The Julia Set

The boundary of the set of initial points $z = x + yi$ in the complex plane for which the orbits are bounded when the complex map $Q_c : \mathbb{C} \to \mathbb{C}$, $Q_c(z) = z^2 + c$ (where $c \in \mathbb{C}$) is iterated, is called a Julia set. The filled Julia set is the set of all initial points that have bounded orbits.

The program julia.ode computes the Julia set by by starting inside the unit circle and iterating backwards as an iterated function system , i.e. iterating the inverse map $\pm\sqrt{z - c}$, randomly choosing between the positive and negative square root.

In the program, atan2(u, v) takes into account the signs of u and v to return the correct argument for the angle θ for $u + iv$. Also the option "trans=100" throws away the first 100 points of the iteration.

After starting up julia.ode, turn off the axes using "Graphics→axes opts" and changing the three entries X-org(1-on) etc. from 1 to 0.

Integrate the equation using "InitialConditions→ Go" to see the Julia Set for the $c = cx + icy$ in the (x, y) plane. Try different values of cx and cy and see if you can see a pattern of how the Julia sets change.

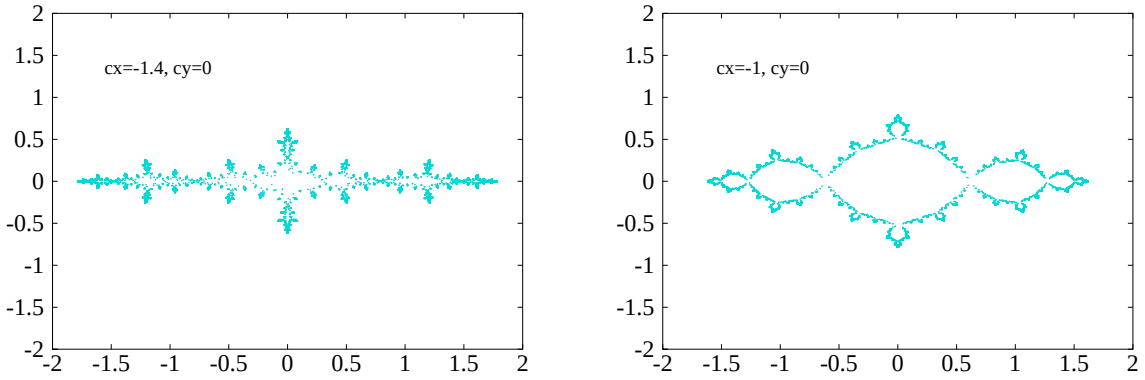

Figure 1: Several Julia Sets.

from: Simulating, Analyzing, and Animating Dynamical Systems, by Bard Ermentrout, SIAM 2002.

To make: Animated Julia Sets

Let's make a whole bunch of Julia sets and animate the result as the parameter varies. We set `cx=0` and vary `cy` from `-1` to `1`. First, change the total number of iterates back to 2000. Click on `Initialconds Range` (**I R**) and fill in the dialog as follows:

`Range over:` cy
`Steps:` 20
`Start:` -1
`End:` 1
`Reset storage:` Y
`Use old ic's:` Y
`Cycle color:` N
`Movie:` Y

Notice that you should type in `Y` in the `Movie` entry since this tells *XPPAUT* you want to save the contents of the window. Also, make sure that no windows other than perhaps the dialog window obscure the main graphics window. Click on **Ok** and 20 Julia sets will be computed. Each screen image is saved in memory so that you can play them back. Now click on `Kinescope Playback` (**K P**) and you can step through each of the plots one at a time by clicking on a mouse button. Click **Esc** to exit. To get a smooth animation of the Julia sets, click on `Kinescope Autoplay` (**K A**). The tell *XPPAUT* how many times you want to repeat the animation (e.g 2) and how many milliseconds between frames (e.g. 100). After this, the animation will be repeated as a smooth movie.

You can save the animations in several ways. One of these requires that you have a separate software program that can encode a series of still images into a movie. An easier way but one which is not quite as efficient is to create an animated GIF. Animated GIF's can be played on most web browsers such as Netscape or Explorer. Click on `Kinescope Make Anigif` and your movie will be replayed. At the same time, a file called `anim.gif` will be produced on your disk. You can view this with `xanim` or other viewing software or just open it with your web browser.