Control System Design

* 2012 Shooter Roller Speed Control System Analysis and Design

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# Introduction

## Summary

Proportion close loop speed control system on our 2012 shooter cannot keep calibrated wheel angular speed from changing at steady state if resistant torque or any system parameters have been changed. As results, shooting distance will vary. To avoid this problem, a proper compensator design of control system is required.

Using the shooter as an example, this paper have modeled and analyzed shooter speed control systems with following control loop

* open loop (Chapter 0)
* proportion close loop (Chapter 3)
* integration close loop (Chapter 4)
* proportion plus integration close loop (Chapter 5)

Mathematical expressions of each control system can explain why constant angular velocity can or cannot be maintained in steady state. These mathematical expressions are summarized in Table 1.

When closed form solutions of these mathematical expressions can be achieved, these solutions can be used for predicting control system performance or design. 2012 shooter, fortunately, is simple enough to have closed form solutions, particularly for steady-state angular velocity.

After modeling and analyzing a control system, more design and tuning work need to be done. Chapter 6 outlines a general methodology for control system design and realization. This methodology has considered other factors such as control loop bandwidth, sample rate, etc., which will affect control system design.

PID compensator (controller) is widely used in control system. But, selecting a proper form of PID compensator or design a more complicated compensator requires advanced control system knowledge which is not in the scope of this paper. Therefore, Chapter 6 only presents compensator design/selection results without further explanation.

The design approach presented in Chapter 6 assumes that a PID routine is available in software or in component firmware. If not, Chapter 7 presents a practical way on how to derive formula for PID compensator.

If you would like to understand why steady state angular velocity of shooter can or cannot be kept constant, please read Chapter2, 3, 4, 5. If you are interested in programming and tuning shooter performance, Chapter2, 6, and 7 covers what you want to know.

In conclusion, if you want to design a high performance control system for shooter, driving train or other system, you have to

1. Know original system to be controlled. Modeling cannot be skipped.
2. Design a proper compensator and control system to match the original system and meet design spec.
3. Since you know your system and your design, the tuning of final system performance become definitive though there are many practical issues waiting for you to solve.

## Assumptions

*Speed Controller Linearization*

Speed controller used on our robot has nonlinear characteristics for increasing and decreasing output speed of actuator (motor and its transmission). This nonlinear characteristic can be minimized by adding sensing device and control program. In this paper, we assume that such linearization process is in place. So, we treat speed controller to be a linear system.

*Code Simplification*

Details of most of SW code implementation are simplified in this analysis since they do not affect carrying out the concept of speed control system. Some other details or parameters such as sample rate are critical for a control system. These parameters will be discussed when needed.

## Requirements of Knowledge

Basic understanding of Newton’s Law and concept of integration and differentiation are required for self-guided reading. Also, be familiar with the shooter components. Control System Mini-Series 1 – 3 will help you understand basic concepts. Otherwise, please go through this paper with mentors or your parents.

Table 1 Summary of Characteristics of Speed Control Systems

|  |  |  |  |
| --- | --- | --- | --- |
| Speed Control System | Governing Differential Equation | Steady State Angular Velocity () | Comments |
| Open loop (original system) | Eq. 2‑8  General Form | Eq. 2‑10 | * 1st order system (inertia system) * Steady-state wheel speed varies with changing of system parameters and resistant force. |
| P – compensation (current speed control system) | Eq. 3‑9  General Form | Eq. 3‑11 | * Same as above * Steady-state wheel speed varies with changing of system parameters and resistant force. |
| I - compensation | Eq. 4‑10  General Form | Eq. 4‑14 | * 2nd order system, possible to achieve faster response * Steady-state wheel speed will not vary with changing of system parameters and resistant force except speed sensor gain. * Only gain can be changed for tuning. |
| PI - compensator | Eq. 5‑10  General Form | Eq. 5‑14 | * Same as above * Gain , can be changed for tuning. |
| Description of symbols | – angular velocity of shooter wheel, rad/s  – first derivative of angular velocity of shooter wheel, angular acceleration, rad/s2  – second derivative of angular velocity of shooter wheel, angular jerk, rad/s3  - commanded angular velocity of shooter wheel, rad/s  – total moment of inertia of rotational components at shooter wheel, including wheel, gears, and motor rotor, kg-m2  – resistant of motor coil, ohm  – total resistant torque acted on shooter wheel, N-m  – gear ratio of gearbox from motor axis to wheel axis  – back electromotive force constant, volt/(rad/s)  – motor force constant, N-m/A  – speed control gain, volt/count (volt/volt)  - forward path gain, count/rad/s (or volt/rad/s)  - speed sensor gains, rad/s/rad/s  – cutoff frequency for 1st order system  – natural frequency for 2nd order system  – circular natural frequency of 2nd order system  – damping ratio of 2nd order system | | |

