Precalculus for Team-Based Inquiry Learning

2024 Development Edition

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Website: Precalculus for Team-Based Inquiry Learning[1]

[1] teambasedinquirylearning.github.io/precalculus/

[2] github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Polynomial and Rational Functions (PR)

Objectives

BIG IDEA for the chapter goes here, in outcomes/main.ptx
By the end of this chapter, you should be able to...

1. Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.
2. Use quadratic models to solve an application problem and establish conclusions.
3. Rewrite a rational function as a polynomial plus a proper rational function.
4. Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.
5. Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.
6. Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

Readiness Assurance. Before beginning this chapter, you should be able to...

a Readiness Outcome 1

- Review:
- Practice:

b Readiness Outcome 2

- Review:
- Practice:

1.1 Graphing Quadratic Functions (PR1)

Objectives

- Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

1.1.1 Activities

Quadratic functions have many different applications in the real world. For example, say we want to identify a point at which the maximum profit or minimum cost occurs. Before we can interpret some of these situations, however, we will first need to understand how to read the graphs of quadratic functions to locate these least and greatest values.

Activity 1.1.1 Use the graph of the quadratic function $f(x) = 3(x-2)^2 - 4$ to answer the questions below.

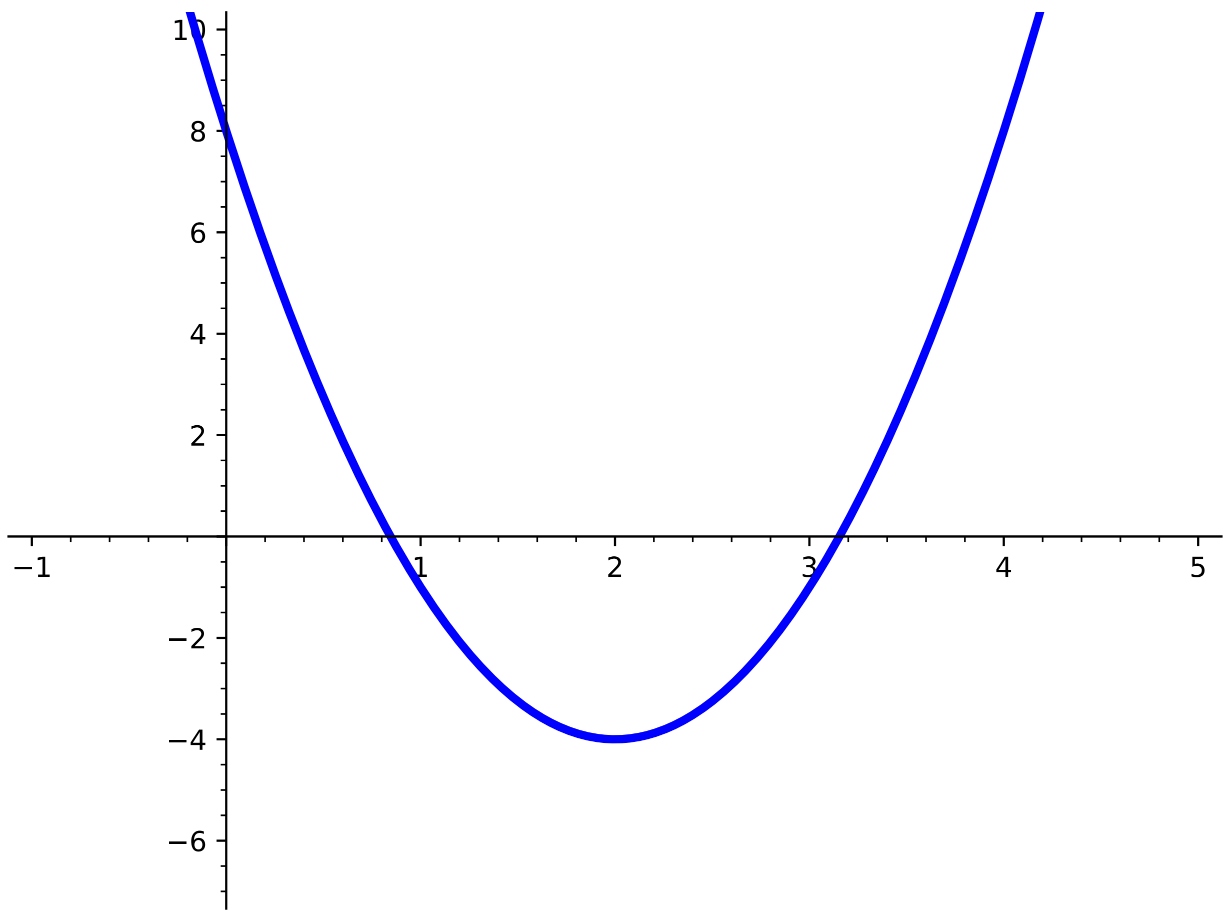


Figure 1.1.2

(a) How would you describe the behavior of this function? What is happening to the y-values as the x-values increase? Do you notice any other patterns of the y-values of the table?

