Precalculus for Team-Based Inquiry Learning

2024 Development Edition

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Website: Precalculus for Team-Based Inquiry Learning[1]

[1] teambasedinquirylearning.github.io/precalculus/

[2] github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Polynomial and Rational Functions (PR)

Objectives

BIG IDEA for the chapter goes here, in outcomes/main.ptx
By the end of this chapter, you should be able to...

1. Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.
2. Use quadratic models to solve an application problem and establish conclusions.
3. Rewrite a rational function as a polynomial plus a proper rational function.
4. Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.
5. Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.
6. Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

1.1 Graphing Quadratic Functions (PR1)

Objectives

- Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

Observation 1.1.1 Quadratic functions have many different applications in the real world. For example, say we want to identify a point at which the maximum profit or minimum cost occurs. Before we can interpret some of these situations, however, we will first need to understand how to read the graphs of quadratic functions to locate these least and greatest values.

Activity 1.1.2 Use the graph of the quadratic function $f(x) = 3(x-2)^2 - 4$ to answer the questions below.

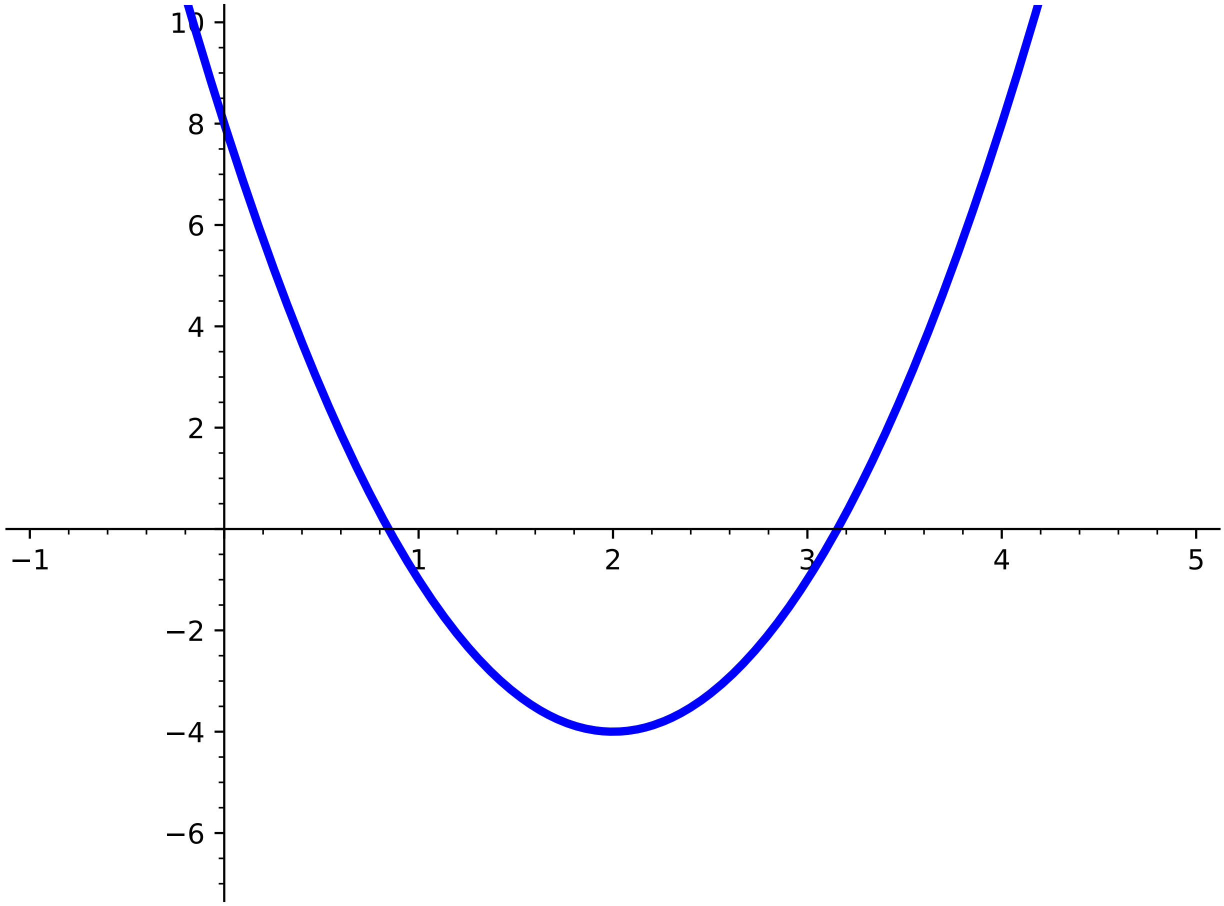


Figure 1.1.3

(a) Make a table for values of $f(x)$ corresponding to the given x-values. What is happening to the y-values as the x-values increase? Do you notice any other patterns of the y-values of the table?

