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Fields

Electrostatic

electric — stationary

· Electric charges (represented by Q or q) are of positive and negative
· Measured through Coulombs (C)

Electric charge is a property
- Some things/particles in the universe may not have this property: Photon

just like how photons do not have mass

neutral does not mean no charge property (it might be net 0; no property means not composed of any charge)

Protons and electrons have the same magnitude of electric charge...

This is called the e = elementary charge

where... $e \approx 1.6 \times 10^{-19}$ C

∴ proton ∘, $+e$ electron ∘, $-e$

Some particles are precisely fractions of the elementary charge

e.g. up quark ∘, $+\frac{1}{3}e$ (subatomic particles are made of quarks!)

For larger things, any object in the universe have a total charge — the sum of all charges it is composed of

E.g. $^{4}_{2}He$

$-e^-$

$p^+ \; p^+$

$n^0 \; n^0$

$-e^-$

$+e+e-e-e = 0$

Neutral!

Charged particles

· when converting particles from neutral to charged, typically the negative charge is changed, not positive

↳ This follows similarly to the atom where electrons are added / removed instead of protons

But, how do we transfer charge?

1. By friction (triboelectric effect)

· By rubbing two neutral materials together, friction allows for the transfer of charge

· Probabistically, one material may attract more charge and the other may lose more charge

It follows the following table:

Table of triboelectric materials

MOST POSITIVE

⋮

more + will likely end up more +

glass

leather

lead

Polystyrene

rubber

plastic wrap

more - will likely end up more -

⋮

MOST NEGATIVE

E.g. when rubber and glass is rubbed together

⇓

glass transfers - charge to rubber

⇓

rubber ends up -, glass ends up +

"Two materials charged by friction end up with opposite charges"

(Transfer of charge

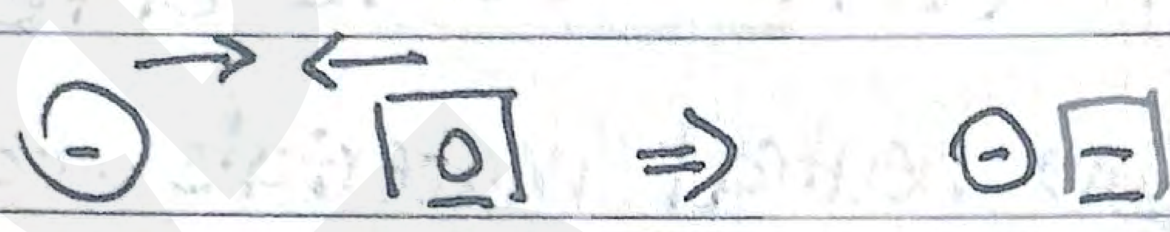
Date. ______ NO. ______

ways of transfer of charge

2. By contact

- When a material of non-neutral charge is in contact with a material
- Charge is transferred such that they end up with the same sign

i.e "Two materials charged by contact end up with the same sign of charge"



- Regarding the distribution of charge after contact, the total charge is conserved but individual charge is based on mass

○ charge = A_0 □ charge = B_0

$$Q_{total} = A_0 + B_0 = A_1 + B_1 = A_2 + B_2$$

0: ○ A_0 □ B_0 $Q_{total} = A_0 + B_0$

1: ○□ A_1 B_1 $Q_{total} = A_1 + B_1$

2: ○ A_2 □ B_2 $Q_{total} = A_2 + B_2$

conserved charge

A_2 and B_2 determined by masses (when equal mass, $A_2 = B_2$)

IB only expects equal mass

Date. NO.

ways of transfer of charge

3. By induction

- Happens in the following timeframes

time 1

rod and sphere separated

time 2

rod and sphere in contact

time 3 grounding

sphere is grounded

⇓

grounded means in contact with earth

⇓

electrons are transfered to the earth in order to neutralize rod-sphere

time 4

sphere ungrounded

time 5

sphere, rod with distinct charges

* Grounding

↳ Tries to neutralize system by forcing - charge in from excess or forcing - charge out

↳ ~~when it can't neutralize~~ completely, it will try to achieve most neutral (close to 0 charge

"Two materials charged by induction end up with different sign charges"

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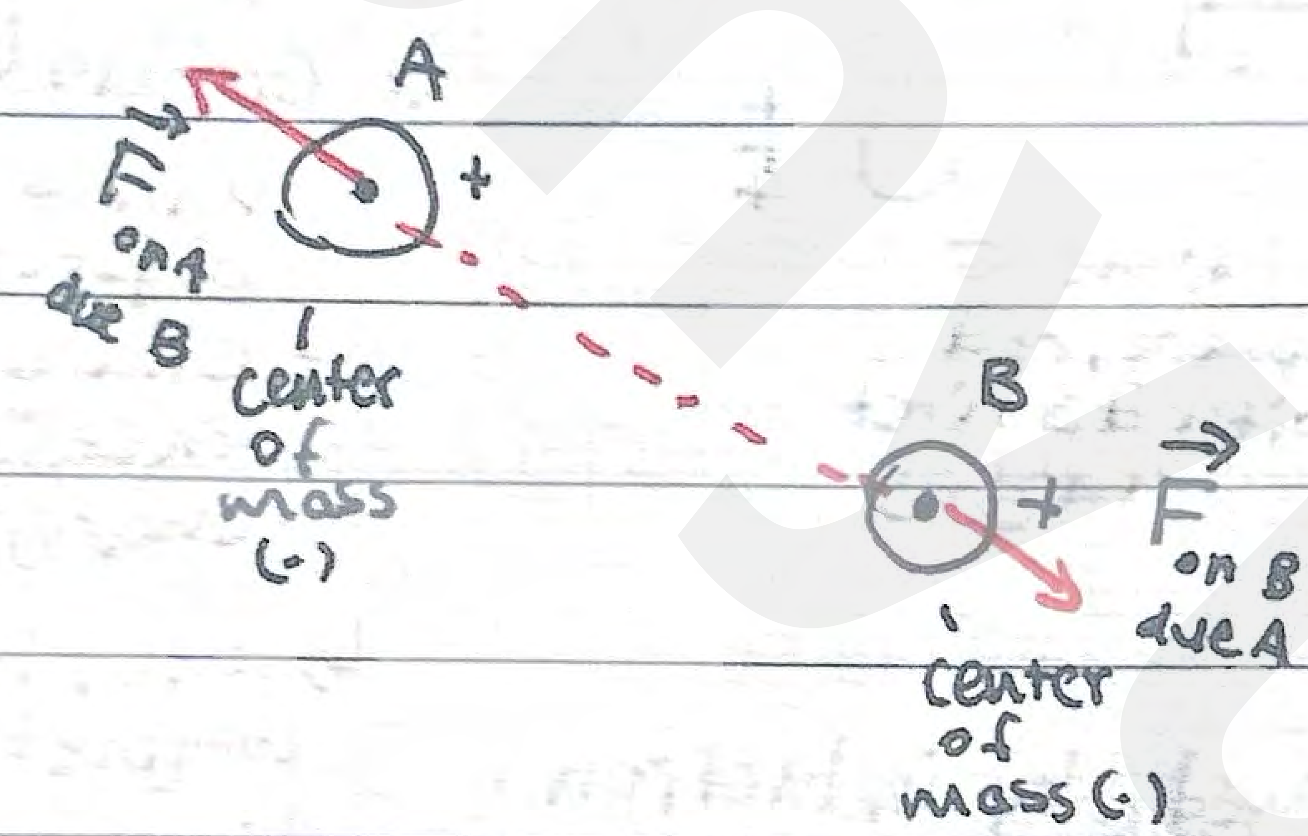
Electric Force - Force of interaction between two or more charged particles

- Opposite charged particles experience a force of attraction.
- Same (sign) charged particles experience a force of repulsion.

How do you find vector?

- The direction is determined by the two charges and their center of masses.

i.e.



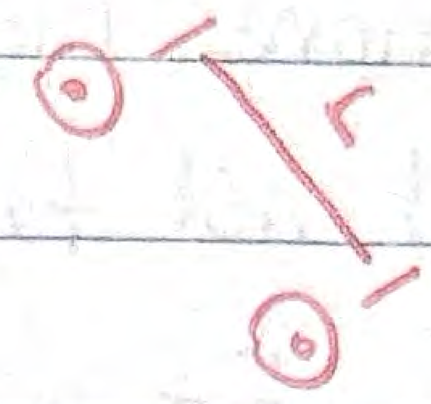
- The magnitude of the forces are equal since Newton's second law (equal and opposite charge)

$$\boxed{\left|\vec{F}_{\text{on A due B}}\right| = \left|\vec{F}_{\text{on B due A}}\right|}$$

The magnitude depends on their two charges, their distance apart.

Electric Force

This magnitude is determined by the following formula...



$$|\vec{F}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2}$$

coulomb's force

epsilon (to do with ambience)

where...

ϵ_0 - permittivity of free space $= 8.85\times10^{-12}\, C^2 \cdot N^{-1} \cdot m^{-2}$

r - distance between two centers of mass

Q_n - magnitude of particle n's charge

Since ϵ_0 is a constant and in turn $\frac{1}{4\pi\epsilon_0}$...

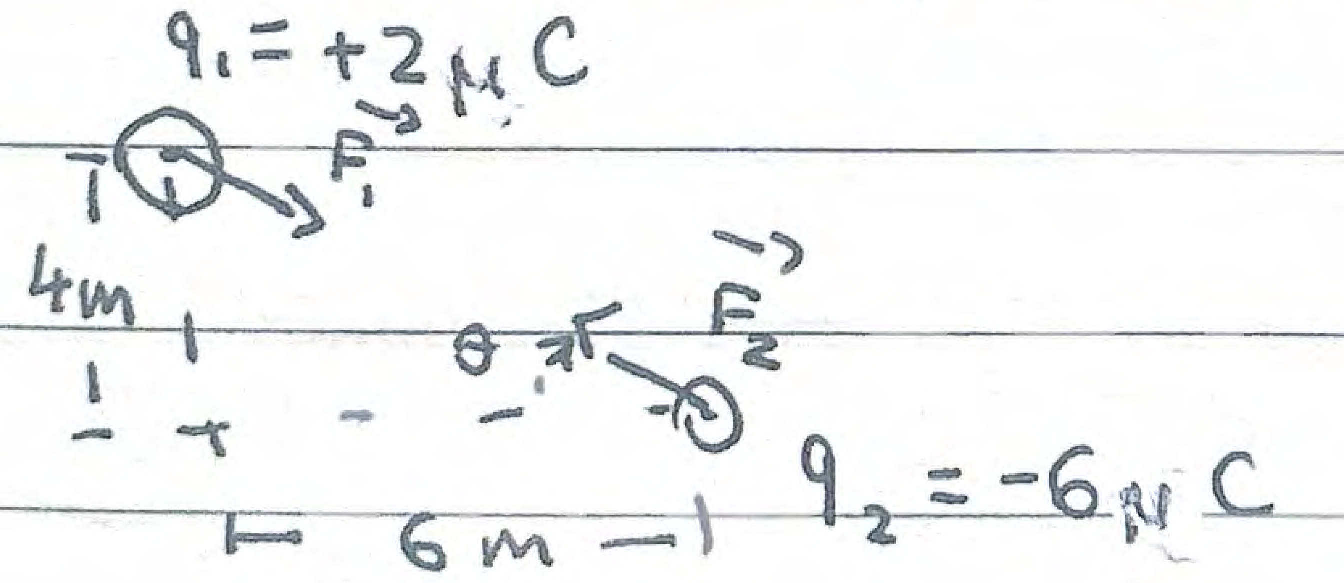
coulomb's constant

$$|\vec{F}| = k \cdot \frac{Q_1 \cdot Q_2}{r^2} \quad \text{such that} \quad k = 8.99\times10^9\, Nm^2C^{-2}$$

Oftentimes, simply calculating the magnitude is not enough, you need the direction.

→ Express direction as an angle with cardinal direction ⇒ E.g. $\theta°$ N of W

E.g.



