

Precalculus for Team-Based Inquiry Learning

2024 Development Edition

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¹teambasedinquirylearning.github.io/precalculus/

²github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Polynomial and Rational Functions (PR)

Objectives

BIG IDEA for the chapter goes here, in outcomes/main.ptx

By the end of this chapter, you should be able to...

1. Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.
2. Use quadratic models to solve an application problem and establish conclusions.
3. Rewrite a rational function as a polynomial plus a proper rational function.
4. Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.
5. Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.
6. Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

1.1 Graphing Quadratic Functions (PR1)

Objectives

- Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

Graphing Quadratic Functions (PR1)

Activity 1.1.1 Use the graph of the quadratic function $f(x) = 3(x - 2)^2 - 4$ to answer the questions below.

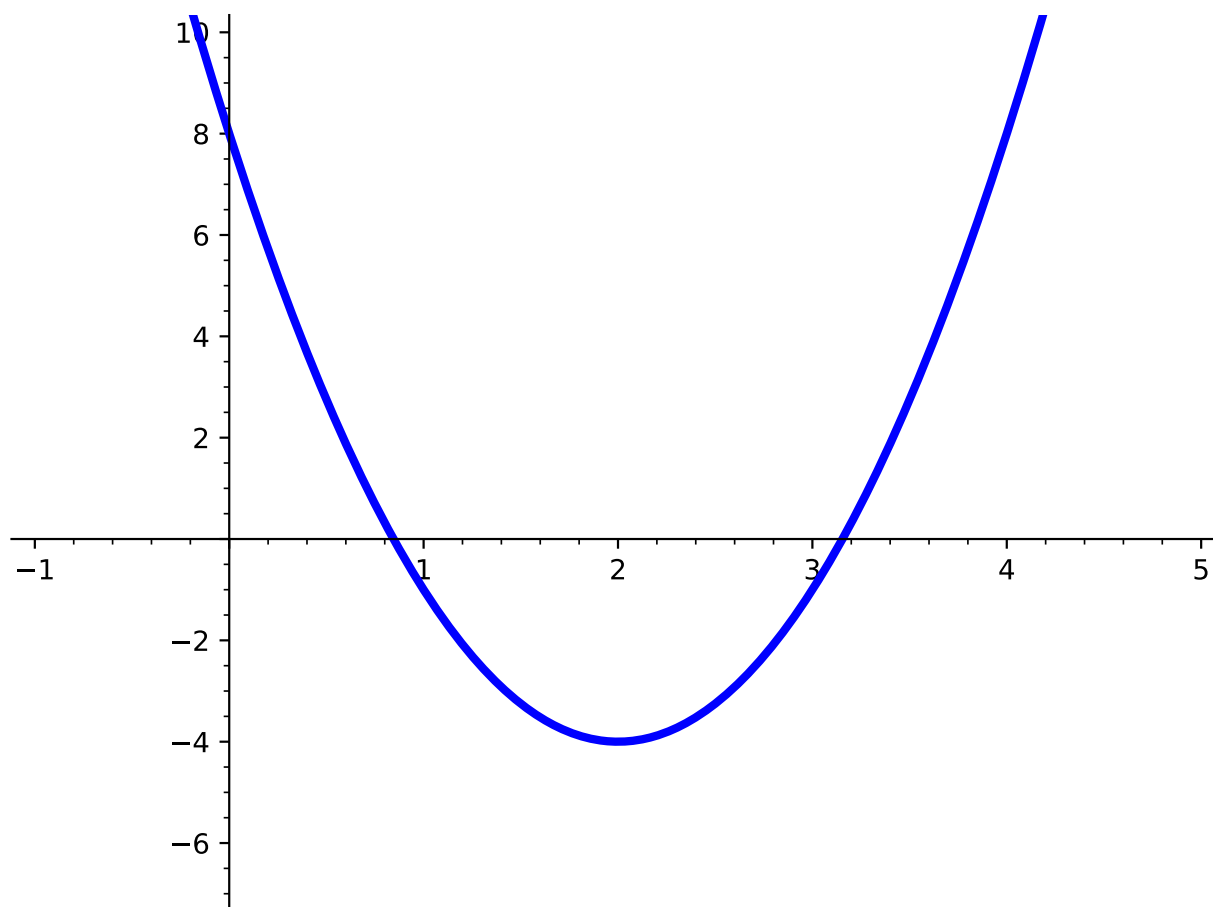


Figure 1.1.2

- (a) How would you describe the behavior of this function? What is happening to the y -values as the x -values increase? Do you notice any other patterns of the y -values of the table?

