2.2.1

$\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

IA Sei $n = 1$ $1^2 = 1 = \frac{1(2)(3)}{6} = 1$ wahr

IB Sei n $\in \mathbb{N}$ beliebig aber fest und $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ gilt

IS zu zeigen $\sum\limits_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$

$= \sum\limits_{k=1}^{n+1} k^2 = \sum\limits_{k=1}^{n} k^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$

$= \frac{n(n+1)(2n+1)+6(n+1)^2}{6}$ |(n+1) raus

$= \frac{n(n+1)((n(2n+1))+6(n+1))^2}{6}$

$= \frac{(n+1)(2n^2+n+6n+6)}{6}$

$= \frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$

q.e.d.

2.2.3

$\sum\limits_{i=0}^{n} (p+i) = \frac{(n+1)(2p+n)}{2}$

IA mit $n = 0$ $(p+0) = \frac{(0+1)(2p+0)}{2} = \frac{2p}{2} = 2$ wahr

IB Sei n $\in \mathbb{N}$ beliebig aber fest und $\sum\limits_{i=0}^{n} (p+i) = \frac{(n+1)(2p+n)}{2}$ gilt

IS zu zeigen das n+1 gilt $\sum\limits_{i=0}^{n+1} (p+i) = \frac{(n+2)(2p+n+1)}{2}$

$\sum\limits_{i=0}^{n+1} (p+i) = \sum\limits_{i=0}^{n} (p+i) + (p+(n+1))$

$\frac{(n+1)(2p+n)+2p+2n+2}{2}$

$= \frac{n^2+2p+2pn+n+2p+2n+2}{2}$

$= \frac{n^2+4p+2pn+n+3n+2}{2}$

$= \frac{(n+2)(2p+n+1)}{2}$

q.e.d.