OPPGAVE 4

Add a sinusoidal pile to the beam. This means adding a function of form $s(x) = -pg\sin(\frac{\pi}{L})$ to the force term $f(x)$. Prove that the solution

$$y(x) = \frac{f}{24EI}x^2(x^2 - 4Lx + 6L^2) - \frac{pgL}{EI\pi}\left(\frac{L^3}{\pi^3}\sin\frac{\pi}{L} - \frac{x^3}{6} + \frac{L}{2}x^2 - \frac{L^2}{\pi^2}x\right) \quad (1)$$

satisfies the EulerBernoulli beam equation and the clamped-free boundary conditions.

Lsning

Skal bevise at:

$$\begin{aligned}
y(x) &= \frac{f}{24EI}x^2(x^2 - 4Lx + 6L^2) - \frac{pgL}{EI\pi}\left(\frac{L^3}{\pi^3}\sin\frac{\pi}{L}x - \frac{x^3}{6} + \frac{L}{2}x^2 - \frac{L^2}{\pi^2}x\right) \\
EIy(x) &= \frac{f}{24}x^2(x^2 - 4Lx + 6L^2) - \frac{pgL}{\pi}\left(\frac{L^3}{\pi^3}\sin\frac{\pi}{L}x - \frac{x^3}{6} + \frac{L}{2}x^2 - \frac{L^2}{\pi^2}x\right) \\
EIy(x) &= \frac{f}{24}(x^4 - 4Lx^3 + 6L^2x^2) - \frac{pgL}{\pi}\left(\frac{L^3}{\pi^3}\sin\frac{\pi}{L}x - \frac{x^3}{6} + \frac{L}{2}x^2 - \frac{L^2}{\pi^2}x\right) \\
EIy'(x) &= \frac{f}{24}(4x^3 - 12Lx^2 + 12L^2x) - \frac{pgL}{\pi}\left(\frac{L^3}{\pi^3}\cos(\frac{\pi}{L}x)\cdot\frac{\pi}{L} - \frac{3x^2}{6} + \frac{2L}{2}x - \frac{L^2}{\pi^2}\right) \\
EIy''(x) &= \frac{f}{24}(12x^2 - 24Lx + 12L^2) - \frac{pgL}{\pi}\left(\frac{L^2}{\pi^2} - \sin(\frac{\pi}{L}x)\cdot\frac{\pi}{L} - x + L\right) \\
EIy'''(x) &= \frac{f}{24}(24x - 24L) - \frac{pgL}{\pi}\left(\frac{L}{\pi} - \cos(\frac{\pi}{L}x)\cdot\frac{\pi}{L} - 1\right) \\
EIy''''(x) &= \frac{f}{24}(24) - \frac{pgL}{\pi}\left(\sin(\frac{\pi}{L}x)\cdot\frac{\pi}{L}\right) \\
EIy''''(x) &= \frac{f24}{24} - pg\cdot\sin(\frac{\pi}{L}x) \\
EIy''''(x) &= f - pg\cdot\sin(\frac{\pi}{L}x)
\end{aligned} \quad (2)$$