Table 1.1.3

x	$f(x)$
0	
1	
2	
3	
4	
5	

(b) At which point (x, y) does the graph reach it's maximum or minimum value? How can you tell from the graph that this is the maximum or minimum value?

(c) Look at the function given and the graph of the function. What do you notice? Is there a faster way to find the maximum or minimum value from the given function?

Definition 1.1.4 The vertex form of a quadratic function is given by $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is the axis of symmetry. ♢

Activity 1.1.5 Use the given the quadratic function, $f(x) = 3(x - 2)^2 - 4$, to answer the following:

(a) Apply the definition to find the vertex of the parabola and the axis of symmetry.

(b) Compare what you got in part a with the values you found in the previous activity. What do you notice?

Definition 1.1.6 Given the standard form of a quadratic function, $f(x) = ax^2 + bx + c$, with real coefficients a, b, and c, the axis of symmetry is defined as $x = \frac{-b}{2a}$ and has a vertex at the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$. ♢

Activity 1.1.7 Use the graph of the quadratic function to answer the questions below.

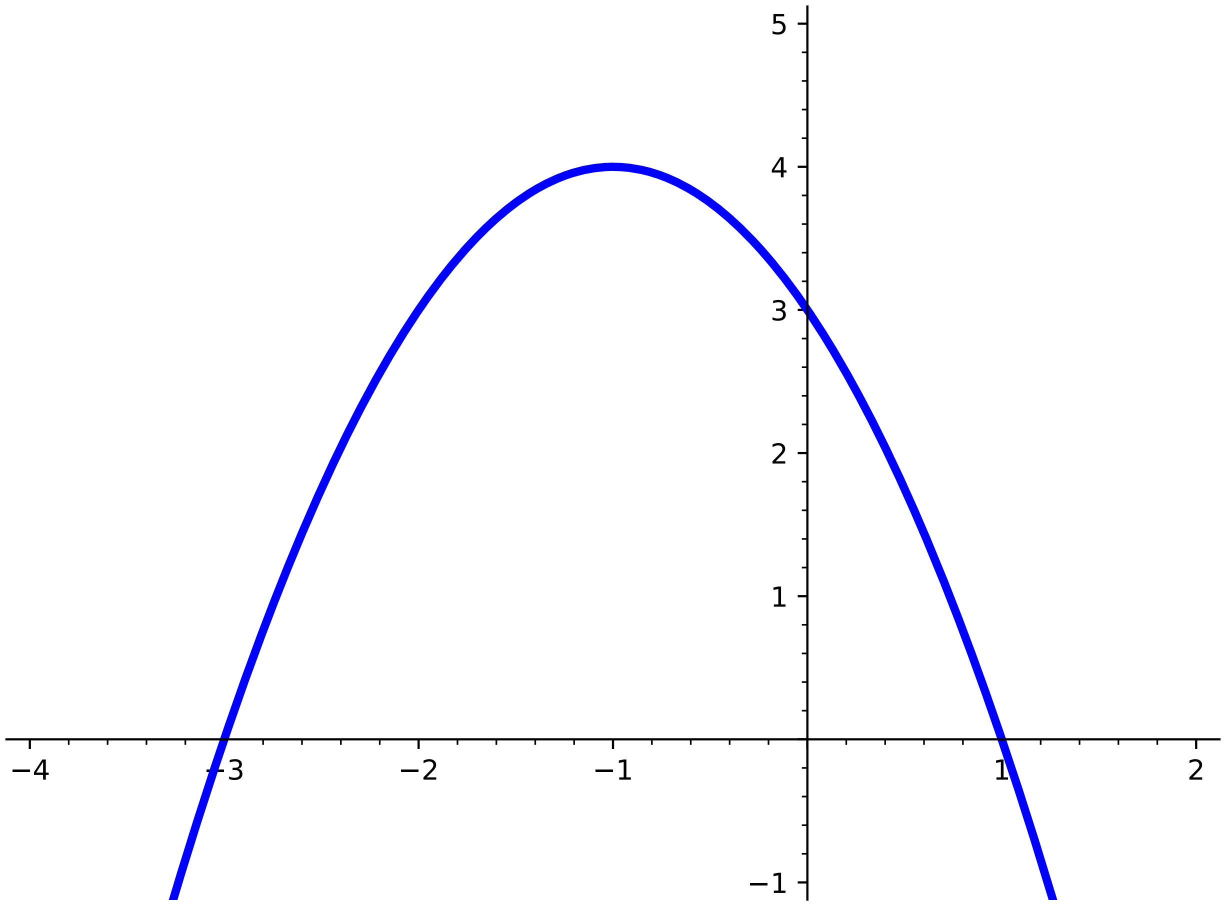


Figure 1.1.8

(a) Which of the following quadratic functions matches the graph shown in the figure?