# Analysis of Open Loop Speed Control System

## Block Diagram

Input & software

Speed Controller

Motor

Gearbox

Shooter Wheel



Figure 2‑1 Original Shooter System

## Analysis

1. Apply Newton’s Law to shooter wheel

Eq. 2‑1

Where – moment of inertia of rotating parts including shooting wheel, gears, and motor rotor.

– angular acceleration of shooting wheel

− Total torque acting on the shooting wheel which equals to the difference of driving torque and resistant torque.

1. Driving torque of shooter wheel is output torque of gearbox. It is amplification of motor torque by gear ratio of gearbox

Eq. 2‑2

1. Motor output torque is proportional to rms current in motor coil. The proportion constant is called motor force constant

Eq. 2‑3

1. RMS current in motor coil depends following factors
   1. Driving voltage applied to ends of motor coil
   2. Back electromotive voltage generated in motor coil when coil crosses magnetic field. is proportional to motor angular velocity by back ElectroMotive Force constant of motor . So,.
   3. And resistor of motor coil *R.*

So, the rms current can be expressed as

Eq. 2‑4

1. Driving voltage comes from PWM speed controller output which is proportional to input value – digitized count.

Eq. 2‑5

1. The digital input of PWM speed controller is proportional to angular velocity command with feed forward gain

Eq. 2‑6

1. Now, let’s put Eq. 2‑1 ~ Eq. 2‑6 above together

Eq. 2‑7

1. Rearrange Eq. 2‑7

Eq. 2‑8

Differential equation Eq. 2‑8 is mathematic presentation of shooter wheel and its speed control circuit.

## Discussion

*General form of Equation of Motion*

Eq. 2‑8 has general form

Eq. 2‑9

where is cutoff frequency (Hz) of open loop speed control system of shooter.

Depending on the value of this cutoff frequency and control signal sample rate, proper compensator of control loop should be designed to achieve best performance.

*Factors Affecting Shooter Behavior*

Solution of Eq. 2‑8 depends on the driving command , resistant torque and other system gains/parameters. Any of them changes during operation or over time period will affect dynamic performance such as accuracy and response time.

This 1st order system (inertia system) will always be stable at steady state although it response to step input is slow. Please refer to Lecture 1 ~ 3 of control system mini-series presented before summer of 2012

*Factors Affecting Shooter Wheel Constant Speed*

When wheel reaches constant speed (steady states), angular acceleration is zero (. Then, from Eq. 2‑8 constant angular velocity will be

Eq. 2‑10

Obviously, if system parameters or resistant torque changes, the steady state angular velocity will change too.

# Analysis of Proportion Close-Loop Speed Control System

## Block Diagram

Proportion Compensator



Speed Sensing and Feedback Path

Figure 3‑1 Proportion Close Loop Speed Control System

## Analysis

The model presented in Figure xx is close loop speed control system. This model is also called proportional close loop control system because only proportional constant is added into open loop control system as a compensator.

