sqrt{2}}{4}\right)}$$

$$\frac{V_{w_2 i}}{V_{w_2 f}} = 1 + \left(\frac{2 - \sqrt{2}}{4}\right)\left[\frac{m_d (1 + k_d)}{k_w m_w}\right]$$

Adding Foam





We have also added a layer of foam opposite of the shooter wheel. This increases the contact angle, making it easier to bring the Frisbee up to speed when it exits the respective wheel. Also, this applies a lower constant force over a larger angle, a force that is dependent on the strength of the Frisbee.

With this found information, we were able to tune our shooter to throw Frisbees of 80 feet.