Find $\vec{F_2}$

$|\vec{F_2}| = (8.99\times10^9)\frac{(12\times10^{-12})}{52}$

$\approx 2.07\times10^{-3}$ N

$\theta = \arctan(\frac{2}{3}) \approx 33.7°$

$\vec{\theta} \approx 33.7°$ N of W

$\therefore \vec{F_2} \approx 2.07\times10^{-3}$ N at $33.7°$ N of W

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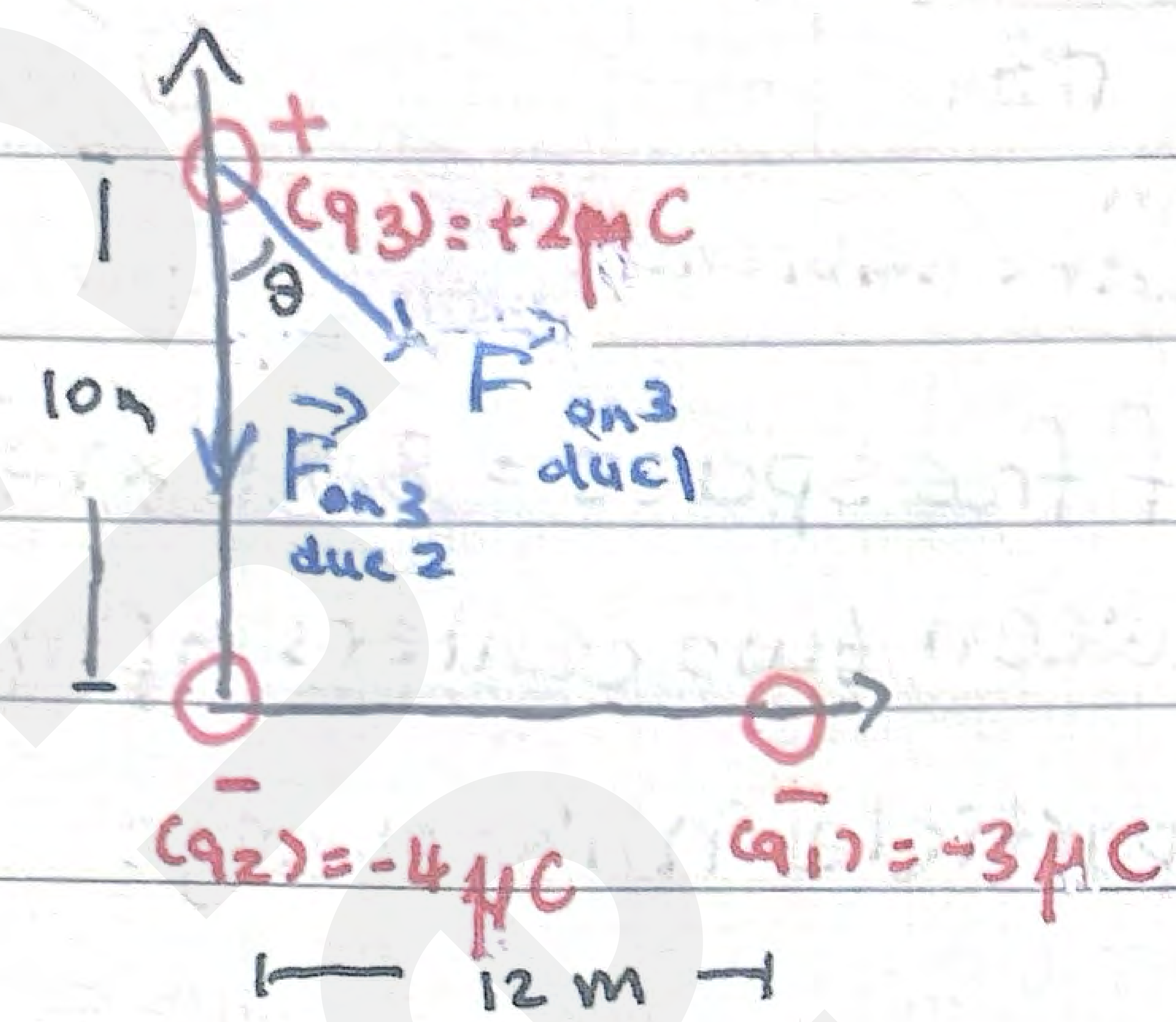
Electric Force

But this formula only works for pairs

For more than 2 particles, do vector addition/subtraction to find net force.

E.g.



$$\left|\vec{F}_{\substack{on3\\due1}}\right| = \frac{k \cdot 6\times10^{-12}}{244} \approx 2.21\times10^{-4}\,N$$

$$\left|\vec{F}_{\substack{on3\\due2}}\right| = \frac{k \cdot 8\times10^{-12}}{100} \approx 7.192\times10^{-4}\,N$$

Find $\vec{F}_3$

$$\therefore \vec{F}_3 \langle \underbrace{\left|\vec{F}_{\substack{on3\\due1}}\right| \times \sin\theta}_{x}, \underbrace{-\left(\left|\vec{F}_{\substack{on3\\due1}}\right| \times \cos\theta + \vec{F}_{\substack{on3\\due2}}\right)}_{y} \rangle$$

$$\theta = \arctan\left(\frac{12}{10}\right) \approx 50.19^\circ$$

$$\therefore \vec{F}_3 = \langle 2.21\times10^{-4} \times \sin(50.19^\circ), -(7.192\times10^{-4} + 2.21\times10^{-4} \times \cos(50.19^\circ)) \rangle$$

$$= \langle 1.69766\times10^{-4}, -8.60694\times10^{-4} \rangle\,N$$

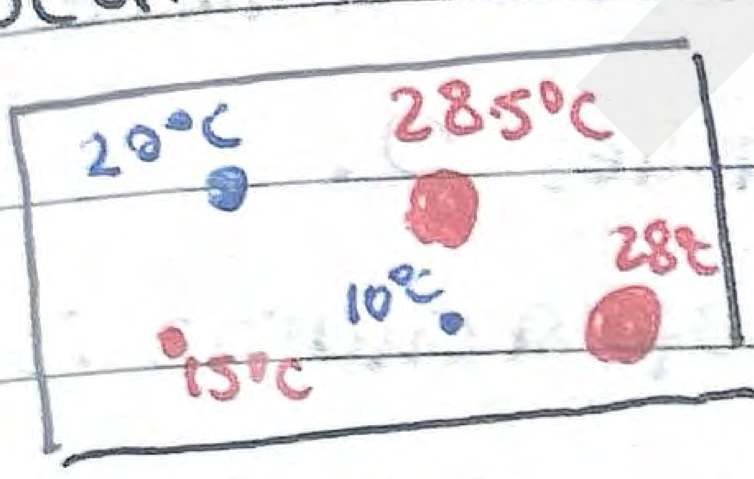
Electric Field - The force per unit charge experienced by a small positive point charge placed at that point

But what is a field?

· A field can be thought of as a matrix of values and their respective direction (if applicable)

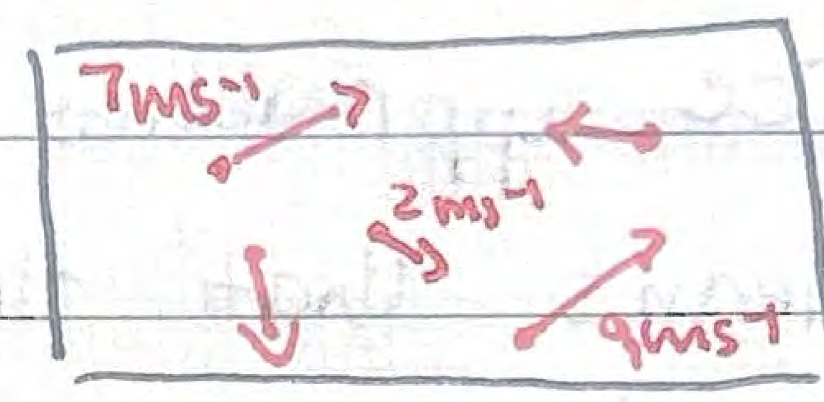
Scalar Field

E.g.



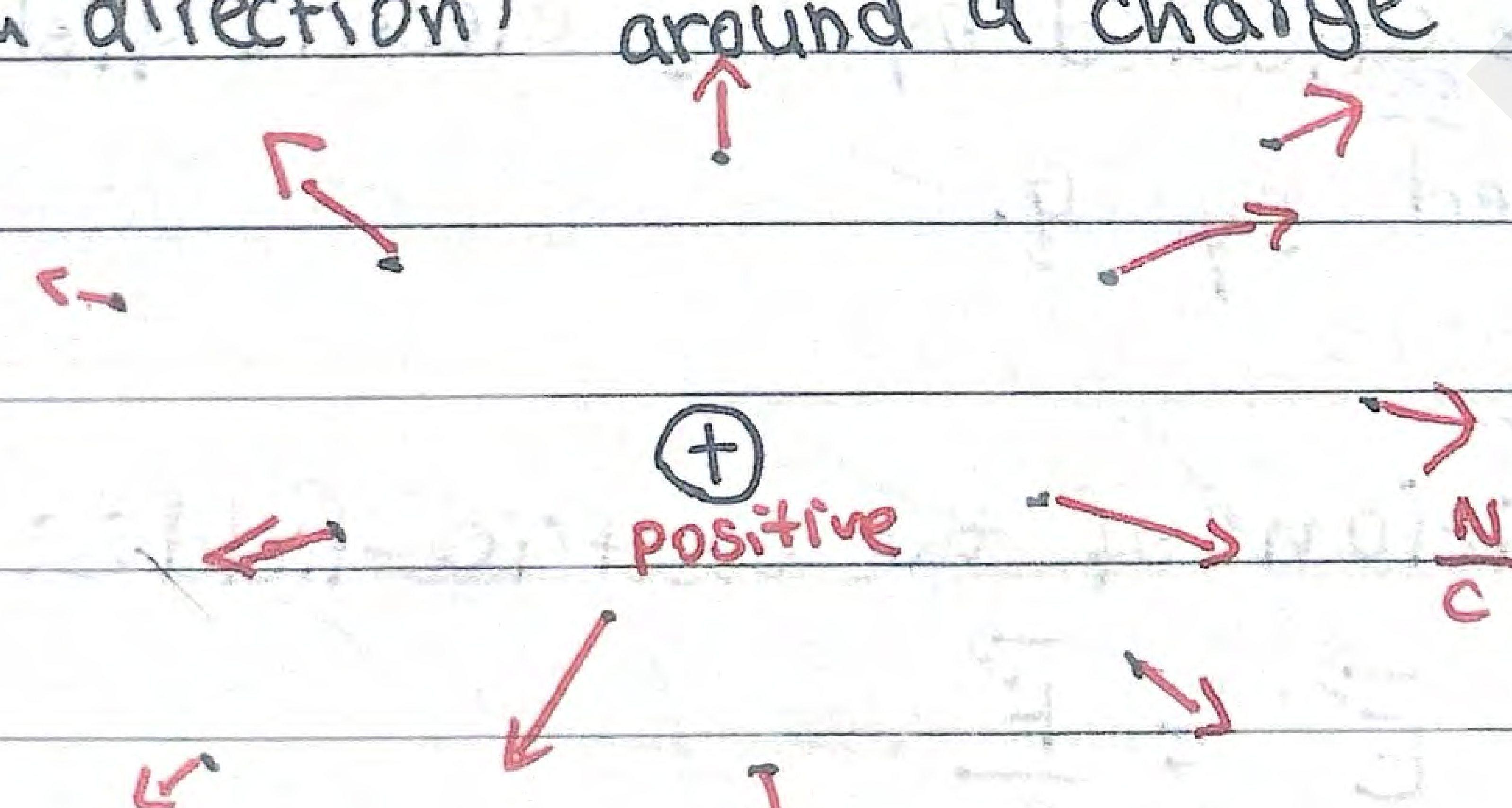
Vector Field

E.g.



The matrix is like a function, you can find the value (and direction) by specifying the coordinate location

Similarly, an electric field indicates the force per charge (with direction) around a charge



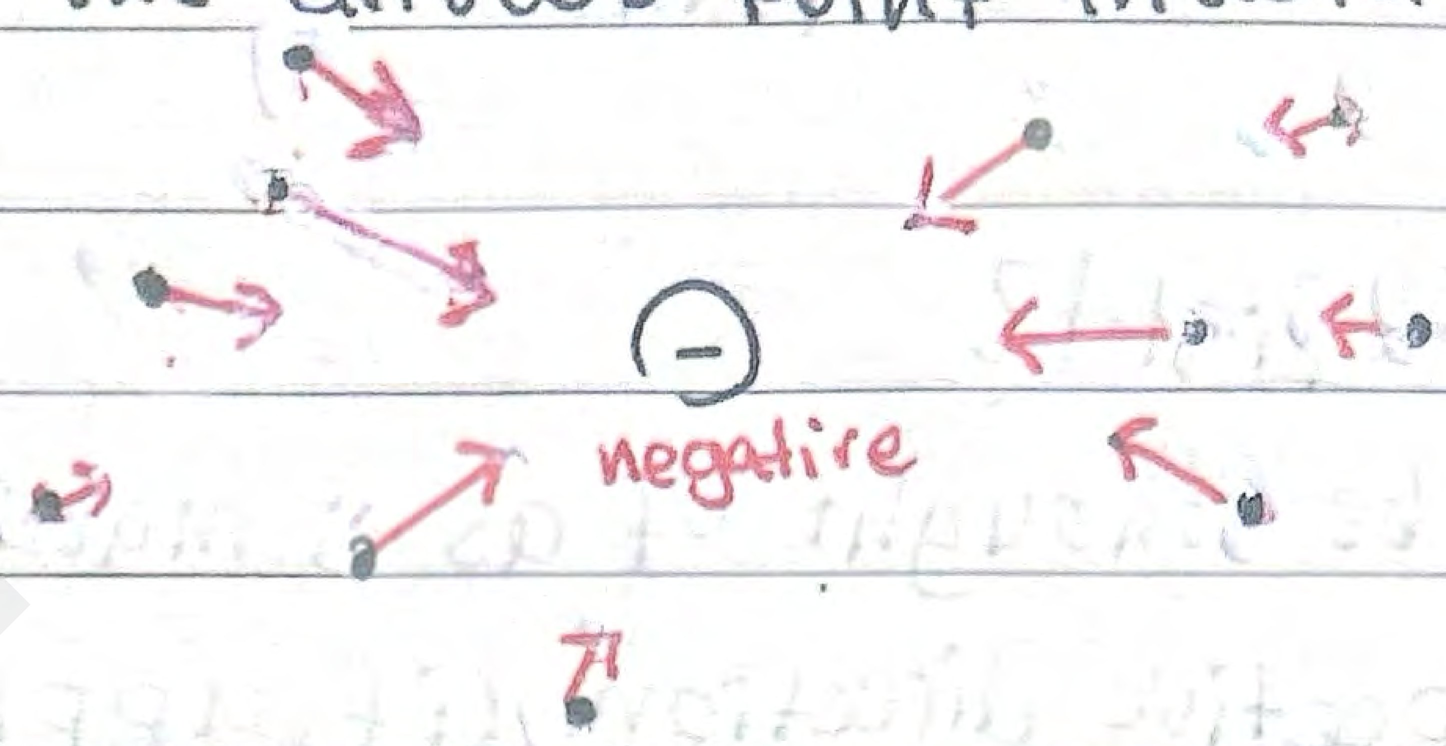
(all arrows point outwards because positive and positive repel, $\frac{N}{C}$ so when $q<0$ arrows are reversed)

Each dot represents a point in space, a possible location for a second charge

Electric field

- An electric field is a vector field; when the center is negative, the arrows point inwards

The test charge is always positive (as to test charge at a point)



The arrows/lines are called Electric Field Lines

As the distance between a charge and another increases, the force approaches but never reaches 0. This means that the electric field of a charge is infinitely large (even though the force might be really small)

The central charge (producer of electric ......) is typically represented by Q.