A. $f(x) = x^2 + 2x + 3$

B. $f(x) = -(x+1)^2 + 4$

C. $f(x) = -x^2 - 2x + 3$

D. $f(x) = -(x+1)^2 + 4$

(b) What is the maximum or minimum value?

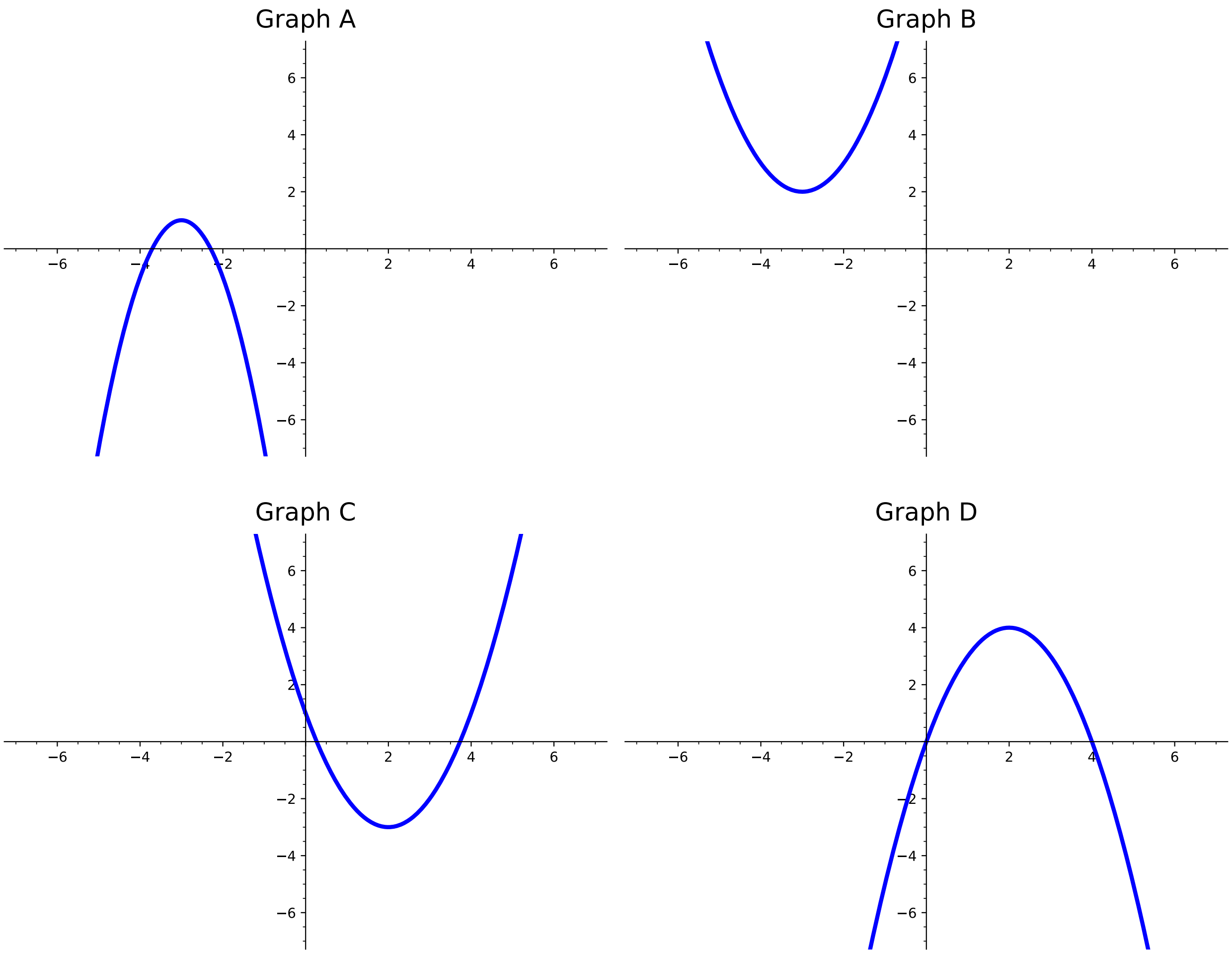
A. -1

B. 4

C. -3

D. 1

Activity 1.1.9 Consider the following four graphs of quadratic functions:



(a) Which of the graphs above have a maximum?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(b) Which of the graphs above have an axis of symmetry of $x = 2$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(c) Which of the graphs above represents the function $f(x) = -(x-2)^2 + 4$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

(d) Which of the graphs above represents the function $f(x) = x^2 - 4x + 1$?

