泰國國際數學競賽 2018（香港賽區）

THAILAND INTERNATIONAL MATHEMATICAL OLYMPIAD 2018 (INDIA REGION)

Senior Secondary Group

Time allowed: 90 minutes

Question Paper

Instructions to Contestants:

1. Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. There are 5 exam areas and 5 questions in each exam area. There is a total of 25 questions in this Question-Answer Book. Each carry 4 marks. Total score is 100 marks. No points are deducted for incorrect answers.

3. All answers should be written on ANSWER SHEET.

4. Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

5. NO calculators can be used during the contest.

6. All figures in the paper are not necessarily drawn to scale.

7. This Question-Answer Book will be collected at the end of the contest.

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

All answers should be written on the ANSWER SHEET.

Rough Work

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Open-Ended Questions (1ˢᵗ ~25ᵗʰ) (4 points for correct answer, no penalty point for wrong answer)

<u>Logical Thinking</u>

1. Find the number of positive integers less than 500, with unit digit 7 and has an even number of factors.

2. There are 34 marbles, Player A and B take marble alternately, starting from A, each player can take 1,2 or 3 marble(s) and then pass to the other player. If the player who takes the last marble wins the game, and A takes 3 marble first, how many marble(s) should Player B take in his first move to ensure he will win the game?

3. 36 children, numbered 1 to 36, sit around a circle in clockwise order. Starting from 1 and counting clockwise, the third child is selected and leaves the circle. Then the process starts repeating from the child on the next to the child just left the circle, the third child is selected and leaves the circle. For the process to continue, what is the number of the last child who remains in the circle?

4. There are many competitors in a chess competition. Each competitor will play 30 games of chess. The scores of each game are allocated in the following ways: 2 points will be awarded for a win, 0 points for a lost, 1 point for a draw game. Find the number of possible scores of a competitor after the competition is completed.

5. Mr. Yellow, Mr. Blue, Mr. Black and Mr. White wear yellow, blue, black and white shirt. Given that each person does not wear the shirt of the same colour with their surname. Mr. White is older than Mr. Blue, the person wearing blue shirt is older than Mr. White. The person who wears white shirt is older than Mr. Yellow and the person wearing the Yellow shirt is the youngest. Which colour of shirt does Mr. White wear?

<u>Algebra</u>

6. Let α and β be the non-zero roots of the equation $x^2 - 20x - 30 = 0$. If $\dfrac{\alpha+\beta}{\alpha^2\beta}$ and $\dfrac{\alpha+\beta}{\alpha\beta^2}$ are the roots of the equation $x^2 - Sx + T = 0$, where S and T are constants. Find the value of $S + T$.

7. Find the coefficient of x^4 in the expansion of $\left(1 - 2x + 3x^2\right)^3$.

8. Express the recursive number $12.0\dot{2}\dot{2}_{(3)}$ in base 3 into simplest fraction.

9. Solve the equation $\log_5\left(2x+7\right) - \log_5 x = 2$.

10. For real value x, find the minimum value of $\dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$.

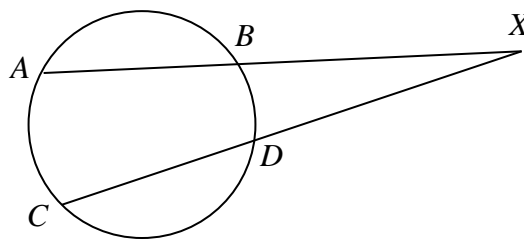All answers should be written on the ANSWER SHEET.

Rough Work

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Number Theory

11. Find the maximum integer which is smaller than $\left(3+\sqrt{8}\right)^{3}$.

12. Find the remainder of $2015^{2018} \div 503$.

13. There are 3 distinct prime numbers. Their product is 11 times of their sum. Find the smallest prime number among them.

14. Find the number of square numbers which are multiples of 44 and not greater than 201800?

15. Given that k is an integer and $5 < k < 20$. The equation of variable x, $x^2 - 2(2k-3)x + 4k^2 - 14k + 8 = 0$ has two integral roots, find the value of k.
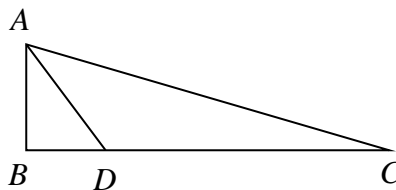
Geometry

16. In the figure below, given that A, B, C and D are points on a circle, *XBA* and *XDC* are straight lines. $XA = 18$, $XB = 10$ and $XC = 24$. Find *XD*.

Question 16

17. In the figure below, given that right-angled triangle *ABC* with $AB = 7$, $BC = 24$ and $AC = 25$. *AD* is the angle bisector of $\angle BAC$. Find the area of $\triangle ADC$.

Question 17

18. For x and y are real number, find the minimum value of $\sqrt{x^2 + y^2} + \sqrt{(x-3)^2 + (y-4)^2}$.

19. Given x is an acute angle, find the maximum value of $9\sin^2 x + 6\cos x + 7$.

20. Find the sum of slope of the two angle bisectors of $2x + y = 0$ and $2x - 4y - 3 = 0$.

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Rough Work

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Combinatorics

21. Find the number of the permutation(s) arranging 5 identical red balls and 3 identical blue balls in a row.

22. There are 3 distinct balls and 5 distinct boxes. We place the 3 balls into the boxes. Each box can contain 0, 1, 2 and 3 balls. No box contain any other boxes inside. Find the probability that 3 balls are in 3 different boxes.

23. A fair 6-faced die is thrown 3 times. Find the probability that the sum of numbers obtained is greater than 5.

24. If a fair 6-faced die is thrown repeatedly and the die is thrown 4 times, the result shown in each throw is always greater than the previous throw. Find the number of possible sequences of the die throw.

25. Given (a,b,c) is a set of positive integers. Find the number of solution set(s) of $a+b+c<10$.

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