The charge affected by the electric field is typically represented by q.

The definition of the electric field is:

$$\vec{E} = \frac{\vec{F}}{q}$$

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Electric field

For point charges (charge at one of the points)

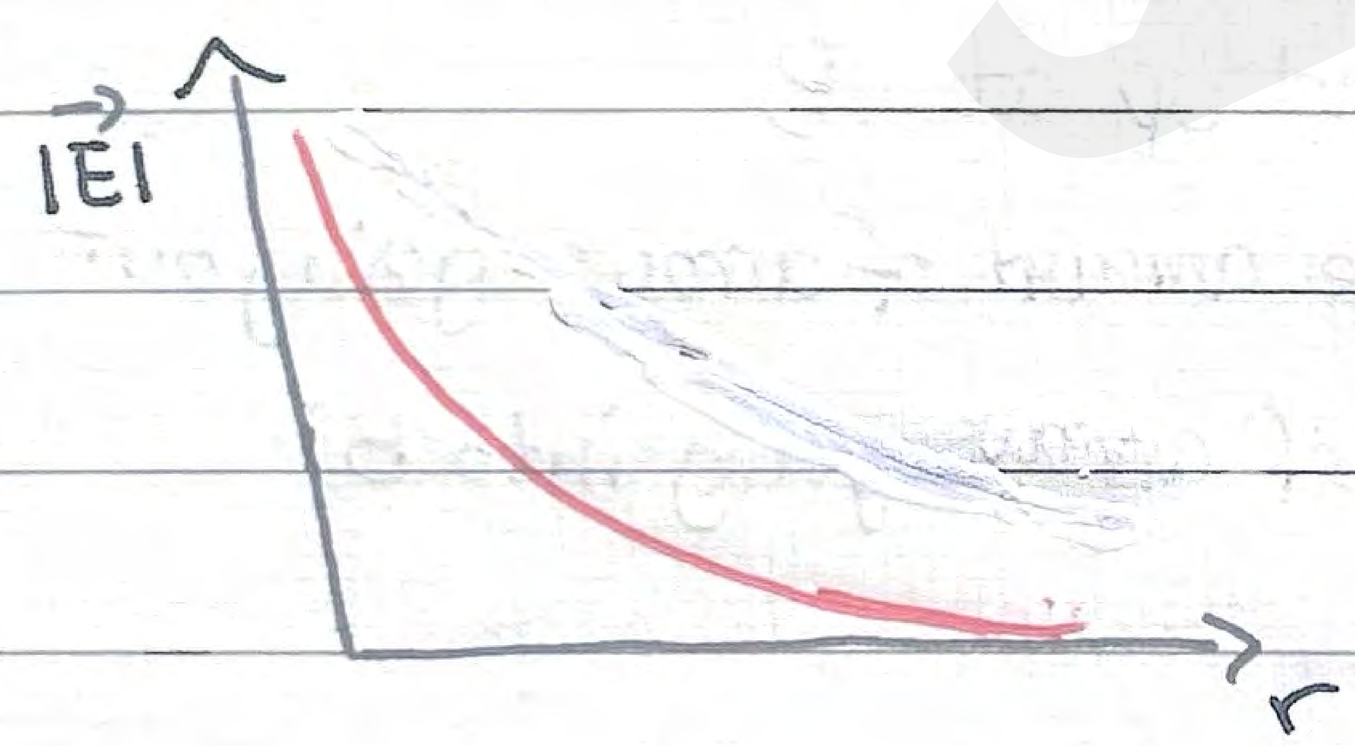
$$|\vec{E}| = \frac{\frac{kQq}{r^2}}{q} = \frac{kQ}{r^2}$$

← makes sense bc $|\vec{F}| = \frac{kQ \cdot q}{r^2}$ where $q = 1\,C$

where...

- $\underline{k}$ - coulomb's constant $(8.99 \times 10^9$
- $\underline{Q}$ - charge of the central charge
- $\underline{r}$ - distance between point charge and central charge

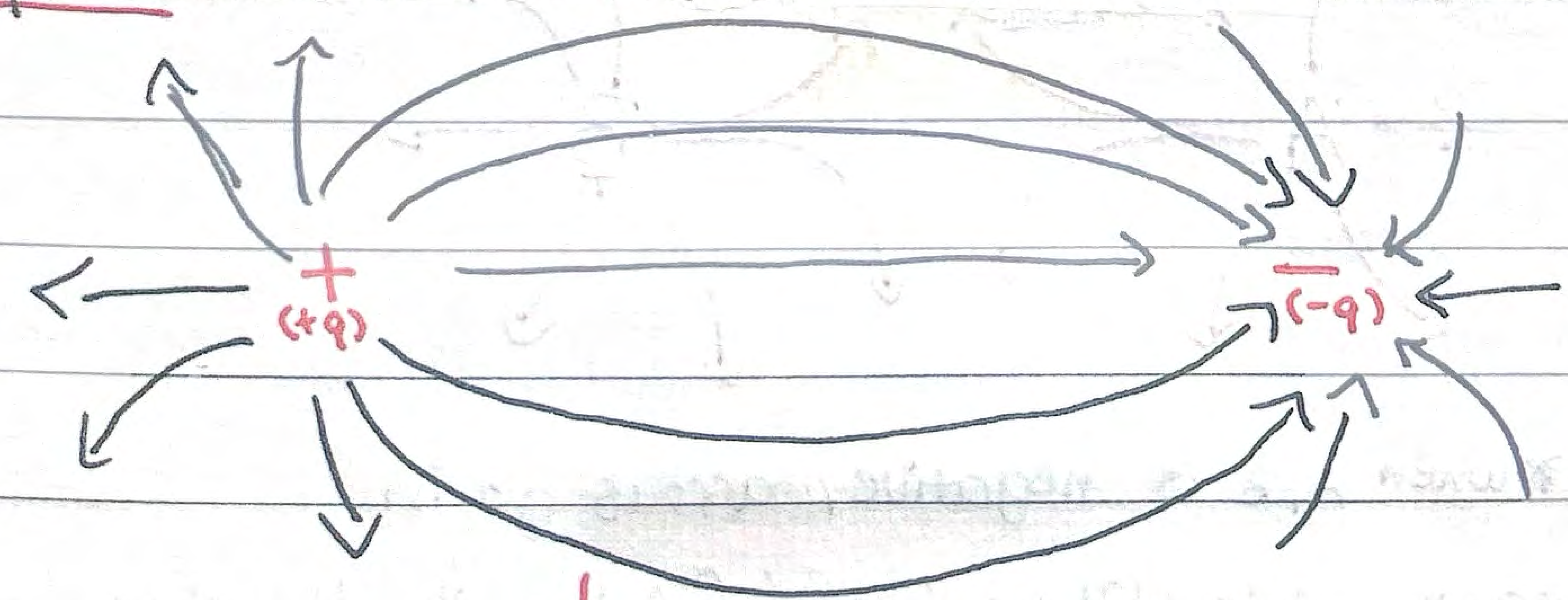
This means that $|\vec{E}|$ increases inversely to r^2



Vertical asymptote at $r=0$

Horizontal asymptote as $r \to \infty$ ($|\vec{E}|$ never disappears)

Dipole - formed between the interactions of 2 electric charges



→ The line on the very edge (seemingly straight) actually curves around the universe

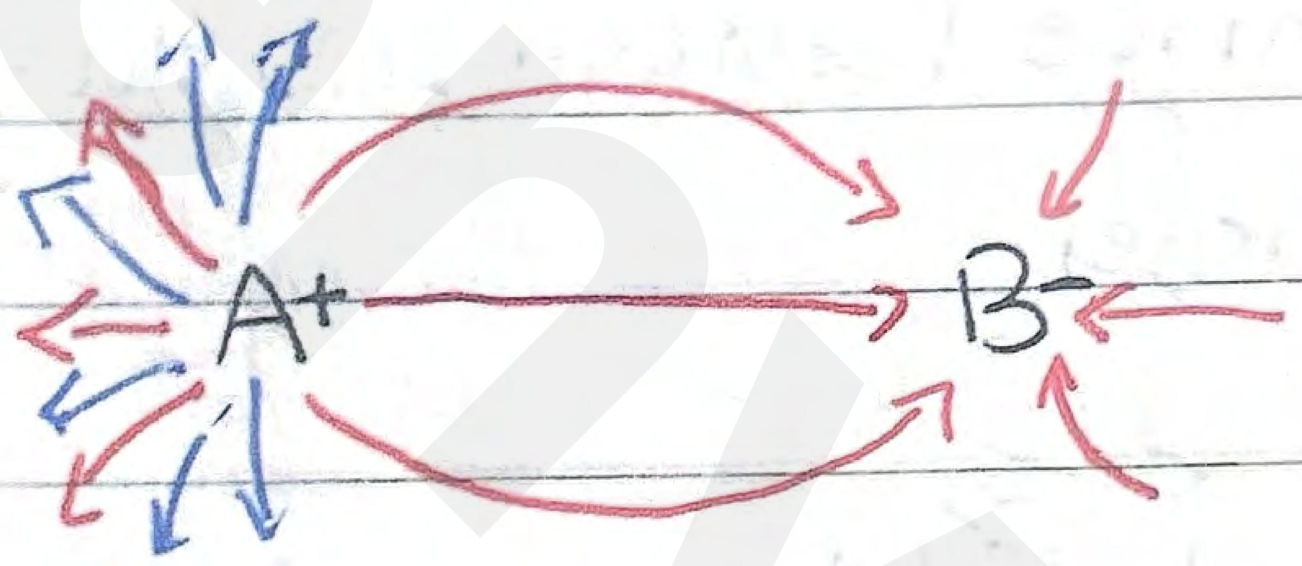
↓ the points on the line results from net force dues to two charges

Electric Field

Drawing Schematics

when two charges are equal in magnitude, # of arrows going into − must equal # of arrows going out of +

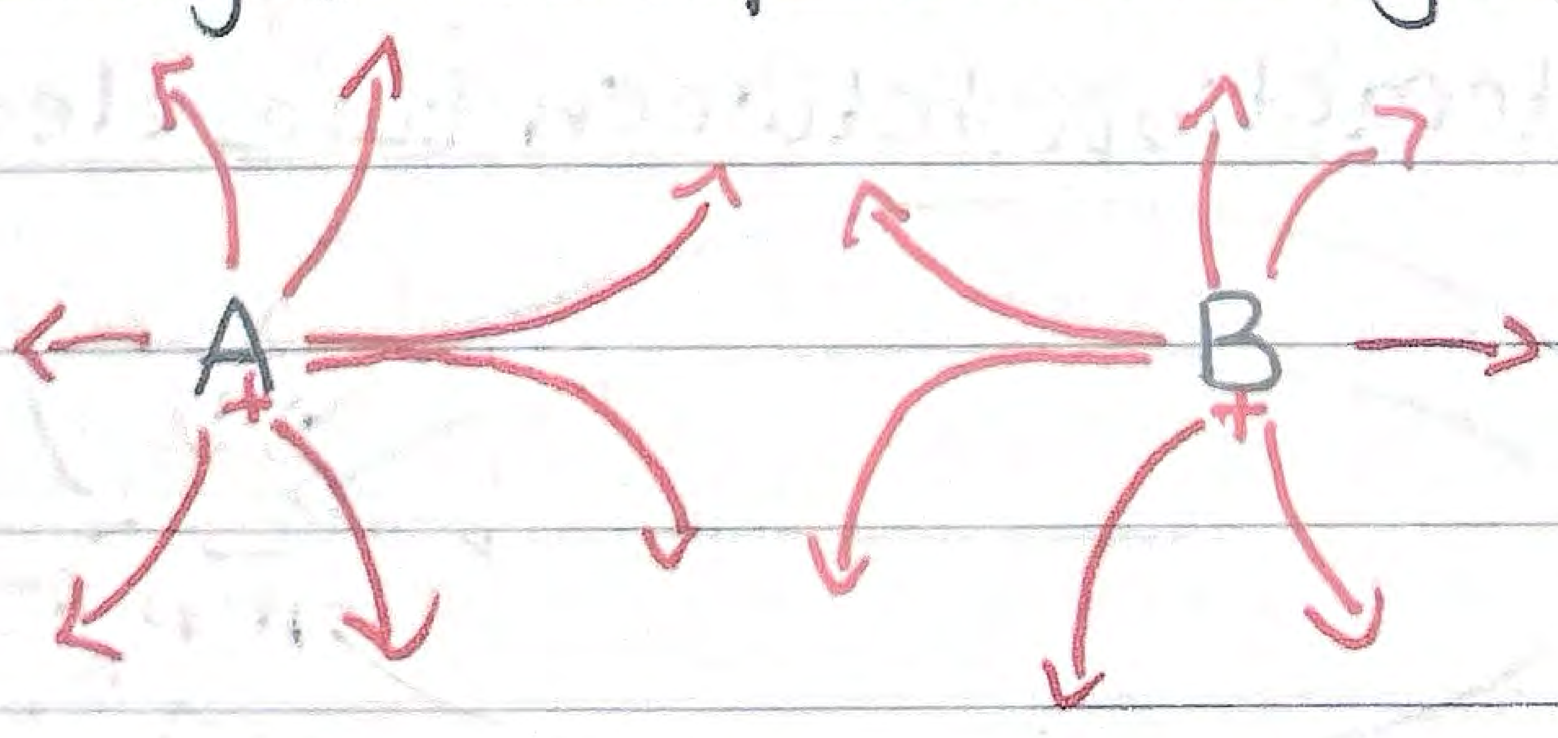
E.g. when charge A has twice the charge as charge B



• lines do not end up at B

(A has twice the amount of arrows going out as the amount of arrows going into B)

When two charges are of the same sign...