1. Since the original system to be controlled has no change, Eq. 2‑1 ~ Eq. 2‑5 can be applied to proportion close loop speed control system. They are listed below

Eq. 3‑1

Eq. 3‑2

Eq. 3‑3

Eq. 3‑4

Eq. 3‑5

1. The digital input of PWM speed controller is proportional to angular velocity error which is the difference between commanded angular velocity and feedback angular velocity

Eq. 3‑6

1. The feedback angular velocity is measured by speed sensor. It should be one-to-one to roller speed. Even though, a conversion constant is put in the feedback loop.

Eq. 3‑7

1. Now, let’s put Eq. 3‑1 ~ Eq. 3‑7 above together

Eq. 3‑8

1. Rearrange Eq. 3‑8, we get a first order differential equation.

Eq. 3‑9

Differential equation Eq. 2‑8 is mathematic presentation of shooter wheel and its speed control circuit.

## Discussion

*General form of the System*

Eq. 3‑8 has general form

Eq. 3‑10

where is cutoff frequency of proportion close loop speed control system of shooter wheel.

Basically, the dynamic characteristics have no difference from open loop control system presentation by Eq. 2‑9.

*Factors Affecting Shooter Behavior*

Solution of Eq. 3‑9 depends on the driving command , resistant torque and other system gains/parameters. Anything changes during operation or over time period will affect shooting wheel motion performance.

*Factors Affecting Shooter Wheel Constant Speed*

When roller reaches constant speed (steady states), angular acceleration is zero (. Then, from Eq. 3‑9 constant angular velocity will be

Eq. 3‑11

Obviously, when resistant torque or any parameters in the system is changed, the shooter wheel speed will be changed with the same commanded angular velocity (). Comparing to Eq. 2‑10, fundamentally, proportion close loop speed control system reduces steady state speed errors through additional gains, but eliminate it.

*Checklist for Speed Calibration*

One way to keep the roller angular speed to be constant is periodical calibration. Particularly, if anything listed below changes, forward path gain has to be recalibrated to keep steady speed to be the same as before.

* Change motor (changing )
* Change speed controller (changing )
* Change motor connection (changing )
* Change/adjust gearbox (changing )
* Change sensor (changing )

*Better Solution to Avoid System Variation Effects*

But, if an integrator is added into forward loop, shooter steady state angular velocity will be immune to all changes listed above except feedback sensor gain. This approach will be discussed in next chapter.

# Analysis of Shooter Speed Control System with Integrator

## Modeling

Integration Compensator



Figure 4‑1 Integration Close Loop Speed Control System

## Analysis

The model presented in Figure xx is integration closed loop speed control system since an integrator is added into forward path as a compensator. The output of an integrator is integration of its input.

1. Since the original system to be controlled has no change, Eq. 2‑1 ~ Eq. 2‑5 can be applied to integration close loop speed control system. They are listed below

Eq. 4‑1

Eq. 4‑2

Eq. 4‑3

Eq. 4‑4

Eq. 4‑5

1. All above steps are same as last chapters. But, the digital input of PWM speed controller is proportional to integration of angular velocity error which is the difference between ideal angular velocity and feedback angular velocity

Eq. 4‑6

1. The feedback angular velocity is measured by speed sensor. It should be one-to-one to wheel speed. But, a conversion constant is left in the feedback loop.

Eq. 4‑7

1. Now, combine Eq. 4‑1 ~ Eq. 4‑7 above together

Eq. 4‑8

1. Assume resistant torque is constant. Then, take derivative to Eq. 4‑8. We get following equation

Eq. 4‑9

1. Rearrange this equation

Eq. 4‑10

## Discussion

*General form of the control system*

Eq. 4‑10 has general form

Eq. 4‑11

Where – circular natural frequency,

Eq. 4‑12

– natural frequency

– damping ratio,

Eq. 4‑13

*Factors Affecting Shooter Behavior*

Apparently, resistant torque is not in equation any more. Other system parameters still have contribution to control system dynamic performance.

