2.2.1

Zu zeigen: $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

IA Sei $n = 1$ $1^2 = 1 = \frac{1(2)(3)}{6} = 1$

IB Sei N $\in \mathbb{N}$ beliebig aber fest und $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ gilt

IS zu zeigen $\sum\limits_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$
$= \sum\limits_{k=1}^{n+1} k^2 = \sum\limits_{k=1}^{n} k^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$
$= \frac{n(n+1)(2n+1)+6(n+1)^2}{6}$ |(n+1) raus
$= \frac{n(n+1)((n(2n+1))+6(n+1))^2}{6}$
$= \frac{(n+1)(2n^2+n+6n+6)}{6}$
$= \frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$
q.e.d

2.2.3