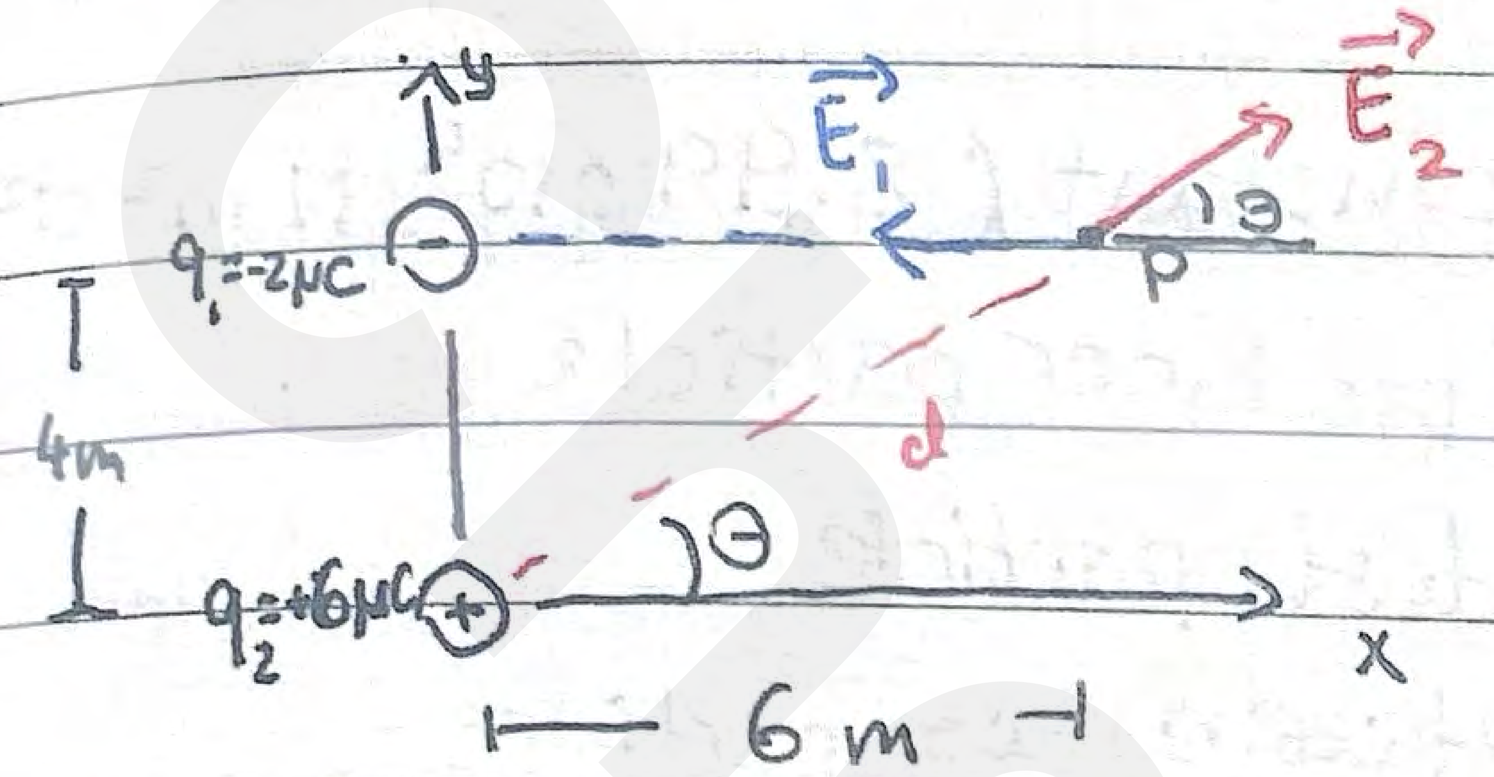


*when A, B is negative, arrows go in

The # of arrows rule still applies (if A has a× the charge as B, then A has a× the amount of arrows as B)

Similar to electric forces, we can also find the net electric field for a point charge influenced by multiple charges

· Pythagorean theorem + horizontal, vertical components



$d=\sqrt{4^2+6^2}=\sqrt{52}$

$\vec{E}_1=\langle -\dfrac{k(2\times10^{-6})}{6^2}, 0\rangle$

$\theta=\arctan\left(\frac{4}{6}\right)\approx 34^\circ$

$\vec{E}_2=\langle \dfrac{k(6\times10^{-6})}{52}\cos(34^\circ), \dfrac{k(6\times10^{-6})}{52}\sin(34^\circ)\rangle$

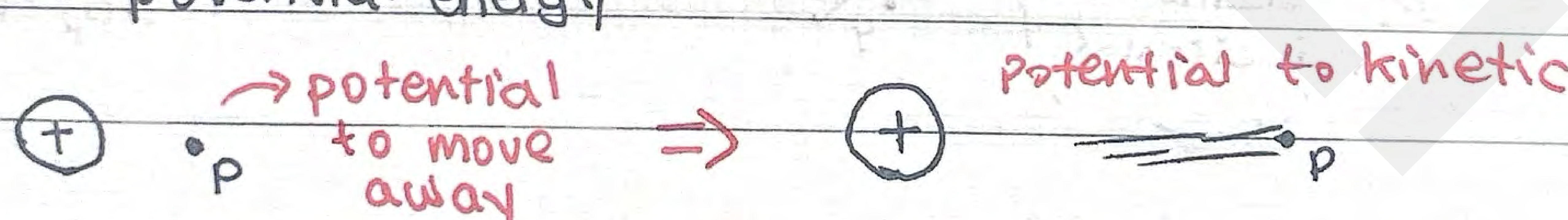
$\vec{E}_{net}=\langle -500+860, 580\rangle NC^{-1}=\langle 360, 580\rangle NC^{-1}$

*It's basically doing Electric force but for charge q = +1C !

Aside from the Electric force and the Electric field, we also have Electric potential energy and electric potential

↳ There's force that leads to a displacement

Electric potential energy

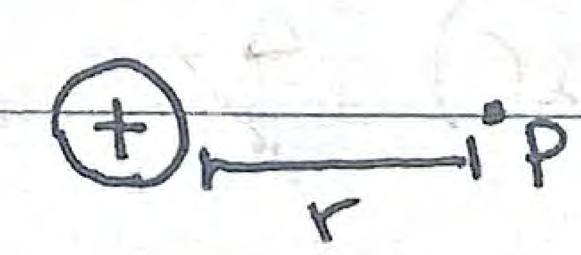


The repulsive force means electric potential energy

$W=\Delta E_p=F\cdot r$

$=\dfrac{kQq}{r^2}\cdot r$

$=\dfrac{kQq}{r}=E_p$



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Electric Potential Energy (pt. charges)

← can be negative

$$E_p = k\frac{Q \cdot q}{r}$$

$\ominus \leftarrow \cdot_{+q}$ (goes in, instead of out)

↑ scalar (no direction, only in/out)

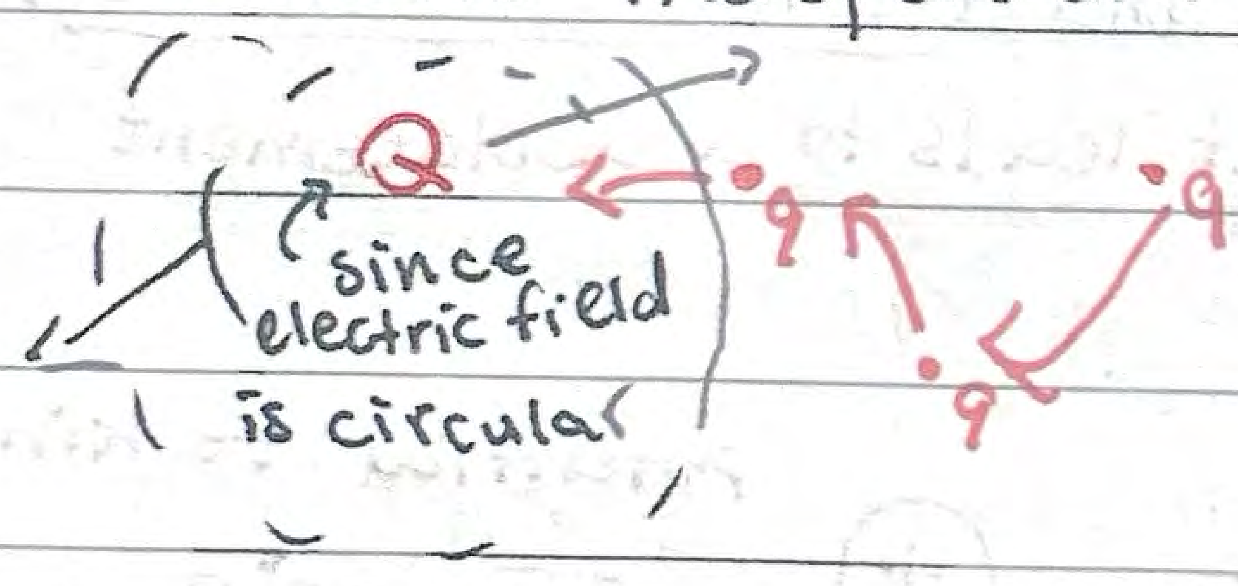
$\boxed{\text{where...}}$

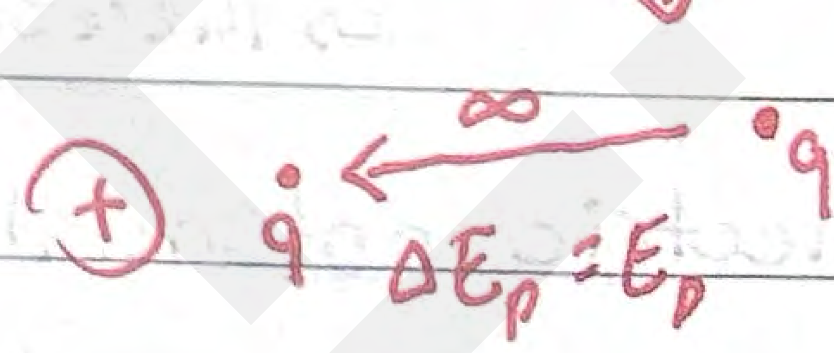
- k - Coulomb's constant ($8.99 \times 10^9\ N m^2 C^{-2}$)
- Q - Charge of producer particle
- q - Charge of test particle
- r - distance between the two particles

"Electric potential energy is defined as the energy that a charge has due to its position in an electric field. This is the work done to move a point charge from infinity to a given position close to a charge Q."

infinity ↓ did not exist before ($E_{p_i} = 0$, unaffected before)

(work is done independent of the path)





Electric potential - At a point P is the amount of work per unit charge as a small test charge q moves from infinity to a point P.

(scalar) ⇑

formally

Electric Potential Difference

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$\swarrow E = -\frac{dV}{dr}$

Electric Potential (voltage)

- Basically Electricfield for electric potential energy
- Electric potential energy per unit charge
- Unit is J/C = Volts = $\boxed{V}$

Just like electric field, electric potential divides by charge

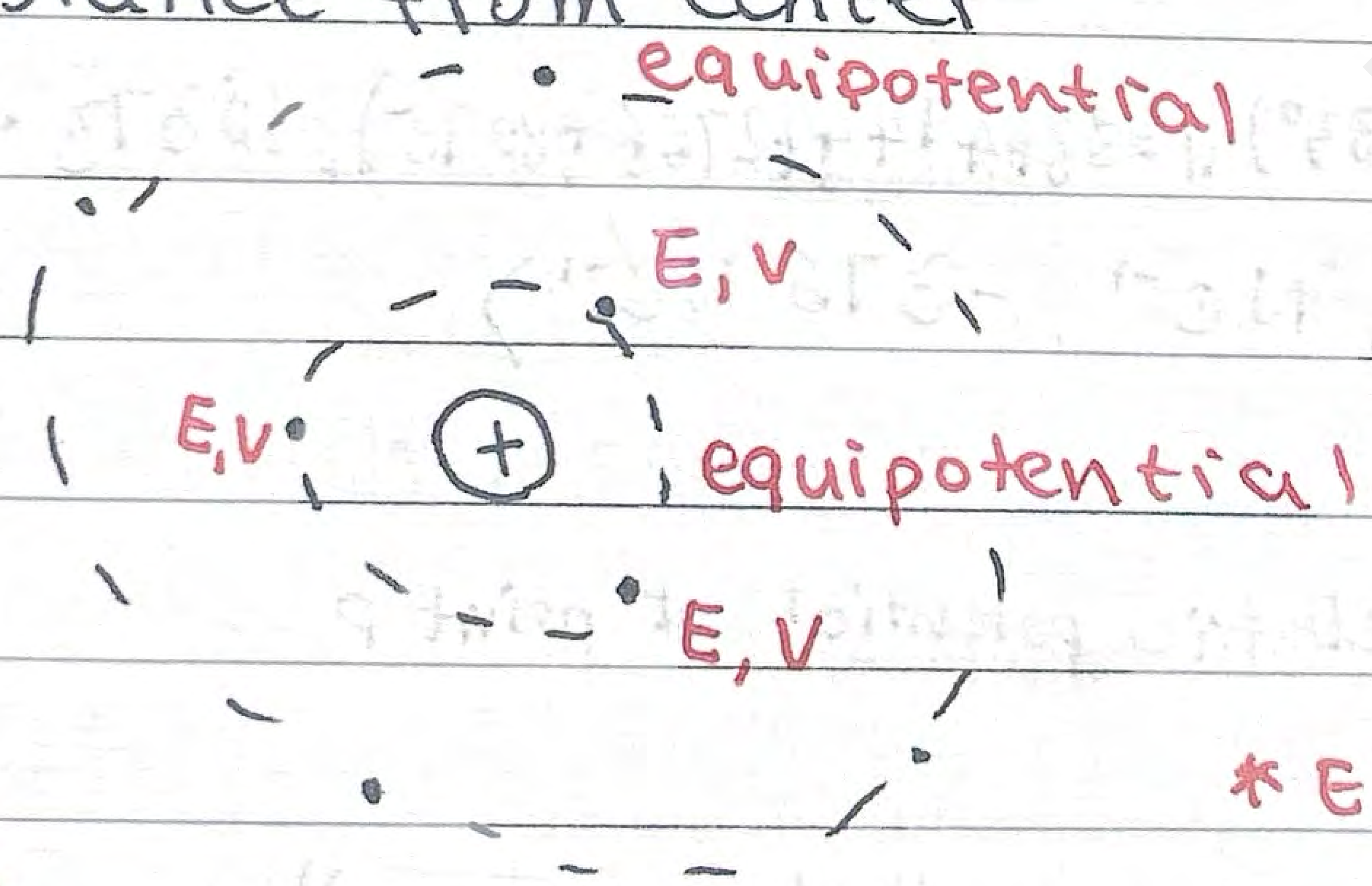
$$\boxed{V = E_p \div q = k\frac{Q}{r}}$$

$\leftarrow$ since V is scalar, $V_{net} = V_1 + V_2 + \dots + V_n$ directly (no need to do vectors)

where...