Table 1.1.4

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	
5	

(b) At which point (x, y) does $f(x)$ have a minimum value? That is, is there a point on the graph that is lower than all other points?

A. The minimum value appears to occur near $(0, 8)$.

B. The minimum value appears to occur near $(-\frac{1}{5}, 10)$.

C. The minimum value appears to occur near $(2, -4)$.

D. There is no minimum value of this function.

(c) At which point (x, y) does $f(x)$ have a maximum value? That is, is there a point on the graph that is higher than all other points?

A. The maximum value appears to occur near $(-2, 44)$.

B. The maximum value appears to occur near $(-\frac{1}{5}, 10)$.

C. The maximum value appears to occur near $(2, -4)$.

D. There is no maximum value of this function.

Definition 1.1.5 The **vertex form** of a quadratic function is given by $f(x) = a(x-h)^2 + k$, where (h, k) is the **vertex** of the parabola and $x = h$ is the **axis of symmetry.** $\Diamond$

Activity 1.1.6 Use the given the quadratic function, $f(x) = 3(x-2)^2 - 4$, to answer the following:

(a) Applying Definition 1.1.5, what is the vertex and axis of symmetry of $f(x)$?

A. vertex: $(2, -4)$; axis of symmetry: $x = 2$

B. vertex: $(-2, 4)$; axis of symmetry: $x = -2$

C. vertex: $(-2, -4)$; axis of symmetry: $x = -2$

D. vertex: $(2, 4)$; axis of symmetry: $x = 2$

(b) Compare what you got in part a with the values you found in Activity 1.1.2. What do you notice?

Definition 1.1.7 Given the **standard form** of a quadratic function, $f(x) = ax^2 + bx + c$, with real coefficients a, b, and c, the **axis of symmetry** is defined as $x = \frac{-b}{2a}$ and has a **vertex** at the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$. $\Diamond$

Activity 1.1.8 Use the graph of the quadratic function to answer the questions below.