*Factors Affecting Shooter Wheel Constant Speed*

When roller reaches constant speed (steady states), angular jerk and acceleration are zero (. Then, from Eq. 4‑10 constant angular velocity will be

Eq. 4‑14

**As long as sensor gain is 1, shooter steady state speed will be same as commanded angular speed.**

*Tuning the System*

From control system mini-series 3, we presented that a well-behaved control system should have damping ratio

and natural frequency

1/s (Hz)

for 50 Hz sample rate control system. From Eq. 4‑12 and Eq. 4‑13, both circular natural frequency and damping ratio are determined by system parameters and share same tunable gain.

However, you may find that with only one tunable gain , and cannot meet above design requirement at same time.

Actually, we need two independent gains to optimize damping ratio and natural frequency . This can be done with PI (proportion and integration) compensator. For details, see next Chapter.

# Analysis of Shooter Speed Control System with Proportion and Integration Compensator

## Modeling

Proportion & Integration Compensator



Figure 5‑1 Proportion and Integration Close Loop Speed Control System

## Analysis

The model presented in Figure xx is close loop speed control system. This model is also called close loop speed control system with proportion and integration compensator.

1. Since the original system to be controlled has no change, Eq. 2‑1 ~ Eq. 2‑5 can be applied to proportion and integration close loop speed control system. They will be listed below

Eq. 5‑1

Eq. 5‑2

Eq. 5‑3

Eq. 5‑4

Eq. 5‑5

1. All above steps are same as last section. But, the digital input of PWM speed controller is proportional to integration of angular velocity error which is the difference between ideal angular velocity and feedback angular velocity

Eq. 5‑6

1. The feedback angular velocity is measured by speed sensor. It should be one-to-one to roller speed. But, a conversion constant is left in the feedback loop.

Eq. 5‑7

1. Now, combine Eq. 5‑1 ~ Eq. 5‑7 above together

Eq. 5‑8

1. Assume resistant torque is constant. Then, take derivative to Eq. 5‑8. We get following equation

Eq. 5‑9

1. Rearrange this equation

Eq. 5‑10

## Discussion

*General form of Equation of Motion*

Eq. 5‑10 has general form

Eq. 5‑11

Where – circular natural frequency, , depending on

Eq. 5‑12

– natural frequency

– damping ratio, , depending on

Eq. 5‑13

*Factors Affecting Shooter Behavior*

Apparently, resistant torque is not in equation any more. Other parameters still have contribution to control system dynamic performance.

*Factors Affecting Shooter Wheel Constant Speed*

When wheel reaches constant speed (steady states), angular jerk and acceleration are zero (. Then, from Eq. 5‑10 constant angular velocity will be

Eq. 5‑14

**As long as sensor gain is 1, shooter steady state speed will be same as commanded angular speed.**

*Tuning the System*

From control system mini-series 3, we presented that a well-behaved control system should have damping ratio

and natural circular frequency

Comparing equation x and x, the circular natural frequency and damping ratio can be calculated with system parameters and adjusted with changing and, respectively.

In control system design, the tuning can be simplified as specifying the ratio .

# Design of Control System

Above chapters present analytical results of various speed control systems. At end of Chapter 3 and 4, it seems that we can design a control system for the speed controller to meet constant speed design requirements. Actually, more factors need to be considered in control system design. This Chapter will give a guideline and example on how to design a control system.

## Design Methodology

1. Define performance requirements

There are many performance requirements, depending on tasks. For examples, for shooter speed control system, constant wheel speed is required before feeding a ball into the shooter. And for aiming target, stable and quick responses of driving train to control command are required.

1. Model plant

Figure 2‑1 presents the model of a system to be controlled. It contains major components which is a basic structure – plant – for control system design.

At this step, individual components need to be model correctly based on data sheets, design, or test results. This part requires knowledge of physics – mechanics, electronics – very well. Some imperfection of real component should be modeled as well.

1. Analyze characteristics of plant.

Establish mathematic description of the every component. Combine all the equations together to form a differential equation for plant.

* Knowing the plant and design requirements, we should ask
* Can the system meet design requirements?
* If not, what kinds of compensators and closed loop control system should be added to the plant?
* And Why?