- E_p - Electric potential energy
- q - point charge
- k - Coulomb's constant ($8.99 \times 10^9\ Nm^2C^{-2}$)
- r - Distance between charge and point charge
- Q - Charge of producer particle

Both Electric field and electric potential are affected by the distance from center



* E = field not energy

Since $|\vec{E}| \propto \frac{1}{r^2}$, $V \propto \frac{1}{r}$ (field: $\vec{E}$)

when V halves, that same distance means $|\vec{E}|$ $\frac{1}{4}$ths.

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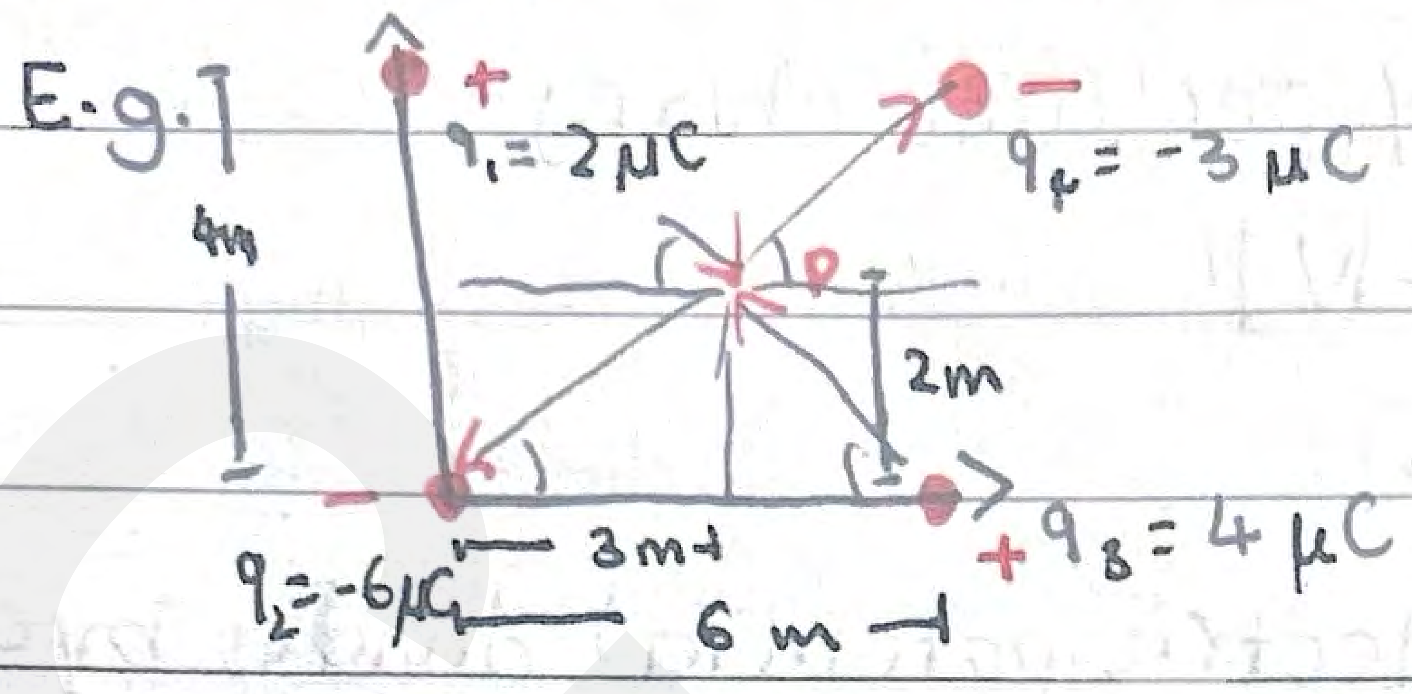
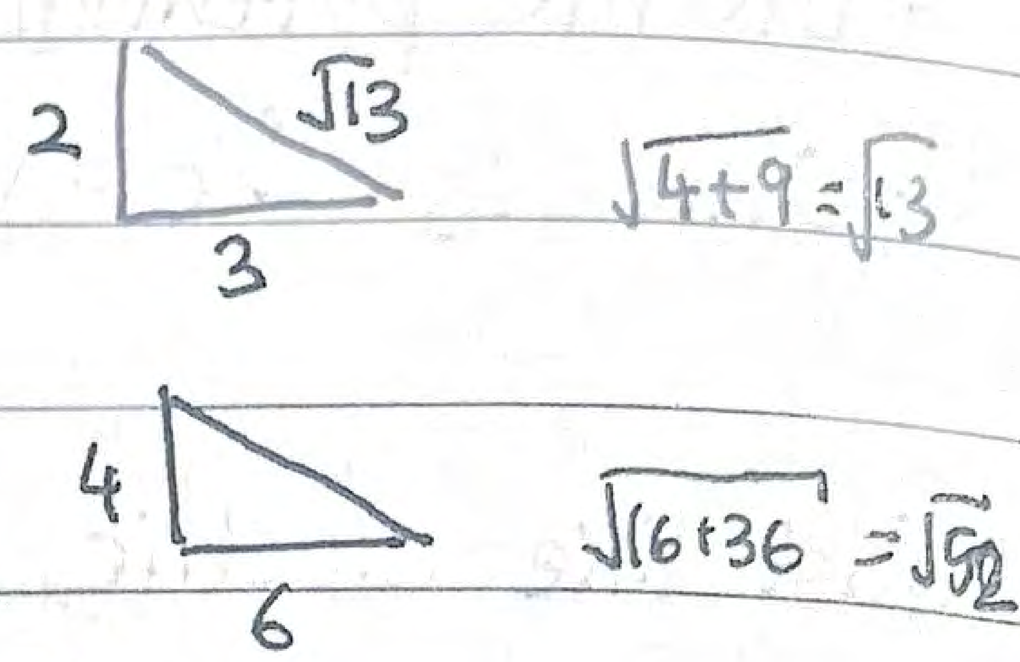
Electrostatic

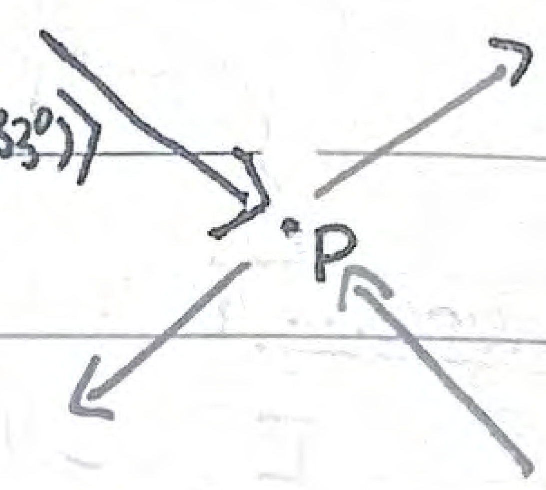
Connecting all of electrostatic together...

E.g.1





a) Find the net Electric Field at point P



$|\vec{E}_1| = \langle k\cdot(2\times10^{-6})/(13)\cdot\cos(33^\circ), k(2\times10^{-6})/13\cdot\sin(33^\circ)\rangle$

$\Downarrow$ 1383

$= \langle -1383\times\cos(33^\circ), -1383\times\sin(33^\circ)\rangle$

$|\vec{E}_2| = \langle -k\cdot(+6\cdot10^{-6})/13\cdot\cos(33^\circ), -4149\cdot\sin(33^\circ)\rangle$

$\Downarrow$ 4149

$|\vec{E}_3| = \langle +k(4\times10^{-6})/13\cdot\cos(33^\circ), +2766\cdot\sin(33^\circ)\rangle$

$\Downarrow$ 2766

$|\vec{E}_4| = \langle +k(3\times10^{-6})/13\cdot\cos(33^\circ), +2075\cdot\sin(33^\circ)\rangle$

$\Downarrow$ 2075

$|\vec{E}_{net}| = \langle \cos(33^\circ)(-3457), \sin(33^\circ)(-691)\rangle$

$= \langle -2899\ NC^{-1}, -376\ NC^{-1}\rangle$

b) Find the net electric potential at point P

$V_{net} = V_1 + V_2 + V_3 + V_4 = \frac{k}{r}(2-6-3+4) \approx -7480132146\ V$

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Electrostatic and Kinematics

Since energy can be connected to work which can be connected to electric potential difference...

We can solve problems with electrostatics and conservation of energy.

$$W = q \cdot V \quad \text{and} \quad W = \Delta E_k$$

$\boxed{\text{so...}}$

$$q \cdot V = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$v_f^2 - v_i^2 = \frac{2 \cdot q \cdot V}{m}$$

$\boxed{\text{or if } v_i = 0}$

$$v_f = \sqrt{\frac{2 \cdot q \cdot V}{m}}$$

Connecting fields and voltages

Electric field is the derivative of electric potential difference with respect to distance

$$E = -\frac{d}{dr}[V] = -\frac{d}{dr}\left[\frac{kQ}{r}\right] = -\left(-\frac{kQ}{r^2}\right) = -\frac{kQ}{r^2}$$

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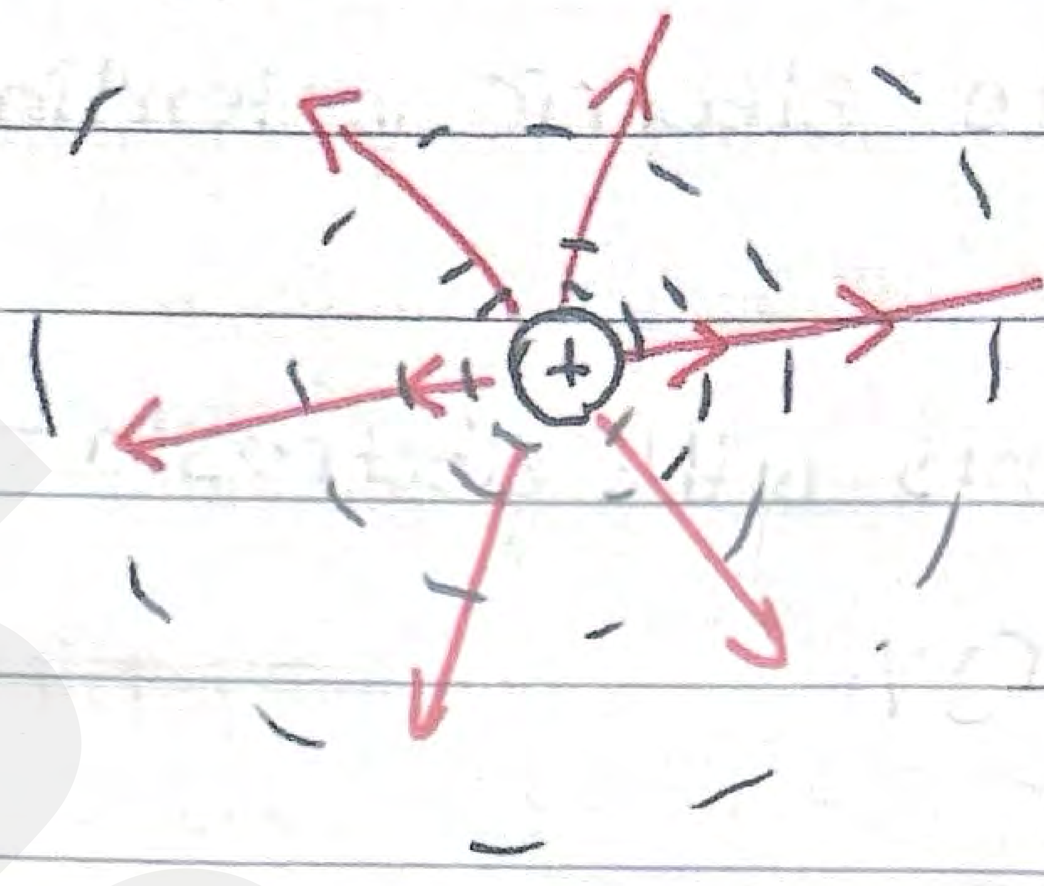
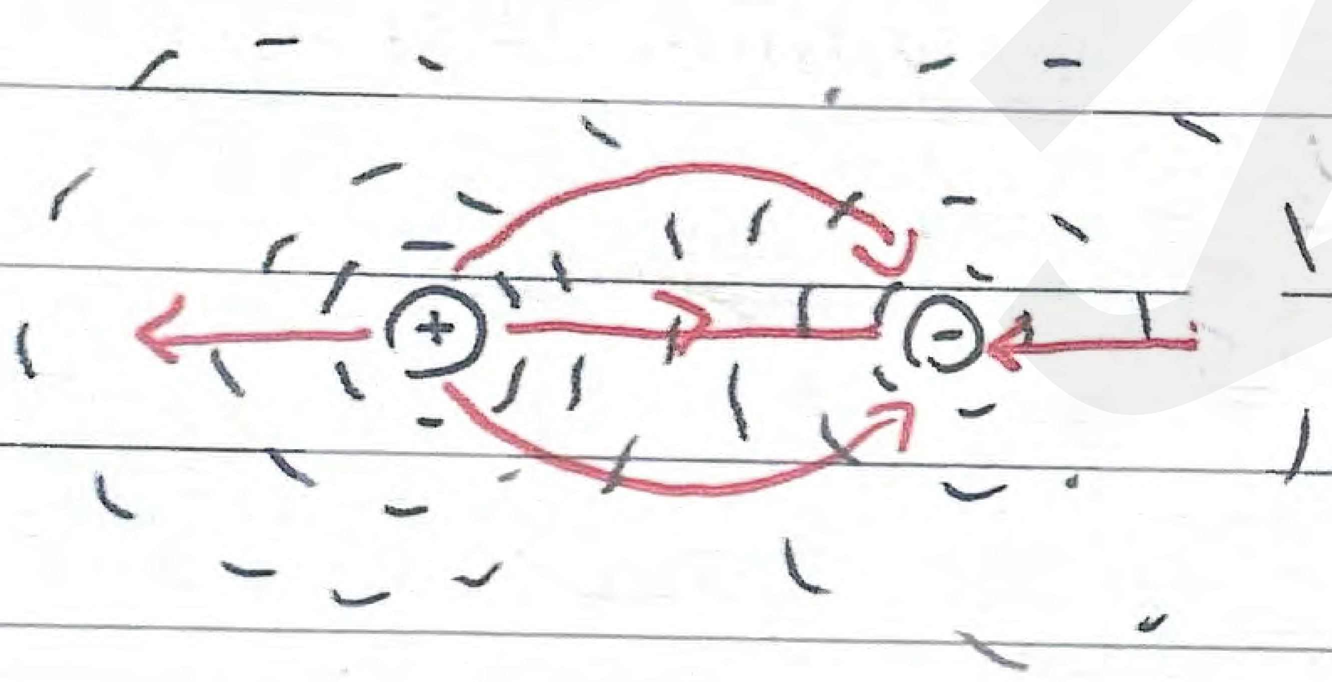
Electric fields and Voltages

