Precalculus for Team-Based Inquiry Learning

2024 Development Edition

Precalculus for Team-Based Inquiry Learning

2024 Development Edition

TBIL Fellows

Editors

Steven Clontz
University of South Alabama

Drew Lewis

Contributing Authors

Tonya DeGeorge
Georgia Gwinnett College

Abby Noble
Middle Georgia State University

Kathy Pinzon
Georgia Gwinnett College

Wendy Sheppard

College of Charleston

October 31, 2023

Website: Precalculus for Team-Based Inquiry Learning[1]

[1] teambasedinquirylearning.github.io/precalculus/

[2] github.com/TeamBasedInquiryLearning/precalculus/blob/main/LICENSE.md

Contents

Chapter 1

Polynomial and Rational Functions (PR)

Objectives

BIG IDEA for the chapter goes here, in outcomes/main.ptx
By the end of this chapter, you should be able to...

1. Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

2. Use quadratic models to solve an application problem and establish conclusions.

3. Rewrite a rational function as a polynomial plus a proper rational function.

4. Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.

5. Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.

6. Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

1.1 Graphing Quadratic Functions (PR1)

Objectives

- Graph quadratic functions and identify their axis of symmetry, and maximum or minimum point.

Activity 1.1.1 Activities may start with an `<introduction>`.

(a) Then we can ask students to do some `<task>`s.

(b) Here's a second `<task>`.

1.2 Quadratic Models and Meanings (PR2)

Objectives

- Use quadratic models to solve an application problem and establish conclusions.

Activity 1.2.1 A water balloon is tossed vertically from a fifth story window. It's height $h(t)$, in meters, at a time t, in seconds, is modeled by the function

$$h(t) = -5t^2 + 20t + 25$$

(a) Complete the following table.

Table 1.2.2

t	$h(t)$
0	
1	
2	
3	
4	
5	

(b) Explain why $h(t)$ is not a linear function.

(c) What is the meaning of $h(0) = 25$?

A. the initial height of the water balloon is 25 meters.

B. the water balloon reaches a maximum height of 25 meters.

C. the water balloon hits the ground after 25 seconds.

D. the water balloon travels 25 meters before hitting the ground.

(d) Find the vertex of the quadratic function.

A. $(0, 25)$

B. $(2, 45)$

C. $(5, 0)$

D. $(1, 40)$

(e) What is the meaning of the vertex?

A. The water balloon reaches a maximum height of 25 meters at the start.

B. After 2 seconds, the water balloon reaches a maximum height of 45 meters.

C. After 5 seconds, the water balloon reaches a maximum height.

D. After 1 second, the water balloon reaches a maximum height of 40 meters.

1.3 Polynomial Long Division (PR3)

Objectives

- Rewrite a rational function as a polynomial plus a proper rational function.

Polynomial Long Division (PR3)

Activity 1.3.1 Using long division, find the quotient and remainder for the given rational function. Rewrite the function as a polynomial plus a proper rational function, given $f(x) = \frac{3x^5-5x^2+2}{x^2+x-1}$.

(a) What is the quotient?

(b) What is the remainder?

(c) What is the divisor?

(d) Write the rational function as a polynomial plus a proper rational function.

(e) How can you check your answer? (Hint: Think of regular long division with positive integers.)

1.4 Zeroes of Polynomial Functions (PR4)

Objectives

- Determine the zeros of a real polynomial function, write a polynomial function given information about its zeros and their multiplicities, and apply the Factor Theorem and the Fundamental Theorem of Algebra.

Activity 1.4.1 Write the polynomial function in factored form using information from the graph below.