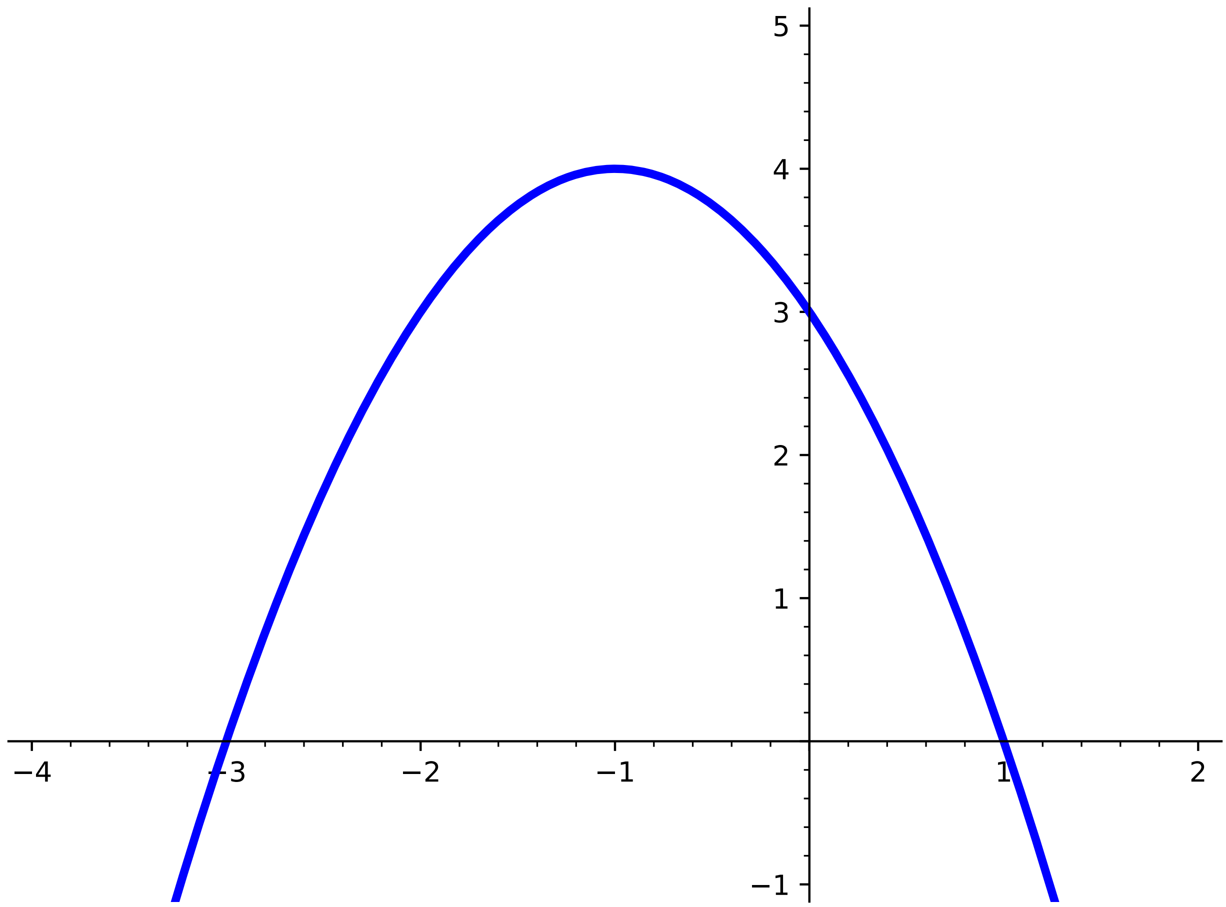


Figure 1.1.9

(a) Which of the following quadratic functions could be the graph shown in the figure?

A. $f(x) = x^2 + 2x + 3$

B. $f(x) = -(x + 1)^2 + 4$

C. $f(x) = -x^2 - 2x + 3$

D. $f(x) = (x + 1)^2 + 4$

(b) What is the maximum or minimum value?