Drawing equipotential lines and field lines

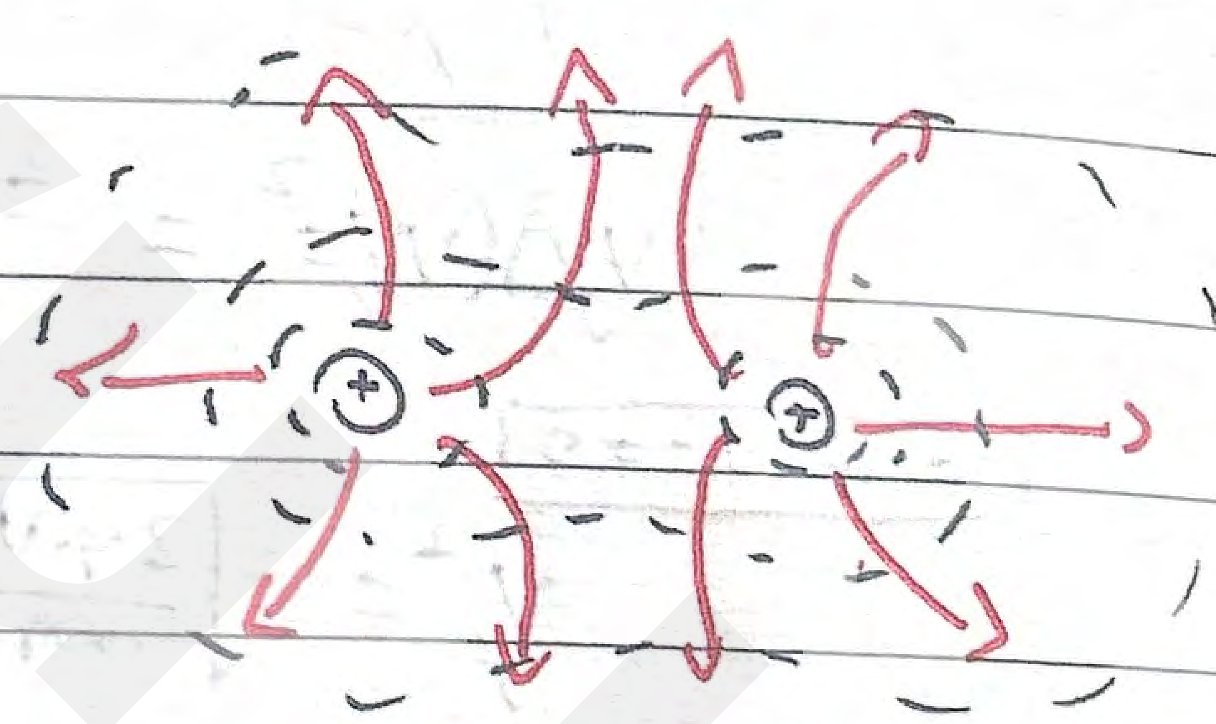


When drawing equipotential lines, make sure the distance between it and the previous line increases each time ($\Delta r \uparrow$ for ΔV is constant)

For two charges...



pos, neg



same sign

Is it possible to achieve uniform electric field?

Notice with two charges

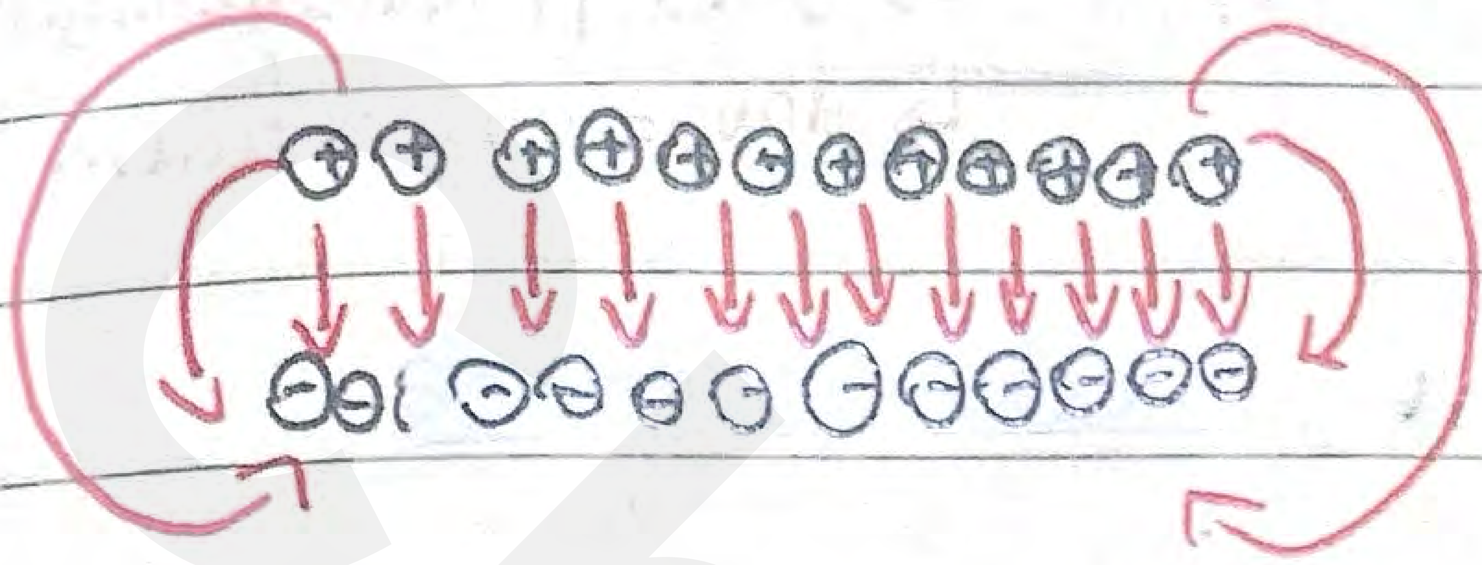
This electricfield line is uniform

so...

Applying to an entire field

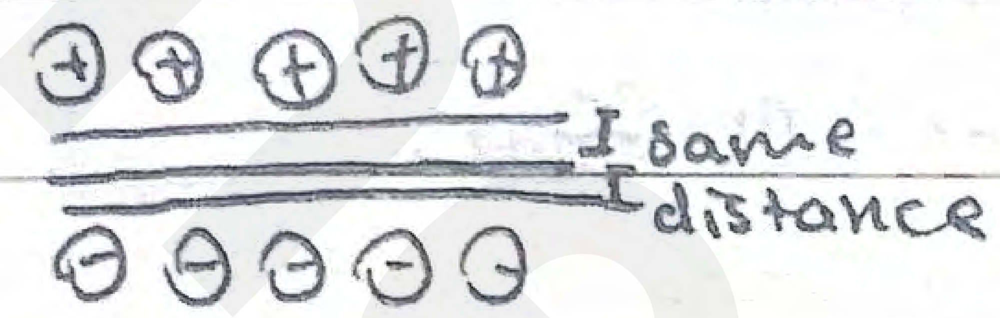
Uniform electric field

Just add more charges!

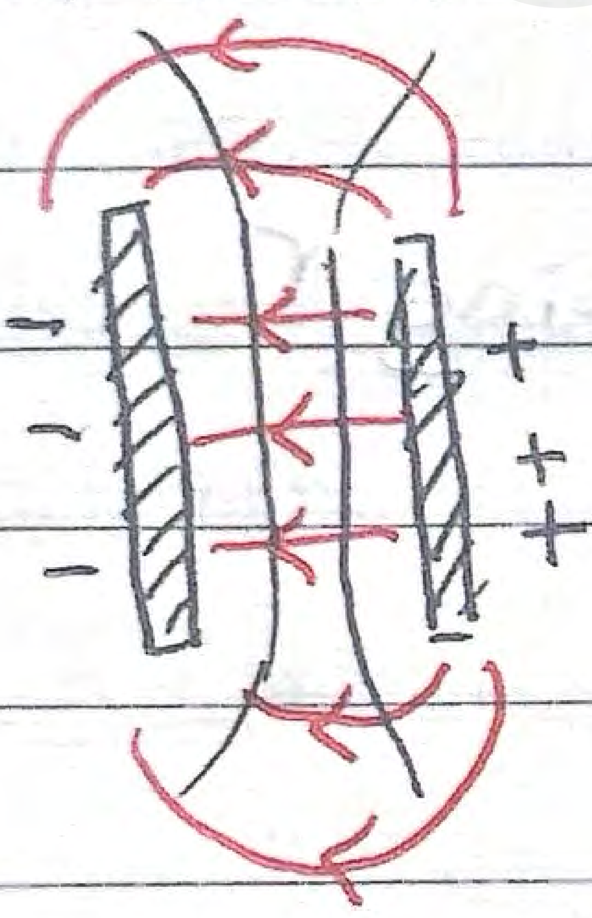


This creates a uniform electric field between

The equipotential lines are also equally separated



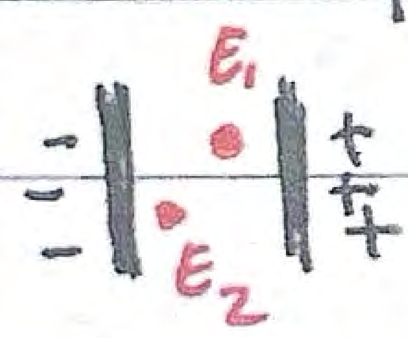
This setup is known as parallel plates



When a charge is placed between the two plates, the charge will accelerate from positive to negative

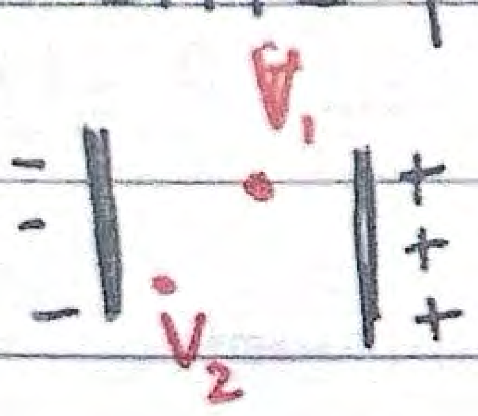
Properties of uniform electric field

1. Electric field strength is CONSTANT



$E_1 = E_2$ (thus force is also constant)

2. Electric potential difference DECREASES (from + to -)



$V_1 > V_2$

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Acceleration and electric field

$$a = \frac{F_{net}}{m} = \frac{F_E}{m} = \frac{qE}{m}$$

$F_{net} = F_E$ because other forces are negligible
↳ gravity: mass negligible
↳ drag force: vacuum

Changing acceleration

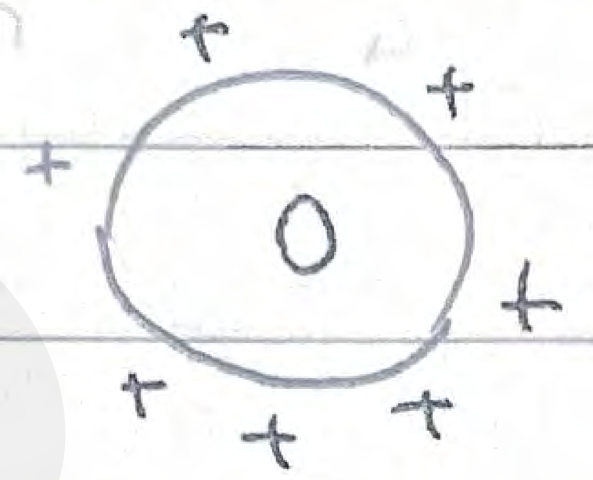
$$a = \frac{\overset{const}{q}E}{\underset{const}{m}} \quad \text{thus...} \quad a \propto E$$

since $E = -\frac{dV}{dr}$ or $-\frac{\Delta V}{\Delta r}$

so increasing V or decreasing r

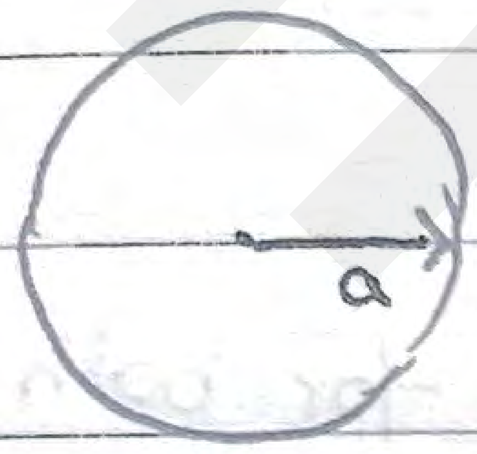
Electric field inside a conductor

- conductor (easy for energy to pass through)
  - ⇒ Electric field is 0, no matter if it's charged
  - ⇒ Charges are distributed across the surface



No charge inside

Electric field and potential



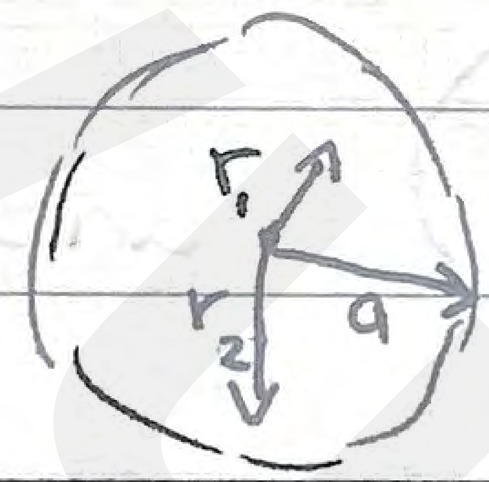
How will their graphs look like?