Table 1.1.3

x	$f(x)$
0	
1	
2	
3	
4	
5	

Graphing Quadratic Functions (PR1)

- (b) At which point (x, y) does the graph reach it's maximum or minimum value? How can you tell from the graph that this is the maximum or minimum value?
- (c) Look at the function given and the graph of the function. What do you notice? Is there a faster way to find the maximum or minimum value from the given function?

Graphing Quadratic Functions (PR1)

Definition 1.1.4 The vertex form of a quadratic function is given by $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is the axis of symmetry. \diamond

Graphing Quadratic Functions (PR1)

Activity 1.1.5 Use the given the quadratic function, $f(x) = 3(x - 2)^2 - 4$, to answer the following:

- (a) Apply the definition to find the vertex of the parabola and the axis of symmetry.
- (b) Compare what you got in part *a* with the values you found in the previous activity. What do you notice?

Graphing Quadratic Functions (PR1)

Definition 1.1.6 Given the standard form of a quadratic function, $f(x) = ax^2 + bx + c$, with real coefficients a, b , and c , the axis of symmetry is defined as $x = \frac{-b}{2a}$ and has a vertex at the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$. \diamond

Graphing Quadratic Functions (PR1)

Activity 1.1.7 Use the graph of the quadratic function to answer the questions below.