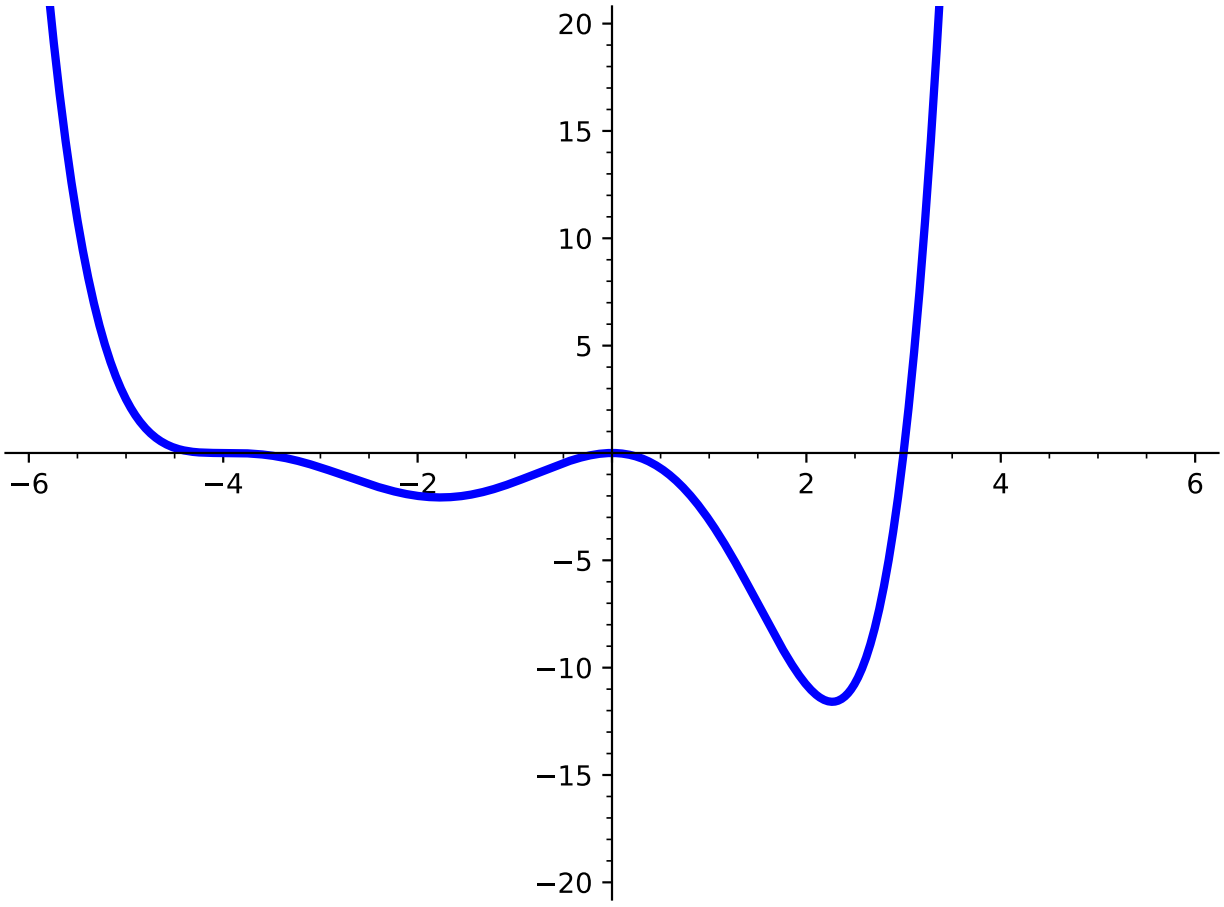


Figure 1.4.2

(a) Using the given graph, what are the real zeros of this function? Select all that apply.

A. 0

B. 1

C. -3

D. 3

E. 4

F. -4

(b) What are the least possible multiplicities for each zero?

(c) What is the least degree of the function?

A. 3

B. 4

C. 5

D. 6

(d) Describe the end behavior of the graph.

A. As $x \to \infty$, $f(x) \to \infty$

B. As $x \to -\infty$, $f(x) \to \infty$

C. As $x \to \infty$, $f(x) \to -\infty$

D. As $x \to -\infty$, $f(x) \to -\infty$

(e) Combining the information in part (d) with the degree of the function, will the leading coefficient be positive or negative?

A. positive

B. negative

(f) Given the point $(2, \frac{-54}{5})$ is on the curve, and using the information in parts (a) through (e), write the function for the graph above in factored form.

1.5 Graphs of Polynomial Functions (PR5)

Objectives

- Find the intercepts, estimated locations of maxima and minima, and end behavior of a polynomial function, and use this information to sketch the graph.

