1.

Part (a): Evaluate $P(A|B)$

The conditional probability of A given B is defined as the probability of A occurring given that B has occurred. This is calculated using the formula for conditional probability:

$$P(A|B) = P(A \cap B) / P(B)$$

Substituting the given values:

$$P(A|B) = 0.4 / 0.5 = 0.8$$

Therefore, the probability of event A occurring given that event B has occurred is 0.8.

Part (b): Evaluate $P(B|A')$

To evaluate the conditional probability of B given the complement of A (denoted as A'), we need to determine the probability of B occurring when A does not occur. This requires two calculations: the probability of the complement of A, and the probability of the intersection of B and A'. The complement rule gives us $P(A')$:

$$P(A') = 1 - P(A) = 1 - 0.8 = 0.2$$

The intersection of B and A' (B occurring without A) can be found by subtracting the intersection of A and B from the probability of B:

$$P(B \cap A') = P(B) - P(A \cap B) = 0.5 - 0.4 = 0.1$$

With these values, we can find $P(B|A')$:

$$P(B|A') = P(B \cap A') / P(A') = 0.1 / 0.2 = 0.5$$

This means that the probability of event B occurring given that event A has not occurred is 0.5.

Part (c): Evaluate $P(A|(A \cup B))$

The conditional probability of A given the union of A and B (A or B or both occurring) is found using the formula for conditional probability. However, since the event of interest (A) is part of the conditioning event ($A \cup B$), this simplifies the calculation. We start by calculating the probability of the union of A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.5 - 0.4 = 0.9$$

Then we use this to find the conditional probability:

$P(A|A \cup B) = P(A) / P(A \cup B) = 0.8 / 0.9 \approx 0.889$

Thus, the probability of event A occurring given that either event A or B (or both) has occurred is approximately 0.889.

2.

Part (a): Probability of Hypertension given Heavy Smoker

The probability that a person is experiencing hypertension given that the person is a heavy smoker is given by the formula:

$P(H|HS) = P(H \cap HS) / P(HS)$

where H is Hypertension, HS is Heavy Smoker, $P(H \cap HS)$ is the probability that a person is both a heavy smoker and has hypertension, and P(HS) is the probability that a person is a heavy smoker.

From the table, $P(H \cap HS)$ is the number of heavy smokers with hypertension (30) divided by the total number of individuals (180), and P(HS) is the number of heavy smokers (49) divided by the total number of individuals (180).

$P(H|HS) = 30 / 49 \approx 0.612$

Therefore, the probability that a person is experiencing hypertension given that the person is a heavy smoker is approximately 0.612 or 61.2%.

Part (b): Probability of Nonsmoker given Nonhypertension

The probability that a person is a nonsmoker given that the person is experiencing no hypertension is given by the formula:

$P(NS|NH) = P(NS \cap NH) / P(NH)$

where NS is Nonsmoker, NH is Nonhypertension, $P(NS \cap NH)$ is the probability that a person is both a nonsmoker and does not have hypertension, and P(NH) is the probability that a person does not have hypertension.

From the table, $P(NS \cap NH)$ is the number of nonsmokers without hypertension (48) divided by the total number of individuals (180), and P(NH) is the number of individuals without hypertension (93) divided by the total number of individuals (180).

$P(NS|NH) = 48 / 93 \approx 0.516$

This means that the probability that a person is a nonsmoker given that the person is experiencing no hypertension is approximately 0.516 or 51.6%.

3.

The problem can be solved by recognizing that the Sheriff's probability of winning is a sum of the probabilities of all the scenarios where he hits the target before Robin Hood. These scenarios form a geometric series because each subsequent scenario is less likely by the same factor: the probability that both Sheriff and Robin Hood miss their previous shots.

The first term 'a' of our geometric series is the Sheriff's probability of hitting the target (0.2), and the common ratio 'r' is the probability that both miss their shots in a single round (0.8 * 0.6 = 0.48).

Using the formula for the sum of an infinite geometric series S = a / (1 - r), we get:

S = 0.2 / (1 - 0.48)

Calculating this gives us the probability that the Sheriff wins the contest. After rounding to the nearest ten-thousandths place, we get:

Probability that Sheriff wins = 0.3846

4.

Part (a): Probability that a defect will be found by at least one inspector

Given that each inspector has a 30% chance of not finding a defect, the probability that none of the three inspectors find a defect in a defective item is (0.3)^3. The probability that at least one inspector finds a defect is the complement of this:

P(defect found by at least one) = 1 - (0.3 * 0.3 * 0.3) = 0.973

Part (b): Probability that a randomly chosen item will not have a defect found

The probability that an item is not found to have a defect by any of the three inspections is the sum of two probabilities:
1. The item is defective, but all inspectors miss the defect.
2. The item is not defective and automatically passes.

The combined probability is:

P(no defect found by any inspector) = (0.1 * (0.3)^3) + (0.9 * 1) = 0.9027

Part (c): Conditional probability an item is defective given it passed inspection

Using Bayes' Rule, we can find the probability that an item is defective given that it passed all three inspections:

P(defective | passed all inspections) = [P(passed all inspections | defective) * P(defective)] / P(passed all inspections)

P(defective | passed all inspections) = [(0.3)^3 * 0.1] / 0.9027 ≈ 0.003

The probability that an item is defective given that it passed all inspections is approximately 0.003 or 0.3% when rounded to the nearest ten-thousandths place.

5.

Part (a): Probability of being infected given a positive test result

The probability of being infected given a positive test result is found using Bayes' Rule:

P(I|+) = [P(+|I) * P(I)] / [P(+|I) * P(I) + P(+|¬I) * P(¬I)]
P(I|+) = [0.95 * 0.01] / [0.95 * 0.01 + 0.04 * 0.99] ≈ 0.1935

The probability that you are infected given that you received a positive result is approximately 0.1935 when rounded to the nearest ten-thousandths place.

Part (b): Probability of both tests producing true positive results if infected, and false positives if healthy

The probability of both tests producing true positive results given that you are infected is simply the square of the probability of a true positive:

P(both true positives|I) = P(+|I) * P(+|I) = 0.95 * 0.95 = 0.9025

The probability of both tests producing false positive results given that you are healthy is the square of the probability of a false positive:

P(both false positives|¬I) = P(+|¬I) * P(+|¬I) = 0.04 * 0.04 = 0.0016

Part (c): Probability of being infected given two positive test results

Using the probabilities found in part (b), we apply Bayes' Rule again to find the probability that you are actually infected given that you tested positive in both tests:

P(I|both +) = [P(both +|I) * P(I)] / [P(both +|I) * P(I) + P(both +|¬I) * P(¬I)]
P(I|both +) = [0.9025 * 0.01] / [0.9025 * 0.01 + 0.0016 * 0.99] ≈ 0.8507

The probability that you are actually infected given that you tested positive in both tests is approximately 0.8507 when rounded to the nearest ten-thousandths place.