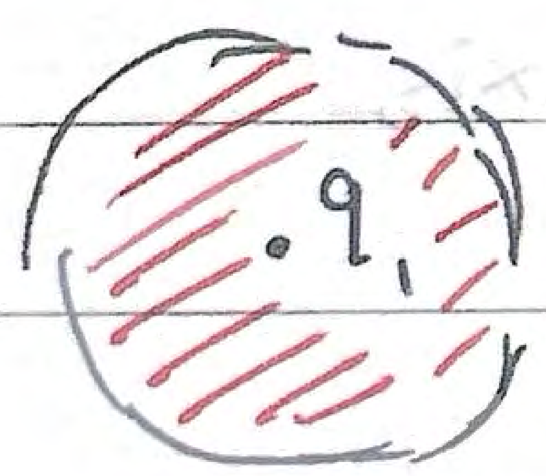
*We assume that the charge is a point charge (all its charge is concentrated in its center)

However, when $r < a$,

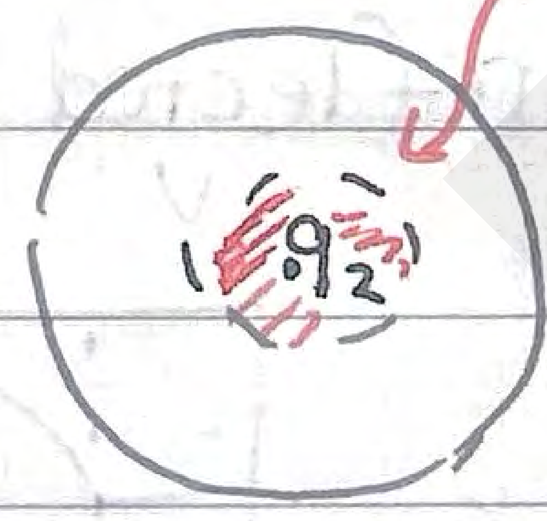
$V = \frac{kq}{r}$ no longer works.

This is because inside, you have less charge...





only a segment



As such, as $r\downarrow$, $q\downarrow$

$\therefore V = \frac{k\, q\downarrow}{r\downarrow}$ which makes V constant inside

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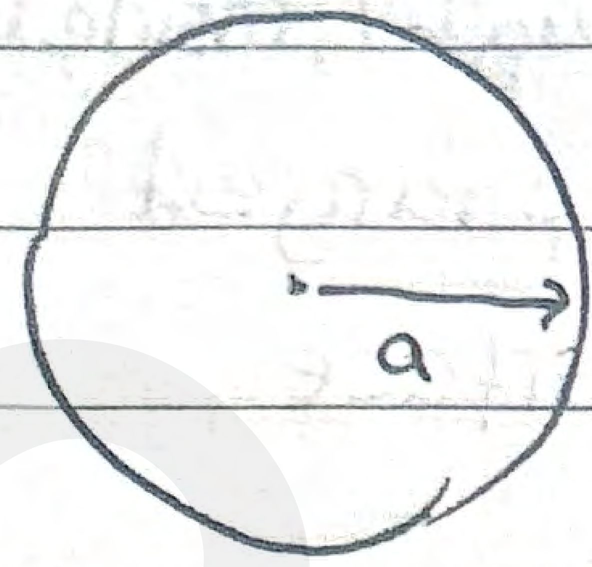
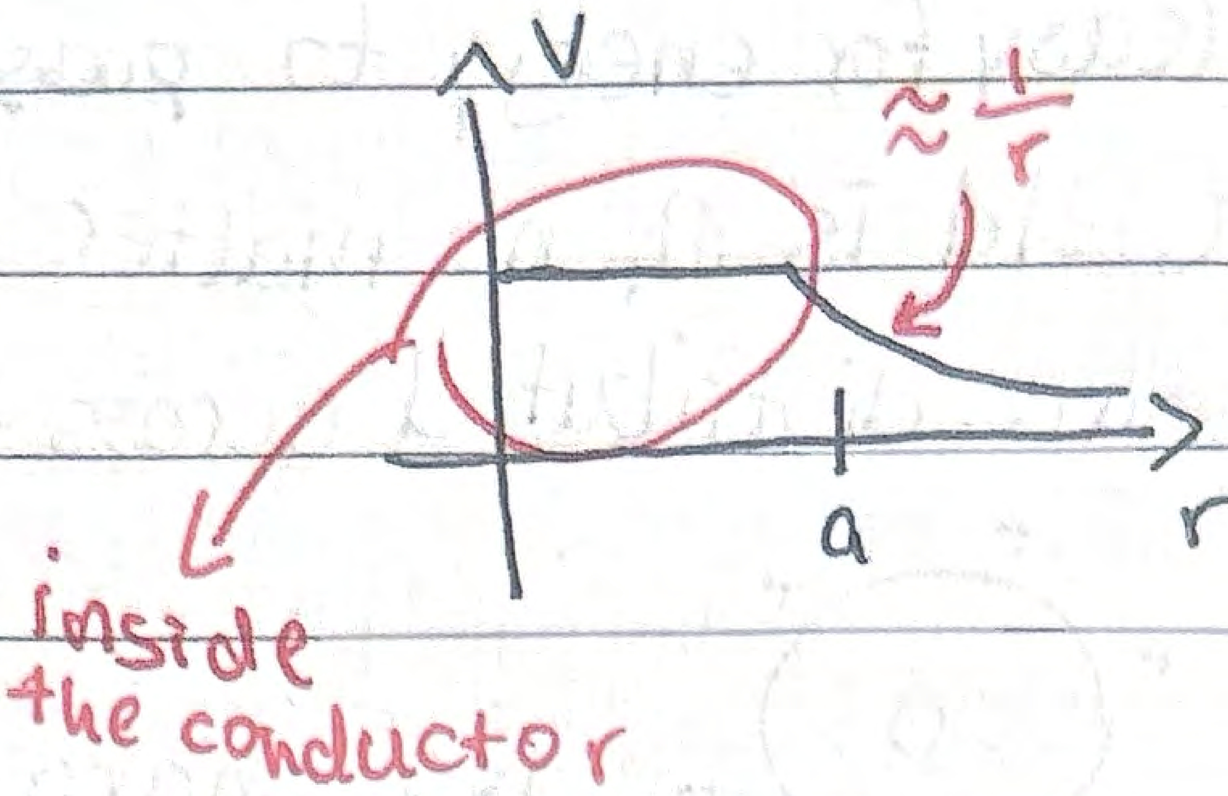
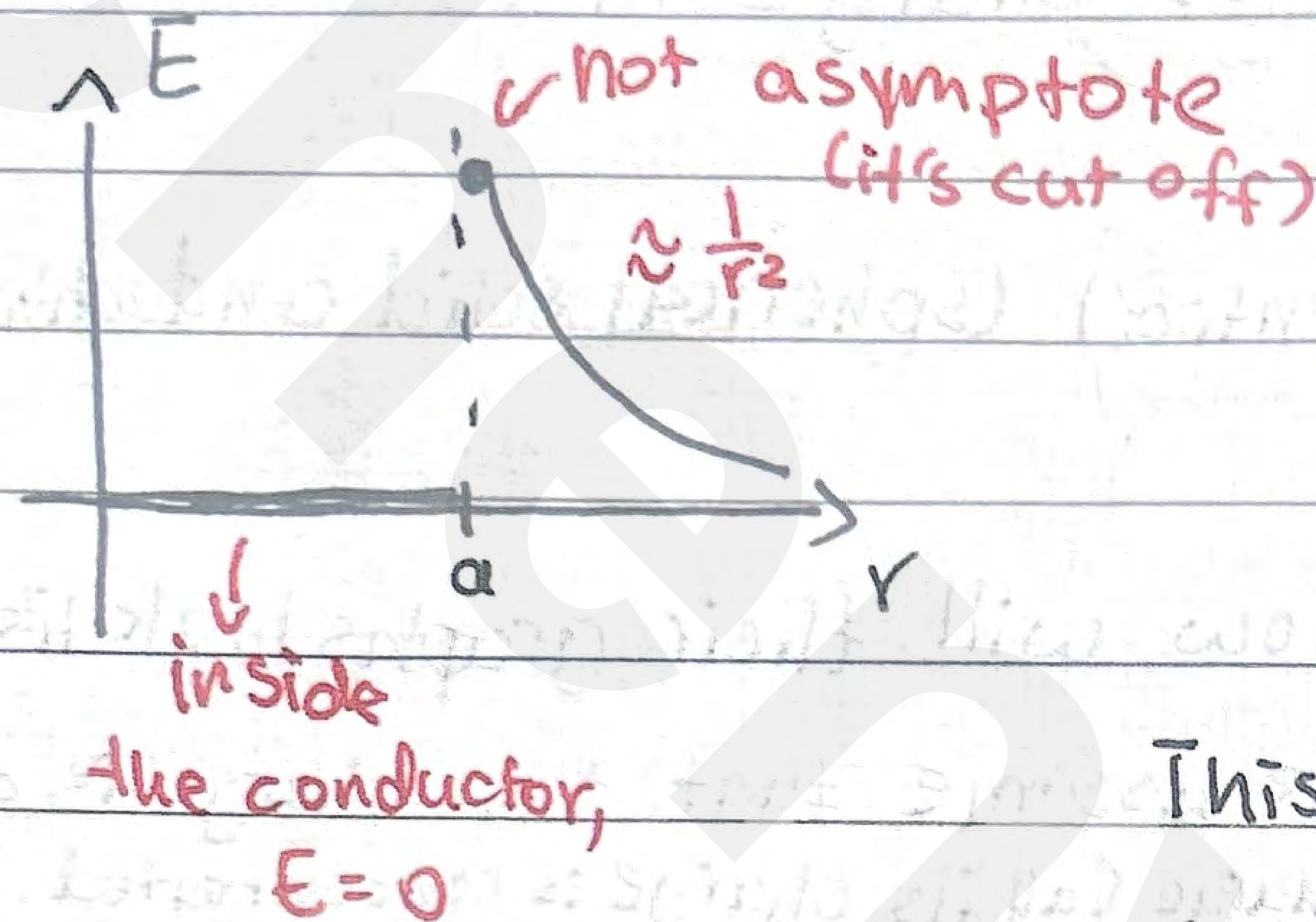
Conductor Electric Field