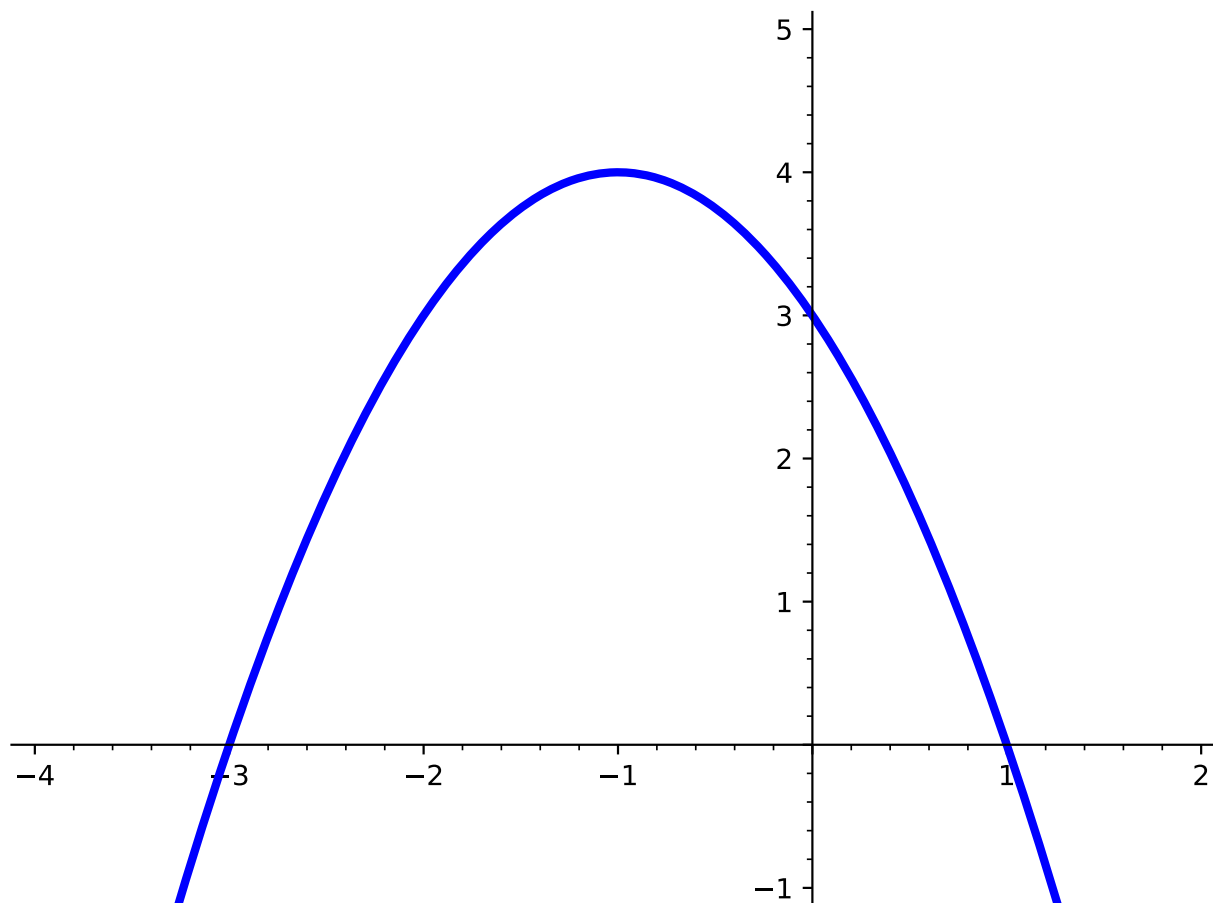


Figure 1.1.8

(a) Which of the following quadratic functions matches the graph shown in the figure?

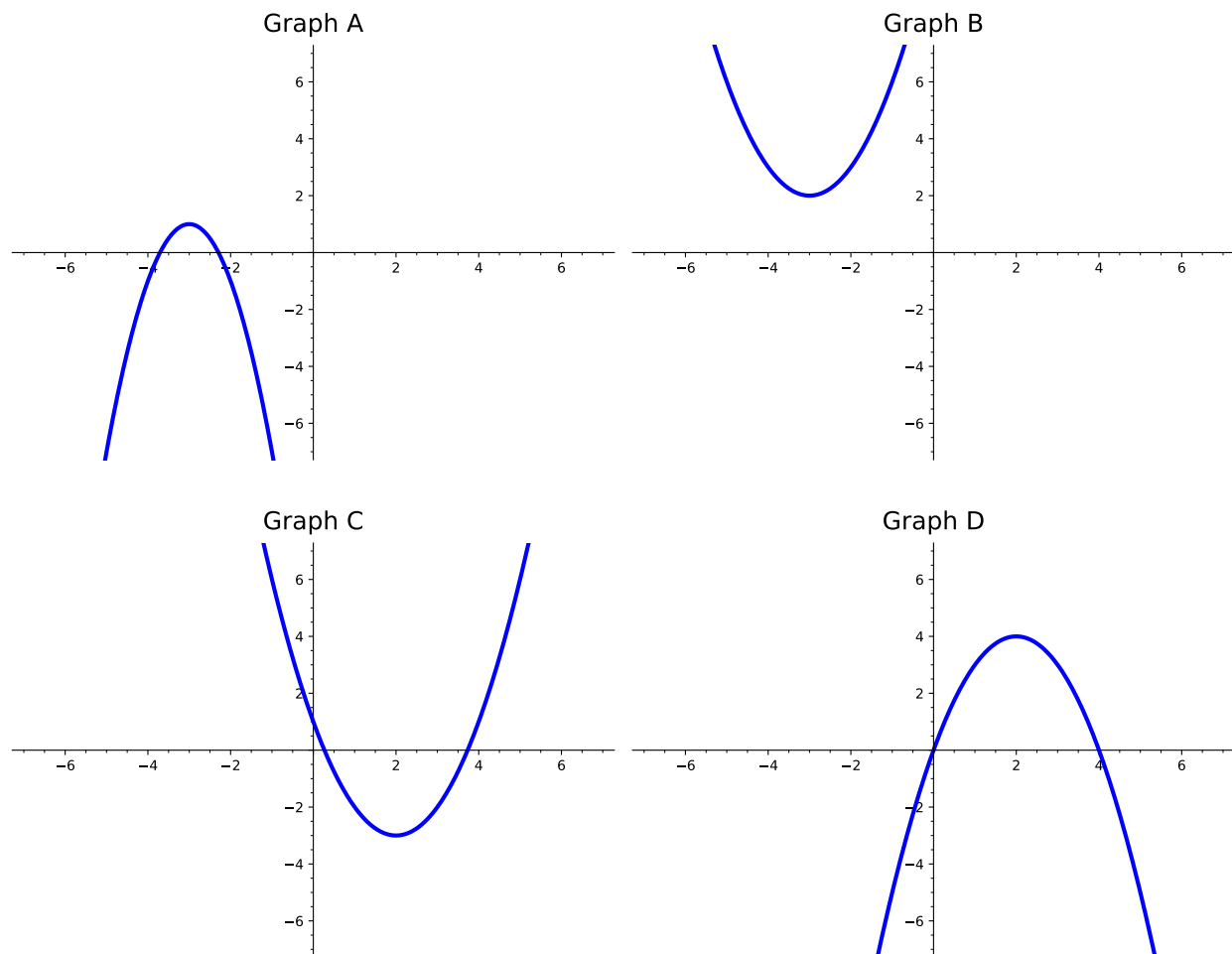
- A. $f(x) = x^2 + 2x + 3$
- B. $f(x) = -(x + 1)^2 + 4$
- C. $f(x) = -x^2 - 2x + 3$
- D. $f(x) = -(x + 1)^2 + 4$

