Sets and Set Theory

CS-1Q

IM Lecture 7

Craig Macdonald

Announcement

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- Staff-Student Liaison Committee
  - Course lecturers and students discuss experiences
- Class reps
  - Class Representative training (2 ½ hours)
  - One per tutorial group, attend liaison committees – once per term
  - Consult with your classmates (in tutorial groups)

Overview

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This lecture

- Sets and Set Theory
- Relations and Predicates

Next lecture (lecture 8)

- Relational Algebra
  - The foundations for SQL

Where to go for more info…

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- Rosen, Discrete Mathematics and Its Applications
  - sets - sections 1.4 & 1.5
  - relations - sections 6.1 & 6.2
  - http://www.mhhe.com/math/advmath/rosen/
- Rolland, section 3.3

Sets and Set Theory

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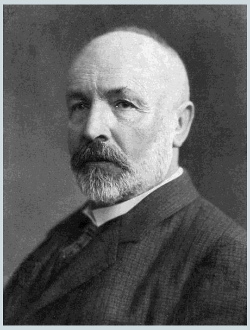
Set theory is the branch of mathematics that studies **sets**

Sets are collections of objects
- The set of all numbers
- the set of all animals with tails
- the set of letters in the alphabet
- the set of all students in a class
- …………………



Georg Cantor (1845-1918)

Sets

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Often all members of a set have similar properties

1 5
3 9
7

Odd numbers less than 10

Susan
Bob Lukas
Steven
Tom Sarah
Clare

Students in a Tutorial Group

Set Theory - Vocabulary

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- Objects in a set are called ***'elements'*** or ***'members'*** of a set
- A set is said to 'contain' its elements

- In databases
  - all employees of a company make up a '***set***' of employees

Describing Sets

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- Describing a set
  - List all the members between braces
    - E.g. **{a, b, c, d}**
    - Represents the set with the four elements a, b, c, and d.

Describing Sets

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- E.g. The set V of all vowels in the English alphabet
- E.g. The set O of positive integers less than 10

Describing Sets

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- E.g. The set V of all vowels in the English alphabet
  - **V = {a, e, i, o, u}**
- E.g. The set O of positive integers less than 10
  - **O = {1, 3, 5, 7, 9 }**
- | | denotes the *cardinality* of a set
  - **|V| = 5, |O| = 5**

Set Equality

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- Two sets are ***equal*** if and only if they have the same elements
  - **Order doesn't matter**
    - {1,3,5} = {1,5,3} = {3,1,5} = {3,5,1}= {5,1,3} ={5,3,1}
  - **Repetition doesn't matter**
    - {1,2} = {1,1,2} = {1,2,2,2,2}

Set Equality

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A = {1,2,3}

B = {3,2,1} C = {1,1,2,2,2,3} D= {1,2,3}

Which set(s) are equal to A?

Set Equality

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A = {1,2,3}

B = {3,2,1} C = {1,1,2,2,2,3} D= {1,2,3}

A = B, C and D

Sets

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- Sets *usually* group together elements with associated properties
  - but seemingly unrelated properties can also be listed as a set
  - {2, e, Fred, Paris} is also a set
    - We just don't know much about exactly how they are related to each other

Predicates and Sets

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- It is sometimes inconvenient or impossible to describe a set by listing all of its elements
  - What is the set of all integers less than 1 million?

Predicates and Sets

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- It is sometimes inconvenient or impossible to describe a set by listing all of its elements
  - What is the set of all integers less than 1 million?
    - {1,2,3,4,5……………………………………!!!!!!!}

Set Builder Notation

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- Characterise all those elements in the set by stating the properties they must have to be members

E.g.

- The set O of all positive integers less than 10 in set builder notation is:
  - O = {X | X is an odd integer less than 10}

Predicates and Sets

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- A **predicate** is sometimes used to indicate **set membership**
- A predicate *P(x)* will be true or false, depending on whether *x* belongs to a set

Predicates and set membership

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An example

{*x* | *x* is a positive integer less than 4}

is the set {1,2,3}

If *t* is an element of the set {*x* | *P*(*x*)}

then the statement *P*(*t*) is *true*

So if P(x) says x/2 = 0

{x|P (x)} contains…. the set of all even numbers

Predicates and set membership

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- Here, *P*(*x*) is referred to as the ***predicate***, and *x* the *subject* of the *proposition*
- Sometimes, *P*(*x*) is also called a propositional function, as each choice of x produces a proposition

Some Notation

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- **a $\in$ A**
  - a is an element of set A
- **a $\notin$ A**
  - a is not an element of set A
- **ø**
  - The empty or null set
  - Also represented by { }

The Power Set

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- Given a set S, the ***power set*** is the set of all subsets of the set S
  - Denoted by P(S)
- E.g. the power set of {0,1,2} is

The Power Set

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- E.g. the power set of {0,1,2} is
  - P({0,1,2}) = {Ø, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }
  - NB - the **empty set** and the **set itself** are members of this set of subsets
- If a set has n elements, its power set has 2^n elements
- The power set does not contain numbers, it contains SETs

