Precalculus for Team-Based Inquiry Learning

2024 Development Edition

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TBIL Fellows

Editors

Steven Clontz
University of South Alabama

Drew Lewis

Contributing Authors

Tonya DeGeorge
Georgia Gwinnett College

Abby Noble
Middle Georgia State University

Kathy Pinzon
Georgia Gwinnett College

Wendy Sheppard
College of Charleston

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Website: Precalculus for Team-Based Inquiry Learning[1]

[1] teambasedinquirylearning.github.io/precalculus/

[2] github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Polynomial and Rational Functions (PR)

Objectives

BIG IDEA for the chapter goes here, in outcomes/main.ptx
By the end of this chapter, you should be able to...

1. Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.
2. Use quadratic models to solve an application problem and establish conclusions.
3. Rewrite a rational function as a polynomial plus a proper rational function.
4. Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.
5. Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.
6. Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

Readiness Assurance. Before beginning this chapter, you should be able to...

a Readiness Outcome 1

- Review:
- Practice:

b Readiness Outcome 2

- Review:
- Practice:

1.1 Graphing Quadratic Functions (PR1)

Objectives

- Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

1.1.1 Activities

Quadratic functions have many different applications in the real world. For example, say we want to identify a point at which the maximum profit or minimum cost occurs. Before we can interpret some of these situations, however, we will first need to understand how to read the graphs of quadratic functions to locate these least and greatest values.

Activity 1.1.1 Use the graph of the quadratic function $f(x) = 3(x-2)^2 - 4$ to answer the questions below.