1. Understand control system limitation

There are many things limiting our design. For example, system signal sample rate. If our system signal sample rate is 50 Hz. Final control systems can only response 5 ~ 10 Hz driving or disturbance signals with reasonable response time and accuracy.

Another example, for our shooter speed control, because speed sensor only provides 2 pulses per revolution, there is no way to control wheel speed within 1/2 of revolution.

1. Design compensator

This is core part of control system design. Basically, we would like to add artificial ‘component’ – software package and sensing system – into the plant to make a new system which will meet design requirements.

Typically, a sensing system is proportion element with gain equal to 1, i.e., truly reflects what sensor sees.

So, the core part of control system is to select compensator.

In general, design of a closed loop control system requires more control system knowledge. Fortunately, from modeling and analytical exercises in Chapter 2 ~ 5, we can conclude that system described in Chapter 4 and 5 will meet design requirements. I – compensator or PI – compensator will do the job.

Which compensator is more suitable for our speed control system? It is determined by results of step 2 ~ 4 and additional control system knowledge which is out of scope of this paper.

1. Tuning the system

Tuning a control system is actually determining a set of gains/parameters based on both design and test. With proper designed compensator, the tuning becomes more certain because you have insight of your control system, instead stepping in the dark. This chapter will also tell you how to tune a system through examples.

1. Solve practically problems

Achieving the goal of system performance is based on how realistically knowing a system when modeling and designing a system. Very possibly, some factors are ignored in the initial design process, then, we have to run tests to figure out what is wrong. Based on new findings, design iteration will be performed. This is perfectly normal. If we do our design by principle and with right methodology, we are a step closer to our goal after every iteration.

1. Fine tuning system

You know your system, and you know your design, you will achieve best performance out of your system. Just be confident.

## Design of Shooter Speed Control System with Proper Compensator

1. Define performance requirements

Shooter performance requirements are listed below

* 1. Shooter wheel speed has to be constant before balls are fed into shooter.
  2. Shooter wheel speed should to be constant when components are replaced, resistant torque varies within tolerance.
  3. Calibration should be done once. Periodical maintenance and checking should be performed before a competition event.

1. Model plant
   1. The shooter has been model as shown in Figure 2‑1 Original Shooter System
   2. Get parameter values (conversion factors) of each component from data sheets, design documents, or test results
   3. Make sure units are correct.
2. Analyzing the open loop system
   1. Establish mathematical description of each component (Eq. 2‑1~ Eq. 2‑6)
   2. Combine all equation together (Eq. 2‑8).
   3. This is 1st order differential equation.
      1. Never oscillate
      2. Response speed is slow.
      3. Steady-state wheel speed will vary with changes of system parameters or resistant torque.
      4. **Proportion close loop speed control system cannot prevent steady-state wheel speed from changing.**
      5. **Cutoff frequency fn = TBD. Need real values plugging into equations.**
3. Knowing limitation
   1. System sample rate is 50 Hz. So, close loop control system band width cannot more than 5 ~ 10 Hz.
   2. Jaguar speed controller has nonlinearity. Linearization software has to be in place to ensure the modeling is right.
   3. Speed sensor sample rate is 2 samples per rev.
4. Design compensator

Compensator selection depends on characteristic of open loop control system, the cutoff frequency. With different value of, we will have following control systems.

*System I*

* 1. If > 50 Hz (> 30 Hz may be fine too), a simple integration compensator will be sufficient. See more analysis in Chapter 4
  2. The tunable gains are and
  3. Design open loop gain . Therefore,
  4. Design to have 5 Hz bandwidth.

*System II*

1. If < 50 Hz (or < 30 Hz), PI - compensator will be required.
2. The tunable gains are , and
3. Design open loop gain . Therefore,
4. Design
5. Design and is little bit tricky, simulation is recommended before coding. Following only is one of scenarios as example. Assume

So, keep as dependent variable of .

1. This design is guaranteed to be stable, but may not have best step response as mention in Lecture 2 of control system mini-series.