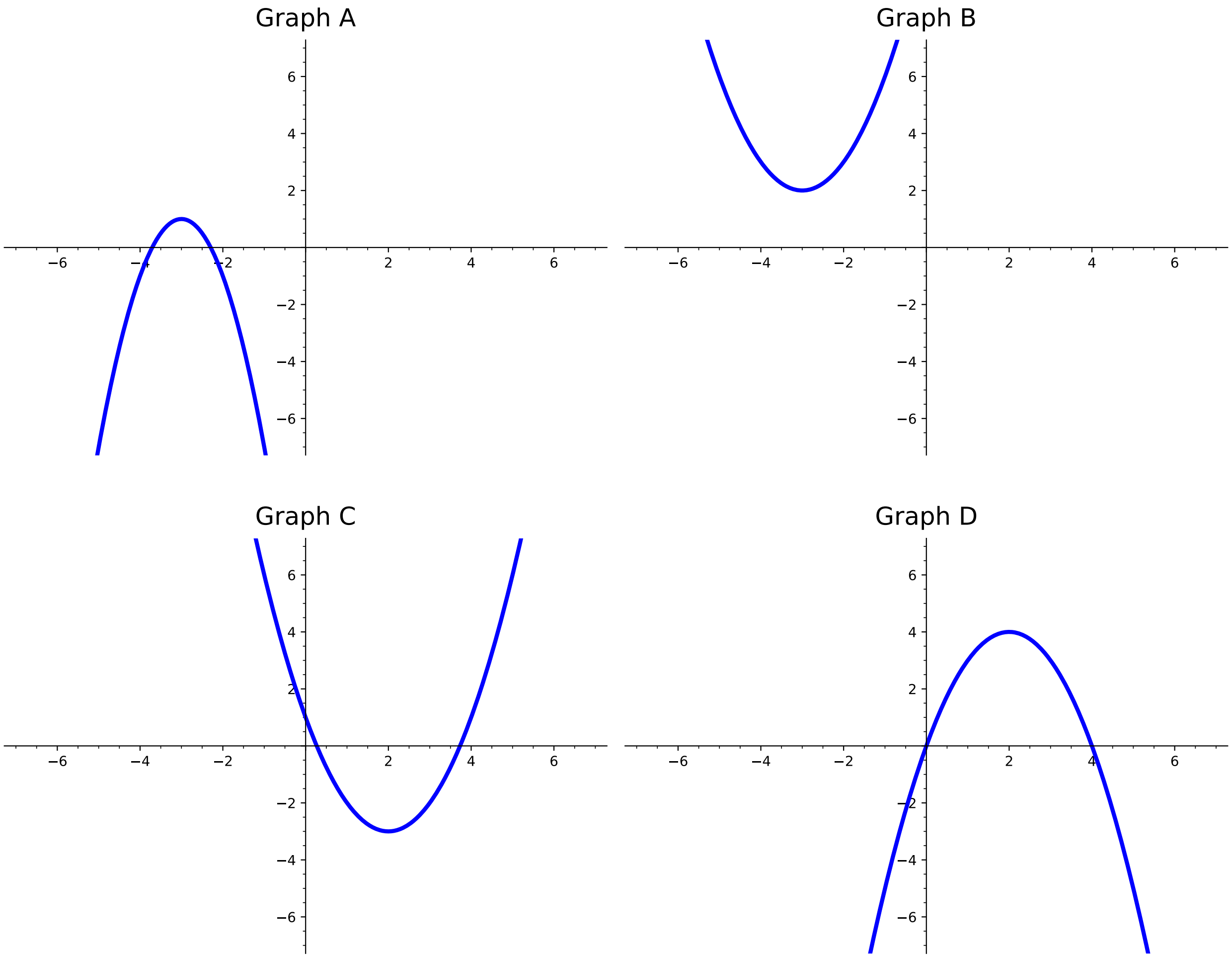
A. -1

B. 4

C. -3

D. 1

Activity 1.1.10 Consider the following four graphs of quadratic functions:



(a) Which of the graphs above have a maximum?

A. Graph A

B. Graph B

C. Graph C

D. Graph D

(b) Which of the graphs above have a minimum?

A. Graph A

B. Graph B

C. Graph C

D. Graph D

(c) Which of the graphs above have an axis of symmetry at $x = 2$?

A. Graph A

B. Graph B

C. Graph C

D. Graph D

(d) Which of the graphs above represents the function $f(x) = -(x-2)^2+4$?

A. Graph A

B. Graph B

C. Graph C

D. Graph D

(e) Which of the graphs above represents the function $f(x) = x^2 - 4x + 1$?

A. Graph A

B. Graph B

C. Graph C

D. Graph D

Remark 1.1.11 Notice that the maximum or minimum value of the quadratic function is the vertex. How can you determine if the vertex is a maximum or minimum?

Activity 1.1.12 A function $f(x)$ has a maximum value at 7 and its axis of symmetry at $x = -2$.

(a) Sketch a graph of a function that meets the criteria for $f(x)$.

(b) Was your graph the only possible answer? Try to sketch another graph that meets this criteria.

Remark 1.1.13 Other points, such as zeros (i.e., x-intercepts), may be helpful in sketching a more accurate graph of a quadratic function.

Activity 1.1.14 Consider the following two quadratic functions $f(x) = x^2 - 4x + 12$ and $g(x) = 2x^2 + 8x - 10$ and answer the following questions:

(a) Applying Definition 1.1.7, what is the vertex and axis of symmetry of $f(x)$?

A. vertex: $(2, -16)$; axis of symmetry: $x = 2$

B. vertex: $(-2, 16)$; axis of symmetry: $x = -2$

C. vertex: $(-2, -16)$; axis of symmetry: $x = -2$

D. vertex: $(2, 16)$; axis of symmetry: $x = 2$

(b) Applying Definition 1.1.7, what is the vertex and axis of symmetry of $g(x)$?

A. vertex: $(2, -16)$; axis of symmetry: $x = 2$

B. vertex: $(-2, 16)$; axis of symmetry: $x = -2$

C. vertex: $(-2, -16)$; axis of symmetry: $x = -2$

D. vertex: $(2, 16)$; axis of symmetry: $x = 2$

(c) What do you notice about $f(x)$ and $g(x)$?