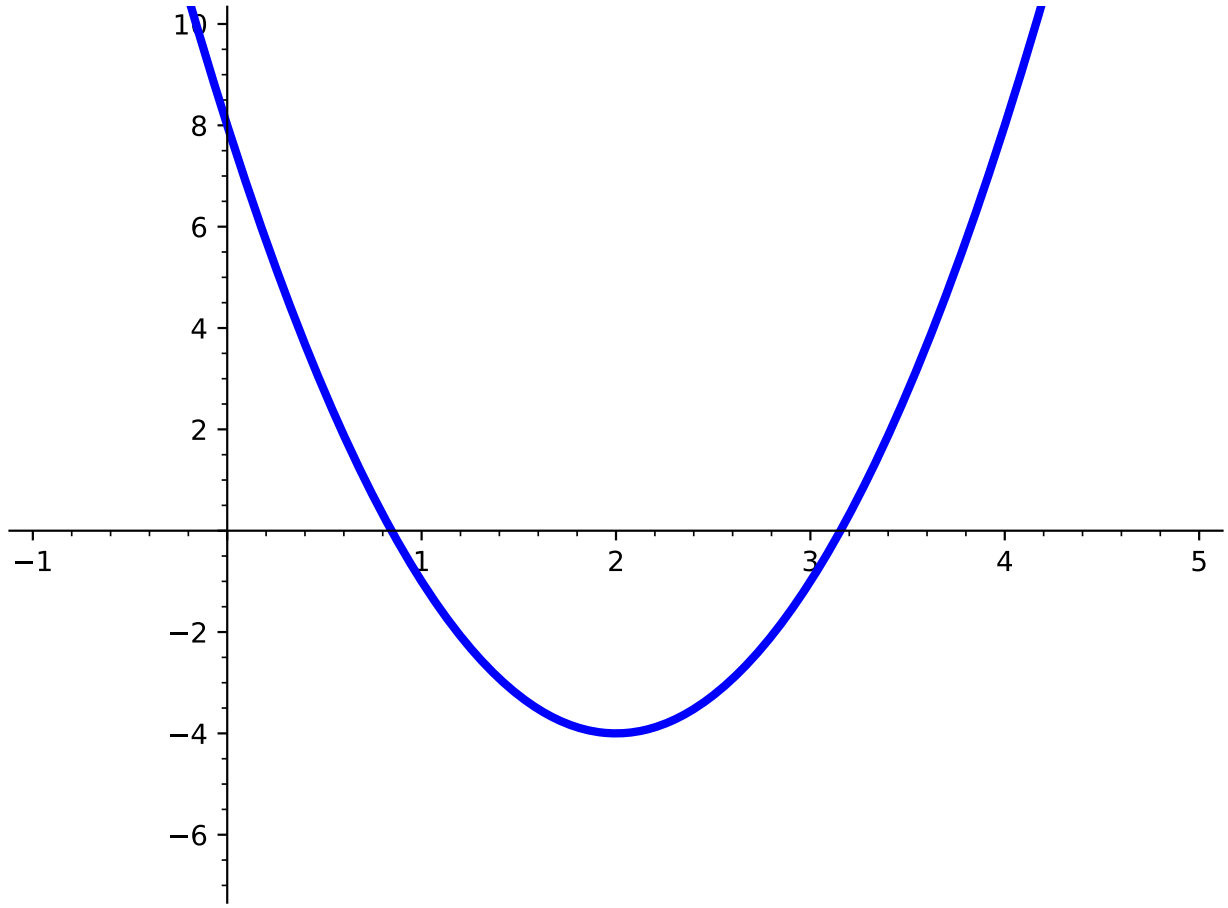


Figure 1.1.2

(a) How would you describe the behavior of this function? What is happening to the y-values as the x-values increase? Do you notice any other patterns of the y-values of

the table?

Table 1.1.3

x	$f(x)$
0	
1	
2	
3	
4	
5	

(b) At which point (x, y) does the graph reach it's maximum or minimum value? How can you tell from the graph that this is the maximum or minimum value?

(c) Look at the function given and the graph of the function. What do you notice? Is there a faster way to find the maximum or minimum value from the given function?

Definition 1.1.4 The vertex form of a quadratic function is given by $f(x) = a(x-h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is the axis of symmetry. $\lozenge$

Activity 1.1.5 Use the given the quadratic function, $f(x) = 3(x-2)^2 - 4$, to answer the following:

(a) Apply the definition to find the vertex of the parabola and the axis of symmetry.

(b) Compare what you got in part a with the values you found in the previous activity. What do you notice?

Definition 1.1.6 Given the standard form of a quadratic function, $f(x) = ax^2 + bx + c$, with real coefficients a, b, and c, the axis of symmetry is defined as $x = \frac{-b}{2a}$ and has a vertex at the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$. $\lozenge$

Activity 1.1.7 Use the graph of the quadratic function to answer the questions below.