Activity 1.5.1 Activities may start with an `<introduction>`.

(a) Then we can ask students to do some `<task>`s.

(b) Here's a second `<task>`.

1.6 Properties of Graphs of Rational Functions (PR6)

Objectives

- Find the domain and range, vertical and horizontal asymptotes, and intercepts of a rational function and use this information to sketch the graph.

Definition 1.6.1 A function r is rational provided that it is possible to write r as the ratio of two polynomials, p and q. That is, r is rational provided that for some polynomial functions p and q, we have

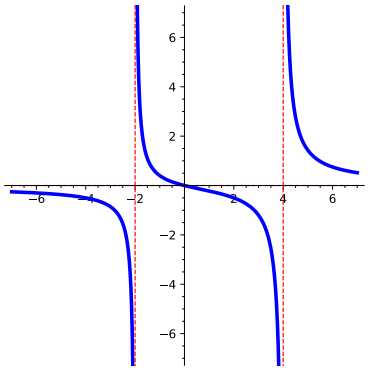
$$r(x) = \frac{p(x)}{q(x)}.$$

$\Diamond$

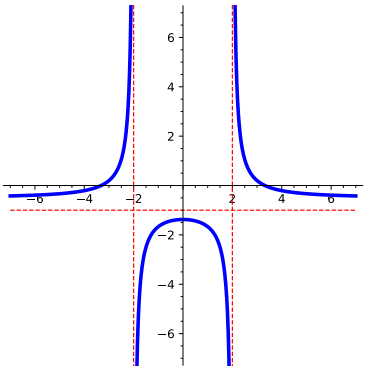
Properties of Graphs of Rational Functions (PR6)

Activity 1.6.2 Consider the following six graphs of rational functions:

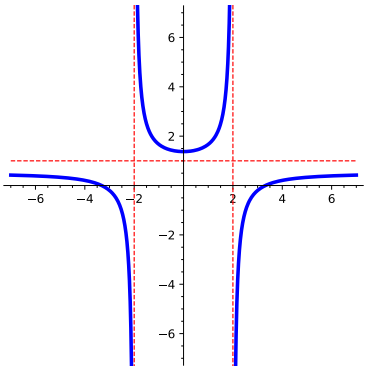
Graph A



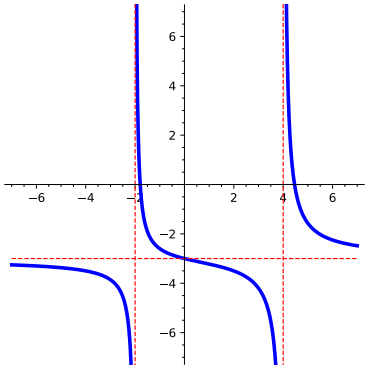
Graph B



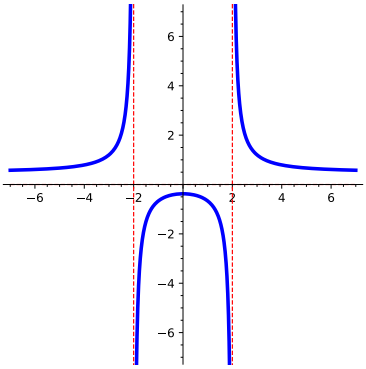
Graph C



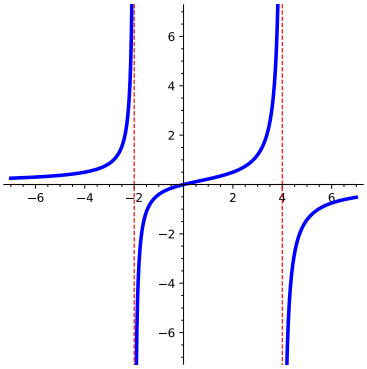
Graph D



Graph E



Graph F



(a) Which of the graphs above represents the function $f(x) = \frac{2x}{x^2-2x-8}$?

(b) Which of the graphs above represents the function $g(x) = \frac{x^2+3}{2x^2-8}$?

Colophon

This book was authored in PreTeXt.