(d) Now graph both $f(x)$ and $g(x)$ and draw a sketch of each graph on one coordinate plane. How are they similar/different?

1.2 Quadratic Models and Meanings (PR2)

Objectives

- Use quadratic models to solve an application problem and establish conclusions.

Activity 1.2.1 A water balloon is tossed vertically from a fifth story window. It's height $h(t)$, in feet, at a time t, in seconds, is modeled by the function

$$h(t) = -16t^2 + 40t + 50$$

(a) Complete the following table. Do all the values have meaning in terms of the model?

Table 1.2.2

t	$h(t)$
0	
1	
2	
3	
4	
5	

(b) Compute the slope of the line joining $t = 0$ and $t = 1$. Then, compute the slope of the line joining $t = 1$ and $t = 2$. What do you notice about the slopes?

(c) What is the meaning of $h(0) = 50$?

A. the initial height of the water balloon is 50 feet.

B. the water balloon reaches a maximum height of 50 feet.

C. the water balloon hits the ground after 50 seconds.

D. the water balloon travels 50 feet before hitting the ground.

(d) Find the vertex of the quadratic function.

A. $(0, 50)$

B. $(1, 74)$

C. $(1.25, 75)$

D. $(3.4, 0)$

(e) What is the meaning of the vertex?

A. The water balloon reaches a maximum height of 50 feet at the start.

B. After 1 second, the water balloon reaches a maximum height of 74 feet.

C. After 1.25 seconds, the water balloon reaches the maximum height.

D. After 3.5 seconds, the water balloon hits the ground.

Activity 1.2.3 The population of a small city is given by the function $P(t) = -50t^2 + 1200t + 32000$, where t is the number of years after 2015.

(a) When will the population of the city reach a maximum?

A. 2020

B. 2022

C. 2025

D. 2027

(b) Determine when the population of the city is increasing and when it is decreasing.

(c) When will the population of the city reach 36,000 people?

A. 2019

B. 2025

C. 2027

D. 2035

Activity 1.2.4 The unit price of an item affects its supply and demand. That is, if the unit price increases, the demand for the item will usually decrease. For example, an online streaming service currently has 84 million subscribers at a monthly charge of \$6. Market research has suggested that if the owners raise the price to \$8, they would lose 4 million subscribers. Assume that subscriptions are linearly related to the price.

(a) Which of the following represents a linear function which relates the price of the streaming service p to the number of subscribers Q?

A. $Q(p) = -2p$

B. $Q(p) = -2p + 84$

C. $Q(p) = -2p - 4$

D. $Q(p) = -2p + 96$

(b) Using the fact that Revenue $= pQ$, which of the following represents the Revenue R in terms of the price p.

A. $R(p) = -2p^2$

B. $R(p) = -2p^2 + 84p$

C. $R(p) = -2p^2 - 4p$

D. $R(p) = -2p^2 + 96p$

(c) What price should the streaming service charge for a monthly subscription to maximize their revenue?

A. \$10

B. \$19.50

C. \$24

D. \$28.25

(d) How many subscribers would the company have at this price?

A. 39.5 million

B. 48 million

C. 57 million

D. 76 million

(e) What is the maximum revenue?

A. 760 million

B. 1112 million

C. 1152 million

D. 1116 million

Activity 1.2.5 The owner of a ranch decides to enclose a rectangular region with 240 feet of fencing. To help the fencing cover more land, he plans to use one side of his barn as part of the enclosed region. What is the maximum area the rancher can enclose?

1.3 Polynomial Long Division (PR3)

Objectives

- Rewrite a rational function as a polynomial plus a proper rational function.

Polynomial Long Division (PR3)

Activity 1.3.1 Using long division, find the quotient and remainder for the given rational function. Rewrite the function as a polynomial plus a proper rational function, given $f(x) = \frac{3x^5 - 5x^2 + 2}{x^2 + x - 1}$.

(a) What is the quotient?

(b) What is the remainder?

(c) What is the divisor?

(d) Write the rational function as a polynomial plus a proper rational function.

(e) How can you check your answer? (Hint: Think of regular long division with positive integers.)

1.4 Zeroes of Polynomial Functions (PR4)

Objectives

- Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.