Graphical representation of sets

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- Sets can be represented graphically using ***Venn diagrams***
- The ***universal set U*** (which contains all of the objects under consideration) is represented by a rectangle
- Inside the rectangle, circles are used to represent sets
- Sometimes points are used to represent the particular elements of the set

A Example Set

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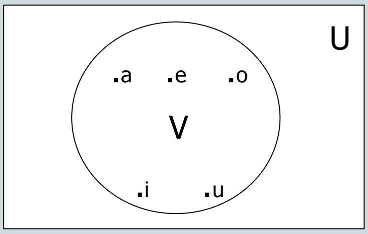

The set V of vowels from all letters U

A Example Set

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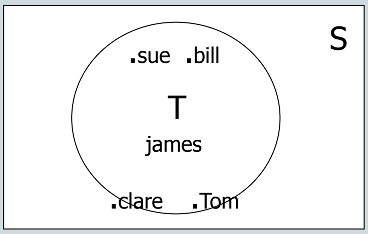

The set T of people in tutorial group from all Students S

Subsets

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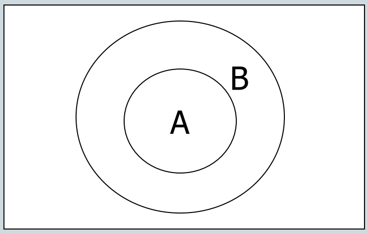

A is a subset of B

$A \subset B$

A test that returns true iff $A \subset B$

Set Operations

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- Two sets can be combined in many different ways
  - The following illustrates some such combinations
    - See Rosen, 1.5 for further explanations

Union

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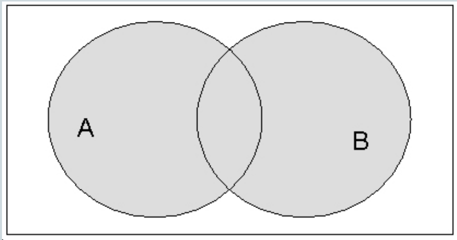

Symbol like Union

The union of A and B

$A \cup B$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

The set that contains those elements that are either in A, B, or in both

Intersection

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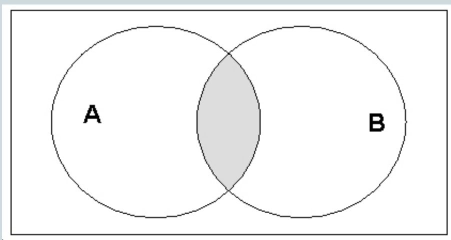

The **intersection** of A and B

$A \cap B$

Symbol like aNd

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

Difference

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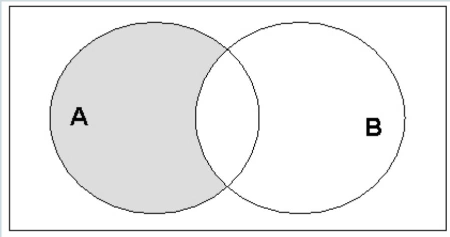

The **difference** of A and B

A - B

$A - B = \{x \mid x \in A \wedge x \notin B\}$

Complement

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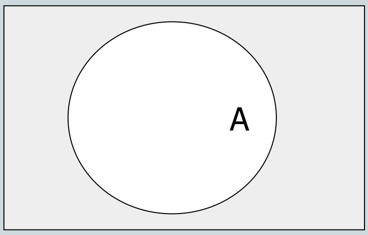

The complement of A

$\overline{A}$

$\overline{A} = \{x \mid x \notin A\}$

Symmetric Difference

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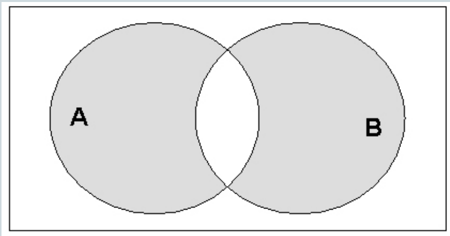

The symmetric difference of A and B

$A \oplus B = (A - B) \cup (B - A)$

$A \oplus B = \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$

Summary

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- What are sets?
- Notation for making sets, comparing sets
- Operators: making new sets from other sets
  - **$\cup$ union**
  - **$\cap$ intersection**
  - **– difference**
  - **$\overline{A}$ complement**
  - **$\oplus$ symmetric difference**

Relations

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- Relationships between elements of sets are represented using the structure called a ***relation***
- Relations are the fundamental data structure used to store information in databases (remember?)
- Relations are used to identify elements in sets with common properties

Where to go for more info...

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- Rosen, Discrete Mathematics and Its Applications
  - Sets - sections 1.4 & 1.5
  - Relations - sections 6.1 & 6.2
  - see http://www.mhhe.com/math/advmath/rosen/
- Rolland, section 3.3