(b) What is the maximum or minimum value?

- | | |
|---------|---------|
| A. -1 | C. -3 |
| B. 4 | D. 1 |

Graphing Quadratic Functions (PR1)

Activity 1.1.9 Consider the following four graphs of quadratic functions:



(a) Which of the graphs above have a maximum?

A. Graph A

C. Graph C

B. Graph B

D. Graph D

(b) Which of the graphs above have an axis of symmetry of $x = 2$?

A. Graph A

C. Graph C

B. Graph B

D. Graph D

(c) Which of the graphs above represents the function $f(x) = -(x-2)^2 + 4$?

A. Graph A

C. Graph C

B. Graph B

D. Graph D

Graphing Quadratic Functions (PR1)

(d) Which of the graphs above represents the function $f(x) = x^2 - 4x + 1$?

A. Graph A

C. Graph C

B. Graph B

D. Graph D

Graphing Quadratic Functions (PR1)

Remark 1.1.10 Notice that the maximum or minimum value of the quadratic function is the vertex. How can you determine if the vertex is a maximum or minimum?

Graphing Quadratic Functions (PR1)

Activity 1.1.11 Sketch the graph of a function $f(x)$ that meets the following criteria:

1. The function $f(x)$ has a maximum at 7.
2. The axis of symmetry is at $x = -2$.

1.2 Quadratic Models and Meanings (PR2)

Objectives

- Use quadratic models to solve an application problem and establish conclusions.

Quadratic Models and Meanings (PR2)

Activity 1.2.1 A water balloon is tossed vertically from a fifth story window. It's height $h(t)$, in meters, at a time t , in seconds, is modeled by the function

$$h(t) = -5t^2 + 20t + 25$$

(a) Complete the following table.

Table 1.2.2

t	$h(t)$
0	
1	
2	
3	
4	
5	

(b) Explain why $h(t)$ is not a linear function.

(c) What is the meaning of $h(0) = 25$?

- A. the initial height of the water balloon is 25 meters.
- B. the water balloon reaches a maximum height of 25 meters.
- C. the water balloon hits the ground after 25 seconds.
- D. the water balloon travels 25 meters before hitting the ground.

(d) Find the vertex of the quadratic function.

- A. (0, 25)
- B. (2, 45)
- C. (5, 0)
- D. (1, 40)

(e) What is the meaning of the vertex?

- A. The water balloon reaches a maximum height of 25 meters at the start.
- B. After 2 seconds, the water balloon reaches a maximum height of 45 meters.
- C. After 5 seconds, the water balloon reaches a maximum height.
- D. After 1 second, the water balloon reaches a maximum height of 40 meters.