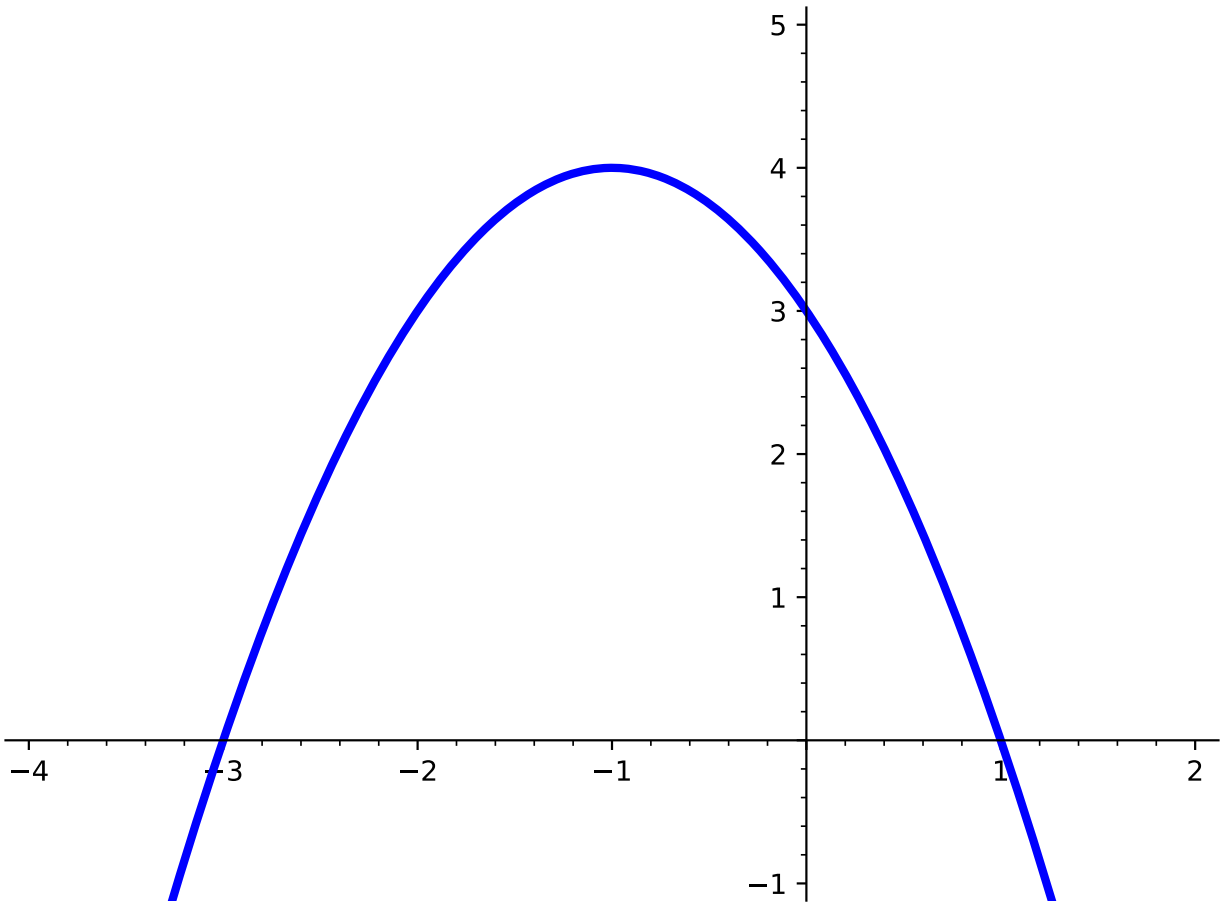


Figure 1.1.8

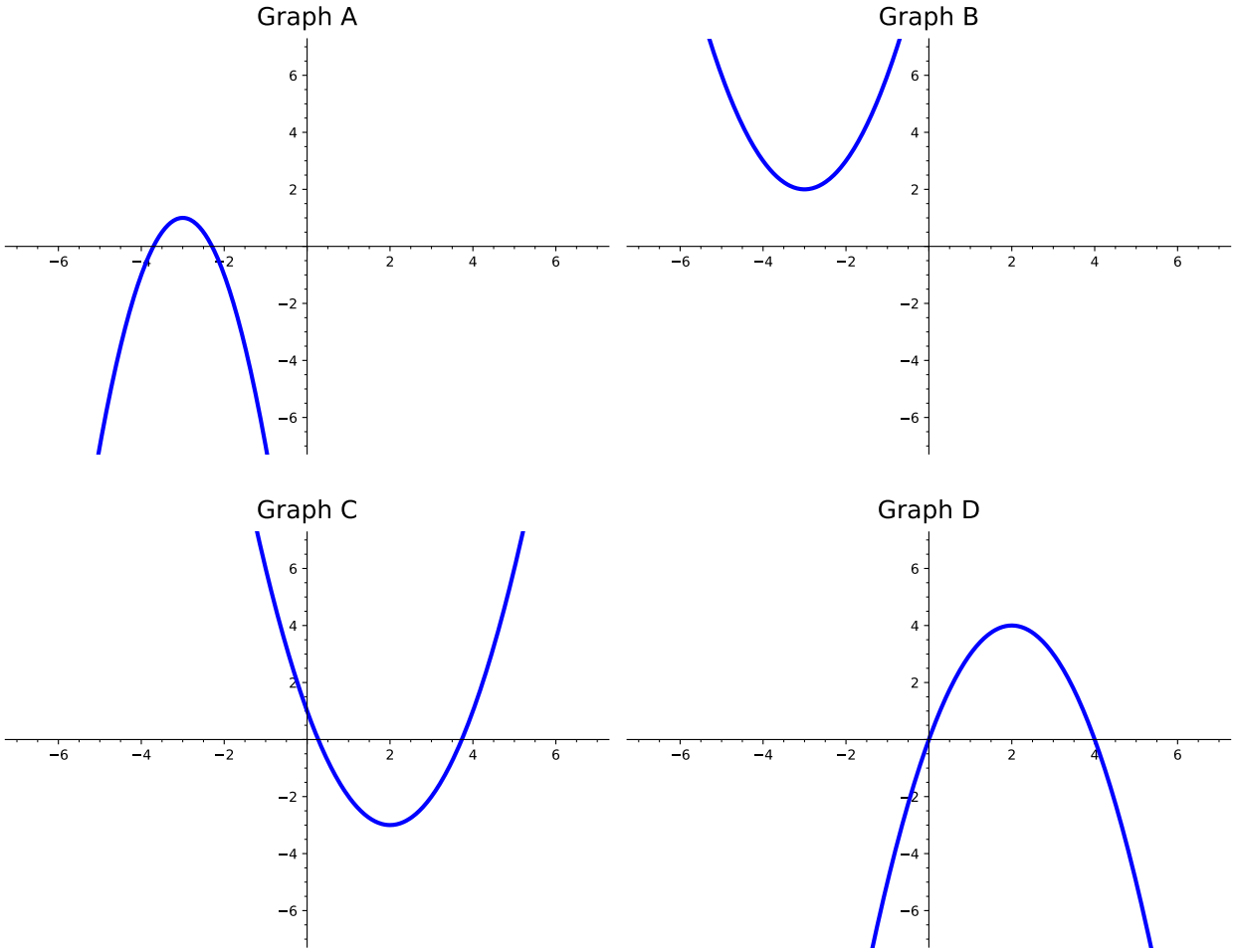
(a) Which of the following quadratic functions matches the graph shown in the figure?

A. $f(x) = x^2 + 2x + 3$
B. $f(x) = -(x+1)^2 + 4$
C. $f(x) = -x^2 - 2x + 3$
D. $f(x) = -(x+1)^2 + 4$

(b) What is the maximum or minimum value?

A. -1
B. 4
C. -3
D. 1

Activity 1.1.9 Consider the following four graphs of quadratic functions:



(a) Which of the graphs above have a maximum?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(b) Which of the graphs above have an axis of symmetry of $x = 2$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(c) Which of the graphs above represents the function $f(x) = -(x-2)^2 + 4$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(d) Which of the graphs above represents the function $f(x) = x^2 - 4x + 1$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

Remark 1.1.10 Notice that the maximum or minimum value of the quadratic function is the vertex. How can you determine if the vertex is a maximum or minimum?

Activity 1.1.11 Sketch the graph of a function $f(x)$ that meets the following criteria:

1. The function $f(x)$ has a maximum at 7.
2. The axis of symmetry is at $x = -2$.

1.1.2 Videos

It would be great to include videos down here, like in the Calculus book!

1.2 Quadratic Models and Meanings (PR2)

Objectives

- Use quadratic models to solve an application problem and establish conclusions.

1.2.1 Activities

Activities go here! Don't forget to put text in <p> tags or it won't show up.

Activity 1.2.1 A water balloon is tossed vertically from a fifth story window. It's height $h(t)$, in meters, at a time t, in seconds, is modeled by the function

$$h(t) = -5t^2 + 20t + 25$$

(a) Complete the following table.

Table 1.2.2

t	$h(t)$
0	
1	
2	
3	
4	
5	

(b) Explain why $h(t)$ is not a linear function.

(c) What is the meaning of $h(0) = 25$?

A. the initial height of the water balloon is 25 meters.

B. the water balloon reaches a maximum height of 25 meters.

C. the water balloon hits the ground after 25 seconds.