Fundamentals of Physics 3 Homework Template

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1 Introduction

You can leave the automatic numbering for sections as is or you can suppress the numbering by using `\section*` instead of `\section`.

Problem 2.7

Find the electric field a distance z from the center of a sphere of uniform surface charge σ with radius R.

Solution The surface element of the sphere is

$$\mathrm{d}a' = (R\,\mathrm{d}\theta')(R\sin\theta'\,\mathrm{d}\phi') \tag{1}$$

$$\begin{aligned}\boldsymbol{r} &= z\,\hat{\boldsymbol{z}}\\ \boldsymbol{r}' &= R\,\hat{\boldsymbol{r}}\\ \boldsymbol{\imath} &= \boldsymbol{r} - \boldsymbol{r}' = z\,\hat{\boldsymbol{z}} - R\,\hat{\boldsymbol{r}}\end{aligned}$$

Let's start by breaking $\hat{\boldsymbol{r}}$ into Cartestian coordinates:

$$\hat{\boldsymbol{r}} = \cos\theta\,\hat{\boldsymbol{z}} + \sin\theta(\cos\phi\,\hat{\boldsymbol{x}} + \sin\phi\,\hat{\boldsymbol{y}}) \tag{2}$$

$$\begin{aligned}\boldsymbol{\imath} &= \boldsymbol{r} - \boldsymbol{r}'\\ &= z\,\hat{\boldsymbol{z}} - R[\cos\theta\,\hat{\boldsymbol{z}} + \sin\theta(\cos\phi\,\hat{\boldsymbol{x}} + \sin\phi\,\hat{\boldsymbol{y}})]\\ &= -R\sin\theta\cos\phi\,\hat{\boldsymbol{x}} - R\sin\theta\sin\phi\,\hat{\boldsymbol{y}} + (z - R\cos\theta)\,\hat{\boldsymbol{z}}\end{aligned} \tag{3}$$

Next, we use the law of cosines to the magnitude of $\boldsymbol{\imath}$.

(4)

$$\begin{aligned}\boldsymbol{E} &= \int \frac{1}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{\imath}}}{\imath^2}\sigma\,\mathrm{d}a'\\ &= \frac{\sigma}{4\pi\epsilon_0}\int \frac{\boldsymbol{\imath}}{\imath^3}R^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\end{aligned}$$