A. Graph A
B. Graph B
C. Graph C
D. Graph D

Remark 1.1.10 Notice that the maximum or minimum value of the quadratic function is the vertex. How can you determine if the vertex is a maximum or minimum?

Activity 1.1.11 Sketch the graph of a function $f(x)$ that meets the following criteria:

1. The function $f(x)$ has a maximum at 7.
2. The axis of symmetry is at $x = -2$.

1.1.2 Videos

It would be great to include videos down here, like in the Calculus book!

1.2 Quadratic Models and Meanings (PR2)

Objectives

- Use quadratic models to solve an application problem and establish conclusions.

1.2.1 Activities

Activities go here! Don't forget to put text in <p> tags or it won't show up.

Activity 1.2.1 A water balloon is tossed vertically from a fifth story window. It's height $h(t)$, in meters, at a time t, in seconds, is modeled by the function

$$h(t) = -5t^2 + 20t + 25$$

(a) Complete the following table.

Table 1.2.2

t	$h(t)$
0	
1	
2	
3	
4	
5	

(b) Explain why $h(t)$ is not a linear function.

(c) What is the meaning of $h(0) = 25$?

A. the initial height of the water balloon is 25 meters.

B. the water balloon reaches a maximum height of 25 meters.

C. the water balloon hits the ground after 25 seconds.

D. the water balloon travels 25 meters before hitting the ground.

(d) Find the vertex of the quadratic function.

A. $(0, 25)$
B. $(2, 45)$
C. $(5, 0)$
D. $(1, 40)$

(e) What is the meaning of the vertex?

A. The water balloon reaches a maximum height of 25 meters at the start.
B. After 2 seconds, the water balloon reaches a maximum height of 45 meters.
C. After 5 seconds, the water balloon reaches a maximum height.
D. After 1 second, the water balloon reaches a maximum height of 40 meters.

Activity 1.2.3 The population of a small city is given by the function $P(t) = -50t^2 + 1200t + 32000$, where t is the number of years after 2015.

(a) When will the population of the city reach a maximum?

A. 2020
B. 2022
C. 2025
D. 2027

(b) Determine when the population of the city is increasing and when it is decreasing.

(c) When will the population of the city reach 36,000 people?

A. 2019
B. 2025
C. 2027
D. 2035

Activity 1.2.4 The unit price of an item affects its supply and demand. That is, if the unit price goes up, the demand for the item will usually decrease. For example, an online streaming service currently has 84 million subscribers at a monthly charge of \$6. Market research has suggested that if the owners raise the price to \$8, they would lose 4 million subscribers. Assume that subscriptions are linearly related to the price.

(a) Which of the following represents a linear function which relates the price of the streaming service p to the number of subscribers Q?

A. $Q(p) = -2p$
B. $Q(p) = -2p + 84$
C. $Q(p) = -2p - 4$
D. $Q(p) = -2p + 96$

(b) Using the fact that Revenue $= pQ$, which of the following represents the Revenue R in terms of the price p.

A. $R(p) = -2p^2$
B. $R(p) = -2p^2 + 84p$
C. $R(p) = -2p^2 - 4p$
D. $R(p) = -2p^2 + 96p$

(c) What price should the streaming service charge for a monthly subscription to maximize their revenue?

A. \$10

B. \$19.50

C. \$24

D. \$28.25

(d) How many subscribers would the company have at this price?

A. 39.5 million

B. 48 million

C. 57 million

D. 76 million

1.2.2 Videos

It would be great to include videos down here, like in the Calculus book!

1.3 Polynomial Long Division (PR3)

Objectives

- Rewrite a rational function as a polynomial plus a proper rational function.

1.3.1 Activities

Activity 1.3.1 Using long division, find the quotient and remainder for the given rational function. Rewrite the function as a polynomial plus a proper rational function, given $f(x) = \frac{3x^5-5x^2+2}{x^2+x-1}$.

(a) What is the quotient?

(b) What is the remainder?

(c) What is the divisor?

(d) Write the rational function as a polynomial plus a proper rational function.

(e) How can you check your answer? (Hint: Think of regular long division with positive integers.)

1.3.2 Videos

It would be great to include videos down here, like in the Calculus book!