Note: Detail explanation of above design is omitted since it requires more knowledge of control system design. The tunable parameters are and . This system is guaranteed to be stable. But response to command will be slower.

A more aggressive compensator is to set n = 1. Then the risk is that the system may be unstable. Since system with n = 10 already meet spec, we skip discussion n = 1 design.

1. Tuning the system and assuming PID controller is available and set gain of D as zero.

*System I*

* 1. In open loop setup as Chapter 2, tune up or down from design value in above step until .
  2. Close the loop and tune up or down slightly to achieve performance meeting spec.
  3. This control system will be stable and have quick response to step input.
  4. If sinusoidal signal generator is available, more quantitative tuning can be specified.

*System II*

* 1. In open loop setup as Chapter 2, tune up or down from design value in above step until .

1. Based on data sheets, design documents or test results to calculate . can be measured if you have sinusoidal signal generator.
2. Program as dependent variable of .
3. Start with setting, arbitrary value for safe start. Tune up to near 1 to get satisfied results.
4. If sinusoidal signal generator is available, more quantitative tuning can be specified.
5. Solve problem

Troubleshoot if results are not as anticipated. This will be situation dependent.

1. Fine tuning system

Run tests. Make fine tune if necessary.

# Discrete PID Compensator

In modern control system, compensator can be formed in software which provides more flexibility and avoids many physical limitations of analog circuitry. In software, the control system states are reevaluated at discrete time, typically at constant time interval. Then, continuous form PID compensator should be transformed to discrete form PID compensator.

This Chapter will present formula for typical discrete PID compensators, which are used for programming.

## PID Compensator

PID compensator contains three parts as shown in Figure 7‑1 a), proportional gain, integrator with gain, and differentiator with gain. This is typical mathematical presentation of PID system in time domain.

In control system design, equivalent expression in frequency domain (Figure 7‑1 b)) will be used for compensator design. But, it is out of comfortable zone for most of team members. For slightly more complicated compensators than typical PID compensator, time domain analysis becomes difficult, close-form solution may be impossible. Fortunately, people has developed frequency domain analysis and design tool for us to use. Here, derivation of the frequency analysis is skipped, and only results are presented here.

Implementation of designed compensator for a control system relies on software. Tustin transformation (Figure 7‑1 c)) is the tool to develop a programmable formula of a compensator from continuous form compensator in frequency domain. Applying the transformation is simple as presented in examples although derivation is skipped.

a)

Continuous PID in time domain

Laplace Transformation

b)

Continuous PID in frequency domain

Tustin Transformation

c)

Discrete PID in time domain

k is current state, k-n is nth previous state, a and b are constants

**Use this form for PID compensator programming**

Figure 7‑1 Continuous and Discrete Form PID Compensators

## Example of Discrete Transformation

By setting one or more value of gains to be zero, different variation PID compensator can be achieved.

1. P – Compensator
2. I – Compensator
3. D – Compensator
4. PI – Compensator
5. PD – Compensator
6. IP – Compensator
7. PID – Compensator

*Example 1 – Full PID Compensator*

Continuous PID compensator has following form

Eq. 7‑1

Take Laplace transformation, the compensator has following expression in frequency domain

Eq. 7‑2

where s – a complex number, .

Apply Tustin transformation, , to above equation

Then,

Eq. 7‑3

where

– current state, – state of last cycle, etc.

– sample rate (sec), for 50 Hz system sample rate

Eq. 7‑3 will be used for programming.

*Example 2 – Integration Compensator*

Continuous integration compensator has following form

Take Laplace transformation, the compensator has following expression in frequency domain

Apply Tustin transformation, , to above equation

In the system-I scenario of shooter speed control system, if bandwidth of control system is ,

*Example 3 – proportion-Integration Compensator*

Continuous integration compensator has following form

Take Laplace transformation, the compensator has following expression in frequency domain

Apply Tustin transformation, , to above equation, we have

Above equation will be used for programming of PI compensator.

As you can see that programming a compensator is quiet easy with discrete expression.