

## The Filled Julia Set

The boundary of the set of initial points  $z = x + yi$  in the complex plane for which the orbits are bounded when the complex map  $Q_c : \mathbb{C} \rightarrow \mathbb{C}$ ,  $Q_c(z) = z^2 + c$  (where  $c \in \mathbb{C}$ ) is iterated, is called a Julia set. The filled Julia set is the set of all initial points that have bounded orbits.

The program filled\_julia.ode computes the filled Julia set:

```
#filled_julia.ode
#drawing the filled Julia set
#
par cx=0.360284,cy=.100376
dx/dt=if(t<=0)then(u^2-v^2+cx)else(x^2-y^2+cx)
dy/dt=if(t<=0)then(2*u*v+cy)else(2*x*y+cy)
du/dt=u
dv/dt=v
# if this crosses 0 we are out of the set
aux amp=x^2+y^2-max(4,cx^2+cy^2)
#
@ xp=u,yp=v
@ maxstor=40000000, meth=discrete
@ xlo=-1.2,xhi=1.2,ylo=-1.2,yhi=1.2,lt=-1
@ rangeover=u,rangereset=no,angelow=-1.2,rangehigh=1.2,rangestep=200
@ poimap=section,poivar=amp,poipln=0,poisgn=1,poistop=1
done
```

The program is similar to the program for computing the Mandelbrot set, however, instead of producing a graph in parameter space  $cx - cy$  where  $c = cx + icy$ , we fix a parameter value  $c$ , and range over the initial conditions in  $u - v$  space.

After starting up julia\_filled.ode, turn off the axes using “Graphics→axes opts” and changing the three entries X-org(1-on) etc. from 1 to 0.

Then follow the instructions for producing the Mandelbrot set, but instead set “Vary1” to “u” and “Vary2” to “v” when the table comes up when you click on InitialConditions→2 par (range). I used Steps 400 in each case. Under Numerics→ColorCode, choose another quantity and accept T then choose “optimize” to get an range for the colour coding. You can then play with this to get a colouring range that you prefer.

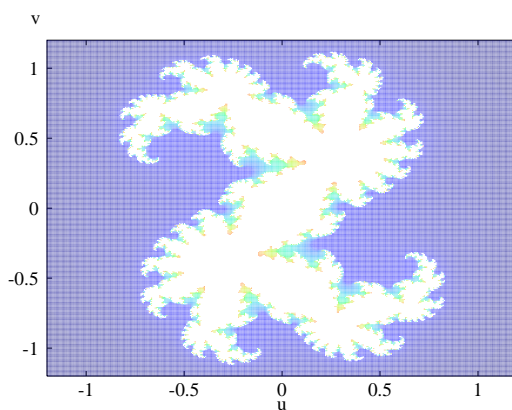


Figure 1: Filled Julia Sets for  $c = 0.360284 + 0.100376i$ .