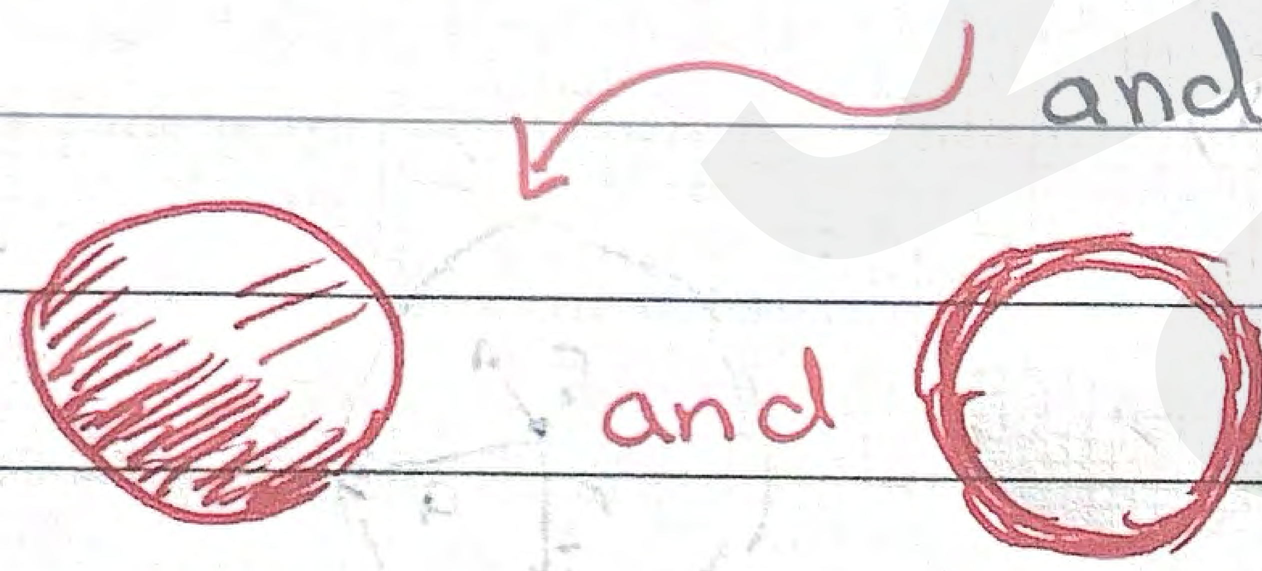




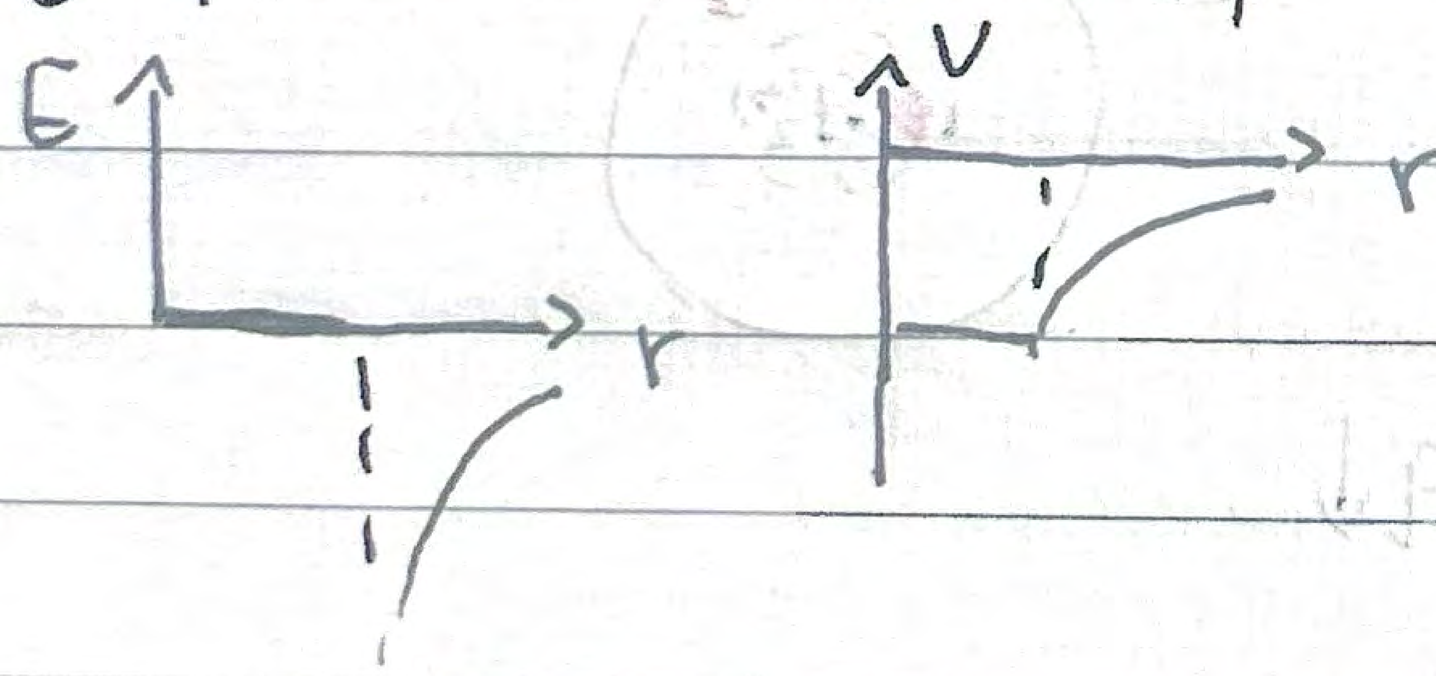


$$E = -\frac{dV}{dr}$$

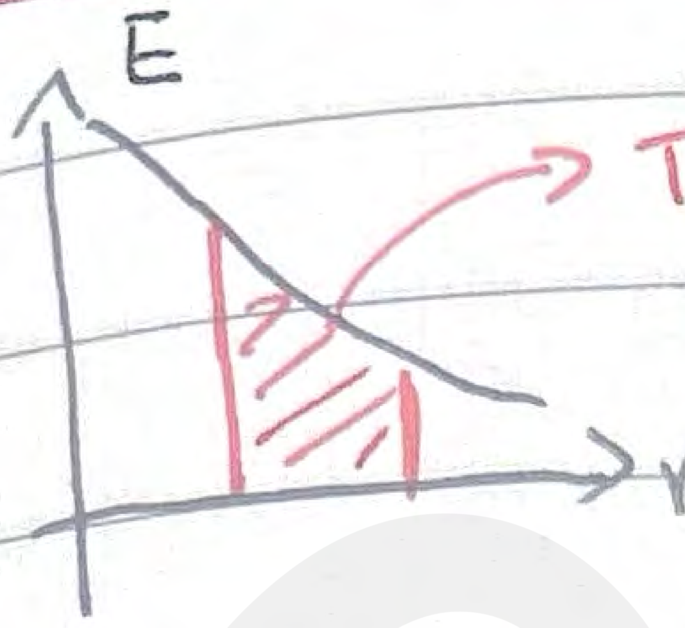
This applies for both conductive spherical solids and shells (hollow)



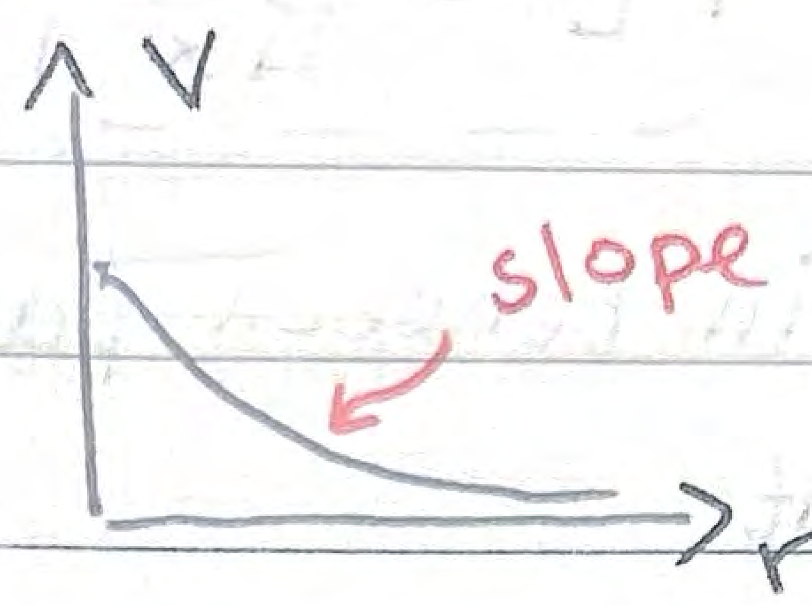
when the point charge is negative, it's the same graphs but reflected by the x axis



Electric Field graph



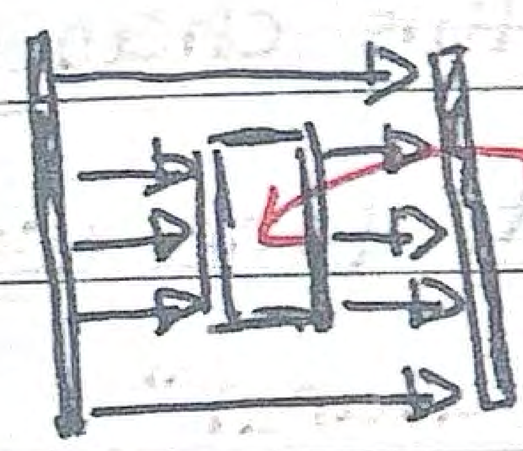
The integral of E is electric potential difference (not work)!



slope is - electric field!

Electricstatic Shielding

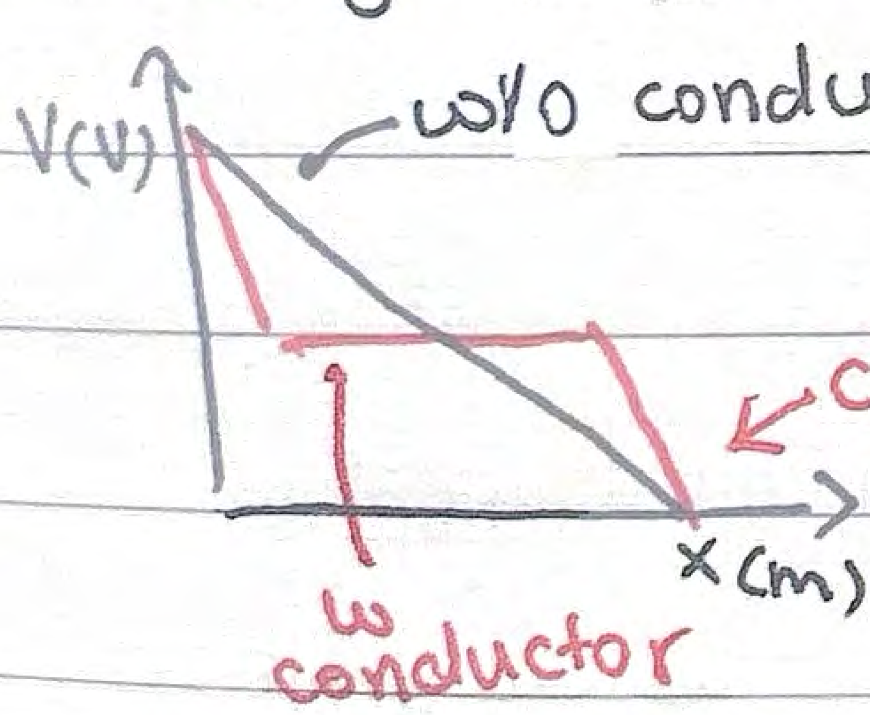
when you place a conductor between a uniform electric field, it creates a space inside the conductor with 0 electric field



nothing inside

This space inside is how the faraday cage works

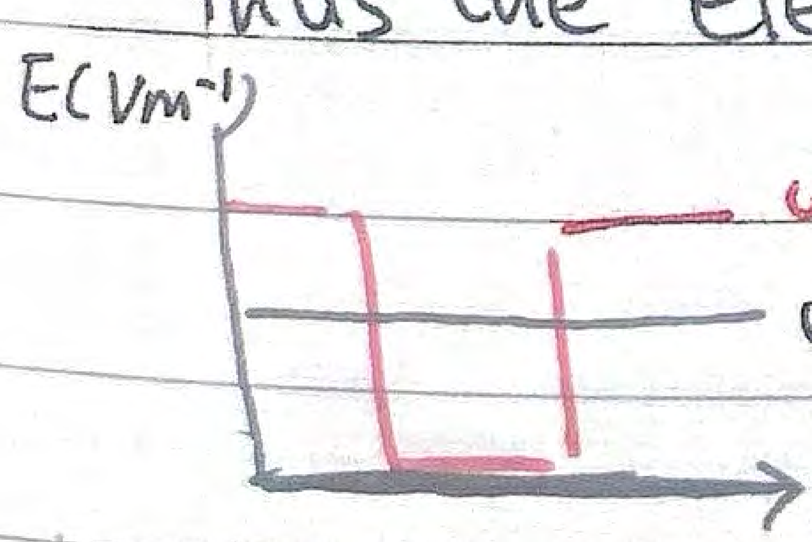
The graph would look like this



constant (uniform electric field)

(w conductor decreases faster because of conductor's effect on potential difference as well)

Thus the electric field would be



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Parallel plates (uniform e-field)

Electric field is defined by

$$\boxed{E = \frac{\Delta V}{\Delta x}}$$

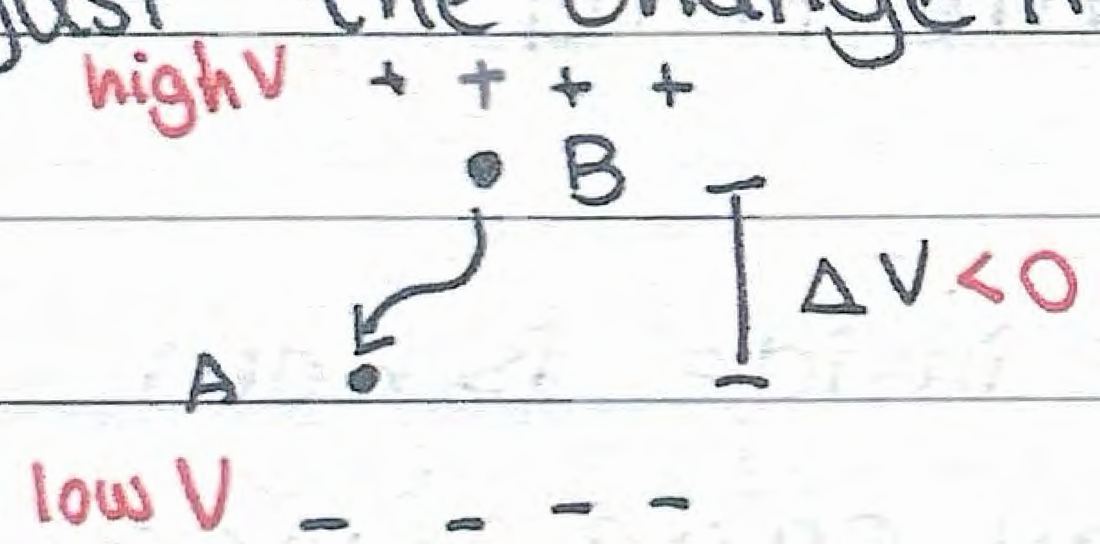
Since in between parallel plates, electric field stays constant.

$$\therefore E = \frac{\Delta V}{d}$$

ΔV ← potential difference between two plates

d ← separation of two plates

Potential difference between two points is just the change in voltage (electric pot. diff.)



↳ You can use this change and charge to find work

work <0 when "natural"

work >0 when "forced"

After finding work, you can use $W = \Delta E_k$ to find energy (kinetic)